① $\displaystyle\int \frac{1}{(x-2)\cdot(x^2+x+4)}\cdot dx \Rightarrow \frac{1}{(x-2)\cdot(x^2+x+4)} = \frac{A}{(x-2)}_{(x^2+x+4)} + \frac{Bx+C}{(x^2+x+4)}_{(x-2)}$

$$\frac{1}{\cancel{(x-2)}\cdot(x^2+\cancel{x+4})} = \frac{A(x^2+x+4)+(Bx+C)\cdot(x-2)}{\cancel{(x-2)}\cdot(x^2+\cancel{x}+4)}$$

$$1 = Ax^2+Ax+4A+Bx^2-2Bx+Cx-2C$$

$$\begin{aligned} 1 &= 4A-2C \\ 0\cancel{x} &= \cancel{x}\cdot(A-2B+C) \\ 0\cancel{x^2} &= \cancel{x^2}\cdot(A+B) \end{aligned} \quad \left\{ \begin{aligned} 4A-2C &= 1 \\ A-2B+C &= 0 \\ A+B &= 0 \end{aligned} \right\} \quad \boxed{A=\frac{1}{10}} \quad \boxed{B=-\frac{1}{10}} \quad \boxed{C=-\frac{3}{10}}$$

$$\int \frac{\frac{1}{10}dx}{(x-2)} + \int \frac{-\frac{1}{10}x-\frac{3}{10}}{(x^2+x+4)}\cdot dx \;\text{ZD}\; \boxed{\frac{1}{10}\int\frac{dx}{(x-2)}}^{*1} - \boxed{\frac{1}{10}\int\frac{x-3}{(x^2+x+4)}\cdot dx}^{*2}$$

$$\boxed{\frac{1}{10}\ln|x-2|}^{*1} - \frac{1}{10}\cdot\left[\frac{1}{2}\int\frac{2x+1}{(x^2+x+4)}\cdot dx - \frac{5}{2}\int\frac{dx}{(x^2+x+4)}\right]^{*2}$$

$$-\frac{1}{10}\left[\frac{1}{2}\ln|x^2+x+4| - \frac{5}{2}\int\frac{dx}{(x+\frac{1}{2})^2+\frac{7}{2}}\right]^{*2}$$

$$-\frac{1}{10}\left[\frac{1}{2}\ln|x^2+x+4| - \frac{5}{2}\int\frac{dx}{(x+\frac{1}{2})^2+(\sqrt{7/2})^2}\right]^{*2} \;\text{ZD}\; -\frac{1}{10}\cdot\left[\frac{1}{2}\ln|x^2+x+4| - \frac{5}{2\sqrt{7/2}}_{(\sqrt{7/2})}\,\text{arctg}\left(\frac{x+1/2}{\sqrt{7/2}}_{(\sqrt{7/2})}\right)\right]^{*2}$$

$$-\frac{1}{10}\left[\frac{1}{2}\ln|x^2+x+4| - \frac{5\sqrt{7/2}}{2\cdot(7/2)}\cdot\text{arctg}\left(\frac{\sqrt{7/2}\,x+\frac{\sqrt{7/2}}{2}}{7/2}\right)\right]^{*2}$$

$$\left[-\frac{1}{20}\ln|x^2+x+4| - \frac{5\sqrt{7/2}\,\text{arctg}\left(\frac{\sqrt{7/2}\,x+\frac{\sqrt{7/2}}{2}}{7/2}\right)}{70}\right]^{*}$$

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Resultado final: $*1 + *2 + C$

$$\frac{1}{10}\ln|x-2| - \frac{1}{20}\ln|x^2+x+4| - \frac{5\sqrt{7/2}\,\text{arctg}\left(\dfrac{\sqrt{7/2}\,x + \frac{\sqrt{7/2}}{x}}{7/2}\right)}{70} + C$$

---

② $\displaystyle\int \frac{9x^2-28x+12}{x^3-5x^2+6x}\cdot dx \Rightarrow \int \frac{9x^2-28x+12}{x\cdot(x^2-5x+6)}dx \Rightarrow \int \frac{9x^2-28x+12}{x\cdot(x-2)\cdot(x-3)}\cdot dx$

$$\frac{9x^2-28x+12}{x\cdot(x-2)\cdot(x-3)} = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$(x-2)\cdot(x-3)$ $\quad (x-3)\cdot x$ $\quad (x-2)\cdot x$

$$\frac{9x^2-28x+12}{\cancel{x\cdot(x-2)\cdot(x-3)}} = \frac{A\cdot(x^2-5x+6)+B\cdot(x^2-3x)+C\cdot(x^2-2x)}{\cancel{x\cdot(x-2)\cdot(x-3)}}$$

$$9x^2-28x+12 = Ax^2-5Ax+6A+Bx^2-3Bx+Cx^2-2Cx$$

$9\cancel{x^2} = \cancel{x^2}\cdot(A+B+C)$ $\quad A+B+C=9$ $\quad \boxed{A=2}$

$-28\cancel{x} = \cancel{x}\cdot(-5A-3B-2C)$ $\quad -5A-3B-2C=-28$ $\quad \boxed{B=4}$

$12 = 6A$ $\quad 6A = 12$ $\quad \boxed{C=3}$

$$\int \frac{2}{x}dx + \int \frac{4}{(x-2)}dx + \int \frac{3}{(x-3)}dx$$

$$2\int \frac{dx}{x} + 4\int \frac{dx}{(x-2)} + 3\int \frac{dx}{(x-3)}$$

$$2\ln|x| + 4\ln|x-2| + 3\ln|x-3| + C$$

$$③ \int \frac{x^2+3x+4}{(x-1)\cdot(x+2)\cdot(x+3)}\,dx \Rightarrow \frac{x^2+3x+4}{(x-1)\cdot(x+2)\cdot(x+3)} = \frac{A}{(x-1)}_{(x+2)\cdot(x+3)} + \frac{B}{(x+2)}_{(x-1)\cdot(x+3)} + \frac{C}{(x+3)}_{(x-1)\cdot(x+2)}$$

$$\frac{x^2+3x+4}{\cancel{(x-1)}\cdot\cancel{(x+2)}\cdot\cancel{(x+3)}} = \frac{A\cdot(x^2+5x+6)+B\cdot(x^2+2x-3)+C\cdot(x^2+x-2)}{\cancel{(x-1)}\cdot\cancel{(x+2)}\cdot\cancel{(x+3)}}$$

$$x^2+3x+4 = Ax^2+5Ax+6A+Bx^2+2Bx-3B+Cx^2+Cx-2C$$

$$\left.\begin{aligned} \cancel{x^2} &= \cancel{x^2}\cdot(A+B+C) \\ 3\cancel{x} &= \cancel{x}\cdot(5A+2B+C) \\ 4 &= 6A-3B-2C \end{aligned}\right\} \begin{aligned} A+B+C&=1 \\ 5A+2B+C&=3 \\ 6A-3B-2C&=4 \end{aligned} \quad \boxed{A=\frac{2}{3}} \quad \boxed{B=-\frac{2}{3}} \quad \boxed{C=1}$$

$$\frac{2}{3}\int\frac{dx}{(x-1)} - \frac{2}{3}\int\frac{dx}{(x+2)} + \int\frac{dx}{(x+3)}$$

$$\frac{2}{3}\ln|x-1| - \frac{2}{3}\ln|x+2| + \ln|x+3|$$

$$\frac{2}{3}\cdot\Big[\ln|x-1| - \ln|x+2|\Big] + \ln|x+3|$$

$$\frac{2}{3}\ln\left|\frac{x-1}{x+2}\right| + \ln|x+3| + C$$

$$④ \int \frac{3x^2-21x+31}{(x-1)\cdot(x-3)^2}\,dx \Rightarrow \frac{3x^2-21x+31}{(x-1)\cdot(x-3)^2} = \frac{A}{x-1}_{(x-3)^2} + \frac{B}{(x-3)}_{(x-1)\cdot(x-3)} + \frac{C}{(x-3)^2}_{(x-1)}$$

$$\frac{3x^2-21x+31}{\cancel{(x-1)}\cdot\cancel{(x-3)^2}} = \frac{A\cdot(x^2-6x+9)+B\cdot(x^2-4x+3)+C\cdot(x-1)}{\cancel{(x-1)\cdot(x-3)^2}}$$

$$3x^2-21x+31 = Ax^2-6Ax+9A+Bx^2-4Bx+3B+Cx-C$$

$$\left.\begin{aligned} 3\cancel{x^2} &= \cancel{x^2}\cdot(A+B) \\ -21\cancel{x} &= \cancel{x}\cdot(-6A-4B+C) \\ 31 &= 9A\,\cancel{(+(C))}-C \end{aligned}\right\} \begin{aligned} A+B&=3 \\ -6A-4B+C&=-21 \\ 9A-C&=31 \end{aligned} \quad \boxed{A=\frac{13}{4}} \quad \boxed{B=-\frac{1}{4}} \quad \boxed{C=-\frac{5}{2}}$$

$$\frac{13}{4}\int\frac{dx}{(x-1)} - \frac{1}{4}\int\frac{dx}{(x-3)} - \frac{5}{2}\int\frac{dx}{(x-3)^2}$$

$$\frac{13}{4}\ln|x-1| - \frac{1}{4}\ln|x-3| - \frac{5}{2}\int(x-3)^{-2}\cdot dx$$

$$\frac{13}{4}\ln|x-1| - \frac{1}{4}\ln|x-3| + \frac{5}{2}\times\left[\frac{1}{(x-3)}\right]$$

$$\frac{13}{4}\ln|x-1| - \frac{1}{4}\ln|x-3| + \frac{5}{(2x-6)} + C_{\shortparallel}$$

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5) $$\int\frac{(x^3+x^2+x-2)}{(x^4+3x^2+2)}\cdot dx$$

$$x^4+3x^2+2$$

$$\begin{array}{ccc} x^2 & & 2 \\ x^2 & & 1 \end{array}$$

$$(x^2+2)\cdot(x^2+1)$$

$$\int\frac{x^3+x^2+x-2}{(x^2+1)\cdot(x^2+2)}\cdot dx$$

CD $$\frac{x^3+x^2+x-2}{(x^2+1)\cdot(x^2+2)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+2)}$$

$$(x^2+2) \qquad (x^2+1)$$

$$\frac{x^3+x^2+x-2}{\cancel{(x^2+1)}\cdot\cancel{(x^2+2)}} = \frac{(Ax+B)\cdot(x^2+2) + (Cx+D)\cdot(x^2+1)}{\cancel{(x^2+2)}\cdot\cancel{(x^2+1)}}$$

$$x^3+x^2+x-2 = Ax^3+2Ax+Bx^2+2B+Cx^3+Cx+Dx^2+D$$

$$\left.\begin{array}{l} \cancel{x^3} = \cancel{x^3}\cdot(A+C) \\ \cancel{x^2} = \cancel{x^2}(B+D) \\ \cancel{x} = \cancel{x}\cdot(2A+C) \\ -2 = 2B+D \end{array}\right\}\left.\begin{array}{l} A+C=1 \\ B+D=1 \\ 2A+C=1 \\ 2B+D=-2 \end{array}\right\}\begin{array}{l} \boxed{A=1} \\ \boxed{B=4} \\ \boxed{C=0} \\ \boxed{D=-3} \end{array}$$

$$\int\frac{(x+4)dx}{(x^2+1)} + \int\frac{-3}{(x^2+2)}\cdot dx$$

$$\int\frac{x\,dx}{(x^2+1)} + 4\int\frac{dx}{(x^2+1)} - 3\int\frac{dx}{(x^2+2)}$$

$$\frac{1}{2}\ln|x^2+1| + 4\,arctg\left(\frac{x}{1}\right) - \frac{3}{\sqrt{2}}\,arctg\left(\frac{x}{\sqrt{2}}\right)$$

$$\frac{1}{2}\ln|x^2+1| + 4\,arctg(x) - \frac{3\sqrt{2}\,arctg\left(\frac{\sqrt{2}x}{2}\right)}{2} + C$$

---

6) $$\int\frac{(3x-1)\cdot dx}{(x^3-8)} \Rightarrow \int\frac{(3x-1)\cdot dx}{(x-2)\cdot(x^2+2x+4)}$$

$$\frac{3x-1}{(x-2)\cdot(x^2+2x+4)} = \frac{A}{(x-2)}_{(x^2+2x+4)} + \frac{Bx+C}{(x^2+2x+4)}_{(x-2)} \Rightarrow \frac{3x-1}{\cancel{(x-2)\cdot(x^2+2x+4)}} = \frac{A(x^2+2x+4)+(Bx+C)\cdot(x-2)}{\cancel{(x-2)\cdot(x^2+2x+4)}}$$

$$3x-1 = Ax^2+2Ax+4A+Bx^2-2Bx+Cx-2C$$

$$\begin{aligned} 0x^2 &= x^2\cdot(A+B) \\ 3\cancel{x} &= \cancel{x}\cdot(2A-2B+C) \\ -1 &= 4A-2C \end{aligned} \quad \begin{cases} A+B=0 \\ 2A-2B+C=3 \\ 4A-2C=-1 \end{cases} \quad \boxed{A=\frac{5}{12}} \quad \boxed{B=-\frac{5}{12}} \quad \boxed{C=\frac{4}{3}} \vee \boxed{C=\frac{16}{12}}$$

$$\boxed{\frac{5}{12}\int\frac{dx}{(x-2)}}^{*1} + \boxed{\frac{1}{12}\int\frac{-5x+16}{(x^2+2x+4)}}^{*2}$$

$$\boxed{\frac{5}{12}\ln|x-2|}^{*1} + \frac{1}{12}\left[-\frac{5}{2}\int\frac{(2x+2)\,dx}{(x^2+2x+4)} + 21\int\frac{dx}{(x+1)^2+3}\right]^{*2}$$

$$\frac{1}{12}\left[-\frac{5}{2}\ln|x^2+2x+4| + 21\int\frac{dx}{(x+1)^2+(\sqrt{3})^2}\right]^{*2}$$

$$\frac{1}{12}\left[-\frac{5}{2}\ln|x^2+2x+4| + \frac{21}{\sqrt{3}}\,arctg\left(\frac{x+1}{\sqrt{3}}\right)\right]^{*2}$$

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$$\frac{1}{12}\left[-\frac{5}{2}\ln|x^2+2x+4| + \frac{\cancel{21}^{7}\sqrt{3}}{\cancel{3}}\operatorname{arctg}\left(\frac{\sqrt{3}x+\sqrt{3}}{3}\right)\right] *2$$

Resultado final: $*1 + *2 + C$ //

$$\frac{5}{12}\ln|x-2| - \frac{5}{24}\ln|x^2+2x+4| + \frac{7\sqrt{3}\operatorname{arctg}\left(\frac{\sqrt{3}x+\sqrt{3}}{3}\right)}{12} + C$$

Inf.

Integrantes:

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- André Jgucamba
- Edivaldo Lijório
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Construção de imagens:

$$2x^2+3x-2 \Rightarrow (2x-1)\cdot(x+2)$$

$$\begin{array}{ccc} 2x & & -1 \\ & \times & \\ x & & +2 \end{array}$$

$$4x - x = 3x$$

3.18 - $\displaystyle\int_2^3 \frac{dx}{(2x-1)\cdot(x+2)} \Rightarrow \int_2^3 \frac{dx}{(x+2)\cdot(2x-1)}$

$$\frac{A}{(x+2)} + \frac{B}{(2x-1)} = \frac{1}{(x+2)\cdot(2x-1)}$$

$$\frac{A(2x-1)+B(x+2)}{(x+2)\cdot(2x-1)} = \frac{1}{(x+2)\cdot(2x-1)}$$

$$2Ax - A + Bx + 2B = 1$$

$$\left.\begin{array}{l} x\cdot(2A+B) = 0 \\ -A+2B = 1 \end{array}\right\} \begin{array}{l} 2A+B=0 \\ 2B-A=1 \end{array}$$

$$\begin{array}{l} \cancel{2A}+B=0 \\ -\cancel{2A}+4B=2 \end{array} \qquad \begin{array}{l} 5B=2 \\ B=\frac{2}{5} \end{array}$$

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$$2A + B = 0$$
$$2A + \frac{2}{5} = 0$$
$$\frac{2A}{1} = -\frac{2}{5}$$
$$10A = -2$$
$$A = -\frac{2}{10}$$
$$\boxed{A = -\frac{1}{5}}$$
$$\boxed{B = \frac{2}{5}}$$

$$-\frac{1}{5}\int_2^3 \frac{dx}{(x+2)} + \frac{2}{5}\int_2^3 \frac{dx}{(2x-1)}$$

$2x-1 = u$
$2dx = du$
$dx = \frac{du}{2}$

$$-\frac{1}{5}\ln|x+2|\Big|_2^3 + \frac{2}{10}\int_3^5 \frac{du}{u}$$

$u: 3 \longrightarrow 5$

$$-\frac{1}{5}\ln|x+2|\Big|_2^3 + \frac{2}{10}\ln|u|\Big|_3^5$$

$$-\frac{1}{5}\cdot\left[\ln|5| - \ln|4|\right] + \frac{2}{10}\left[\ln|5| - \ln|3|\right]$$

$$\frac{2}{10}\ln\left|\frac{5}{3}\right| - \frac{1}{5}\ln\left|\frac{5}{4}\right|$$

$$\frac{1}{5}\ln\left|\frac{5}{3}\right| - \frac{1}{5}\ln\left|\frac{5}{4}\right|$$

$$\frac{1}{5}\cdot\ln\left|\frac{20}{15}\right| \Rightarrow \frac{1}{5}\ln\left|\frac{4}{3}\right|$$

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