Zero Truncated Poisson Distribution

R Data Analysis Examples: Zero-Truncated Poisson Regression

Zero-truncated poisson regression is used to model count data for which the value zero cannot occur.

Zero Truncated Poisson Distribution

The zero-truncated Poisson (ZTP) distribution is a certain discrete probability distribution whose support is the set of positive integers. This distribution is also known as the conditional Poisson distribution[1] or the positive Poisson distribution.[2] It is the conditional probability distribution of a Poisson-distributed random variable, given that the value of the random variable is not zero. Thus it is impossible for a ZTP random variable to be zero. Consider for example the random variable of the number of items in a shopper's basket at a supermarket checkout line. Presumably a shopper does not stand in line with nothing to buy (i.e. the

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Since the ZTP is a truncated distribution with the truncation stipulated as k ¿ 0, one can derive the probability mass function g(k;) from a standard Poisson distribution f(k;) as follows: [4]

$$g(k;\lambda) = P(X = k \mid k > 0) = \frac{f(k;\lambda)}{1 - f(0;\lambda)} = \frac{\lambda^k e^{-\lambda}}{k!\,(1 - e^{-\lambda}}$$

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Distribution Parameters

The mean is

$$\mathrm{E}[X] = \frac{\lambda}{1 - e^{-\lambda}} = \frac{\lambda e^{\lambda}}{e^{\lambda} - 1}$$

and the variance is

$$\mathrm{Var}[X] = \frac{\lambda}{1 - e^{-\lambda}} - \frac{\lambda^2 e^{-\lambda}}{(1 - e^{-\lambda})^2}$$

$$= \frac{\lambda e^{\lambda}}{e^{\lambda} - 1} \left[1 - \frac{\lambda}{e^{\lambda} - 1} \right]$$

Examples of zero-truncated Poisson regression

Example 1.

- A study of length of hospital stay, in days, as a function of age, kind of health insurance and whether or not the patient died while in the hospital.
- Length of hospital stay is recorded as a minimum of at least one day.

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Example 2.

- A study of the number of journal articles published by tenured faculty as a function of discipline (fine arts, science, social science, humanities, medical, etc).
- To get tenure faculty must publish, therefore, there are no tenured faculty with zero publications.

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Example 3.

- A study by the county traffic court on the number of tickets received by teenagers as predicted by school performance, amount of driver training and gender.
- Only individuals who have received at least one citation are in the traffic court files.