- This talks is about regression methods in which the dependent variable takes nonnegative integer or count values.
- The dependent variable is usually the number of times an event occurs.

- Linear regression is used to model and predict continuous measurement variables.
- Poisson regression is used to model and predict discrete count variables.

Poisson regression assumes the response variable Y has a Poisson distribution, and assumes the logarithm of its expected value can be modeled by a linear combination of unknown parameters. A Poisson regression model is sometimes known as a log-linear model, especially when used to model contingency tables.

Examples of Poisson regression

(1) The number of persons killed by mule or horse kicks in the Prussian army per year. Ladislaus Bortkiewicz collected data from 20 volumes of *Preussischen Statistik*. These data were collected on 10 corps of the Prussian army in the late 1800s over the course of 20 years.

(2) The number of people in line in front of you at the grocery store. Predictors may include the number of items currently offered at a special discounted price and whether a special event (e.g., a holiday, a big sporting event) is three or fewer days away.

(3) The number of awards earned by students at one high school. Predictors of the number of awards earned include the type of program in which the student was enrolled (e.g., vocational, general or academic) and the score on their final exam in math.

Overview

Some examples of event counts are:

- number of claims per year on a particular car owners insurance policy,
- number of workdays missed due to sickness of a dependent in a one-year period,
- number of papers published per year by a researcher.

Poisson Distribution

- The number of persons killed by mule or horse kicks in the Prussian army per year.
- Ladislaus Bortkiewicz collected data from 20 volumes of Preussischen Statistik.
- These data were collected on 10 corps of the Prussian army in the late 1800s over the course of 20 years, giving a total of 200 observations of one corps for a one year period. The period or module of observation is thus one year.

Poisson Distribution: Prussian Cavalary

- The total deaths from horse kicks were 122, and the average number of deaths per year per corps was thus $122/200 = 0.61$.
- In any given year, we expect to observe, well, not exactly 0.61 deaths in one corps
- Here, then, is the classic Poisson situation: a rare event, whose average rate is small, with observations made over many small intervals of time.

```
rpois(200,lambda=0.61)
> X
[1]   1 2 0 1 0 3 0 0 1 0 0 4 0 0 0 1 0 1 0 2
[21]  0 0 0 2 2 0 0 0 1 0 0 0 0 1 0 0 0 1 2 0
[41]  0 0 1 0 1 0 1 0 0 1 1 0 1 0 0 1 0 0 3 1
.......
.......
[141] 0 0 0 0 1 2 0 1 0 1 0 0 0 0 0 0 0 1 0 0
[161] 1 0 1 0 0 0 0 1 0 0 0 0 0 1 1 1 0 2 0 1
[181] 0 0 2 0 2 0 0 1 0 0 3 1 0 0 0 1 1 0 0 0
>
> mean(X)
[1] 0.53
> var(X)
[1] 0.5317588
```

Overview

- Poisson regression is main technique used to model count variables.
- Poisson Distribution : Mean and Variance are equal

$$\mathrm{E}(X) = \mathrm{Var}(X)$$

- Sometimes conventional Poisson Regression is not an appropriate technique, and alternative or variant techniques are used instead.
- For example, Negative Binomial regression is for modelling count variables, usually for over-dispersed count outcome variables.

Generalized Linear Models

- In statistics, the problem of modelling count variables is an example of generalized linear modelling.
- Generalized linear models are fit using the `glm()` function.
- The form of the `glm` function is

```
glm(formula, family=familytype(link=linkfunction),
  data=dataname)
```

Generalized Linear Models

Family	Default Link Function
binomial	`(link = "logit")`
gaussian	`(link = "identity")`
Gamma	`(link = "inverse")`
inverse.gaussian	`(link = "1/`mu^2`")`
poisson	`(link = "log")`
quasibinomial	`(link = "logit")`
quasipoisson	`(link = "log")`

Texts on GLMs

- Dobson, A. J. (1990) An Introduction to Generalized Linear Models. (*London: Chapman and Hall.*)
- Hastie, T. J. and Pregibon, D. (1992) Generalized linear models. Chapter 6 of Statistical Models in S eds J. M. Chambers and T. J. Hastie, Wadsworth & Brooks/Cole.
- McCullagh P. and Nelder, J. A. (1989) Generalized Linear Models. (*London: Chapman and Hall.*)
- Venables, W. N. and Ripley, B. D. (2002) Modern Applied Statistics with S. *New York: Springer.*

glm2: Fitting Generalized Linear Models

Author(s): Ian Marschner

Fits generalized linear models using the same model specification as glm in the stats package, but with a modified default fitting method that provides greater stability for models that may fail to converge using glm

VGAM: Vector Generalized Linear and Additive Models

Author(s): Thomas W. Yee (t.yee@auckland.ac.nz)

URL: http://www.stat.auckland.ac.nz/$\sim$ yee/VGAM

Vector generalized linear and additive models, and associated models (Reduced-Rank VGLMs, Quadratic RR-VGLMs, Reduced-Rank VGAMs).

This package fits many models and distribution by maximum likelihood estimation (MLE) or penalized MLE. Also fits constrained ordination models in ecology.