Poisson Regression with R

glm function output

- The output begins with echoing the function call. The information on deviance residuals is displayed next.
- Deviance residuals are approximately normally distributed if the model is specified correctly.
- Here it shows a little bit of skeweness since median is not quite zero.

glm function output

- The Poisson regression coefficients for each of the variables along with the standard errors, z-scores, p-values and 95% confidence intervals for the coefficients.
- The coefficient for math is 0.07.
- This means that the expected log count for a one-unit increase in math is 0.07.

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glm function output

- The indicator variable **progAcademic** compares between **prog** $=$ **Academic** and **prog** $=$ **"General"** , the expected log count for **prog** $=$ **Academic** increases by about 1.1.
- The indicator variable **prog.Vocational** is the expected difference in log count (≈ 0.37) between **prog** $=$ **"Vocational"** and the reference group (**prog** $=$ **"General"**).

Deviance

- In statistics, deviance is a quality of fit statistic for a model that is often used for statistical hypothesis testing.
- It is a generalization of the idea of using the sum of squares of residuals in ordinary least squares to cases where model-fitting is achieved by maximum likelihood.

glm function output

- The information on deviance is also provided.
- We can use the residual deviance to perform a goodness of fit test for the overall model.
- The residual deviance is the difference between the deviance of the current model and the maximum deviance of the ideal model where the predicted values are identical to the observed.

glm function output

- Therefore, if the residual difference is small enough, the goodness of fit test will not be significant, indicating that the model fits the data. We conclude that the model fits reasonably well because the goodness-of-fit chi-squared test is not statistically significant.
- If the test had been statistically significant, it would indicate that the data do not fit the model well.
- We could try to determine if there are omitted predictor variables, if our linearity assumption holds and/or if there is an issue of

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```
with(m1, cbind(res.deviance = deviance,
df = df.residual,
        p = pchisq(deviance, df.residual,
            lower.tail=FALSE)))

      res.deviance  df      p
 [1,]        189.4 196 0.6182
```

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- We can also test the overall effect of prog by comparing the deviance of the full model with the deviance of the model excluding prog.
- The two degree-of-freedom chi-square test indicates that prog, taken together, is a statistically significant predictor of num_awards.

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```
# update m1 model dropping prog
m2 <- update(m1, . ~ . - prog)
# test model differences with chi square test
anova(m2, m1, test="Chisq")
```

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```
Analysis of Deviance Table

Model 1: num_awards ~ math
Model 2: num_awards ~ prog + math
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1       198        204
2       196        189  2     14.6  0.00069 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
```

Incident Rate Ratios

- Sometimes, we might want to present the regression results as **incident rate ratios** (IRRs) and their standard errors, together with the confidence interval.
- To compute the standard error for the incident rate ratios, we will use the **Delta method** (Numerical Computation Method).
- To this end, we make use the function `deltamethod` implemented in R package **msm**.

Incident Rate Ratios

A rate ratio (sometimes called an incidence density ratio) in epidemiology, is a relative difference measure used to compare the incidence rates of events occurring at any given point in time. A common application for this measure in analytic epidemiologic studies is in the search for a causal association between a certain risk factor and an outcome.[1]

$$\text{Incidence Rate Ratio} = \frac{\text{Incidence Rate 1}}{\text{Incidence Rate 2}}$$

Incident Rate Ratios

Incidence rate is the occurrence of an event over person-time, for example person-years.

$$\text{Incidence Rate} = \frac{\text{events}}{\text{Person Time}}$$

Note: the same time intervals must be used for both incidence rates.

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```
s <- deltamethod(list(~ exp(x1), ~ exp(x2),
#exponentiate old estimates dropping the p v
rexp.est <- exp(r.est[, -3])

# replace SEs with estimates for exponentiat
rexp.est[, "Robust SE"] <- s
```

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```
rexp.est

                 Estimate Robust SE       LL      UL
 (Intercept)     0.005263   0.00340 0.001484 0.01867
 progAcademic    2.956065   0.94904 1.575551 5.54620
 progVocational 1.447458   0.57959 0.660335 3.17284
 math           1.072672   0.01119 1.050955 1.09484
```

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- The output above indicates that the incident rate for prog = "Academic" is 2.96 times the incident rate for the reference group (prog = "General").
- Likewise, the incident rate for prog = "Vocational" is 1.45 times the incident rate for the reference group holding the other variables at constant.

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- The percent change in the incident rate of num_awards is by 7% for every unit increase in math.

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- Sometimes, we might want to look at the expected marginal means.
- For example, what are the expected counts for each program type holding math score at its overall mean?
- To answer this question, we can make use of the predict function.
- First off, we will make a small data set to apply the predict function to it.

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```
(s1 <- data.frame(math = mean(p$math),
  prog = factor(1:3, levels = 1:3,
  labels = levels(p$prog))))

    math       prog
1 52.65    General
2 52.65   Academic
3 52.65 Vocational
```

Poisson Regression with R

```
predict(m1, s1, type="response", se.fit=TRUE)

 $fit
      1      2      3
 0.2114 0.6249 0.3060

 $se.fit
       1       2       3
 0.07050 0.08628 0.08834

 $residual.scale
 [1] 1
```

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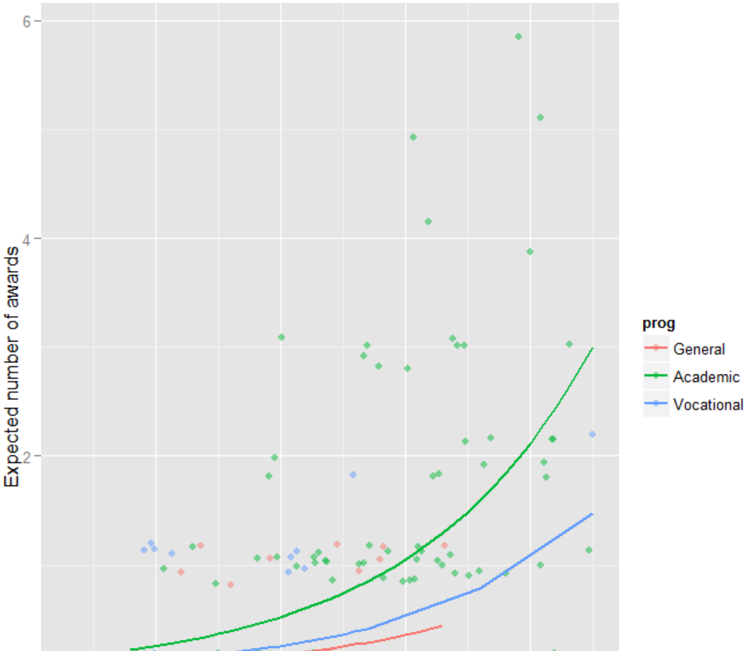
- In the output above, we see that the predicted number of events for level 1 of prog is about 0.21, holding math at its mean.
- The predicted number of events for level 2 of prog is higher at 0.62, and the predicted number of events for level 3 of prog is about .31.
- The ratios of these predicted counts ($\frac{0.625}{0.211} = 2.96$, $\frac{0.306}{0.211} = 1.45$) match what we saw looking at the IRR.


6
4
2
Expected number of awards
prog
General
Academic
Vocational

Poisson Regression with R

- We can also graph the predicted number of events with the commands below.
- The graph indicates that the most awards are predicted for those in the academic program (prog $= 2$), especially if the student has a high math score.
- The lowest number of predicted awards is for those students in the general program (prog $=$ 1).
- The graph overlays the lines of expected values onto the actual points, although a small amount of random noise was added vertically to lessen overplotting.

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```
# Calculate and store predicted values
p$phat <- predict(m1, type="response")

# order by program and then by math
p <- p[with(p, order(prog, math)), ]
```

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```
ggplot(p, aes(x = math, y = phat, colour = prog)) +
  geom_point(aes(y = num_awards), alpha=.5, position=
  geom_line(size = 1) +
  labs(x = "Math Score", y = "Expected number of awar
```