A Quick Introduction to MATLAB for Mathematical Statistics

Antonio Prgomet, Lund University

June, 2020

Cumulative Distribution Function, CDF

- Recall that the CDF is defined as: $P(X \leq x) = F_X(x)$
- Example: Let $X \sim \mathcal{N}(2, 4)$, calculate $P(X \leq 6.4)$.

  Begin with standardizing:
  $P(X \leq 6.4) \iff P(\frac{X-2}{4} \leq \frac{6.4-2}{4}) = F_{\mathcal{N}(0,1)}(1.1)$

  Analytically, we have:

$$F_{N(0,1)}(1.1) = \int_{-\infty}^{1.1} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

- Solve this with MATLAB, table or calculator.

Lower Quantile (Inverse CDF) - Percentil (Swedish)

- The CDF takes a value as input and returns a probability.
  Ex: normcdf(-**0.5244**,0,1) = **0.3**
- The Lower Quantile function takes a probability as input and returns a value. Ex: norminv(**0.3**,0,1) = -**0.5244**
- So, the Lower Quantile function is the inverse function to the CDF (heuristic argument).

- Lower Quantile = Quantile = Inverse CDF = Percentil (in Swedish).
- The α lower quantile is, for a continuous distribution, defined as the x_α such that the following equality holds ("Area to the left"):

$$P(X \leq x_\alpha) = \alpha$$

Upper Quantile - Kvantil (Swedish)

- The α upper quantile is, for a continuous distribution, defined as the x_α such that the following equality holds ("Area to the right"):

$$P(X > x_\alpha) = \alpha$$

- Upper Quantile = Quantile = Kvantil (in Swedish)

- Source of confusion: In English quantile often refers to the lower quantile but can be used for upper quantile aswell. So, make sure you know what the author means by "quantile" when reading the litterature!

Finding the upper quantile

- In MATLAB, **norminv** is the Lower Quantile. If α probability to the right $\iff$ $1 - \alpha$ probability to the left, so it is easy to obtain the upper quantile with help of the lower quantile.