$$f(x) \approx P_{1}(x) = \frac{x-b}{a-b}f(a) + \frac{x-a}{b-a}f(b)$$

$$I = \int_{a}^{b} f(x)dx \cong \int_{a}^{b} P_{1}(x)dx$$

$$I = \int_{a}^{b} x-b f(a) + \frac{x-a}{b-a}f(b) dx$$

$$I = \int_{a}^{b} (x-b) \left(\frac{1}{a-b}f(a)\right) + (x-a)\left(\left(\frac{1}{b-a}\right)f(b)\right) dx$$

$$I = \int_{a}^{b} (x-b) \left(\frac{1}{a-b}f(a)\right) + (x-a)\left(\left(\frac{1}{b-a}\right)f(b)\right) dx$$

$$I = \int_{a}^{b} (x-b) \left(\frac{1}{a-b}f(a)\right) + (x-a)f(b) dx$$

$$I = \int_{a}^{b} (x-b) f(a) - (x-a)f(b) dx$$

$$I = \frac{1}{a-b} \int_{a}^{b} ((x-b)f(a) - (x-a)f(b)) dx$$

$$I = \frac{1}{a-b} \int_{a}^{b} f(a)x - f(a)a - f(b)x + f(b)a dx$$

$$I = \frac{1}{a-b} \int_{a}^{b} f(a)x - f(a)b - f(b)x + f(b)a dx$$

$$I = \frac{1}{a-b} \int_{a}^{b} f(a)x - f(a)b - f(b)x + f(b)a dx$$

$$I = \frac{1}{a-b} \left[(f(a)-f(b)) \frac{x^{2}}{2} + (f(a)b-f(b)a) x \right]_{a}^{b}$$

$$I = \frac{1}{a-b} \left[(f(a)-f(b)) \left(\frac{(b-a)(b+a)}{2} \right) + (f(a)b-f(b)a) (b-a) \right]$$

$$I = \frac{1}{a-b} \left[(f(a)-f(b)) \left(\frac{(b-a)(b+a)}{2} \right) + (f(a)b-f(b)a) (b-a) \right]$$

$$I = -\left[\left((f(a)-f(b)) \left(\frac{(b+a)}{2} \right) + (f(a)b-f(b)a) \right) \right]$$

$$I = -\left[\left((f(a)-f(b)) \left(\frac{(b+a)}{2} \right) + (f(a)b-f(b)a) \right) \right]$$

$$I = \frac{(f(b)-f(a))(b+a)}{2} + (f(a)b-f(b)a)$$

$$I = \frac{f(a)b-f(a)a+f(b)b-f(a)a}{2}$$

$$I = \frac{f(a)b-f(a)a+f(b)b-f(a)a}{2}$$