

Poisson regression with VGAM

Packages

Useful packages: *lmtest*, *MASS*, *pscl*, *sandwich*, *VGAM*

```
install.packages("lmtest", "MASS",  
  "pscl", "sandwich", "VGAM")
```

The vgamdata data set

```
> head(vgamdata)
```

	X1	X2	Y
1	-5.120966	0.7158503	7
2	-3.925800	2.3375171	7
3	-2.115866	-2.8088281	0
4	-3.659427	6.6888561	4
5	-1.859439	2.6380000	1
6	1.758675	4.8020512	0

Using `vglm()` from package VGAM

```
library(VGAM)
summary(vglmFit <- vglm(Y ~ X1 + X2,
family=poissonff, data=vgamdata))
# not shown
```

Analyse event rates

offset is the exposure $\ln(t)$

```
Nt    <- 100  
Ti    <- sample(20:40, Nt, replace=TRUE)  
Xt    <- rnorm(Nt, 100, 15)  
Yt    <- rbinom(Nt, size=Ti, prob=0.5)
```

```
glm(Yt ~ Xt, family=poisson(link="log"), offset=log(Ti))
```

```
Call: glm(formula = Yt ~ Xt, family = poisson(link = "log")
```

Coefficients:

(Intercept)	Xt
-0.5466	-0.0017

Degrees of Freedom: 99 Total (i.e. Null); 98 Residual

Null Deviance: 51.74

Residual Deviance: 50.73 AIC: 502.6

Overdispersion

Adjusted Poisson-regression

Same parameter estimates as in Poisson model, but different standard errors, hence different p-values

```
glmFitQP <- glm(Y ~ X1 + X2,  
               family=quasipoisson(link="log"),  
               data=vgamdata)  
summary(glmFitQP)
```

Call:

```
glm(formula = Y ~ X1 + X2, family = quasipoisson(link = "log",  
data = vgamdata)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-3.3237	-1.2454	-0.2932	0.8200	2.9249

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept)	1.932e-01	1.297e-01	1.490	0.138
X1	-2.549e-01	2.839e-02	-8.979	<2e-16 ***
X2	-8.653e-05	1.443e-02	-0.006	0.995

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for quasipoisson family taken to be 1

Null deviance: 494.5 on 199 degrees of freedom

Residual deviance: 356.2 on 197 degrees of freedom

AIC: NA

Number of Fisher Scoring iterations: 5

Using `vglm()` from package VGAM

```
library(VGAM)
vglm(Y ~ X1 + X2, family=quasipoissonff, data=vgamdata)
# not shown
```

Heteroscedasticity consistent standard errors

Same parameter estimates as in Poisson model, but different standard errors, hence different p-values

```
library(sandwich)
hcSE <- vcovHC(glmFitP, type="HC0")
```

```
library(lmtest)
coeftest(glmFitP, vcov=hcSE)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.19316888	0.13268996	1.4558	0.1455
X1	-0.25491612	0.02698458	-9.4467	<2e-16 ***
X2	-0.00008653	0.01319493	-0.0066	0.9948

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Negative binomial regression with VGAM

Using glm.nb() from package MASS

```
library(MASS)  
glmFitNB <- glm.nb(Y ~ X1 + X2, data=vgamdata)
```

Negative binomial regression with VGAM

```
summary(glmFitNB)
```

Call:

```
glm.nb(formula = Y ~ X1 + X2, data = vgamdata, init.theta =  
link = log)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-2.7695	-1.0533	-0.2315	0.6472	2.4427

Negative binomial regression with VGAM

Coefficients:

	Estimate	Std. Error	z	value	Pr(> z)
(Intercept)	0.166025	0.125963	1.318		0.187
X1	-0.261257	0.029119	-8.972		<2e-16 ***
X2	0.002651	0.014735	0.180		0.857

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Negative binomial regression with VGAM

(Dispersion parameter for Negative Binomial(5.1819) family

Null deviance: 345.33 on 199 degrees of freedom

Residual deviance: 256.60 on 197 degrees of freedom

AIC: 824.58

Negative binomial regression with VGAM

Number of Fisher Scoring iterations: 1

Theta: 5.18 Std. Err.: 1.79

2 x log-likelihood: -816.58

Negative binomial regression with VGAM

Using `vglm()` from package VGAM

```
library(VGAM)
vglm(Y ~ X1 + X2, family=negbinomial, data=vgamdata)
# not shown
```

Test the negative binomial model against the Poisson model

Test the negative binomial model against the Poisson model

```
library(psc1)  
odTest(glmFitNB)
```

Likelihood ratio test of H0: Poisson, as restricted NB model: n.b., the distribution of the test-statistic under H0 is non-standard e.g., see `help(odTest)` for details/references

Critical value of test statistic at the $\alpha = 0.05$ level: 2.7055

Chi-Square Test Statistic = 15.8533 p-value = 3.422e-05

Zero-inflated Regression models with VGAM

Zero-inflated Poisson regression

```
library(pscl)
ziFitP <- zeroinfl(Y ~ X1 + X2 | 1, dist="poisson", data=vg
summary(ziFitP)
```

Zero-inflated Poisson regression with VGAM

Call:

```
zeroinfl(formula = Y ~ X1 + X2 | 1, data = vgamdata, dist =
```

Pearson residuals:

Min	1Q	Median	3Q	Max
-1.6788	-0.8887	-0.1755	0.7678	3.3165

Zero-inflated Poisson regression with VGAM

Count model coefficients (poisson with log link):

Estimate Std. Error z value Pr(>|z|)

(Intercept)	0.456827	0.124391	3.672	0.00024 ***
X1	-0.216984	0.025879	-8.385	< 2e-16 ***
X2	-0.002068	0.012180	-0.170	0.86516

Zero-inflated Poisson regression with VGAM

Zero-inflation model coefficients (binomial with logit link):

Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-1.8868	0.2927	
	-6.446	1.15e-10	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Number of iterations in BFGS optimization: 10 Log-likelihood:
-402.4 on 4 Df

Zero-inflated Poisson regression with VGAM

Using `vglm()` from package VGAM

```
library(VGAM)
```

```
vglm(Y ~ X1 + X2, family=zipoissonff, data=vgamdata)
```

```
# not shown
```


Vuong Test

- ▶ The Vuong test compares the zero-inflated model with an ordinary poisson regression model.
- ▶ A significant z-test indicates that the zero-inflated model is better.

Vuong-Test

Using `vuong()` from package **pscl**: Poisson model against zero-inflated Poisson model

```
library(pscl)
```

```
vuong(ziFitP, glmFitP)
```

```
Vuong Non-Nested Hypothesis Test-Statistic: -0.6038871
```

```
(test-statistic is asymptotically distributed  $N(0,1)$  under  
null that the models are indistinguishable)
```

```
in this case:
```

```
model2 > model1, with p-value 0.27296
```

Zero-inflated negative binomial regression with VGAM

```
ziFitNB <- zeroinfl(Y ~ X1 + X2 | 1, dist="negbin", data=vga  
summary(ziFitNB)
```

Call:

```
zeroinfl(formula = Y ~ X1 + X2 | 1, data = vgamdata, dist =
```

Zero-inflated negative binomial regression with VGAM

Pearson residuals:

Min	1Q	Median	3Q	Max
-1.6352	-0.8709	-0.1633	0.7424	3.3172

Count model coefficients (negbin with log link):

Estimate	Std. Error	z value	Pr(> z)		
(Intercept)	0.426760	0.136718	3.121	0.0018	**
X1	-0.222478	0.028374	-7.841	4.48e-15	***
X2	-0.001403	0.012926	-0.109	0.9135	
Log(theta)	3.474664	1.443439	2.407	0.0161	*

Zero-inflated negative binomial regression with VGAM

Zero-inflation model coefficients (binomial with logit link)

Estimate	Std. Error	z value	Pr(> z)
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(Intercept)	-1.973	0.341	-5.786	7.2e-09 ***
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Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Theta = 32.287

Number of iterations in BFGS optimization: 26

Log-likelihood: -402.1 on 5 Df

Zero-inflated negative binomial regression with VGAM

Using `vglm()` from package VGAM

```
library(VGAM)
vglm(Y ~ X1 + X2, family=zinegbinomial, data=vgamdata)
# not shown
```

Zero-inflated negative binomial regression with VGAM

Vuong-Test using `vuong()` from package `pscl`: negative binomial model against zero-inflated negative binomial model

```
library(pscl)
vuong(ziFitNB, glmFitNB)
Vuong Non-Nested Hypothesis Test-Statistic: -1.037018
(test-statistic is asymptotically distributed  $N(0,1)$  under
null that the models are indistinguishable)
in this case:
model2 > model1, with p-value 0.14986
```