Modelling Count Variables with R

Introduction

Negative binomial regression is for modeling count variables, usually for over-dispersed count outcome variables.

Negative Binomial regression with R

- Negative binomial regression can be used for over-dispersed count data, that is when the conditional variance exceeds the conditional mean.
- It can be considered as a generalization of Poisson regression since it has the same mean structure as Poisson regression and it has an extra parameter to model the over-dispersion.

Negative Binomial regression with R

- If the conditional distribution of the outcome variable is over-dispersed, the confidence intervals for Negative binomial regression are likely to be narrower as compared to those from a Poisson regression.

Examples of negative binomial regression

- **Example 1** School administrators study the attendance behavior of high school juniors at two schools.
  Predictors of the number of days of absence include the type of program in which the student is enrolled and a standardized test in math.
- **Example 2** A health-related researcher is studying the number of hospital visits in past 12 months by senior citizens in a community based on the characteristics of the individuals and the types of health plans under which each one is covered.

Negative Binomial Regression with R

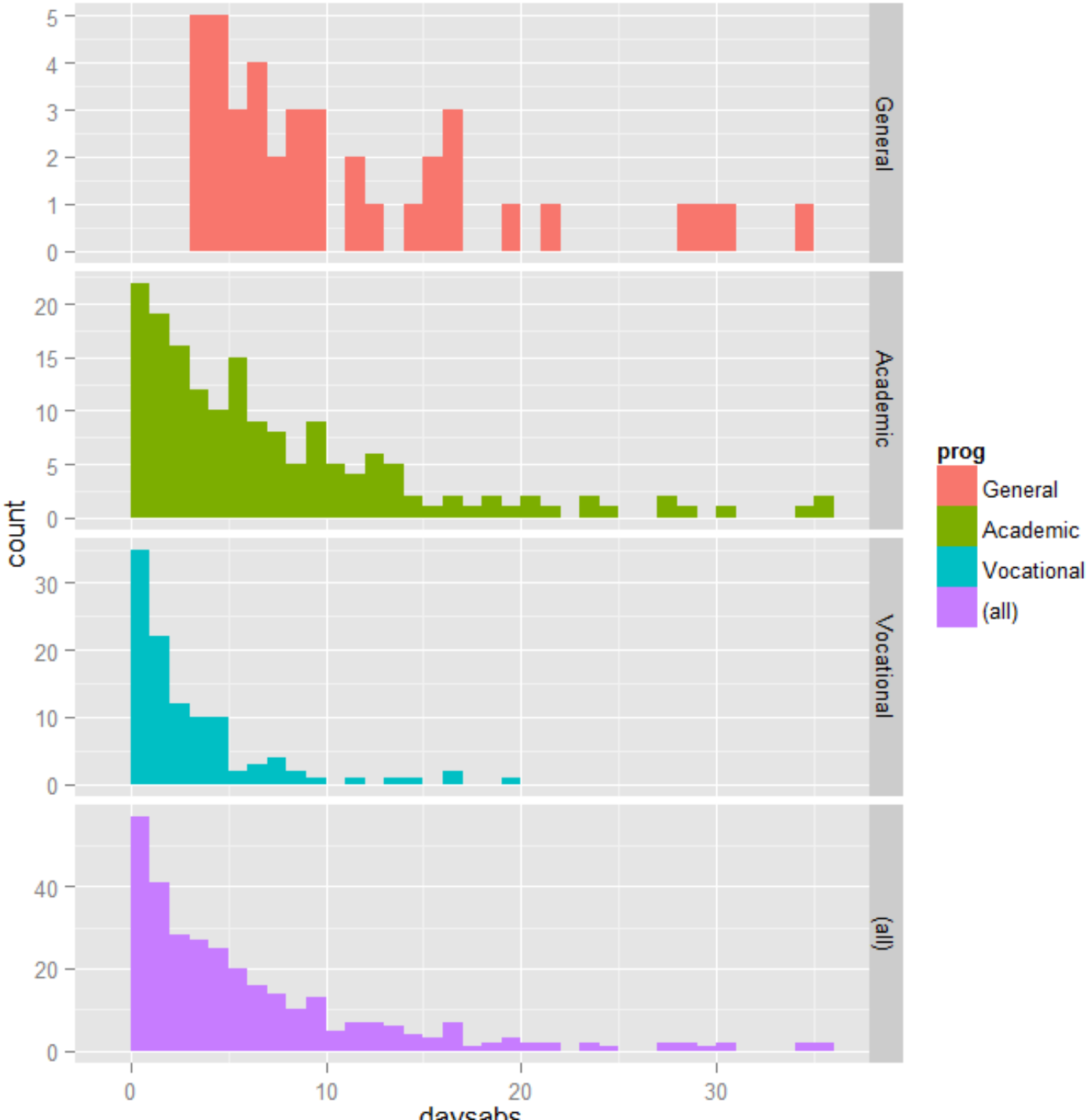
Description of the data Let's pursue Example 1 from above.

- We have attendance data on 314 high school juniors from two urban high schools in the file **negbin.csv** .
- The response variable of interest is days absent, **daysabs**.
- The variable **math** gives the standardized math score for each student.
- The variable **prog** is a three-level nominal variable indicating the type of instructional program in which the student is enrolled.

Negative Binomial Regression with R

Exploratory Data Analysis

```
summary(dat)
       id          gender       math          daysabs
  1001   :  1   female:160   Min.   : 1.0   Min.   : 0.00
  1002   :  1   male  :154   1st Qu.:28.0   1st Qu.: 1.00
  1003   :  1                Median :48.0   Median : 4.00
  1004   :  1                Mean   :48.3   Mean   : 5.96
  1005   :  1                3rd Qu.:70.0   3rd Qu.: 8.00
  1006   :  1                Max.   :99.0   Max.   :35.00
  (Other):308
        prog
  General   : 40
  Academic  :167
  Vocational:107
```


General
Academic
Vocational
(all)
count
daysabs
0
10
20
30
prog
General
Academic
Vocational
(all)

Negative Binomial Regression with R

Each variable has 314 valid observations and their distributions seem quite reasonable. The unconditional mean of our outcome variable is much lower than its variance.

Data Set

- The table below shows the average numbers of days absent by program type and seems to suggest that program type is a good candidate for predicting the number of days absent, our outcome variable, because the mean value of the outcome appears to vary by **prog**.

Data Set

- The variances within each level of prog are higher than the means within some of the levels.
- These are the conditional means and variances. These differences suggest that over-dispersion is present and that a Negative Binomial model would be appropriate.

Negative Binomial Regression with R

```
with(dat, tapply(daysabs, prog, function(x) {
sprintf("M (SD) = %1.2f (%1.2f)", mean(x), sd(x))
}))
##                   General
## "M (SD) = 10.65 (8.20)"
##                  Academic
## "M (SD) = 6.93 (7.45)"
##                Vocational
## "M (SD) = 2.67 (3.73)"
```

Negative Binomial Regression with R

Negative binomial regression analysis

We will use the `glm.nb` function from the MASS package to estimate a negative binomial regression.

```
summary(m1 <- glm.nb(daysabs ~ math + prog,
 data = negbinom))
##
## Call:
## glm.nb(formula = daysabs ~ math + prog,
##         data = dat, init.theta = 1.032713156,
##      link = log)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -2.155  -1.019  -0.369   0.229   2.527
```

Negative Binomial Regression with R

- R first displays the call and the deviance residuals.
- Next, we see the regression coefficients for each of the variables, along with standard errors, z-scores, and p-values.

Negative Binomial Regression with R

```
Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)      2.61527    0.19746   13.24  < 2e-16 ***
math            -0.00599    0.00251   -2.39    0.017 *
progAcademic    -0.44076    0.18261   -2.41    0.016 *
progVocational -1.27865    0.20072   -6.37  1.9e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

Negative Binomial Regression with R

- The variable **math** has a coefficient of -0.006, which is statistically significant.
- This means that for each one-unit increase in **math**, the expected log count of the number of days absent decreases by 0.006.
- The indicator variable shown as **progAcademic** is the expected difference in log count between group 2 and the reference group (prog=1).

Negative Binomial Regression with R

```
## (Dispersion parameter for Negative Binomial(1.033) famil
##
##     Null deviance: 427.54  on 313  degrees of freedom
## Residual deviance: 358.52  on 310  degrees of freedom
## AIC: 1741
##
## Number of Fisher Scoring iterations: 1
##
##
##                Theta:  1.033
##            Std. Err.:  0.106
##
##  2 x log-likelihood:  -1731.258
```

Negative Binomial Regression with R

- The expected log count for level 2 of prog is 0.44 lower than the expected log count for level 1.
- The indicator variable for progVocational is the expected difference in log count between group 3 and the reference group.

Negative Binomial Regression with R

- The expected log count for level 3 of prog is 1.28 lower than the expected log count for level 1.
- To determine if prog itself, overall, is statistically significant, we can compare a model with and without prog.
- The reason it is important to fit separate models, is that unless we do, the overdispersion parameter is held constant.

Negative Binomial Regression with R

```
m2 <- update(m1, . ~ . - prog)
anova(m1, m2)
## Likelihood ratio tests of Negative Binomial Models
##
## Response: daysabs
##          Model  theta Resid. df    2 x log-lik.   Test
## 1         math 0.8559         312           -1776
## 2 math + prog 1.0327         310           -1731 1 vs 2
##     Pr(Chi)
## 1
## 2 1.652e-10
```

Negative Binomial Regression with R

- The two degree-of-freedom chi-square test indicates that prog is a statistically significant predictor of daysabs.
- The null deviance is calculated from an intercept-only model with 313 degrees of freedom.
- Then we see the residual deviance, the deviance from the full model.
- We are also shown the AIC and 2*log likelihood.

Negative Binomial Regression with R

- The theta parameter shown is the dispersion parameter.
- Note that R parameterizes this differently from SAS, Stata, and SPSS.
- The R parameter (theta) is equal to the inverse of the dispersion parameter (alpha) estimated in these other software packages.
- Thus, the theta value of 1.033 seen here is equivalent to the 0.968 value seen in the Stata Negative Binomial Data Analysis Example because $1/0.968 = 1.033$.

Checking model assumption

- As we mentioned earlier, negative binomial models assume the conditional means are not equal to the conditional variances.
- This inequality is captured by estimating a dispersion parameter (not shown in the output) that is held constant in a Poisson model.
- Thus, the Poisson model is actually nested in the negative binomial model.
- We can then use a likelihood ratio test to compare these two and test this model assumption.
- To do this, we will run our model as a Poisson.

Negative Binomial Regression with R

```
m3 <- glm(daysabs ~ math + prog,
          family = "poisson", data = dat)
pchisq(2 * (logLik(m1) - logLik(m3)),
          df = 1, lower.tail = FALSE)
## 'log Lik.' 2.157e-203 (df=5)
```

Negative Binomial Regression with R

- In this example the associated chi-squared value is 926.03 with one degree of freedom.
- This strongly suggests the negative binomial model, estimating the dispersion parameter, is more appropriate than the Poisson model.

Negative Binomial Regression with R

We can get the confidence intervals for the coefficients by profiling the likelihood function.

```
(est <- cbind(Estimate = coef(m1), confint(m1)))
## Waiting for profiling to be done...
##                  Estimate   2.5 %    97.5 %
## (Intercept)      2.615265  2.2421  3.012936
## math            -0.005993 -0.0109 -0.001067
## progAcademic    -0.440760 -0.8101 -0.092643
## progVocational -1.278651 -1.6835 -0.890078
```

Incidence Rate Ratios

- We might be interested in looking at incident rate ratios rather than coefficients.
- To do this, we can exponentiate our model coefficients. The same applies to the confidence intervals.

Negative Binomial Regression with R

```
exp(est)
##                  Estimate  2.5 %  97.5 %
## (Intercept)       13.6708 9.4127 20.3470
## math               0.9940 0.9892  0.9989
## progAcademic       0.6435 0.4448  0.9115
## progVocational     0.2784 0.1857  0.4106
```

Negative Binomial Regression with R

- The output above indicates that the incident rate for prog = 2 is 0.64 times the incident rate for the reference group (prog = 1).
- Likewise, the incident rate for prog = 3 is 0.28 times the incident rate for the reference group holding the other variables constant.
- The percent change in the incident rate of **daysabs** is a 1% decrease for every unit increase in math.

Negative Binomial Regression with R

- The form of the model equation for negative binomial regression is the same as that for Poisson regression.
- The log of the outcome is predicted with a linear combination of the predictors:

$$ln(\widehat{daysabs_i}) = Intercept + b_1(prog_i = 2) + b_2(prog_i = 3) + b_3 math_i$$

Negative Binomial Regression with R

$$\widehat{daysabs}_i = e^{Intercept + b_1(prog_i=2) + b_2(prog_i=3) + b_3 math_i}$$

$$= e^{Intercept} e^{b_1(prog_i=2)} e^{b_2(prog_i=3)} e^{b_3 math_i}$$

The coefficients have an additive effect in the $ln(y)$ scale and the IRR have a multiplicative effect in the y scale. The dispersion parameter in negative binomial regression does not effect the expected counts, but it does effect the estimated variance of the expected counts.

Predicted values

- For assistance in further understanding the model, we can look at predicted counts for various levels of our predictors.
- Below we create new datasets with values of math and prog and then use the predict command to calculate the predicted number of events.

Negative Binomial Regression with R

- First, we can look at predicted counts for each value of prog while holding math at its mean.
- To do this, we create a new dataset with the combinations of prog and math for which we would like to find predicted values, then use the `predict` command.

Negative Binomial Regression with R

```
newdata1 <- data.frame(math = mean(dat$math),
prog = factor(1:3, levels = 1:3,
labels = levels(dat$prog)))

newdata1$phat <- predict(m1, newdata1,
 type = "response")

newdata1
    math        prog   phat
 1 48.27     General 10.237
 2 48.27    Academic  6.588
 3 48.27 Vocational  2.850
```

Negative Binomial Regression with R

- In the output above, we see that the predicted number of events (e.g., days absent) for a general program is about 10.24, holding math at its mean.
- The predicted number of events for an academic program is lower at 6.59, and the predicted number of events for a vocational program is about 2.85.

Negative Binomial Regression with R

Below we will obtain the mean predicted number of events for values of math across its entire range for each level of prog and graph these.

Negative Binomial Regression with R

```
newdata2 <- data.frame(
math = rep(seq(from = min(dat$math),
       to = max(dat$math), length.out = 100), 3),
prog = factor(rep(1:3, each = 100), levels = 1:3, lab
levels(dat$prog)))
```

Negative Binomial Regression with R

```
newdata2 <- cbind(newdata2, predict(m1, newdata2, type
newdata2 <- within(newdata2, {
DaysAbsent <- exp(fit)
LL <- exp(fit - 1.96 * se.fit)
UL <- exp(fit + 1.96 * se.fit)
})
```

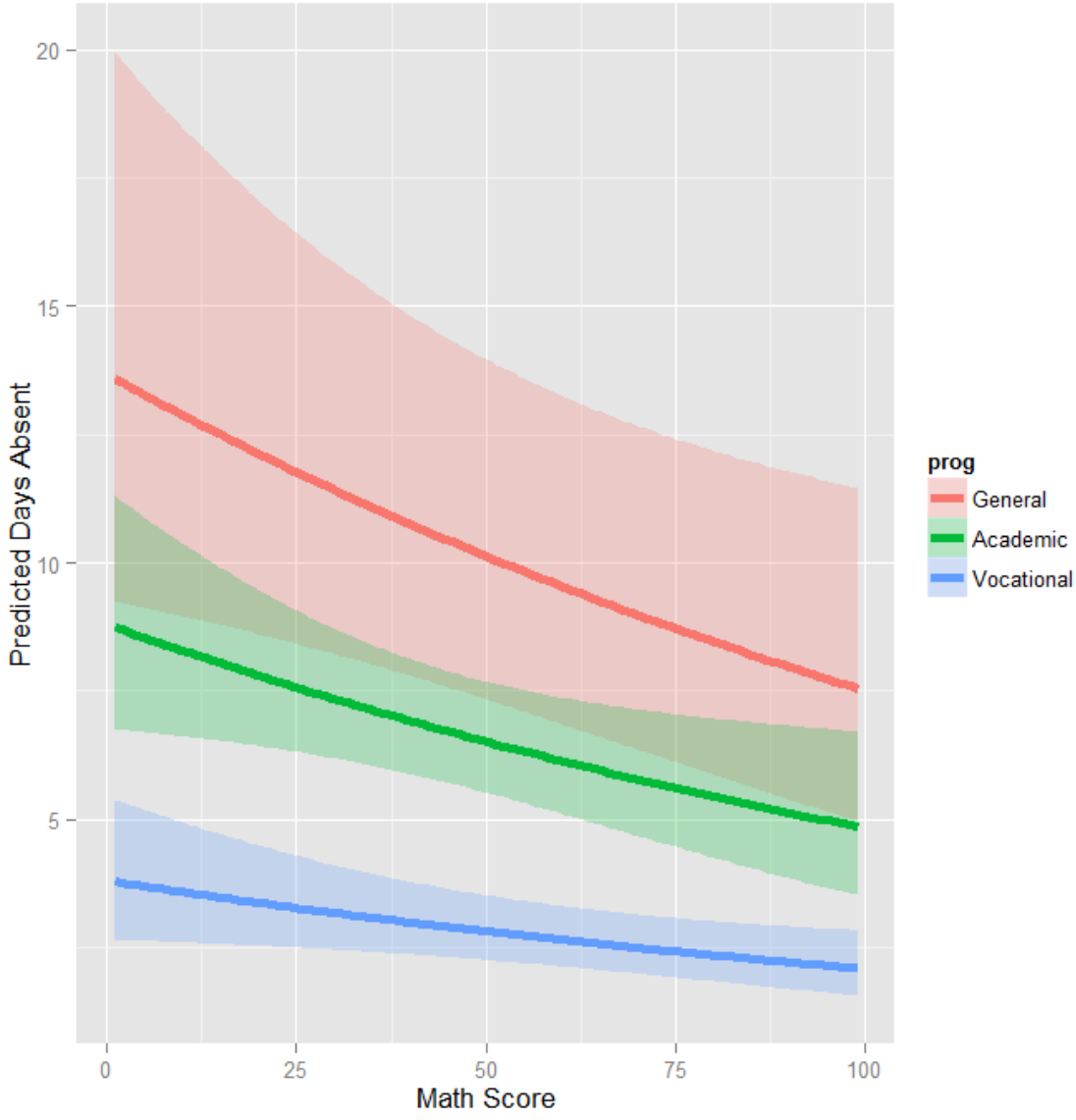

20
15
10
5
Predicted Days Absent
0
25
50
75
100
Math Score
prog
General
Academic
Vocational

Figure:

Negative Binomial Regression with R

- The graph shows the expected count across the range of math scores, for each type of program along with 95 percent confidence intervals.
- Note that the lines are not straight because this is a log linear model, and what is plotted are the expected values, not the log of the expected values.

Things to consider

- It is not recommended that negative binomial models be applied to small samples.
- One common cause of over-dispersion is excess zeros by an additional data generating process. In this situation, **zero-inflated model** should be considered.
- If the data generating process does not allow for any 0s (such as the number of days spent in the hospital), then a **zero-truncated model** may be more appropriate.

Negative Binomial Regression with R

- Count data often have an exposure variable, which indicates the number of times the event could have happened.
- This variable should be incorporated into your negative binomial regression model with the use of the `offset` option.

Negative Binomial Regression with R

- The outcome variable in a negative binomial regression cannot have negative numbers.
- You will need to use the `m1$resid` command to obtain the residuals from our model to check other assumptions of the negative binomial model