2020

CFA® EXAM REVIEW

COVERS ALL TOPICS IN LEVEL II

LEVEL II CFA®

FORMULA SHEETS

WILEY

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.

Published simultaneously in Canada.

Quantitative Methods

Correlation and Introduction to Linear Regression

Linear Regression with One Independent Variable

$$\text{Regression model equation} = Y_i = b_0 + b_1X_i + \varepsilon_i,\ i = 1,\ldots,n$$

- b_1 and b_0 are the regression coefficients.
- b_1 is the slope coefficient.
- b_0 is the intercept term.
- ε is the error term that represents the variation in the dependent variable that is not explained by the independent variable.

$$\text{Regression line equation} = \hat{Y}_i = \hat{b}_0 + \hat{b}_1X_i,\ i = 1,\ldots,n$$

Regression Residuals

$$\sum_{i=1}^{n}[Y_i - (\hat{b}_0 + \hat{b}_1X_i)]^2$$

where:
Y_i = Actual value of the dependent variable
$\hat{b}_0 + \hat{b}_1X_i$ = Predicted value of dependent variable

The Standard Error of Estimate

$$\text{SEE} = \left(\frac{\sum_{i=1}^{n}(Y_i - \hat{b}_0 - \hat{b}_1X_i)^2}{n-2}\right)^{1/2} = \left(\frac{\sum_{i=1}^{n}(\hat{\varepsilon}_i)^2}{n-2}\right)^{1/2} = \left(\frac{\text{SSE}}{n-2}\right)^{1/2}$$

The Coefficient of Determination

Total variation = Unexplained variation + Explained variation

$$R^2 = \frac{\text{Explained variation}}{\text{Total variation}} = \frac{\text{Total variation} - \text{Unexplained variation}}{\text{Total variation}}$$
$$= 1 - \frac{\text{Unexplained variation}}{\text{Total variation}}$$

Hypothesis Tests on Regression Coefficients

CAPM: $R_{ABC} = R_F + \beta_{ABC}(R_M - R_F)$

$R_{ABC} - R_F = \alpha + \beta_{ABC}(R_M - R_F) + \varepsilon$

- The intercept term for the regression, b_0, is α.
- The slope coefficient for the regression, b_1, is β_{ABC}.

The regression sum of squares (RSS)

$$\text{RSS} = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 \rightarrow \text{Explained variation}$$

The sum of squared errors or residuals (SSE)

$$\text{SSE} = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 \rightarrow \text{Unexplained variation}$$

Confidence Interval for a Regression Coefficient

$$\hat{b}_n \pm t_C s_{\hat{b}_n}$$

F-Statistic

$$F = \frac{\text{Mean regression sum of squares}}{\text{Mean squared error}} = \frac{RSS / k}{SSE / [n-(k+1)]} = \frac{MSR}{MSE}$$

where k = the number of slope coefficients (b values)

ANOVA Table

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares
Regression (explained)	k	RSS	$\text{MSR} = \frac{\text{RSS}}{k} = \frac{\text{RSS}}{1} = \text{RSS}$
Error (unexplained)	$n - (k + 1)$	SSE	$\text{MSE} = \frac{\text{SSE}}{n-2}$
Total	$\boldsymbol{n - 1}$	**SST**	

k = the number of slope coefficients in the regression.

Prediction Intervals

$$s_f^2 = s^2 \left[1 + \frac{1}{n} + \frac{(\text{X} - \bar{\text{X}})^2}{(n-1)s_x^2} \right]$$

$$\hat{Y} \pm t_c s_f$$

MULTIPLE REGRESSION

Multiple Regression Equation

$$\text{Multiple regression equation} = Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \ldots + b_kX_{ki} + \varepsilon_i,\ i = 1, 2, \ldots, n$$

Y_i = the ith observation of the dependent variable Y
X_{ji} = the ith observation of the independent variable X_j, $j = 1,2,\ldots, k$
b_0 = the intercept of the equation
$b_1, \ldots, b_k$ = the slope coefficients for each of the independent variables
ε_i = the error term for the ith observation
n = the number of observations

Residual Term

$$\hat{\varepsilon}_i = Y_i - \hat{Y}_i = Y_i - (\hat{b}_0 + \hat{b}_1X_{1i} + \hat{b}_2X_{2i} + \ldots + \hat{b}_kX_{ki})$$

Confidence Intervals

$$\hat{b}_j \pm (t_c \times s_{\hat{b}_j})$$

estimated regression coefficient ± (critical t-value)(coefficient standard error)

F-statistic

$$F\text{-stat} = \frac{\text{MSR}}{\text{MSE}} = \frac{\text{RSS}/k}{\text{SSE}/[n-(k+1)]}$$

R^2 and Adjusted R^2

$$R^2 = \frac{\text{Total variation} - \text{Unexplained variation}}{\text{Total variation}} = \frac{\text{SST} - \text{SSE}}{\text{SST}} = \frac{\text{RSS}}{\text{SST}}$$

$$\text{Adjusted } R^2 = \bar{R}^2 = 1 - \left(\frac{n-1}{n-k-1}\right)(1-R^2)$$

Testing for Heteroskedasticity—The Breusch-Pagan (BP) Test

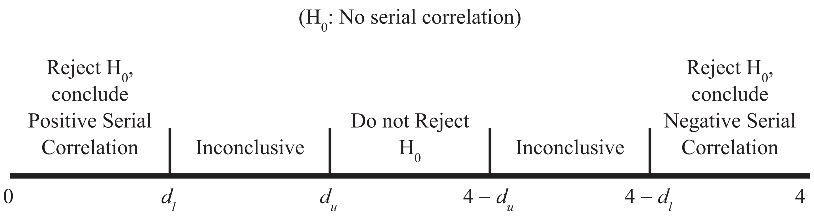
$$\chi^2 = n\text{R}^2 \text{ with } k \text{ degrees of freedom.}$$

n = Number of observations
R^2 = Coefficient of determination of the **second regression** (the regression when the squared residuals of the original regression are regressed on the independent variables)
k = Number of independent variables

Testing for Serial Correlation–The Durbin-Watson (DW) Test

DW ≈ 2(1 − r); where r is the sample correlation between squared residuals from one period and those from the previous period.

Value of Durbin-Watson Statistic



Problems in Linear Regression and Solutions

Problem	Effect	Solution
Heteroskedasticity	Incorrect standard errors	Use robust standard errors (corrected for conditional heteroskedasticity)
Serial correlation	Incorrect standard errors (additional problems if a lagged value of the dependent variable is used as an independent variable)	Use robust standard errors (corrected for serial correlation)
Multicollinearity	High R^2 and low t-statistics	Remove one or more independent variables; often no solution based in theory

Logit Model

$$\ln\left(\frac{p}{1-p}\right) = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + \varepsilon$$

$$\hat{p} = \frac{\exp\left[\hat{b}_0 + \hat{b}_1X_1 + \hat{b}_2X_2 + \hat{b}_3X_3\right]}{1 + \exp\left[\hat{b}_0 + \hat{b}_1X_1 + \hat{b}_2X_2\right]}$$

where:

p = probability of event

TIME-SERIES ANALYSIS

Linear Trend Models

$$y_t = b_0 + b_1 t + \varepsilon_t, \quad t = 1, 2, \ldots, T$$

where:
y_t = the value of the time series at time t (value of the dependent variable)
b_0 = the y-intercept term
b_1 = the slope coefficient/trend coefficient
t = time, the independent or explanatory variable
ε_t = a random-error term

Log-Linear Trend Models

A series that grows exponentially can be described using the following equation:

$$y_t = e^{b_0 + b_1 t}$$

where:
y_t = the value of the time series at time t (value of the dependent variable)
b_0 = the y-intercept term
b_1 = the slope coefficient
t = time =1, 2, 3, . . . , T

We take the natural logarithm of both sides of the equation to arrive at the equation for the log-linear model:

$$\ln y_t = b_0 + b_1 t + \varepsilon_t, \quad t = 1, 2, \ldots, T$$

AUTOREGRESSIVE (AR) TIME-SERIES MODELS

$$x_t = b_0 + b_1 x_{t-1} + \varepsilon_t$$

A pth order autoregressive model is represented as:

$$x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-2} + \ldots + b_p x_{t-p} + \varepsilon_t$$

Detecting Serially Correlated Errors in an AR Model

$$t\text{-stat} = \frac{\text{Residual autocorrelation for lag}}{\text{Standard error of residual autocorrelation}}$$

where:
Standard error of residual autocorrelation $=1/\sqrt{T}$
T = Number of observations in the time series

Mean Reversion

$$x_t = \frac{b_0}{1 - b_1}$$

Multiperiod Forecasts and the Chain Rule of Forecasting

$$\hat{x}_{t+1} = \hat{b}_0 + \hat{b}_1 x_t$$

Random Walks

$$x_t = x_{t-1} + \varepsilon_t, \mathrm{E}(\varepsilon_t) = 0, \mathrm{E}(\varepsilon_t^2) = \sigma^2, \mathrm{E}(\varepsilon_t \varepsilon_s) = 0 \text{ if } t \neq s$$

The first difference of the random walk equation is given as:

$$y_t = x_t - x_{t-1} = x_{t-1} + \varepsilon_t - x_{t-1} = \varepsilon_t,\ \mathrm{E}(\varepsilon_t) = 0,\ \mathrm{E}(\varepsilon_t^2) = \sigma^2,\ \mathrm{E}(\varepsilon_t \varepsilon_s) = 0 \text{ for } t \neq s$$

Random Walk with a Drift

$$\begin{aligned} &x_t = b_0 + b_1 x_{t-1} + \varepsilon_t \\ &b_1 = 1,\ b_0 \neq 0, \text{ or} \\ &x_t = b_0 + x_{t-1} + \varepsilon_t,\ \mathrm{E}(\varepsilon_t) = 0 \end{aligned}$$

The first-difference of the random walk with a drift equation is given as:

$$y_t = x_t - x_{t-1},\ y_t = b_0 + \varepsilon_t,\ b_0 \neq 0$$

The Unit Root Test of Nonstationarity

$$\begin{aligned} &x_t = b_0 + b_1 x_{t-1} + \varepsilon_t \\ &x_t - x_{t-1} = b_0 + b_1 x_{t-1} - x_{t-1} + \varepsilon_t \\ &x_t - x_{t-1} = b_0 + (b_1 - 1) x_{t-1} + \varepsilon_t \\ &x_t - x_{t-1} = b_0 + g_1 x_{t-1} + \varepsilon_t \end{aligned}$$

Seasonality

$$x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-n} + \varepsilon_t$$

Where n = number of periods in the seasonal pattern

Autoregressive Moving Average (ARMA) Models

$$x_t = b_0 + b_1 x_{t-1} + \ldots + b_p x_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$
$$E(\varepsilon_t) = 0,\ E(\varepsilon_t^2) = \sigma^2,\ E(\varepsilon_t \varepsilon_s) = 0 \text{ for } t \neq s$$

Autoregressive Conditional Heteroskedasticity Models (ARCH Models)

$$\hat{\varepsilon}_t^2 = a_0 + \hat{a}_1 \hat{\varepsilon}_{t-1}^2 + u_t$$

The error in period t+1 can then be predicted using the following formula:

$$\hat{\sigma}_{t+1}^2 = \hat{a}_0 + \hat{a}_1 \hat{\varepsilon}_t^2$$

Machine Learning and Big Data Projects

ML Algorithm Type	Supervised/ Unsupervised	When to Use?
Classification and Regression Tree (CART)	Supervised	Most commonly applied to binary classification or regression.
Deep Learning Net	Both	A form of neural network with three or more ”hidden” layers
Ensemble Learning	Supervised	The use of a combination of algorithms to describe the data.
Hierarchical	Unsupervised	A form of clustering data (separating observations into groups) into different and final levels of clusters based on relationships between clusters.
K-Means	Unsupervised	A form of clustering data into a predetermined number of groups.
K-Nearest Neighbor (KNN)	Supervised	Mainly used for classification, by classifying new observations based on existing data.
LASSO	Supervised	A type of penalized regression that also eliminates the least important features of the regression model.
Neural Networks	Both	Commonly used for regression and classification in which input features (similar to regression independent variables) are connected to the output (target) variable by “hidden” layers of relationships.
Penalized Regression	Supervised	Regression technique to avoid overfitting by penalizing data features that make insufficient contribution to the regression model.
Principal Components Analysis (PCA)	Unsupervised	Used to help reduce the features in a data set to a manageable level.
Random Forest	Supervised	Type of ensemble learning using collection of decision trees.
Reinforcement Learning	Unsupervised	An algorithm that uses the experience of millions of trials and errors to maximize future success.
Support Vector Machine (SVM)	Supervised	Used for classification, regression, and outlier detection by finding the optimal boundary between sets of data points.

LASSO Penalized Regression Constraint

$$\sum_{i=1}^{n}\left(Y_i - Y_2\right) + \lambda\sum_{k=1}^{K}\left|\hat{b}_k\right|$$

where:
λ = hyperparameter set by researcher prior to learning
b_k = regression coefficient of kth feature (factor)

Normalization

$$X_{i\ (normalized)} = \frac{X_i - X_{min}}{X_{max} - X_{min}}$$

Standardization

$$X_{i\ (standardized)} = \frac{X_i - \mu}{\sigma}$$

Excerpt from "Probabilistic Approaches: Scenario Analysis, Decision Trees, and Simulation"

Table 2-1: Risk Types and Probabilistic Approaches

Discrete/ Continuous	Correlated/ Independent	Sequential/ Concurrent	Risk Approach
Discrete	Independent	Sequential	Decision tree %
Discrete	Correlated	Concurrent	Scenario analysis
Continuous	Either	Either	Simulations

ECONOMICS

CURRENCY EXCHANGE RATES: UNDERSTANDING EQUILIBRIUM VALUE

Currency Cross Rates

For example, given the USD/EUR and JPY/USD exchange rates, we can calculate the cross rate between the JPY and the EUR, JPY/EUR as follows:

$$\frac{\text{JPY}}{\text{EUR}} = \frac{\text{JPY}}{\text{USD}} \times \frac{\text{USD}}{\text{EUR}}$$

Cross Rate Calculations with Bid-Ask Spreads

$\text{USD/EUR}_{\text{bid}} = 1.3802$

- Represents the price of **EUR** (base currency).
- An investor can **sell** EUR for USD at this price (as it is the **bid** price quoted by the dealer).

$\text{USD/EUR}_{\text{ask}} = 1.3806$

- Represents the price of **EUR**
- An investor can **buy** EUR with USD at this price.

Determining the $\text{EUR/USD}_{\text{bid}}$ cross rate:

$$\text{EUR/USD}_{\text{bid}} = 1/(\text{USD/EUR}_{\text{ask}})$$

Determining the $\text{EUR/USD}_{\text{ask}}$ cross rate:

$$\text{EUR/USD}_{\text{ask}} = 1/(\text{USD/EUR}_{\text{bid}})$$

Forward exchange rates (*F*)—One-year Horizom

$$F_{FC/DC} = S_{FC/DC} \times \frac{(1+i_{FC})}{(1+i_{DC})} \qquad F_{PC/BC} = S_{PC/BC} \times \frac{(1+i_{PC})}{(1+i_{BC})}$$

Forward exchange rates (F)—Any Investment Horizom

$$F_{FC/DC} = S_{FC/DC} \times \frac{1+(i_{FC} \times \text{Actual}/360)}{1+(i_{DC} \times \text{Actual}/360)}$$

$$F_{PC/BC} = S_{PC/BC} \times \frac{1+(i_{PC} \times \text{Actual}/360)}{1+(i_{BC} \times \text{Actual}/360)}$$

Currencies Trading at a Forward Premium/Discount

$$F_{FC/DC} - S_{FC/DC} = S_{FC/DC}\left(\frac{(i_{FC} - i_{DC}) \times \text{Actual}/360}{1+(i_{DC} \times \text{Actual}/360)}\right)$$

$$F_{PC/BC} - S_{PC/BC} = S_{PC/BC}\left(\frac{(i_{PC} - i_{BC}) \times \text{Actual}/360}{1+(i_{BC} \times \text{Actual}/360)}\right)$$

Covered Interest Rate Parity

$$F_{PC/BC} = S_{PC/BC} \times \frac{1 + (i_{PC} \times \text{Actual}/360)}{1 + (i_{BC} \times \text{Actual}/360)}$$

The forward premium (discount) on the base currency can be **expressed** as a percentage as:

$$\text{Forward premium (discount) as a \%} = \frac{F_{PC/BC} - S_{PC/BC}}{S_{PC/BC}}$$

The forward premium (discount) on the base currency can be **estimated** as:

$$\text{Forward premium (discount) as a \%} \approx \frac{F_{PC/BC} - S_{PC/BC}}{S_{PC/BC}} = \frac{F_{PC/BC}}{S_{PC/BC}} - 1 \approx i_{PC} - i_{BC}$$

Uncovered Interest Rate Parity

Expected future spot exchange rate:

$$S^{e}{}_{FC/DC} = S_{FC/DC} \times \frac{(1 + i_{FC})}{(1 + i_{DC})}$$

The expected percentage change in the spot exchange rate can be **calculated** as:

$$\text{Expected \% change in spot exchange rate} = \%\Delta S^{e}{}_{PC/BC} = \frac{S^{e}{}_{PC/BC} - S_{PC/BC}}{S_{PC/BC}}$$

The expected percentage change in the spot exchange rate can be **estimated** as:

$$\text{Expected \% change in spot exchange rate} \approx \%\Delta S^{e}{}_{PC/BC} \approx i_{PC} - i_{BC}$$

Purchasing Power Parity (PPP)

$$\text{Law of one price}: P^{X}{}_{FC} = P^{X}{}_{DC} \times S_{FC/DC}$$

$$\text{Law of one price}: P^{X}{}_{PC} = P^{X}{}_{BC} \times S_{PC/BC}$$

Absolute Purchasing Power Parity (Absolute PPP)

$$S_{FC/DC} = GPL_{FC} / GPL_{DC}$$

$$S_{PC/BC} = GPL_{PC} / GPL_{BC}$$

Relative Purchasing Power Parity (Relative PPP)

$$\text{Relative PPP: } E(S^{T}{}_{FC/DC}) = S^{0}{}_{FC/DC}\left(\frac{1 + \pi_{FC}}{1 + \pi_{DC}}\right)^{T}$$

Where π = inflation rate.

Ex Ante Version of PPP

Ex ante PPP: $\%\Delta S^{e}_{FC/DC} \approx \pi^{e}_{FC} - \pi^{e}_{DC}$

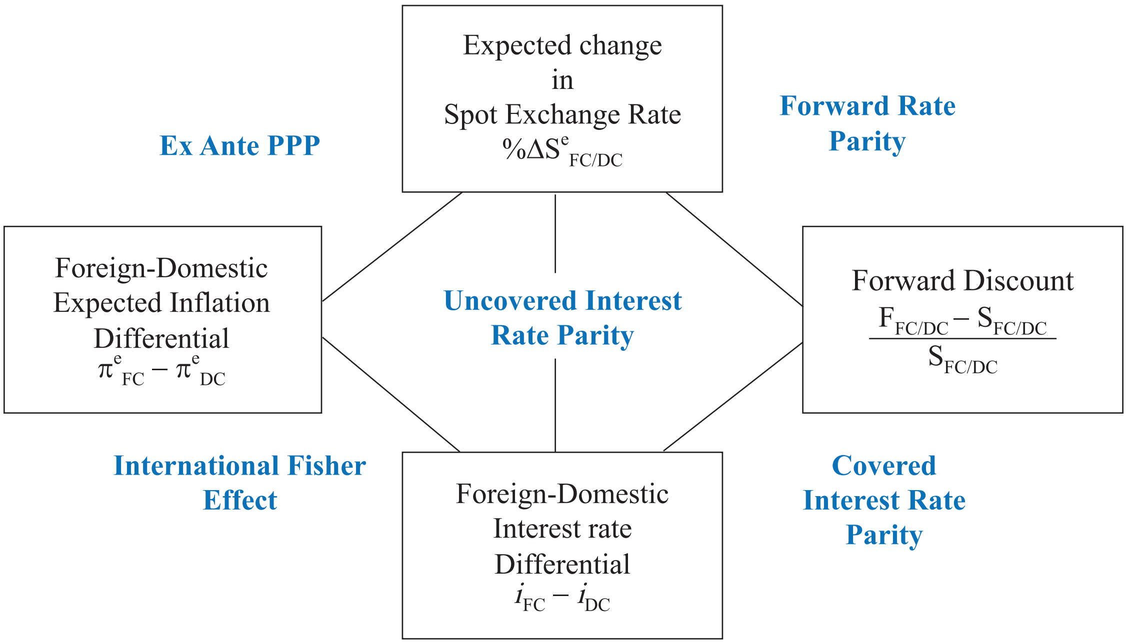
Ex ante PPP: $\%\Delta S^{e}_{PC/BC} \approx \pi^{e}_{PC} - \pi^{e}_{BC}$

The Fisher Effect

Fisher Effect: $i = r + \pi^{e}$

International Fisher Effect: $(i_{FC} - i_{DC}) = (\pi^{e}_{FC} - \pi^{e}_{DC})$

Figure 1: Spot Exchange Rates, Forward Exchange Rates, and Interest Rates



Balance of Payment

Current account + Capital account + Financial account = 0

ECONOMIC GROWTH AND THE INVESTMENT DECISION

Relationship between Economic Growth and Stock Prices

$$P = GDP\left(\frac{E}{GDP}\right)\left(\frac{P}{E}\right)$$

P = Aggregate price or value of earnings.
E = Aggregate earnings.

This equation can also be expressed in terms of growth rates:

$$\Delta P = \Delta(GDP) + \Delta(E/GDP) + \Delta(P/E)$$

Production Function

$$Y = AK^{\alpha}L^{1-\alpha}$$

Y = Level of aggregate output in the economy.
L = Quantity of labor.
K = Quantity of capital.
A = Total factor productivity. Total factor productivity (TFP) reflects the general level of productivity or technology in the economy. TFP is a scale factor i.e., an increase in TFP implies a proportionate increase in output for any combination of inputs.
α = Share of GDP paid out to capital.
$1 - \alpha$ = Share of GDP paid out to labor.

$$y = Y/L = A(K/L)^{\alpha}(L/L)^{1-\alpha} = Ak^{\alpha}$$

y = Y/L = Output per worker or labor productivity.
k = K/L = Capital per worker or capital-labor ratio.

Cobb-Douglas Production Function (Growth Accounting Equation)

$$\Delta Y/Y = \Delta A/A + \alpha\Delta K/K + (1 - \alpha)\Delta L/L$$

Potential GDP (Labor Productivity Growth Accounting Equation)

Growth rate in potential GDP = Long-term growth rate of labor force
+ Long-term growth rate in labor productivity

Labor Supply

Total number of hours available for work = Labor force × Average hours worked per worker

Neoclassical Model (Solow's Model)

$$\frac{Y}{K} = \left(\frac{1}{s}\right)\left[\left(\frac{\theta}{(1-\alpha)}\right) + \delta + n\right] \equiv \Psi$$

s = Fraction of income that is saved
θ = Growth rate of TFP
α = Elasticity of output with respect to capital
δ = Constant rate of depreciation on physical stock
n = Labor supply growth rate
Ψ = Equilibrium output-to-capital ratio

Savings/Investment Equation:

$$sy = \left[\left(\frac{\theta}{(1-\alpha)}\right) + \delta + n\right]k$$

Growth Rates of Output Per Capita and the Capital-Labor Ratio

$$\frac{\Delta y}{y} = \frac{\theta}{(1-\alpha)} + \alpha s\ \frac{Y}{K} - \Psi$$

$$\frac{\Delta k}{k} = \left(\frac{\theta}{(1-\alpha)}\right) + s\left(\frac{Y}{K} - \Psi\right)$$

Production Function in the Endogenous Growth Model

$$y_e = f(k_e) = ck_e$$

INTERCORPORATE INVESTMENTS

Summary of Accounting Treatment for Investments

	In Financial Assets	In Associates	Business Combinations	In Joint Ventures
Influence	Not significant	Significant	Controlling	Shared Control
Typical percentage interest	Usually < 20%	Usually 20%–50%	Usually > 50%	Varies
Current Financial Reporting Treatment (prior to IFRS 9 taking effect)	Classified as • Held-to-maturity. • Available-for-sale. • Fair value through profit or loss (held for trading or designated as fair value). • Loans and receivables.	Equity method	Consolidation	IFRS: Equity method or proportionate consolidation.
New Financial Reporting Treatment (post IFRS 9 taking effect)	Classified as • Fair value through profit or loss. • Fair value through other comprehensive income. • Amortized cost.	Equity method	Consolidation	IFRS: Equity method

Types of Business Combination

Combination	Description
Merger	Company A + Company B = Company A
Acquisition	Company A + Company B = (Company A + Company B)
Consolidation	Company A + Company B = Company C

Adjusted Values Upon Reclassification of Sale of Receivables:

CFO	Lower
CFF	Higher
Total cash flow	Same
Current assets	Higher
Current liabilities	Higher
Current ratio (Assuming it was greater than 1)	Lower

Impact of Different Accounting Methods on Financial Ratios

	Equity Method	Acquisition Method
Leverage	Better (lower) as liabilities are lower and equity is the same	Worse (higher) as liabilities are higher and equity is the same
Net Profit Margin	Better (higher) as sales are lower and net income is the same	Worse (lower) as sales are higher and net income is the same
ROE	Better (higher) as equity is lower and net income is the same	Worse (lower) as equity is higher and net income is the same
ROA	Better (higher) as net income is the same and assets are lower	Worse (lower) as net income is the same and assets are higher

EMPLOYEE COMPENSATION: POST-EMPLOYMENT AND SHARE-BASED

Final year's salary = Current salary × [(1 + Annual compensation increase)$^{\text{years until retirement}}$]

$$\boxed{\text{Lump sum at retirement} = \text{Final salary} \times \text{Benefit formula} \times \text{Years of service}}$$

Estimated annual payment = (Estimated final salary × Benefit formula) × Years of service

Annual unit credit = Value at retirement / Years of service

Types of Post-Employment Benefits

Type of Benefit	Amount of Post-Employment Benefit to Employee	Obligation of Sponsoring Company	Sponsoring Company's Pre-funding of Its Future Obligation
Defined contribution pension plan	Amount of future benefit is not defined. Actual future benefit will depend on investment performance of plan assets. Investment risk is borne by employee.	Amount of the company's obligation (contribution) is defined in each period. The contribution, if any, is typically made on a periodic basis with no additional future obligation.	Not applicable.
Defined benefit pension plan	Amount of future benefit is defined, based on the plan's formula (often a function of length of service and final year's compensation). Investment risk is borne by company.	Amount of the future obligation, based on the plan's formula, must be estimated in the current period.	Companies typically pre-fund the DB plans by contributing funds to a pension trust. Regulatory requirements to pre-fund vary by country.
Other post-employment benefits (e.g., retirees' health care)	Amount of future benefit depends on plan specifications and type of benefit.	Eventual benefits are specified. The amount of the future obligation must be estimated in the current period.	Companies typically do not pre-fund other post-employment benefit obligations.

A company's pension obligation will *increase* as a result of:
- Current service costs.
- Interest costs.
- Past service costs.
- Actuarial losses.

A company's pension obligation will *decrease* as a result of:
- Actuarial gains.
- Benefits paid.

Reconciliation of the Pension Obligation:

Pension obligation at the beginning of the period
+ Current service costs
+ Interest costs
+ Past service costs
+ Actuarial losses
– Actuarial gains
– Benefits paid
Pension obligation at the end of the period

The fair value of assets held in the pension trust (plan) will *increase* as a result of:

- A positive actual dollar return earned on plan assets; and
- Contributions made by the employer to the plan.

The fair value of plan assets will *decrease* as a result of:

- Benefits paid to employees.

Reconciliation of the Fair Value of Plan Assets:

Fair value of plan assets at the beginning of the period
+ Actual return on plan assets
+ Contributions made by the employer to the plan
– Benefits paid to employees
Fair value of plan assets at the end of the period

Balance Sheet Presentation of Defined Benefit Pension Plans

$$\boxed{\text{Funded status} = \text{Fair value of plan assets} - \text{Pension obligation}}$$

Where pension obligation is either pension benefit obligation (US GAAP) or the present value of the defined benefit obligation (IFRS).

- If Pension obligation > Fair value of plan assets:
  Plan is underfunded → Negative funded status → Net pension liability.
- If Pension obligation < Fair value of plan assets:
  Plan is overfunded → Positive funded status → Net pension asset.

Calculating Periodic Pension Cost

$$\boxed{\begin{array}{l}\text{Net periodic pension cost} = \text{Ending net pension liability} - \text{Beginning net pension liability} \\ \qquad + \text{ Employer contributions}\end{array}}$$

$$\boxed{\begin{array}{l}\text{Periodic pension cost} = \text{Current service costs} + \text{Interest costs} + \text{Past service costs} \\ \qquad + \text{ Actuarial losses} - \text{Actuarial gains} - \text{Actual return on plan assets}\end{array}}$$

Under the **corridor method**, if the net cumulative amount of unrecognized actuarial gains and losses at the beginning of the reporting period exceeds 10% of the greater of (1) the defined benefit obligation or (2) the fair value of plan assets, then the *excess* is amortized over the expected average remaining working lives of the employees participating in the plan and included as a component of periodic *pension expense* on the P&L.

Components of a Company's Defined Benefit Pension Periodic Costs

IFRS Component	IFRS Recognition	U.S. GAAP Component	U.S. GAAP Recognition
Service costs	Recognized in P&L.	Current service costs Past service costs.	Recognized in P&L. Recognized in OCI and subsequently amortized to P&L over the service life of employees.
Net interest income/expense	Recognized in P&L as the following amount: Net pension liability or asset × interest rate.[(a)]	Interest expense on pension obligation Expected return on plan assets.	Recognized in P&L. Recognized in P&L as the following amount: Plan assets × expected return.
Remeasurements: Net return on plan assets and actuarial gains and losses	Recognized in OCI and not subsequently amortized to P&L. • Net return on plan **assets** = Actual return − (Plan assets × Interest rate). • Actuarial gains and losses = Changes in a company's pension obligation arising from changes in actuarial assumptions.	Actuarial gains and losses including differences between the actual and expected returns on plan assets.	Recognized immediately in P&L or, more commonly, recognized in OCI and subsequently amortized to P&L using the corridor or faster recognition method.[(b)] • Difference between expected and actual return on assets = Actual return − (Plan assets × Expected return). • Actuarial gains and losses = Changes in a company's pension obligation arising from changes in actuarial assumptions.

(a) The interest rate used is equal to the discount rate used to measure the pension liability (the yield on high-quality corporate bonds.)
(b) If the cumulative amount of unrecognized actuarial gains and losses exceeds 10 percent of the greater of the value of the plan assets or of the present value of the DB obligation (under U.S. GAAP, the projected benefit obligation), the difference must be amortized over the service lives of the employees.

Impact of Key Assumptions on Net Pension Liability and Periodic Pension Cost

Assumption	Impact of Assumption on Net Pension Liability (Asset)	Impact of Assumption on Periodic Pension Cost and Pension Expense
Higher discount rate	Lower obligation	Pension cost and pension expense will both typically be lower because of lower opening obligation and lower service costs.
Higher rate of compensation increase	Higher obligation	Higher service and interest costs will increase periodic pension cost and pension expense.
Higher expected return on plan assets	No effect, because fair value of plan assets are used on balance sheet	Not applicable for IFRS. No effect on periodic pension cost under U.S. GAAP. Lower periodic pension expense under U.S. GAAP.

Multinational Operations

- The **presentation currency (PC)** is the currency in which the parent company reports its financial statements. It is typically the currency of the country where the parent is located. For example, U.S. companies are required to present their financial results in USD, German companies in EUR, Japanese companies in JPY, and so on.
- The **functional currency (FC)** is the currency of the primary business environment in which an entity operates. It is usually the currency in which the entity primarily generates and expends cash.
- The **local currency (LC)** is the currency of the country where the subsidiary operates.

Table 1

		Foreign Currency	
Transaction	**Type of Exposure**	**Strengthens**	**Weakens**
Export sale	Asset (account receivable)	Gain	Loss
Import purchase	Liability (account payable)	Loss	Gain

Methods for Translating Foreign Currency Financial Statements of Subsidiaries

Current Rate/ Temporal Method	Local Currency	T	Functional Currency	CR	Presentation Currency
Temporal Method	Local Currency	T	Functional Currency	=	Presentation Currency
Current Rate Method	Local Currency	=	Functional Currency	CR	Presentation Currency

- The current rate is the exchange rate that exists on the balance sheet date.
- The average rate is the average exchange rate over the reporting period.
- The historical rate is the actual exchange rate that existed on the original transaction date.

Rules for Foreign Currency Translation

	Current Rate Method FC = LC	**Temporal Method FC = PC**
Income Statement Component	**Exchange Rate Used**	
Sales	Average rate	Average rate
Cost of goods sold	Average rate	Historical rate
Selling expenses	Average rate	Average rate
Depreciation expense	Average rate	Historical rate
Amortization expense	Average rate	Historical rate
Interest expense	Average rate	Average rate
Income tax	Average rate	Average rate
Net income before translation gain (loss)		Computed as Rev – Exp
Translation gain (loss)	N/A	Plug in Number
Net income	Computed as Rev – Exp	Computed as ΔRE +
Less: Dividends	Historical rate	Dividends
Change in retained earnings	Computed as NI – Dividends	Historical rate
	Used as input for translated B/S	From B/S
Balance Sheet Component	**Exchange Rate Used**	
Cash	Current rate	Current rate
Accounts receivable	Current rate	Current rate
Monetary assets	Current rate	Current rate
Inventory	Current rate	Historical rate
Nonmonetary assets measured at current value	Current rate	Current rate
Property, plant and equipment	Current rate	Historical rate
Less: Accumulated depreciation	Current rate	Historical rate
Nonmonetary assets measured at historical cost	Current rate	Historical rate
Accounts payable	Current rate	Current rate
Long-term notes payable	Current rate	Current rate
Monetary liabilities	Current rate	Current rate
Nonmonetary liabilities:		
Measured at current value	Current rate	Current rate
Measured at historical cost	Current rate	Historical rate
Capital stock	Historical rate	Historical rate
Retained earnings	From I/S	To balance Used as input for translated I/S
Cumulative translation adjustment	Plug in Number	N/A

Balance Sheet Exposure

	Foreign Currency (FC)	
Balance Sheet Exposure	Strengthens	Weakens
Net asset	Positive translation adjustment	Negative translation adjustment
Net liability	Negative translation adjustment	Positive translation adjustment

Effects of Exchange Rate Movements on Financial Statements

	Temporal Method, Net Monetary Liability Exposure	Temporal Method, Net Monetary Asset Exposure	Current Rate Method
Foreign currency strengthens relative to parent's presentation currency	↑Revenues ↑Assets ↑Liabilities ↓Net income ↓Shareholders' equity Translation loss	↑Revenues ↑Assets ↑Liabilities ↑Net income ↑Shareholders' equity Translation gain	↑Revenues ↑Assets ↑Liabilities ↑Net income ↑Shareholders' equity Positive translation adjustment
Foreign currency weakens relative to parent's presentation currency	↓Revenues ↓Assets ↓Liabilities ↑Net income ↑Shareholders' equity Translation gain	↓Revenues ↓Assets ↓Liabilities ↓Net income ↓Shareholders' equity Translation loss	↓Revenues ↓Assets ↓Liabilities ↓Net income ↓Shareholders' equity Negative translation adjustment

Evaluating Quality of Financial Reports

Relationship between Financial Reporting Quality and Earnings Quality

		Financial Reporting Quality	
		Low	**High**
Earnings (Results) Quality	High	LOW financial reporting quality impedes assessment of earnings quality and impedes valuation.	HIGH financial reporting quality enables assessment. HIGH earnings quality increases company value.
	Low		HIGH financial reporting quality enables assessment. LOW earnings quality decreases company value.

Accounting Warning Signs[1]

Potential Issues	Possible Actions/Choices	Warning Signs
• Overstatement or non-sustainability of operating income and/or net income ○ Overstated or accelerated revenue recognition ○ Understated expenses ○ Misclassification of revenue, gains, expenses, or losses	• Contingent sales with right of return, "channel stuffing" (the practice of inducing customers to order products they would otherwise not order or order at a later date through generous terms), "bill and hold" sales (encouraging customers to order goods and retain them on seller's premises) • Lessor use of finance (capital) leases • Fictitious (fraudulent) revenue • Capitalizing expenditures as assets • Lessee use of operating leases • Classifying non-operating income or gains as part of operations • Classifying ordinary expenses as non-recurring or non-operating • Reporting gains through net income and losses through other comprehensive income	• Growth in revenue higher than that of industry or peers • Increases in discounts to and returns from customers • Higher growth rate in receivables than revenue • Large proportion of revenue in final quarter of year for a non-seasonal business • Cash flow from operations is much lower than operating income • Inconsistency over time in the items included in operating revenues and operating expenses • Increases in operating margin • Aggressive accounting assumptions, such as long, depreciable lives • Losses in non-operating income or other comprehensive income and gains in operating income or net income • Compensation largely tied to financial results
• Misstatement of balance sheet items (may affect income statement) ○ Over- or understatement of assets ○ Over- or understatement of liabilities ○ Misclassification of assets and/or liabilities	• Choice of models and model inputs to measure fair value • Classification from current to non-current • Over- or understating reserves and allowances • Understating identifiable assets and overstating goodwill	• Models and model inputs that bias fair value measures • Inconsistency in model inputs when measuring fair value of assets compared with that of liabilities • Typical current assets, such as accounts receivable and inventory, included in non-current assets • Allowances and reserves that fluctuate over time or are not comparable with peers • High goodwill value relative to total assets • Use of special purpose vehicles • Large changes in deferred tax assets and liabilities • Significant off-balance-sheet liabilities

[1]2015 CFA Program Curriculum Volume 2, page 305, Exhibit 4.

Potential Issues	Possible Actions/Choices	Warning Signs
• Overstatement of cash flow from operations	• Managing activities to affect cash flow from operations • Misclassifying cash flows to positively affect cash flow from operations	• Increase in accounts payable and decrease in accounts receivable and inventory • Capitalized expenditures in investing activities • Sales and leaseback • Increases in bank overdrafts

The Beinish Model

$$\begin{aligned}\text{M-score} = &-4.84 + 0.920(\text{DSR}) + 0.528(\text{GMI}) + 0.404(\text{AQI}) + 0.892(\text{SGI})\\ &+ 0.115(\text{DEPI}) - 0.172(\text{SGAI}) + 4.670(\text{Accruals}) - 0.327(\text{LEVI})\end{aligned}$$

Where

M-score = score indicating probability of earnings manipulation

DSR (days sales receivable index) = $(\text{Receivables}_t/\text{sales}_t)$ / $(\text{Receivables}_{t-1}/\text{sales}_{t-1})$

GMI (gross margin index) = $\text{Gross margin}_{t-1}$ / Gross margin_t

$$\text{AQI(asset quality index)} = \frac{1-(\text{PPE}_t + \text{CA}_t)}{\text{TA}_t} \Big/ \frac{1-(\text{PPE}_{t-1} + \text{CA}_{t-1})}{\text{TA}_{t-1}}$$

SGI (sales growth index) = $\text{sales}_t/\text{sales}_{t-1}$

DEPI (depreciation index) = $\text{Depreciation rate}_{t-1}/\text{Depreciation rate}_t$

Depreciation rate = Depreciation/(PPE + Depreciation)

SGAI (SG&A index) = $(\text{SGA}_t/\text{Sales}_t)/(\text{SGA}_{t-1}/\text{Sales}_{t-1})$

Accruals = (income before extraordinary items – CFO)/Total assets

LEVI (leverage index) = $\text{Leverage}_t/\text{Leverage}_{t-1}$

Leverage = debt/assets

Altman Model

$$\begin{aligned}\text{Z-score} = &\ 1.2(\text{Net working capital/Total assets}) + 1.4(\text{Retained earnings/Total assets})\\ &+ 3.3(\text{EBIT/Total assets}) + 0.6(\text{Market value of equity/Book value of liabilities})\\ &+ 1.0(\text{Sales/Total assets})\end{aligned}$$

Integration Of Financial Statement Analysis Techniques

A Financial Statement Analysis Framework:

	Phase	Sources of Information	Examples of Output
1.	Define the purpose and context of the analysis.	• The nature of the analyst's function, such as evaluating an equity or debt investment or issuing a credit rating. • Communication with client or supervisor on needs and concerns. • Institutional guidelines related to developing specific work product.	• Statement of the purpose or objective of analysis. • A list (written or unwritten) of specific questions to be answered by the analysis. • Nature and content of report to be provided. • Timetable and budgeted resources for completion.
2.	Collect input data.	• Financial statements, other financial data, questionnaires, and industry/economic data. • Discussions with management, suppliers, customers, and competitors. • Company site visits (e.g., to production facilities or retail stores).	• Organized financial statements. • Financial data tables. • Completed questionnaires, if applicable.
3.	Process input data, as required, into analytically useful data.	• Data from the previous phase.	• Adjusted financial statements. • Common-size statements. • Forecasts.
4.	Analyze/interpret the data.	• Input data and processed data.	• Analytical results.
5.	Develop and communicate conclusions and recommendations (e.g., with an analysis report).	• Analytical results and previous reports. • Institutional guidelines for published reports.	• Analytical report answering questions posed in Phase 1. • Recommendations regarding the purpose of the analysis, such as whether to make an investment or grant credit.
6.	Follow-up.	• Information gathered by periodically repeating above steps as necessary to determine whether changes to holdings or recommendations are necessary.	• Update reports and recommendations.

DuPont Analysis

ROE = Tax Burden × Interest burden × EBIT margin × Total asset turnover × Financial leverage

$$\text{ROE} = \frac{\text{NI}}{\text{EBT}} \times \frac{\text{EBT}}{\text{EBIT}} \times \frac{\text{EBIT}}{\text{Revenue}} \times \frac{\text{Revenue}}{\text{Average Asset}} \times \frac{\text{Average Asset}}{\text{Average Equity}}$$

Corporate Finance

Capital Budgeting

Expansion Project

Initial investment outlay for a new investment = FCInv + NWCInv

NWCInv = ΔNon-cash current assets − ΔNon-debt current liabilities

Annual after-tax operating cash flows (CF)

$$\boxed{CF = (S - C - D)(1 - t) + D \quad \text{or} \quad CF = (S - C)(1 - t) + tD}$$

Terminal year after-tax non-operating cash flow (TNOCF):

$$\boxed{TNOCF = Sal_T + NWCInv - t(Sal_T - BV_T)}$$

Replacement Project

Investment outlays:

$$\boxed{\text{Initial investment for a replacement project} = FCInv + NWCInv - Sal_0 + t(Sal_0 - BV_0)}$$

Annual after-tax operating cash flow:

$$\boxed{CF = (S - C)(1 - t) + tD}$$

Terminal year after-tax non-operating cash flow:

$$\boxed{TNOCF = Sal_T + NWCInv - t(Sal_T - B_T)}$$

Mutually Exclusive Projects with Unequal Lives

1. Least Common Multiple of Lives Approach

   In this approach, both projects are repeated until their “chains” extend over the same time horizon. Given equal time horizons, the NPVs of the two project chains are compared and the project with the higher chain NPV is chosen.

2. Equivalent Annual Annuity Approach (EAA)

   This approach calculates the annuity payment (equal annual payment) over the project’s life that is equivalent in present value (PV) to the project’s NPV. The project with the higher EAA is chosen.

SML

$$\boxed{R_i = R_F + \beta_i[E(R_M) - R_F]}$$

R_i = Required return for project or asset i
R_F = Risk-free rate of return
β_i = Beta of project or asset i
$[E(R_M) - R_F]$ = Market risk premium

Economic Income

Economic income = After-tax operating cash flow + Change in market value
Economic income = After-tax operating cash flow + (Ending market value – Beginning market value)

OR

Economic income = After-tax operating cash flow – (Beginning market value – Ending market value)

Economic income = After-tax cash flows – Economic depreciation

Economic Profit

Economic profit = [EBIT (1 – Tax rate)] – \$WACC

Economic profit = NOPAT – \$WACC

NOPAT = Net operating profit after tax
\$WACC = Dollar cost of capital = Cost of capital (%) × Invested capital

Under this approach, a project's NPV is calculated as the sum of the present values of economic profit earned over its life discounted at the cost of capital.

$$\text{NPV} = \text{MVA} = \sum_{t=1}^{\infty} \frac{\text{EP}_t}{(1+\text{WACC})^t}$$

Residual Income

$$RI_t = NI_t - r_e B_{t-1}$$

Where
RI_t = residual income in period t
NI_t = net income in period t
$r_e B_{t-1}$ = equity charge against beginning book value

The RI approach calculates value from the perspective of equity holders only. Therefore, future residual income is discounted at the **required rate of return on equity** to calculate NPV.

$$\text{NPV} = \sum_{t=1}^{\infty} \frac{\text{RI}_t}{(1+r_E)^t}$$

Claims Valuation

- Separate cash flows available to debt and equity holders
- Discount them at their respective required rates of return.
  - Cash flows available to debt holders are discounted at the cost of debt.
  - Cash flows available to equity holders are discounted at the cost of equity.
- Add the present values of the two cash flow streams to calculate total company/asset value.

Capital Structure

The Capital Structure Decision

$$r_{WACC} = \left(\frac{D}{V}\right) r_D (1 - t) + \left(\frac{E}{V}\right) r_E$$

r_D = Marginal cost of debt
r_E = Marginal cost of equity
t = Marginal tax rate
D = Market value of the company's outstanding debt
E = Market value of shareholders' equity
V = D + E = Value of the company

MM Proposition II without Taxes: Higher Financial Leverage Raises the Cost of Equity

$$r_{WACC} = \left(\frac{D}{V}\right) r_D + \left(\frac{E}{V}\right) r_E = r_0$$

Company's cost of equity (r_E) under MM Proposition II without taxs is calculated as:

$$\underbrace{r_E}_{\text{Dependent variable}} = \overbrace{r_0}^{\text{Intercept}} + \underbrace{(r_0 - r_D)}_{\text{Slope}}\overbrace{\frac{D}{E}}^{\text{Independent variable}}$$

The total value of the company is calculated as:

$$V = \frac{\text{Interest}}{r_D} + \frac{\text{EBIT} - \text{Interest}}{r_E}$$

The systematic risk (β) of the company's assets can be expressed as the weighted average of the systematic risk of the company's debt and equity.

$$\beta_A = \left(\frac{D}{V}\right)\beta_D + \left(\frac{E}{V}\right)\beta_E$$

This formula can also be expressed as:

$$\beta_E = \beta_A + (\beta_A - \beta_D)\left(\frac{D}{E}\right)$$

L2-2017-FS-04-CF.indd 34 12/09/19 2:18 AM

Relaxing the Assumption of no Taxes

$$\boxed{V_L = V_U + tD}$$

The WACC is then calculated as:

$$\boxed{r_{WACC} = \left(\frac{D}{V}\right) r_D (1-t) + \left(\frac{E}{V}\right) r_E}$$

And the cost of equity is calculated as:

$$\boxed{r_E = r_0 + (r_0 - r_D)(1-t)\left(\frac{D}{E}\right)}$$

Modigliani and Miller Propositions

	Without Taxes	With Taxes
Proposition I	$V_L = V_U$	$V_L = V_U + tD$
Proposition II	$r_E = r_0 + (r_0 - r_D)\frac{D}{E}$	$r_E = r_0 + (r_0 - r_D)(1-t)\left(\frac{D}{E}\right)$

The Optimal Capital Structure: The Static Trade-Off Theory

$$\boxed{V_L = V_U + tD - PV(\text{Costs of financial distress})}$$

Dividends and Share Repurchases

The expected decrease in share price when it goes ex-dividend can be calculated using the following equation:

$$P_W - P_X = \frac{1 - T_D}{1 - T_{CG}} \times D$$

P_w = Share price with the right to receive the dividend
P_x = Share price without the right to receive the dividend
D = Amount of dividend
T_D = Tax rate on dividends
T_{CG} = Tax rate on capital gains

Double Taxation System

$$ETR = CTR + [(1 - CTR) \times MTR_D]$$

ETR = Effective tax rate
CTR = Corporate tax rate
MTR_D = Investor's marginal tax rate on dividends

Split-Rate Tax System

$$ETR = CTR_D + [(1 - CTR_D) \times MTR_D]$$

CTR_D = Corporate tax rate on earnings distributed as dividends.

Stable Dividend Policy

The expected increase in dividends is calculated as:

$$\text{Expected dividend increase} = (\text{Expected earnings} \times \text{Expected payout ratio} - \text{Previous dividend}) \times \text{Adjustment factor}$$

$$\text{Expected dividend} = \text{Previous dividend} + (\text{Expected earnings} \times \text{Expected payout ratio} - \text{Previous dividend}) \times \text{Adjustment factor}$$

Adjustment factor = l/N
N = Number of years over which the adjustment is expected to occur

EPS Effects of Share Buyback

$$\text{EPS after buyback} = \frac{(\text{Earnings} - \text{After-tax cost of funds})}{\text{Shares outstanding after buyback}}$$

Analysis of Dividend Safety

$$\boxed{\text{Dividend payout ratio} = (\text{dividends} / \text{net income})}$$

$$\boxed{\text{Dividend coverage ratio} = (\text{net income} / \text{dividends})}$$

$$\boxed{\text{FCFE coverage ratio} = \text{FCFE} / [\text{Dividends} + \text{Share repurchases}]}$$

Mergers and Acquisition

Post-merger EPS

$$EPS_{Post} = \frac{E_{A,\text{Pre}} + E_{T,\text{Pre}}}{S_{A,\text{Pre}} + S_{A,Issued}}$$

$$S_{A,Issued} = \frac{MV_T}{P_A}$$

Where
E_A = Acquirer's pre-merger earnings
E_T = Target's pre-merger earnings
$S_{A,\text{Pre}}$ = Acquirer's pre-merger number of shares outstanding
$S_{A,\text{Issued}}$ = Acquirer's shares issued to purchase target
MV_T = Market value of target company's shares
P_A = Price of acquirer's shares

Mergers and the Industry Life Cycle

Industry Life Cycle Stage	Industry Description	Motives for Merger	Types of Merger
Pioneering development	• Low but slowly increasing sales growth. • Substantial development costs.	• Younger, smaller companies may sell themselves to larger firms in mature or declining industries to enter into a new growth industry. • Young companies may merge with firms that allow them to pool management and capital resources.	• Conglomerate • Horizontal
Rapid accelerating growth	• High profit margins. • Low competition.	• To meet substantial capital requirements for expansion.	• Conglomerate • Horizontal
Mature growth	• Decrease in the entry of new competitors. • Growth potential remains.	• To achieve economies of scale, savings, and operational efficiencies.	• Horizontal • Vertical

Industry Life Cycle Stage	Industry Description	Motives for Merger	Types of Merger
Stabilization and market maturity	• Increasing capacity constraints • Increasing competition.	• To achieve economies of scale in research, production, and marketing to match low costs and prices of competitors. • Large companies may buy smaller companies to improve management and provide a broader financial base.	• Horizontal
Deceleration of growth and decline	• Overcapacity. • Eroding profit margins.	• Horizontal mergers to ensure survival. • Vertical mergers to increase efficiency and profit margins. • Conglomerate mergers to exploit synergy. • Companies in the industry may acquire companies in young industries.	• Horizontal • Vertical • Conglomerate

Source: Adapted from J. Fred Weston, Kwang S. Chung, and Susan E. Hoag, *Mergers, Restructuring, and Corporate Control* (New York: Prentice Hall, 1990, p. 102) and Bruno Solnik and Dennis McLeavy, *International Investments,* 5th edition (Boston: Addison Wesley, 2004, pp. 264–265).

Major Differences of Stock versus Asset Purchases

	Stock Purchase	Asset Purchase
Payment	Target shareholders receive compensation in exchange for their shares.	Payment is made to the selling company rather than directly to shareholders.
Approval	Shareholder approval required.	Shareholder approval might not be required.
Tax: Corporate	No corporate-level taxes.	Target company pays taxes on any capital gains.
Tax: Shareholder	Target company's shareholders are taxed on their capital gain.	No direct tax consequence for target company's shareholders.
Liabilities	Acquirer assumes the target's liabilities.	Acquirer generally avoids the assumption of liabilities.

Herfindahl-Hirschman Index (HHI)

$$\sum_{i}^{n}\left(\frac{\text{Sales or output of firm i}}{\text{Total sales or output of market}}\times 100\right)^2$$

HHI Concentration Levels and Possible Government Response

Post-Merger HHI	Concentration	Change in HHI	Government Action
Less than 1,000	Not concentrated	Any amount	No action
Between 1,000 and 1,800	Moderately concentrated	100 or more	Possible challenge
More than 1,800	Highly concentrated	50 or more	Challenge

FCFF is estimated by:

	Net income
+	Net interest after tax
=	Unlevered net income
+	Changes in deferred taxes
=	NOPLAT (net operating profit less adjusted taxes)
+	Net noncash charges (depreciation)
–	Change in net working capital
–	Capital expenditures (capex)
	Free cash flow to the firm (FCFF)

Net interest after tax = (Interest expense – Interest income) (1 – tax rate)
Working capital = Current assets (excl. cash and equivalents) – Current liabilities (excl. short-term debt)

Comparable Company Analysis

$$TP = \frac{(DP - SP)}{SP}$$

TP = Takeover premium
DP = Deal price per share
SP = Target's stock price per share

Bid Evaluation

Target shareholders' gain = Takeover premium = $P_T - V_T$

Acquirer's gain = Synergies – Premium
$= S - (P_T - V_T)$

S = Synergies created by the merger transaction

The **post-merger value of the combined company** is composed of the pre-merger value of the acquirer, the pre-merger value of the target, and the synergies created by the merger. These sources of value are adjusted for the cash paid to target shareholders to determine the value of the combined post-merger company.

$$V_{A*} = V_A + V_T + S - C$$

V_{A*} = Value of combined company
C = Cash paid to target shareholders

EQUITY

Equity Valuation: Applications And Processes

Perceived mispricing:

Perceived mispricing = True mispricing + Error in the estimate of intrinsic value.

$$\boxed{V_E - P = (V - P) + (V_E - V)}$$

V_E = Estimate of intrinsic value
P = Market price
V = True (unobservable) intrinsic value

Return Concepts

Holding Period Return

$$\text{Holding period return} = \frac{P_H - P_0 + D_H}{P_0}$$

P_H = Price at the end of the holding period
P_0 = Price at the beginning of the period
D_H = Dividend

$$r = \frac{P_H - P_0}{P_0} + \frac{D_H}{P_0}$$
$$= \text{Capital gain yield} + \text{Dividend yield}$$

Required Return

When the investor's estimate of intrinsic value (V_0) is different from the current market price (P_0), the investor's expected return has two components:

1. The required return (r_T) earned on the asset's current market price; and
2. The return from convergence of price to value [$(V_0 - P_0)/P_0$].

Expected Alpha

$$E(\alpha) = E(r_{i,HP}) - r_i$$

Expected alpha is also known as *ex-ante alpha* or *expected abnormal return*. Expected return for the holding period will exceed required return when perceived value is greater than market price.

Realized Alpha

$$\text{Realized alpha} = \text{HPR} - r_{i,HP}$$

Realized alpha is also known as *ex-post alpha*, and equals actual holding period return *HPR* less required return.

Internal Rate of Return

$$\text{Intrinsic Value} = \frac{\text{Next year's expected dividend}}{\text{Required return} - \text{Expected dividend growth rate}}$$

$$V_0 = \frac{D_1}{k_e - g}$$

Assuming efficient pricing (i.e. the market price equals its intrinsic value), IRR equals required return on equity. Therefore, IRR can be estimated as:

$$\boxed{\text{Required return (IRR)} = \frac{\text{Next year's dividend}}{\text{Market price}} + \text{Expected dividend growth rate}}$$

$$\boxed{k_e(\text{IRR}) = \frac{D_1}{P_0} + g}$$

Equity Risk Premium

Required Return (CAPM)

$$\boxed{r_i = r_f + \beta_{i,M}(r_M - r_f)}$$

Where
r_i = required return on an individual asset
r_f = the nominal risk-free rate
$\beta_{i,M}$ = the sensitivity of stock i to changes in the equity market
$(r_M - r_f)$ = the market risk premium; i.e., extra amount required for investors to hold equities rather than a risk-free asset

Required Return (Buildup Approach)

$$\boxed{r_i = r_f + \text{equity risk premium} + \text{other risk premiums/(discounts)}}$$

Other risk premiums are used primarily in describing compensation for the additional risk of closely held stock (e.g., lack of marketability or liquidity) while discounts describe a reduction in compensation for control over the business and other situations.

Gordon Growth Model (GGM) Estimates

Gordon Growth Model (GGM) Risk Premium Estimate

$$\boxed{ERP_{GGM} = \frac{D_1}{P_0} + g_e - Y_{LTGB}}$$

Where
D_1/P_0 = current market dividend yield
g_e = long-term earnings growth rate
Y_{LTGB} = yield on long-term government bonds

Supply Side Estimate (Ibbotson-Chen)

$$\boxed{\text{Equity risk premium} = \{[(1 + \text{EINFL})\,(1 + \text{EGREPS})\,(1 + \text{EGPE}) - 1] + \text{EINC}\} - \text{Expected RF}}$$

Where
EINFL = Expected Inflation
EGREPS = Expected earnings growth in rate real earnings per share
EGPE = Expected growth rate in the P/E ratio
EINC = Expected income component (e.g., dividends and reinvestment)
ERFR = Expected risk-free rate

$$\text{Expected inflation} = \frac{1 + \text{YTM of 20-year maturity T-bonds}}{1 + \text{YTM of 20-year maturity TIPS}} - 1$$

The Fama-French Model

$$r_i = RF + \beta_i^{mkt} RMRF + \beta_i^{size} SMB + \beta_i^{value} HML$$

$RMRF = R_M - R_F$; the market risk premium
β_{mkt} = Market beta
β_{size} = Company size beta
β_{value} = Value beta

The Pastor-Stambaugh model (PSM)

$$r_i = R_F + \beta_i^{mkt} RMRF + \beta_i^{size} SMB + \beta_i^{value} HML + \beta_i^{liq} LIQ$$

"SMB = small minus big" CR "HML = high minus low
β^{liq} = Liquidity beta

BIRR model

r_i = T-bill rate + (Sensitivity to confidence risk × Confidence risk)
+ (Sensitivity to time horizon risk × Time horizon risk)
+ (Sensitivity to inflation risk × Inflation risk)
+ (Sensitivity to business cycle risk × Business cycle risk)
+ (Sensitivity to market timing risk × Market timing risk)

Build-up method

r_i = Risk-free rate + Equity risk premium + Size premium + Specific-company premium

For companies with publicly-traded **debt**, the bond-yield plus risk premium approach can be used to calculate the cost of equity:

BYPRP cost of equity = YTM on the company's long-term debt + Risk premium

Adjusting Beta for Beta Drift

Adjusted beta = (2/3) (Unadjusted beta) + (1/3) (1.0)

Estimating the Asset Beta for the Comparable Publicly Traded Firm:

B_{ASSET} reflects only business risk of the comparable company. Therefore it is used as a proxy for business risk of the project being studied.

$$\beta_{ASSET} = \beta_{EQUITY}\left[\frac{1}{1+\left((1-t)\frac{D}{E}\right)}\right]$$

where:
D/E = debt-to-equity ratio of the comparable company.
t = marginal tax rate of the comparable company.

Adjust the asset beta of the comparable for the capital structure (financial risk) of the project or company being evaluated:

$B_{PROJECT}$ reflects business and financial risk of the project.

$$\beta_{PROJECT} = \beta_{ASSET}\left[1+(1-t)\frac{D}{E}\right]$$

where:
D/E = debt-to-equity ratio of the subject company.
t = marginal tax rate of the subject company.

Country Spread Model

$$\text{ERP estimate} = \text{ERP for a developed market} + \text{Country premium}$$

Weighted Average Cost of Capital (WACC)

$$\text{WACC} = \frac{\text{MVD}}{\text{MVD}+\text{MVCE}} r_d(1-\text{Tax rate}) + \frac{\text{MVCE}}{\text{MVD}+\text{MVCE}} r$$

MVD = Market value of the company's debt
r_d = Required rate of return on debt
MVCE = Market value of the company's common equity
r = Required rate of return on equity

Industry and Company Analysis

Growth Relative to GDP Growth Approach

$$g_S = \beta_{S,GDP} \times g_{GDP}$$

A company's sales growth rate based on sensitivity of its growth rate to the country growth rate.

Market Growth and Market Share Approach

$$g_S = (1 + g_M)(1 + g_{MS}) - 1$$

A company's sales growth rate based on market growth and growth of the company's share in that market.

Porter's Five Forces

- Threat of substitutes
- Rivalry – Intensity of competition
- Bargaining power of suppliers
- Bargaining power of customers
- Threat of new entrants – Based on profitability of the market, barriers to entry, etc.

Return on Capital Employed

$$ROCE = \frac{\text{Operating profit}}{\text{operating assets} - \text{operating liabilities}} = \frac{\text{Operating profit}}{\text{cash} + \text{cash equivalents} + \text{net working capital} + \text{net fixed asssets}}$$

ROCE (a form of pre-tax ROIC) is useful for comparing companies across countries with different tax structures.

Discounted Dividend Valuation

One-Period DDM

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{P_1}{(1+r)^1} = \frac{D_1 + P_1}{(1+r)^1}$$

V_0 = The value of the stock today (t = 0)
P_1 = Expected price of the stock after one year (t = 1)
D_1 = Expected dividend for Year 1, assuming it will be paid at the end of Year 1 (t = 1)
r = Required return on the stock

Multiple-Period DDM

$$V_0 = \frac{D_1}{(1+r)^1} + \ldots + \frac{D_n}{(1+r)^n} + \frac{P_n}{(1+r)^n}$$

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

Expression for Calculating Value of a Share of Stock

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

Gordon Growth Model

$$V_0 = \frac{D_0(1+g)}{(r-g)}, \text{ or } V_0 = \frac{D_1}{(r-g)}$$

Present Value of Growth Opportunities

$$V_0 = \frac{E_1}{r} + \text{PVGO}$$

P/E Ratio

$$\text{Justified leading P/E ratio} = \frac{P_0}{E_1} = \frac{D_1/E_1}{r-g} = \frac{(1-b)}{r-g}$$

$$\text{Justified trailing P/E} = \frac{P_0}{E_0} = \frac{D_1/E_0}{r-g} = \frac{D_0(1+g)/E_0}{r-g} = \frac{(1-b)(1+g)}{r-g}$$

Where
CR “b = retention rate”
CR “(1 - b) = payout rate”

Value of Fixed-Rate Perpetual Preferred Stock

$$V_0 = \frac{D}{r}$$

Two-Stage Dividend Discount Model

$$V_0 = \sum_{t=1}^{n} \frac{D_0(1+g_S)^t}{(1+r)^t} + \frac{D_0(1+g_S)^n(1+g_L)}{(1+r)^n(r-g_L)}$$

g_s = Short-term supernormal growth rate
g_L = Long-term sustainable growth rate
r = required return
n = Length of the supernormal growth period

The H-Model

$$V_0 = \frac{D_0(1+g_L)}{r-g_L} + \frac{D_0H(g_s-g_L)}{r-g_L}$$

g_s = Short-term high growth rate
g_L = Long-term sustainable growth rate
r = Required return
H = Half-life = 0.5 times the length of the high growth period

The H-model equation can be rearranged to calculate the required rate of return as follows:

$$r = \left(\frac{D_0}{P_0}\right)[(1+g_L)+H(g_s-g_L)]+g_L$$

The Gordon growth formula can be rearranged to calculate the required rate of return given the other variables.

$$r = \frac{D_1}{P_0} + g$$

Sustainable growth rate (SGR)

$$g = b \times ROE$$

b = Earnings retention rate, calculated as 1 – Dividend payout ratio

Basic DuPont Analysis

$$ROE = \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Assets}} \times \frac{\text{Assets}}{\text{Shareholders' equity}}$$
$$= \text{Profit margin} \times \text{Asset turnover} \times \text{Financial leverage}$$

PRAT model

$$g = b \times ROE$$
$$= \frac{\text{Net income} - \text{dividends}}{\text{Net income}} \times ROE$$

$$g = \frac{\text{Net income} - \text{Dividends}}{\text{Net income}} \times \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Shareholders' equity}}$$

Free Cash Flow Valuation

FCFF/FCFE

$$\text{Firm Value} = \sum_{t=1}^{\infty} \frac{\text{FCFF}_t}{(1+\text{WACC})^t}$$

$$\text{WACC} = \frac{\text{MV(Debt)}}{\text{MV(Debt)} + \text{MV(Equity)}} r_d (1 - \text{Tax Rate}) + \frac{\text{MV(Equity)}}{\text{MV(Debt)} + \text{MV(Equity)}} r$$

$$\text{Equity Value} = \text{Firm Value} - \text{Market value of debt}$$

$$\text{Equity Value} = \sum_{t=1}^{\infty} \frac{\text{FCFE}_t}{(1+r_e)^t}$$

Computing FCFF from Net Income

$$\text{FCFF} = \text{NI} + \text{NCC} + \text{Int}(1 - \text{Tax Rate}) - \text{FCInv} - \text{WCInv}$$

Investment in fixed capital (FCInv)

$$\text{FCInv} = \text{Capital expenditures} - \text{Proceeds from sale of long-term assets}$$

Investment in working capital (WCInv)

$$\text{WCInv} = \text{Change in working capital over the year}$$

$$\text{Working capital} = \text{Current assets (exc. cash)} - \text{Current liabilities (exc. short-term debt)}$$

Table: Noncash Items and FCFF

Noncash Item	Adjustment to NI to Arrive at FCFF
Depreciation	Added back
Amortization and impairment of intangibles	Added back
Restructuring charges (expense)	Added back
Restructuring charges (income resulting from reversal)	Subtracted
Losses	Added back
Gains	Subtracted
Amortization of long-term bond discounts	Added back
Amortization of long-term bond premiums	Subtracted
Deferred taxes	Added back but requires special attention

Computing FCFF from CFO

Table: IFRS versus U.S. GAAP Treatment of Interest and Dividends

	IFRS	U.S. GAAP
Interest received	CFO or CFI	CFO
Interest paid	CFO or CFF	CFO
Dividend received	CFO or CFI	CFO
Dividends paid	CFO or CFF	CFF

$$\boxed{\text{FCFF} = \text{CFO} + \text{Int}(1 - \text{Tax rate}) - \text{FCInv}}$$

Computing FCFF from EBIT

$$\boxed{\text{FCFF} = \text{EBIT}(1 - \text{Tax rate}) + \text{Dep} - \text{FCInv} - \text{WCInv}}$$

Computing FCFF from EBITDA

$$\boxed{\text{FCFF} = \text{EBITDA}(1 - \text{Tax rate}) + \text{Dep}(\text{Tax rate}) - \text{FCInv} - \text{WCInv}}$$

Computing FCFE from FCFF

$$\boxed{\text{FCFE} = \text{FCFF} - \text{Int}(1 - \text{Tax rate}) + \text{Net borrowing}}$$

Computing FCFE from Net Income

$$\boxed{\text{FCFE} = \text{NI} + \text{NCC} - \text{FCInv} - \text{WCInv} + \text{Net Borrowing}}$$

Computing FCFE from CFO

$$\boxed{\text{FCFE} = \text{CFO} - \text{FCInv} + \text{Net borrowing}}$$

Computing FCFE from EBIT

$$\boxed{\text{FCFE} = \text{EBIT}(1 - \text{Tax rate}) - \text{Int}(1 - \text{Tax rate}) + \text{Dep} - \text{FCInv} - \text{WCInv} + \text{Net borrowing}}$$

Computing FCFE from EBITDA

$$\boxed{\text{FCFE} = \text{EBITDA}(1 - \text{Tax rate}) - \text{Int}(1 - \text{Tax rate}) + \text{Dep}(\text{Tax rate}) - \text{FCInv} - \text{WCInv} + \text{Net borrowing}}$$

Uses of FCFF

Increases in cash balances
Plus: Net payments to providers of debt capital
+ Interest expense (1 – tax rate)
+ Repayment of principal
– New borrowing
Plus: Net payments to providers of equity capital
+ Cash dividends
+ Share repurchases
– New equity issues
= Uses of FCFF

Uses of FCFE

Increases in cash balances
Plus: Net payments to providers of equity capital
+ Cash dividends
+ Share repurchases
– New equity issues
= Uses of FCFE

Constant Growth FCFF Valuation Model

$$\text{Value of the firm} = \frac{FCFF_1}{WACC - g} = \frac{FCFF_0(1+g)}{WACC - g}$$

WACC = Weighted average cost of capital
g = Long-term constant growth rate in FCFF

Constant Growth FCFE Valuation Model

$$\text{Value of equity} = \frac{FCFE_1}{r - g} = \frac{FCFE_0(1+g)}{r - g}$$

r = Required rate of return on equity
g = Long-term constant growth rate in FCFE

An International Application of the Single-Stage Model

$$\text{Value of equity} = \frac{FCFE_0(1 + g_{real})}{r_{real} - g_{real}}$$

General expression for the two-stage FCFF model:

$$\text{Firm value} = \sum_{t=1}^{n} \frac{\text{FCFF}_t}{(1+\text{WACC})^t} + \frac{\text{FCFF}_{n+1}}{(\text{WACC}-g)} \frac{1}{(1+\text{WACC})^n}$$

Firm value = PV of FCFF in Stage 1 + Terminal value × Discount Factor

General expression for the two-stage FCFE model:

$$\text{Equity value} = \sum_{t=1}^{n} \frac{\text{FCFE}_t}{(1+r)^t} + \frac{\text{FCFF}_{n+1}}{(r-g)} \frac{1}{(1+r)^n}$$

Equity value = PV of FCFE in Stage 1 + Terminal value × Discount Factor

Determining Terminal Value

Terminal value in year n = Justified Trailing P/E × Forecasted Earnings in Year n
Terminal value in year n = Justified Leading P/E × Forecasted Earnings in Year n + 1

Non-operating Assets and Firm Value

Value of the firm = Value of operating assets + Value of non-operating assets

Market-based Valuation: Price And Enterprise Value Multiples

Price to Earnings Ratio

$$\text{Trailing P/E ratio} = \frac{\text{Current Stock Price}}{\text{Last year's EPS}}$$

$$\text{Forward P/E ratio} = \frac{\text{Current Stock Price}}{\text{Expected EPS}}$$

Price to Book Ratio

$$\text{P/B ratio} = \frac{\text{Market price per share}}{\text{Book value per share}}$$

$$\text{P/B ratio} = \frac{\text{Market value of common shareholders' equity}}{\text{Book value of common shareholders' equity}}$$

Book value of equity = Common shareholders' equity
= Shareholders' equity – Total value of equity claims that are senior to common stock

$$\text{Book value of equity} = \text{Total assets} - \text{Total liabilities} - \text{Preferred stock}$$

Price to Sales Ratio

$$\text{P/S ratio} = \frac{\text{Market price per share}}{\text{Sales per share}}$$

Relationship between the P/E ratio and the P/S ratio

$$\text{P/E} \times \text{Net profit margin} = (\text{P / E}) \times (\text{E / S}) = \text{P/S}$$

Price to Cash Flow Ratio

$$\text{P/CF ratio} = \frac{\text{Market price per share}}{\text{Free cash flow per share}}$$

Dividend Yield

Justified trailing dividend yield

$$\text{Trailing dividend yield} = \text{Last year's dividend / Current price per share}$$

Justified leading dividend yield

$$\text{Leading dividend yield} = \text{Next year's dividend / Current price per share}$$

Justified P/E Multiple Based on Fundamentals

Justified leading P/E multiple

$$\boxed{V_0 = \frac{D_1}{(r-g)}}$$

$$\boxed{\text{Justified leading P/E} = \frac{P_0}{E_1} = \frac{D_1/E_1}{r-g} = \frac{(1-b)}{r-g}}$$

(1 – b) is the payout ratio.

Justified trailing P/E multiple

$$\boxed{\text{Justified trailing P/E} = \frac{P_0}{E_0} = \frac{D_1/E_0}{r-g} = \frac{D_0(1+g)/E_0}{r-g} = \frac{(1-b)(1+g)}{r-g}}$$

Justified P/B Multiple Based on Fundamentals

$$\boxed{\frac{P_0}{B_0} = \frac{ROE-g}{r-g}}$$

ROE = Return on equity
r = required return on equity
g = Sustainable growth rate

Justified P/S Multiple Based on Fundamentals

$$\boxed{\frac{P_0}{S_0} = \frac{(E_0/S_0)(1-b)(1+g)}{r-g}}$$

E_0/S_0 = Net profit margin
1 – b = Payout ratio

Justified P/CF Multiple Based on Fundamentals

$$\boxed{V_0 = \frac{FCFE_0(1+g)}{(r-g)}}$$

Justified Dividend Yield

$$\boxed{\frac{D_0}{P_0} = \frac{r-g}{1+g}}$$

P/E-to-growth (PEG) ratio

$$\text{PEG} = \frac{\text{P/E}}{\text{Growth (\%)}}$$

Justified P/E – Earnings Yield Models (Fed Model)

$$\frac{P_0}{E_0} = \frac{1}{Y_{T10}} \quad \text{OR} \quad Y_{T10} = \frac{E_0}{P_0}$$

The Fed model assumes the equity market's earnings yield (E/P) should be closely linked to the 10-year Treasury rate. When $Y_{T10} > E/P$, the market is overvalued.

Justified P/E – Earnings Yield Models (Yardeni Model)

$$CEY = CBY - b \times LTEG + residual$$
$$\frac{P}{E} = \frac{1}{CBY - b \times LTEG}$$

Where
CEY = Corporate earnings yield = E/P
CBY = Corporate bond yield
b = Confidence in long-term earnings forecast
LTEG = 5-year earnings growth rate forecast

The advantage of the Yardeni model over the Fed model is consideration of default risk inherent in using corporate bonds, but does not entirely capture equity risk which may run 300-400 bp higher than corporate bond yield.

Own historical P/E

$$\text{Justified Price} = \text{Benchmark Historical P/E} \times \text{Current Earnings}$$

Terminal price based on fundamentals

$$TV_n = \text{Justified leading P/E} \times \text{Forecasted earnings}_{n+1}$$
$$TV_n = \text{Justified trailing P/E} \times \text{Forecasted earnings}_n$$

Terminal price based on comparables

$$TV_n = \text{Benchmark leading P/E} \times \text{Forecasted earnings}_{n+1}$$
$$TV_n = \text{Benchmark trailing P/E} \times \text{Forecasted earnings}_n$$

EV/EBITDA Multiple

Enterprise value = Market value of common equity + Market value of preferred stock + Market value of debt – Value of cash and short-term investments

EBITDA = Net income + Interest + Taxes + Depreciation and amortization

Alternative Denominators in Enterprise Value Multiples

Free Cash Flow to the Firm =	Net Income	plus Interest Expense	minus Tax Savings on Interest	plus Depreciation	plus Amortization	less Investment in Working Capital	less Investment in Fixed Capital
EBITDA=	Net Income	plus Interest Expense	plus Taxes	plus Depreciation	plus Amortization		
EBITA =	Net Income	plus Interest Expense	plus Taxes		plus Amortization		
EBIT =	Net Income	plus Interest Expense	plus Taxes				

Justified forward P/E after accounting for Inflation

$$\frac{P_0}{E_1} = \frac{1}{\rho + (1 - \lambda)I}$$

λ = The percentage of inflation in costs that the company can pass through to revenue.
ρ = Real rate of return
I = Rate of inflation

Unexpected earnings (UE)

$$UE_t = EPS_t - E(EPS_t)$$

Standardized unexpected earnings (SUE)

$$SUE_t = \frac{EPS_t - E(EPS_t)}{\sigma[EPS_t - E(EPS_t)]}$$

EPS_t = Actual EPS for time t
$E(EPS_t)$ = Expected EPS for time t
$\sigma[EPS_t - E(EPS_t)]$ = Standard deviation of $[EPS_t - E(EPS_t)]$

Residual Income Valuation

Net Income

	EBIT (Operating earnings or profit)
Less:	Interest expense
	Pretax income
Less:	Income tax expense
	Net income

Residual Income – Equity Charge Method

$$\text{Residual income} = \text{Net income} - \text{Equity charge}$$
$$\text{Equity charge} = BV_{CE} \times r_{CE}$$

Where BV_{CE} = Beginning-of-period book value of common equity and r_{CE} = Required return on common equity

Residual Income – Capital Charge Method

$$\text{Residual income} = \text{EBIT(i} - \text{Tax rate)} - \text{Capital charge}$$
$$\text{Capital charge} = (BV_{CE} \times r_{CE}) + [BV_D \times r_D(1-t)]$$

Where BV_{CE} = Beginning of period book value of debt, r_D = Required return on debt, and t = effective tax rate

The equity method and capital method yield the *same* residual income if 1) after-tax interest expense for net income equals that used for the capital charge, and 2) capital charge weights for debt and equity are derived from beginning of period book values. Using WACC based on target values to calculate the capital charge will not yield the same residual income under the capital charge method as under the equity charge method.

Economic Value Added

$$\text{EVA} = \text{NOPAT} - (\text{C\%} \times \text{TC})$$

NOPAT = Adjusted net operating profit after tax = EBIT (1 – Tax rate)
C% = Cost of capital (WACC using book value weights)
TC = Beginning-of-period adjusted total capital

Both NOPLAT and total capital are adjusted to capitalize and amortize rather than expense R&D, suspend the capital charge on strategic investments until they yield revenue, add LIFO reserve back to capital, eliminate deferred taxes, treat operating leases as capital leases, and adjust non-recurring items.

Market Value Added

$$\text{MVA} = \text{Market value of the company} - \text{Accounting book value of total capital}$$

$$\text{Market value of company} = \text{Market value of debt} + \text{Market value of equity.}$$

The Residual Income Model

$$\boxed{RI_t = E_t - (r \times B_{t-1})}$$

RI_t = Residual income at time t
E_t = Earnings at time t
r = Required rate of return on equity
B_{t-1} = Book value at time t–1

Intrinsic value of a stock:

$$\boxed{V_0 = B_0 + \sum_{i=1}^{\infty} \frac{RI_t}{(1+r)^t} = B_0 + \sum_{i=1}^{\infty} \frac{E_t - rB_{t-1}}{(1+r)^t}}$$

V_0 = Intrinsic value of the stock today
B_0 = Current book value per share of equity
B_t = Expected book value per share of equity at any time t
r = Required rate of return on equity
E_t = Expected EPS for period t
RI_t = Expected residual income per share

Residual Income Model (Alternative Approach)

$$\boxed{RI_t = EPS_t - (r \times B_{t-1})}$$

$$\boxed{RI_t = (ROE_t - r)B_{t-1}}$$

Where ROE_t is calculated using net income during period t, and beginning-of-period book value rather than average equity over the period.

$$\boxed{V_0 = B_0 + \sum_{t=1}^{\infty} \frac{(ROE_t - r)B_{t-1}}{(1+r)^t}}$$

$$\boxed{V_0 = B_0 + \frac{ROE - r}{r - g} B_0}$$

Clean Surplus Relation

$$\boxed{B_t = B_{t-1} + E_t - D_t}$$

This relationship describes all changes to equity as having gone through the income statement.

Tobin's q

$$\text{Tobin's q} = \frac{\text{Market value of debt and equity}}{\text{Replacement cost of total assets}}$$

Multi-Stage Residual Income Valuation

$$V_0 = B_0 + \sum_{t=1}^{T} \frac{(E_t - rB_{t-1})}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T}$$

When residual income fades over time as ROE declines towards the required return on equity, the intrinsic value of a stock is calculated using the following formula:

$$V_0 = B_0 + \sum_{t=1}^{T-1} \frac{(E_t - rB_{t-1})}{(1+r)^t} + \frac{E_T - rB_{T-1}}{(1+r-\omega)(1+r)^{T-1}}$$

ω = Persistence factor.

Implied Growth Rate

$$g = r - \left[\frac{(ROE - r) \times B_0}{V_0 - B_0}\right]$$

Private Company Valuation

The Capitalized Cash Flow Method

$$V_f = \frac{FCFF_1}{WACC - g_f}$$

V_f = Value of the firm
$FCFF_1$ = Free cash flow to the firm for next twelve months
WACC = Weighted average cost of capital
g_f = Sustainable growth rate of free cash flow to the firm

$$V = \frac{FCFE_1}{r - g}$$

V = Value of equity
$FCFE_1$ = Free cash flow to the equity for next twelve months
r = Required return on equity
g = Sustainable growth rate of free cash flow to equity

Methods Used to Estimate the Required Rate of Return for a Private Company

Capital Asset Pricing Model

Required return on equity = Risk-free rate + (Beta × Market risk premium)

Expanded CAPM

Required return on equity = Risk-free rate + (Beta × Market risk premium) + Small stock premium + Company-specific risk premium

Build-Up Approach

Required return on equity = Risk-free rate + Equity risk premium + Small stock premium + Company-specific risk premium + Industry risk premium

Discount for Lack of Control (DLOC)

$$DLOC = 1 - \left[\frac{1}{1 + \text{Control premium}}\right]$$

Discount for Lack of Marketability (DLOM)

$$DLOM = \left[1 - \frac{1}{1 + \text{Marketability premium}}\right]$$

Combining Discounts

$$\boxed{V_{EI} = V_{EO}(1 - DLOC)(1 - DLOM)}$$

Where V_{EI} = value of equity indicated after discounts and V_{EO} = value of equity in operations

NOTE: These calculations must be performed to first recognize the lack of control in a position and then whether the equity is not easily marketable.

Commodities and Commodity Derivatives: An Introduction

Backwardation and Contango

$$\boxed{F_0 = S_0 e^{rT}}$$

$$\boxed{F_0 = S_0 e^{(r+U)T}}$$

$$\boxed{F_0 = S_0 e^{(r+U-Y)T}}$$

Return Components of Commodity Futures Investments

Rebalancing (diversification) return:

Total return = Spot return + Roll return + Collateral return + Rebalancing return

Spot return:

$$\boxed{RS = \frac{S_t - S_{t-1}}{S_{t-1}}}$$

Roll return:

$$\boxed{R_r = \frac{F_{t,T} - F_{t-1,t}}{F_{t-1,t}} = \frac{F_{t,T} - S_t}{S_t}}$$

Total return:

$$\boxed{\begin{aligned}\text{Total return} &= \text{Collateral return} + \text{Futures return}\\ &= \text{Collateral return} + \text{Spot return} + \text{Roll Return}\end{aligned}}$$

Excess return:

$$\boxed{\text{Excess return} = \text{Spot return} + \text{Roll return} = \text{Futures return}}$$

Fixed Income

The Term Structure and Interest Rate Dynamics

Discount Factor

$$P(T) = \frac{1}{[1 + r(T)]^T}$$

Where P(T) is a discount factor used to determine the present value of a payment received at time T, r(T) is the yield to maturity of the payment, also known as a *spot rate,* and T is the tenor (number of periods to maturity).

Forward Contract Price

$$F(T^*,T) = \frac{1}{[1 + f(T^*,T)]^T}$$

Where T* is the number of periods until initiation of the forward contract, and *f*(T*,T) is the implied t-period forward rate T* periods in the future.

Forward Pricing Model

$$P(T^* + T) = P(T^*)F(T^*,T)$$

Forward Rate Model

$$[1 + r(T^* + T)]^{T^*+T} = [1 + r(T^*)]^{T^*}[1 + f(T^*,T)]^T$$

$$r(T^* + T) = \left\{[1 + r(T^*)]^{T^*}[1 + f(T^*,T)]^T\right\}^{1/(T^*+T)} - 1$$

The forward rate model is the forward pricing model expressed in terms of rates, and shows the T-period forward rate at period T* as a function of any two spot rates.

Relationship between the Spot Rate and One-Period Forward Rates

$$[1 + r(T)]^T = [1 + r(1)][1 + f(1,1)][1 + f(2,1)]\ldots[1 + f(T-1,1)]$$

$$r(T) = \left\{[1 + r(1)][1 + f(1,1)][1 + f(2,1)]\ldots[1 + f(T-1,1)]\right\}^{1/T} - 1$$

Swap Rates

$$\sum_{t=1}^{T}\frac{s(T)}{[1+r(t)]^{t}}+\frac{1}{[1+r(t)]^{t}}-1=0$$
$$\sum_{t=1}^{T}\frac{s(T)}{[1+r(t)]^{t}}+\frac{1}{[1+r(t)]^{t}}=1$$

Where the swap rate, s, solves the equation given the spot rate at initiation, $r(t)$. (The value of the swap at initiation is 0; therefore, the floating rate side of the equation always equals 1.)

Swap Spread

$$\text{Swap spread} = \text{Swap rate} - Y_T$$

The swap spread is the difference between the fixed rate side of a swap and most recently issued government security with equivalent maturity. Swap spreads help investors determine the time value, liquidity risk, and credit risk components of a bond's yield.

The Arbitrage-Free Valuation Framework

Arbitrage-Free Value of an Option-Free, Fixed-Rate Coupon Bond

$$B_n = 0.50 \times \left[\frac{VH + C}{(1+i)} + \frac{VL + C}{(1+i)} \right]$$

Where B_n = the bond value at any node, *n, VH* and *VL* are the high and low values assumed by the bond corresponding to node *n* based on the high and low forward rates, and *C* is the fixed-rate coupon.

Interest Rates

$$r_u = r_d \times e^{2\sigma\sqrt{t}}$$

Where σ = interest rate volatility (standard deviation).

Valuation and Analysis: Bonds With Embedded Options

Value of callable bond = Value of straight bond – Value of embedded call option

Value of embedded call option = Value of straight bond – Value of callable bond

Value of putable bond = Value of straight bond + Value of embedded put option

Value of embedded put option = Value of putable bond – Value of straight bond

Effective Duration:

$$\text{Effective Duration} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\ \text{Curve}) \times PV_0}$$

ΔCurve = the magnitude of the parallel shift in the benchmark yield curve (in decimal).

PV_- = Full price of the bond when the benchmark yield curve is shifted down by ΔCurve.

PV_+ = Full price of the bond when the benchmark yield curve is shifted up by ΔCurve.

PV_0 = Current full price of the bond (i.e., with no shift).

Properties of Effective Durations of Cash and Common Types of Bonds

Type of Bond	Effective Duration
Cash	0
Zero-coupon bond	$\approx$ Maturity
Fixed-rate bond	$<$ Maturity
Callable bond	$\leq$ Duration of straight bond
Putable bond	$\leq$ Duration of straight bond
Floater (Libor flat)	$\approx$ Time (in years) to next reset

Effective Convexity

$$\text{Effective Convexity} = \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta\ \text{Curve})^2 \times PV_0}$$

Floating-Rate Securities

Value of capped floater = Value of uncapped floater – Value of embedded cap.

Value of embedded cap = Value of uncapped floater – Value of capped floater

Value of floored floater = Value of non-floored floater + Value of embedded floor.

Value of embedded floor = Value of floored floater – Value of non-floored floater

Convertible Bonds

$$\text{Conversion value} = \text{Market price of common stock} \times \text{Conversion ratio}$$

$$\text{Market conversion price} = \frac{\text{Market price of convertible security}}{\text{Conversion ratio}}$$

$$\text{Market conversion premium per share} = \text{Market conversion price} - \text{Current market price}$$

$$\text{Market conversion premium ratio} = \frac{\text{Market conversion premium per share}}{\text{Market price of common stock}}$$

$$\text{Premium over straight value} = \frac{\text{Market price of convertible bond}}{\text{Straight value}} - 1$$

Valuation of a convertible bond that is not callable or putable:

$$\text{Convertible security value} = \text{Straight Value} + \text{Value of the call option on the stock}$$

Valuation of a convertible bond that is callable but not putable:

$$\begin{aligned}\text{Convertible callable bond value} = {} & \text{Straight value} + \text{Value of the call option on the stock} \\ & - \text{Value of the call option on the bond}\end{aligned}$$

Valuation of a convertible bond that is callable and putable:

$$\begin{aligned}\text{Convertible callable and putable bond value} = {} & \text{Straight value} \\ & + \text{Value of the call option on the stock} \\ & - \text{Value of the call option on the bond} \\ & + \text{Value of the put option on the bond}\end{aligned}$$

Credit Analysis Models

Change in Bond Price

– Modified duration × (New credit rating credit spread – Original credit rating credit spread)

Structural Models

$$D(T) + E(T) = A(T)$$
$$E(T) = \text{Max}[A(T) - K,0]$$
$$D(T) = A(T) - \text{Max}[A(T) - K,0] \text{ or } D(T) = A(T) - E(T)$$

Derivatives Instruments—Valuation and Strategies

CREDIT DEFAULT SWAPS

Settlement Protocols

$$\begin{aligned}\text{Payout amount} &= \text{Payout ratio} \times \text{notional}\\ &= (1 - \%\ \text{recovery rate}) \times \text{notional}\end{aligned}$$

Where payout ratio is an estimate of the percentage credit loss on the CDS

CDS Pricing

$$\begin{aligned}P_{CDS}\ \text{per 100 par} &= 100 - \text{Upfront premium }\%\\ &\approx 100 - \left[(\text{Credit spread} - \text{Fixed coupon}) \times \text{Duration in years}\right]\end{aligned}$$

CDS Profit/(Loss)

$$\begin{aligned}\text{Protection buyer's } \Pi &= \%\Delta P_{CDS} \times \text{Notional}\\ &= \Delta S_{bps} \times D \times \text{Notional}\end{aligned}$$

The percentage price change in the CDS equals the bp change in spread multiplied by duration.

Pricing and Valuation of Forward Commitments

Valuing a Forward Contract at Expiration (t = T)

Value of long position:

$$\boxed{V_T(T) = S_T - F_0(T)}$$

Value of short position:

$$\boxed{V_T(T) = F_0(T) - S_T}$$

Valuing a Forward Contract at Initiation (t = 0)

$$\boxed{V_0(T) = S_0 - [F_0(T) / (1 + r)^T]}$$

Valuing a Forward Contract during Its Life (T = t)

$$\boxed{V_t(T) = S_t - [F_0(T) / (1 + r)^{T-t}]}$$

$$\boxed{V_t(T) = \text{PV of differences in forward prices} = PV_{t,T}[F_t(T) - F_0(T)]}$$

where $PV_{t,T}(\ )$ means the present value at time t of an amount paid in T – t years (or at time T).

Table 1-1: Value of a Forward Contract

Time	Long Position Value	Short Position Value
At initiation	Zero, as the contract is priced to prevent arbitrage	Zero, as the contract is priced to prevent arbitrage
During life of the contract	$S_t - \left[\frac{F_0(T)}{(1+r)^{T-t}}\right]$	$\left[\frac{F_0(T)}{(1+r)^{T-t}}\right] - S_t$
At expiration	$S_T - F_0(T)$	$F_0(T) - S_T$

Carry Arbitrage Model When the Underlying Has Cash Flows

$$\boxed{F_0(T) = (S_0 - \gamma_0 + \theta_0)\ (1+r)^T}$$

or

$$\boxed{F_0(T) = S_0(1+r)^T - (\gamma_0 - \theta_0)\ (1+r)^T}$$

Valuing a Forward Contract When the Underlying Has Carry Benefits/Costs

$$\boxed{V_t(T) = \text{PV of differences in forward prices} = PV_{t,T}[F_t(T) - F_0(T)]}$$

With continuous compounding, the forward price is calculated as:

$$\boxed{F_0(T) = S_0 e^{(r_c + \theta_c - \gamma_c)T}}$$

Pricing a Forward Rate Agreement

$$\text{FRA}(0,h,m) = \left[\frac{1 + L_0(h+m)\left(\frac{h+m}{360}\right)}{1 + L_0(h)\left(\frac{h}{360}\right)} - 1\right]$$

where:

FRA(0,h,m) = The annualized rate on an FRA initiated at Day 0, expiring on Day h, and based on m-day Libor

h = Number of days until FRA expiration

m = Number of days in underlying hypothetical loan

h + m = Number of days from FRA initiation until end of term of underlying hypothetical loan

L_0 = (Unannualized) Libor rate today

A generic formula used to compute the settlement payment of an FRA to the long position is:

$$\text{FRA payoff} = \frac{\text{NP} \times [(\text{Market Libor} - \text{FRA rate}) \times \text{No. of days in the loan term}/360]}{= 1 + [\text{Market Libor} \times (\text{No. of days in the loan term}/360)]}$$

Fixed-Income Forward and Futures Contracts

$$\text{Accrued interest} = \text{Accrual period} \times \text{Periodic coupon amount}$$

$$\text{AI} = (\text{NAD/NTD}) \times (\text{C/n})$$

where:

NAD = Number of accrued days since the last coupon payment.

NTD = Number of total days during the coupon payment period.

C = Stated annual coupon amount.

n = Number of coupon payments per year.

Markets Where Accrued Interest Is Included in the Bond Price Quote

$F_0(T)$ = Future value of underlying adjusted for carry cash flows

$$F_0(T) = (S_0 - \gamma_0 + \theta_0)\ (1 + r)^T$$

Markets Where Accrued Interest Is Not Included in the Bond Price Quote

$$F_0(T) = QF_0(T) \times CF(T) \text{ and } QF_0(T) = 1/CF(T) \times F_0(T)$$

$$F_0(T) = (S_0 - PVCI_{0,T}) \times (1+r)^T = [B_0(T+Y) + AI_0 - PVCI_{0,T}] \times (1+r)^T$$

The futures price of the bond is calculated as:

$$F_0(T) = [B_0(T+Y) + AI_0 - PVCI_{0,T}] \times (1+r)^T - AI_T$$

The quoted futures price of the bond is calculated as:

$$QF_0(T) = 1/CF(T) \times [B_0(T+Y) + AI_0 - PVCI_{0,T}] \times (1+r)^T - AI_T$$

Currency Forward Contracts

$$\text{Interest rate parity: } F_{0,PC/BC} = \frac{S_{0,PC/BC} \times (1+r_{PC})^T}{(1+r_{BC})^T}$$

r_{PC} = Price-currency risk-free rate
r_{BC} = Base-currency risk-free rate
T = Length of the contract in years

Valuing a Currency Forward Contract

$$V_t(T) = (F_{t,PC/BC} - F_{0,PC/BC}) / (1+r_{PC})^{T-t}$$

Plain-Vanilla Interest Rate Swaps

$$\text{Net fixed rate payment}_t = [\text{Swap fixed rate} - (\text{Libor}_{t-1} + \text{Spread})] * (\text{No. of days}/360) * \text{Notional principal}$$

Pricing a Swap: Determining the Swap Fixed Rate

$$100 = \frac{C}{1+\left(\text{Libor-90} * \frac{90}{360}\right)} + \frac{C}{1+\left(\text{Libor-180} * \frac{180}{360}\right)} + \frac{C}{1+\left(\text{Libor-270} * \frac{270}{360}\right)} + \frac{C}{1+\left(\text{Libor-360} * \frac{360}{360}\right)} + \frac{100}{1+\left(\text{Libor-360} * \frac{360}{360}\right)}$$

Valuing Currency Swaps

$$\boxed{V_a = B_a - S_0 B_b}$$

Where the currency *a* value of the swap equals the difference between the currency *a* fixed bond and the currency *a* value of the currency b bond.

$$\text{Swap fixed rate} = \left[\frac{1 - B_0(N)}{B_0(1) + B_0(2) + B_0(3) + \ldots + B_0(N)}\right] \times 100$$

Valuing Equity Swaps

pay-fixed, receive-return-on-equity

[(1 + Return on equity) * Notional amount] – PV of the remaining fixed-rate payments

pay-floating, receive-return-on-equity

[(1 + Return on equity) * Notional amount] – PV (Next coupon payment + Par value)

pay-return-on-one-equity-instrument, receive-return-on-another-equityinstrument

[(1 + Return on Index 2) * Notional amount] – [(1 + Return on Index 1) * Notional amount]

Valuation of Contingent Claims

Binomial Option Valuation – No Arbitrage Approach

$$u = \frac{S^+}{S}, \; d = \frac{S^-}{S}$$

Where up u and down d describe factors that can be applied to a price to find the new price after an up or down movement.

$$h = \frac{c^+ - c^-}{S^+ - S^-} > 0, \; h = \frac{p^+ - p^-}{S^+ - S^-} < 0$$

Where the hedge ratio h identifies the long position in the underlying to be purchased to offset the call or put price fluctuations.

$$c = hS + PV(-hS^- + c^-)$$

$$p = hS + PV(-hS^+ + p^+)$$

For a two-period binomial model, terminal values for the options are:

$c_{++} = \text{Max}(0, S_{++} - X) = \text{Max}(0, u^2S - X)$,
$c_{+-} = \text{Max}(0, S_{+-} - X) = \text{Max}(0, udS - X)$
$c_{--} = \text{Max}(0, S_{--} - X) = \text{Max}(0, d^2S - X)$

$p_{++} = \text{Max}(0, X - S_{++}) = \text{Max}(0, X - u^2S)$,
$p_{+-} = \text{Max}(0, X - S_{+-}) = \text{Max}(0, X - udS)$
$p_{--} = \text{Max}(0, X - S_{--}) = \text{Max}(0, X - d^2S)$

And put or call value will be the present value of the terminal value for the option. The risk-free rate is used for discounting, and must be compounded over two periods.

Binomial Option Valuation – Expectations Approach

$$E(c_1) = \pi c^{+} + (1-\pi)c^{-}$$
$$E(p_1) = \pi p^{+} + (1-\pi)p^{-}$$
$$\pi = [FV(1) - d]/(u-d) = (1+r-d)/(u-d)$$

Where π is the probability of an up move.

$$c = PV_r[E(c_1)]$$
$$p = PV_r[E(p_1)]$$

Where r is the risk-free rate.

For a two-period binomial model, expected option payoffs are:

$$E(c_2) = \pi^2 c^{++} + 2\pi(1-\pi)c^{+-} + (1-\pi)^2 c^{--}$$
$$E(p_2) = \pi^2 p^{++} + 2\pi(1-\pi)p^{+-} + (1-\pi)^2 p^{--}$$

The present value of the payoffs determines the call and put values:

$$c = PV[E(c_2)]$$
$$p = PV[E(p_2)]$$

The risk-free rate is used for discounting, and must be compounded over two periods.

The Black-Scholes-Merton (BSM) Option Valuation Model

$$c = SN(d_1) - e^{-rT}XN(d_2)$$
$$p = e^{-rT}XN(-d_2) - SN(-d_1)$$

where:

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

σ, = The annualized standard deviation of the continuously compounded return on the stock.
r = The continuously compounded risk-free rate of return.
$N(d_1)$ = Cumulative normal probability of d_1. For example, if d_1 is 0.8252, then $N(d_1)$ will be 0.7954 (look up the normal distribution table).

Table 4-1: BSM and Binomial Option Valuation Model Comparison

	Call Option		Put Option	
Option Valuation Model Terms	**Underlying**	**Financing**	**Underlying**	**Financing**
Binomial model	hS	$PV(-hS^- + c^-)$	hS	$PV(-hS^- + p^-)$
BSM model	$N(d_1)S$	$-N(d_2)e^{-rT}X$	$-N(-d_1)S$	$N(-d_2)e^{-rT}X$

The carry-benefit-adjusted BSM model is as follows:

$$c = Se^{-\gamma T}N(d_1) - e^{-rT}XN(d_2)$$
$$p = e^{-rT}XN(-d_2) - Se^{-\gamma T}N(-d_1)$$

where:

$$d_1 = \frac{\ln(S/X) + \left(r - \gamma + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Carry-benefit-adjusted put-call parity:

$$p + Se^{-\gamma T} = c + e^{-rT}X$$

The Black Model

Under the Black model, European-style options on futures are valued as:

$$c = e^{-rT}\left[F_0(T)N(d_1) - XN(d_2)\right]$$
$$p = e^{-rT}\left[XN(-d_2) - F_0(T)N(-d_1)\right]$$

where:

$$d_1 = \frac{\ln\left[F_0(T)/X\right] + \left(\sigma^2/2\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Futures option put-call parity can be expressed as:

$$c = e^{-rT}[F_0(T) - X] + p$$

Interest Rate Options

Under the standard market model, the prices of interest rate call and put options can be expressed as:

$$c = (AP)e^{-r(t_{j-1}+t_m)}\left[FRA(0,t_{j-1},t_m)N(d_1) - R_X(d_2)\right]$$

$$p = (AP)e^{-r(t_{j-1}+t_m)}\left[R_X N(-d_2) - FRA(0,t_{j-1},t_m)N(-d_1)\right]$$

where:
AP = Accrual period in years

$$d_1 = \frac{\ln\left[FRA(0,t_{j-1},t_m)/R_X\right] + \left(\sigma^2/2\right)t_{j-1}}{\sigma\sqrt{t_{j-1}}}$$

$$d_2 = d_1 - \sigma\sqrt{t_{j-1}}$$

Swaptions

$$\text{Payer swaption} = (AP)PVA\ [R_{FIX}N(d_1) - R_X N(d_2)]$$

$$\text{Receiver swaption} = (AP)PVA[R_X N(-d_2) - R_{FIX}N(-d_1)]$$

where:
PVA = PV of annuity matching the forward swap payment based on a notional amount of 1.
AP = Accrual period.

$$d_1 = \frac{\ln(R_{FIX}/R_X) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

R_{FIX} = Market swap fixed rate (annualized) at the time of swaption expiration (t = T).
R_X = The swaption exercise rate starting at Time T, again quoted on an annual basis. As before, we will assume a notional amount of 1.
σ = Volatility of the forward swap rate. Specifically, it represents the annualized standard deviation of continuously compounded percentage changes in the forward swap rate.

Option Greeks and Implied Volatility

$$\text{Call option delta} = e^{-\delta T}N(d_1)$$

$$\text{Put option delta} = -e^{-\delta T}N(-d_1)$$

Estimating Option Value Using Delta

$$\text{For calls: } \hat{c} - c \cong \text{Delta}_c(\hat{S} - S)$$

$$\text{For puts: } \hat{p} - p \cong \text{Delta}_p(\hat{S} - S)$$

where $\hat{c}$, $\hat{p}$, and $\hat{S}$ denote some new value for the call, put, and stock, respectively.

$$\text{Gamma}_c = \text{Gamma}_p = \frac{e^{-\delta T}}{S\sigma\sqrt{T}} n(d_1).$$

Estimating Option Value Using Delta and Gamma

$$\text{For calls: } \hat{c} - c \approx \text{Delta}_c(\hat{S} - S) + \frac{\text{Gamma}_c}{2}(\hat{S} - S)^2$$

$$\text{For puts: } \hat{p} - p \approx \text{Delta}_p(\hat{S} - S) + \frac{\text{Gamma}_p}{2}(\hat{S} - S)^2$$

where $\hat{c}$, $\hat{p}$, and $\hat{S}$ denote new values for the call, put, and stock, respectively.

Alternative Investments

Private Real Estate Investments

Net Operating Income

Rental income at full occupancy
+ Other income (such as parking)
= Potential gross income (PGI)
– Vacancy and collection loss
= Effective gross income (EGI)
– Operating expenses (OE)
= Net operating income (NOI)

The Direct Capitalization Method

$$\text{Cap rate} = \text{Discount rate} - \text{Growth rate}$$

The cap rate can be defined as the current yield on an investment:

$$\text{Capitalization rate} = \frac{\text{NOI}_1}{\text{Value}}$$

Rearranging the above equation, we can estimate the value of a property by dividing its first-year NOI by the cap rate.

$$\text{Value} = \frac{\text{NOI}_1}{\text{Cap rate}}$$

An estimate of the appropriate cap rate for a property can be obtained from the selling price of similar or comparable properties.

$$\text{Cap rate} = \frac{\text{NOI}}{\text{Sale price of comparable property}}$$

The cap rate derived by dividing rent by recent sales prices of comparables is known as the all risks yield (ARY). The value of a property is then calculated as:

$$\text{Market value} = \frac{\text{Rent}_1}{\text{ARY}}$$

Other Forms of the Income Approach

$$\text{Gross income multiplier} = \frac{\text{Selling price}}{\text{Gross income}}$$

$$\text{Value of subject property} = \text{Gross income multiplier} \times \text{Gross income of subject property}$$

The Discounted Cash Flow Method (DCF)

$$\text{Value} = \frac{\text{NOI}_1}{(\text{r} - \text{g})}$$

The Terminal Capitalization Rate

$$\text{Terminal value} = \frac{\text{NOI for the first year of ownership for the next investor}}{\text{Terminal cap rate}}$$

Cost Approach

$$\text{Appraised value} = \text{Land value} + \text{Building value}$$

$$\begin{aligned}\text{Building value} = {} & \text{Replacement cost} + \text{Developer's profit} \\ & - \text{Curable deterioration} \\ & - \text{Incurable deterioration} \\ & - \text{Functional obsolescence} \\ & - \text{Recent locational obsolescence}\end{aligned}$$

$$\text{Replacement cost} = \text{Building costs psf} \times \text{Total area sf}$$

$$\text{Incurable deterioration} = \frac{\text{Effective age}}{\text{Total economic life}} \times \text{Value after curable deterioration}$$

Sales Comparison Approach

$$\text{Sale price psf} = \frac{\text{Sales price}}{\text{sf}}$$

$$\begin{aligned}\text{Adj Sale price psf} = {} & \text{Sale price psf} \times (1 + \text{\%age adj}) \times (1 + \text{condition adj}) \\ & \times (1 + \text{location adj}) \times (1 + \text{sale date adj})\end{aligned}$$

$$\text{Avg Price psf} = \frac{\sum_{p=1}^{n} \text{Adj sale price psf}_p}{n}$$

$$\text{Appraised value} = \text{Avg Price psf} \times \text{Target property psf}$$

Appraisal-Based Indices

$$\text{Return} = \frac{\text{NOI} - \text{Capital expenditures} + (\text{Ending market value} - \text{Beginning market value})}{\text{Beginning market value}}$$

Loan to Value ratio

$$\boxed{\text{LTV ratio} = \frac{\text{Loan amount}}{\text{Appraised value}}}$$

Debt Service Coverage ratio

$$\boxed{\text{DSCR} = \frac{\text{NOI}}{\text{Debt service}}}$$

Equity dividend rate/Cash-on-cash return

$$\boxed{\text{Equity dividend rate} = \frac{\text{First year cash flow}}{\text{Equity investment}}}$$

Publicly Traded Real Estate Securities

VALUATION: NET ASSET VALUE APPROACH

Capitalization rate

$$\text{Capitalization rate} = \frac{\text{NOI of a comparable property}}{\text{Total value of comparable property}}$$

Net Asset Value per Share

$$\text{NAVPS} = \frac{\text{Net Asset Value}}{\text{Shares outstanding}}$$

VALUATION: RELATIVE VALUATION (PRICE MULTIPLE) APPROACH

Funds from operations (FFO)

Accounting net earnings
Add: Depreciation charges on real estate
Add: Deferred tax charges
Add (Less): Losses (gains) from sales of property and debt restructuring
Funds from operations

Adjusted funds from operations (AFFO)

Funds from operations
Less: Non-cash rent
Less: Maintenance-type capital expenditures and leasing costs
Adjusted funds from operations

AFFO is preferred over FFO as it takes into account the capital expenditures necessary to maintain the economic income of a property portfolio.

PRIVATE EQUITY VALUATION

Quantitative Measures of Return

- PIC (paid in capital): Ratio of paid in capital to date to committed capital.
- DPI (distributed to paid-in) or cash-on-cash return: Value of cumulative distributions paid to LPs as a proportion of cumulative invested capital.
  - (DPI = Cumulative distributions / PIC)
- RVPI (residual value to paid-in): Value of LPs' shareholdings held with the fund as a proportion of cumulative invested capital.
  - RVPI = NAV after distributions / PIC
- TVPI (total value to paid-in): Value of portfolio companies' distributed (realized) and undistributed (unrealized) value as a proportion of cumulative invested capital.
  - TVPI = DPI + RVPI

NAV before distributions = Prior year's NAV after distributions + Capital called down − Management Fees + Operating results

NAV after distributions = NAV before distributions − Carried interest − Distributions

Total Exit Value

Exit value = Initial cost + Earnings growth + Multiple expansion + Debt reduction

Post-money valuation (POST)

$$\text{POST} = \text{PRE} + \text{I}$$

Proportionate ownership of the VC investor

$$= \text{I} / \text{POST}$$

Post-money value

$$\text{Post-money value} = \frac{\text{Exit value}}{(1 + \text{Required rate of return})^{\text{Number of years to exit}}}$$

Required wealth

$$\text{Required wealth} = \text{Investment} \times (1 + \text{IRR})^{\text{Number of years to exit}}$$

Ownership proportion

$$\text{Ownership proportion} = \text{Required wealth} / \text{Exit value}$$

Shares to be issued

$$\text{Shares to be issued} = \frac{\text{Proportion of venture capitalist investment} \times \text{Shares held by company founders}}{\text{Proportion of investment of company founders}}$$

Price per share

$$\text{Price per share} = \frac{\text{Amount of venture capital investment}}{\text{Number of shares issued to venture capital investment}}$$

Adjusted discount rate

$$\text{Adjusted discount rate} = \frac{1+r}{1-q} - 1$$

r = Discount rate unadjusted for probability of failure.
q = Probability of failure.

Backwardation

$$F_t < S_0$$

Most futures contracts will be worth less than the corresponding spot rate to recognize the present value of the future benefit. Futures prices will converge to spot as the contract approaches maturity, at which time $F_t = S_0$.

Contango

$$F_t > S_0$$

Futures contracts have greater value than the spot price.

Components of Futures Returns

$$\text{Spot price return} = (S_t - S_{t-1}) / PS_{t-1}$$

Roll return = [(Near-term futures contract closing price – Farther-term futures contract closing price)/Near-term futures contract closing price] × Percentage of the position in the futures contract being rolled. Roll return is positive for markets in backwardation where the spot rate at maturity for one instrument can be rolled into a lower-priced forward for farther out expiration. Roll yield is negative for markets in contango as the spot price will be lower than the new futures price.

Collateral return is the yield (e.g., interest rate) for the bonds or cash used to maintain the investor's futures position(s).

Portfolio Management

An Introduction to Multifactor Models

Arbitrage Pricing Theory and the Factor Model

$$E(R_p) = R_F + \lambda_1\beta_{p,1} + \ldots + \lambda_K\beta_{p,K}$$

$E(R_p)$ = Expected return on the portfolio p
R_F = Risk-free rate
λ_j = Risk premium for factor j
$\beta_{p,j}$ = Sensitivity of the portfolio to factor j
K = Number of factors

Active Return

$$\text{Active return} = R_p - R_B$$

$$\text{Active return} = \text{Return from factor tilts} + \text{Return from asset selection}$$

Active Risk

$$TE = s(R_p - R_B)$$

Where TE = tracking error

$$\text{Active risk squared} = s^2(R_p - R_B)$$

$$\text{Active risk squared} = \text{Active factor risk} + \text{Active specific risk}$$

$$\text{Active specific risk} = \sum_{i=1}^{n} w_i^a \sigma_{\varepsilon_i}^2$$

Where:
w_i^a = The ith asset's active weight in the portfolio (i.e., the difference between the asset's weight in the portfolio and its weight in the benchmark)
$\sigma_{\varepsilon_i}^2$ = The residual risk of the ith asset (i.e., the variance of the ith asset's returns that is not explained by the factors)

$$\text{Active factor risk} = \text{Active risk squared} - \text{Active specific risk}$$

The Information Ratio

$$IR = \frac{\bar{R}_p - \bar{R}_B}{s(R_p - R_B)}$$

Measuring and Managing Market Risk

Estimating VaR—Parametric Method

$$z = \frac{R - \mu}{\sigma}$$

$$E(R_P) = \sum_{i=1}^{n} w_i R_i$$

$$\sigma_P = \sqrt{w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2 w_i \sigma_i w_j \sigma_j \rho_{i,j}}$$

Equity Exposure—CAPM

$$E(r_a) = r_f + \beta \left[E(r_M) - r_f \right]$$

Fixed Income Exposure

First- and second-order yield effects on bond price:

$$\frac{\Delta B}{B} = -D \frac{\Delta y}{1+y} + \frac{1}{2} C \frac{(\Delta y)^2}{(1+y)^2}$$

Delta

$$\text{Delta} = \frac{\Delta c}{\Delta S}$$

Gamma

$$\Gamma = \text{gamma} = \frac{\Delta \text{delta}}{\Delta S}$$

New call price:

$$c + \Delta c \approx c + \Delta_c \Delta S + \frac{1}{2} \Gamma_c (\Delta S)^2$$

Vega

$$\text{Vega} \approx \frac{\Delta c}{\Delta \sigma_S}$$

New call price:

$$c + \Delta c \approx c + \Delta_c \Delta S + \frac{1}{2} \Gamma_c (\Delta S)^2 + \text{vega} \Delta \sigma_S$$

Economics and Investment Markets

Taylor Rule

$$pr_t = \iota_t + \pi_t + 0.5(\pi_t - \pi_t^*) + 0.5(Y_t - Y_t^*)$$

Where
pr_t = policy rate at time t
ι_t = real short-term interest rates that balance saving and borrowing
π_t = inflation
π_t^* = the inflation target
Y_t and Y_t^* = logarithmic levels of actual and potential real GDP, respectively

Analysis of Active Portfolio Management

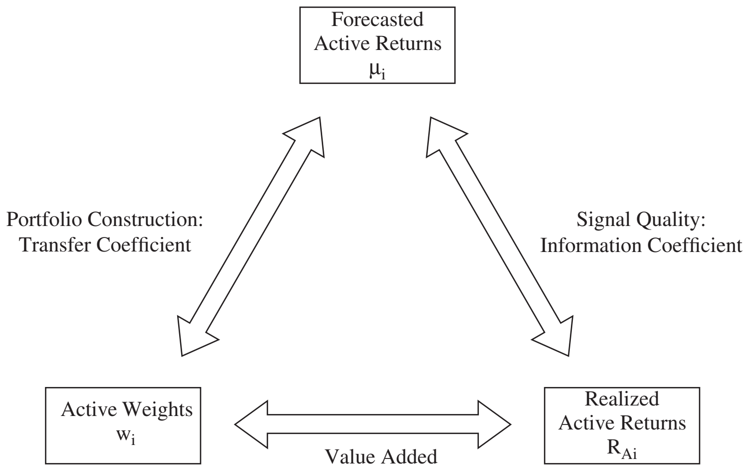
Sharpe Ratio

$$SR_P = \frac{R_P - R_f}{STD(R_P)}$$

Optimal Level of Risk

$$\sigma^*(R_A) = \frac{IR}{SR_B}\sigma(R_B)$$

Figure 2-1: The Correlation Triangle



Mean-Variance-Optimal Active Security Weights

$$\Delta w_i^* = \frac{\mu_i}{\sigma_i^2}\frac{\sigma_A}{IC\sqrt{BR}}$$

where:
Δw_i^* = Active security weight
μ_i = Active return forecast
σ_A = Active portfolio risk
σ_i = Forecasted volatility of the active return on security i
IC = Information coefficient
BR = Breadth

Ex-Ante (Expected) Risk-Weighted Correlation

$$IC = COR\left(\frac{R_{Ai}}{\sigma_i}, \frac{\mu_i}{\sigma_i}\right)$$

The Basic Fundamental Law

$$\boxed{E(R_A)^* = IC\sqrt{BR}\,\sigma_A}$$

The Full Fundamental Law

$$\boxed{E(R_A) = TC\ IC\ \sqrt{BR}\,\sigma_A}$$

Ex-Post (Realized) Risk-Weighted Correlation

$$\boxed{E(R_A|IC_R) = (TC)(IC_R)\sqrt{BR}s_A}$$

TRADING COSTS AND ELECTRONIC MATERIALS

Spreads

$$\begin{aligned} \text{Bid–ask spread} &= \text{Ask} - \text{Bid} \\ \text{Effective spread (buy order)} &= \text{Trade price} - [(\text{Bid} + \text{Ask}/2)] \\ \text{Effective spread (sell order)} &= \text{Trade price} - [(\text{Bid} + \text{Ask}/2)] \end{aligned}$$

Note: A trade at the bid/ask midpoint would have an effective spread of 0. Better prices (lower for buys, higher for sells) would have negative effective spread implicit costs and worse prices would have positive costs.

Volume-Weighted Average Price (VWAP) Transaction Cost Estimate

$$\begin{aligned} \text{VWAP Cost (buy order)} &= \text{Trade VWAP} - \text{Benchmark VWAP} \\ \text{VWAP Cost (sell order)} &= \text{Trade VWAP} - \text{Benchmark VWAP} \end{aligned}$$