Modelling and evaluation of building thermal performance from smart meter readings

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1 Introduction

This article focus on data-driven statistical modelling for assessment of thermal performance and dynamics of buildings. The presented methods can be used to extract valuable information from smart meter readings. For example the methods can be used to provide an automatic and objective energy labelling of buildings Mortensen and Nielsen [2011], and computer-generated suggestions for building refurbishment and thereby form the basis for optimization of energy efficiency in buildings. Furthermore, they can enable the use of buildings for thermal energy buffering providing a flexible load for integration of renewables and load peak-shaving functionality.

Considerations on the choice of modelling method is given. Selecting the methods which can be successfully applied depends on the quality of the available data, regarding sampling frequency and accuracy. The article includes an outline of two methods.

The first method for describing the main energy performance characteristics of a building is based on frequent measurements from smart meters in single family houses, and a climate station located within a few kilometers from the buildings. The main thermal performance characteristics estimated are the response of a building to changes in ambient temperature (UA-value), solar radiation (gA-value), and wind (wA-value). The effect of the wind could be characterized both in terms of the wind speed and the wind direction, implying that wA-values are estimated for different wind directions. Especially, the UA- and wA-values are directly related to the insulation and air sealing of the house. The gA-values are related to the ability of the house to passively use solar heating.

The second method estimates the same characteristics and in addition heat capacities of different parts of the building, providing detailed information about the heat dynamics of the buildings and a model which can form a basis for MPC control as presented in Halvgaard et al. [2012].

The methods can be used to supply users with valuable information about the thermal performance of their house. The thermal characteristics can be presented via web pages or smart phone apps. Here the user can interactively gain information about the energy status of their house and provide information such as their typical indoor temperature, the use of nightly setback and wood burning stove. In addition to the interactive use the methods can also be used by e.g. district heating companies in order to screen for households with an unusual high consumption. In Denmark this is of interest to district heating companies since these are obliged to implement energy savings and from 2010 also energy savings in the network can be formally included in the total energy savings of a given district heating company.

The outline of the article is as follows. In Section 2 the perspectives of application of the results found in the two presented studies are reflected on for an end user or a district heating company. In Section 3 the choices of modelling method is discussed. In Section 4 the first study in which daily average values are analyzed is presented and finally in Section 5 the second study in which a grey-box model for the heat dynamics of a building is identified.

The remaining of this section will give examples of applicable methods in the range from daily values and down to five minute values. Starting with a an overview of useful statistical methods and time series modelling. This is followed by a presentation of a method for thermal charaterisation of buildings based on daily values. The data was recorded in 56 household in Denmark with different composition of users. For this sampling frequency almost only little consideration of the dynamics is needed, but it is showed how the model is extended to include the dynamics of the building when it is applied on four-hourly values. Then follows a section on modelling of heat dynamics with continuous time grey-box modelling of the heat dynamics of a building using five-minutes values. Finally a perspective on applications of dynamical methods to thermal characterisation of buildings are given.

2 Perspectives

The perspectives of the work presented in this article points towards an automatic estimation of the main thermal characteristics of a building. Such calculations can be used to supply users with valuable information about their house via web pages or smart phones or to help e.g. district heating or energy retrofit companies determining house owners who might be worth-while contacting as a service for indicating potentials for energy savings.

First, consider interactive services where measurements of heat consumption is available from smart meters. The results presented in this article show that daily values of heat consumption are sufficient for estimating the non-dynamic thermal characteristics, which are the most important ones with relation to the overall energy consumption of the building.

Based on measurements from the heating season **2009/2010** your typical indoor temperature during the heating season has been estimated to **24** ^oC. If this is not correct you can change it here $\boxed{24}$ ^oC.

If your house has been left empty in longer periods with a partly reduced heat supply you have the possibility of specifying the periods in this $\boxed{\text{calendar}}$.

According to BBR the area of your house is **155** m^2 and from **1971**.

Based on BBR information it is assumed that **you do not use any supplementary heat supply**. If this is not correct you can specify the type and frequency of use here:

- Wood burning stove used $\boxed{0}$ times per week in cold periods.
- Solar heating $\boxed{\text{y/n}}$, approximate size of solar panel $\boxed{0} \times \boxed{0}$ meters.

Based on the indoor temperature **24** ^oC, the use of a wood burning stove **0** times per week, and **no** solar heating installed, the response of your house to climate is estimated as:

- The response to outdoor temperature is estimated to **200** $W/^oC$ **which given the size and age of your house is expectable**[a].
- On a windy day the above value is estimated to increase with **60** $W/^oC$ **when the wind blows from easterly directions. This response to wind is relatively high and indicates a problem related to the air sealing on the eastern side of the house.**
- On a sunny day during the heating season the house is estimated to receive **800** W as an average over 24 hours. **This value is quite expectable.**

[a]Many kind of different recommendations can be given here.

Figure 1: Main elements in a possible user interaction. **Bold** entries indicate information specific for the user and $\boxed{\text{boxed}}$ fields indicate information which the user has the ability to enter. Assuming measurements is available on a time scale of 4 hours the above could be supplemented with the dynamic characteristics of the response, see more in the Report ENFOR [2010b].

The information to the user could be extended with behavioral information. The *actual* heating season can be detected and if this is unreasonable long the user can be advised to turn of the heating system during e.g. summer periods. Also, if the user specify the overall indoor temperature and if data from the summer period is used the consumption used for hot tab water can be estimated.

Figure 1 shows a text-based sketch of the main elements in a user interface of a possible application based on the work presented in this article. As indicated such a user interaction should indicate clearly to the user what the estimated values mean in terms of the insulation of the house. Two parallel approaches are possible in this aspect:

(i) The values are related to a database of expert knowledge regarding what energy class the building belongs to.

(ii) The values are related to values estimated for other users and the system can automatically inform the user about the thermal performance of the particular house compared to other houses.

As the number of users increases the information sharing ability of the application will be increasingly valuable.

3 Considerations on modelling method

A wide range of methods exists for thermal characterisation of buildings based on measured data. Naturally the choice of method depends on the purpose of the modelling, but generally speaking two important factors determines which methods can be applied with success:

- Frequency of data: The resolution is both a matter of accuracy of the measurement equipment and sampling frequency of the measurements. Most installed energy meters measure accumulated values with a resolution which is apllicaple for modelling with daily values. Modelling with daily values requires little inclusion of dynamical effects [Mortensen and Nielsen, 2011], whereas modelling with a higher sampling frequency requires dynamical methods, as the second method presented in this paper. Smart meters typically measure with a time resolution of 10 to 15 minutes.

- The available variables: As a minimum the energy consumption for heating and the outdoor temperature is needed. Preferably also solar radiation. Other important climate variables are wind speed and direction. More variables are: seperate heat consumption for heating and hot tab water, electricity consumption, indoor temperature.

The method needs to account for uncertaincy caused by several other effects, such as, the distance from where the climate measurements are recorded to the building, this will have an effect on the accuracy. Clearly one other challenge is dealing with measurements from inhabitet buildings, where the effect of users behaviour in the building needs to be accounted for.

3.1 Time series modelling

Modelling of time series data from dynamical systems requires proper use of statistical modelling techniques as for example presented by Madsen [2007]. The available methods are ranging from simple linear regression models where no dynamics are included, over classical linear time series models (ARMAX type of models), to grey-box models based on stochastic differential equations. Furthermore a set of measures for evaluating model performance and providing insights into un-modelled features, which should be taken into account. Very important are the auto-correlation and the cross-correlation functions.

3.1.1 Linear regression models

The most fundamental and widespread statistical models are based on linear regression. An output variable is modelled as a linear function of some input variables and the coefficients are estimated by minimizing the squared errors, which makes maximum likelihood estimates if the error is normal distributed. This type of models are perfectly suited for modelling physical phenomenons, for example the response in energy consumption to the ambient temperature and other climate variables as carried out in the first study presented in Section 4. The linear regression techniques can applied and extended in many ways, for example by time-varying estimation of the coefficients, inclusion of dynamical effects and non-linear dependencies, which is also important parts of the modelling in the first study. The application of linear regression models, only including the dynamics of a system in a very crude manner, is useful for daily or lower time resolution values.

3.1.2 Linear time series models

ARMAX models can be applied for modelling dynamical systems and can be very effective for modelling heat dynamics of buildings. ARMAX models include a linear transfer functions for modelling dynamical effects and the coefficients can be estimated with maximum likelihood techniques. If the models are restricted to ARX models, they can be fitted as simple linear regression models, making robust and fast parameter estimation possible. Furthermore, these models can also be made time adaptive and non-linear, making them very useful also for modelling complicated effects, such as solar gains in buildings. The steady-state properties, e.g. the UA- and gA-value can be estimated with this type of models, together with the time constants of the system. It is noted that important statistical time series techniques must be used in the model evaluation and selection, such as the auto-correlation and cross-correlation functions, see for example Madsen [2007] for more details. For examples of applying ARMAX type of models see [Jiménez and Madsen, 2008], [Jiménez et al., 2008] and [Bacher et al., 2012].

3.1.3 Grey-box models of a dynamic system

A grey-box model is established using a combination of prior physical knowledge and statistics, i.e. information embedded in data. The prior physical knowledge is formulated by a set of stochastic differential equations formulated on state space form. The equations describe a lumped model of the heat dynamics of the building. The physical model is coupled with a data-driven part in which the information embedded in observed data is used for parameter estimation. The data-driven part is represented by a discrete time measurement equation. The measurement error is assumed to be a white noise process and this assumption enables evaluation and tests of the performance of the model, since if the assumption is met it is an indication that the physical model is consistent with the observed heat dynamics of the building. For more on grey-box modelling see for example [Madsen and Holst, 1995], [Andersen et al., 2000], [Kristensen et al., 2004] and Bacher and Madsen [2011].

4 Thermal characterisation of buildings using data from smart meters

The next session is concerned with estimating thermal characteristics of single family houses based on measurements of energy consumption and climate. The main thermal characteristics describe how the building respond to: temperature differences between indoor and outdoor environment (UA-value), solar radiation (gA-value), and wind (wA-value). The effect of the wind can be characterized both in terms of the wind speed and the wind direction, implying that wA-values are estimated for different wind directions. Especially, the UA and wA-values are directly related to the insulation and air sealing of the building. The gA-values are related to the ability of the building to passively use solar heating. The estimated thermal characteristics have been analyzed with respect to background information regarding the households. The information is obtained via questionnaires and via the Danish Building Register (BBR). The significant effects are the ground area of the building, the year of construction, and the number of times per week a wood burning stove is used. This analysis is found in the Report [ENFOR, 2010b].

Further characterization of the building is the dynamic response to changes in climate variables. This is carried out by ENFOR [2010b], where the dynamic response is characterized by time constants of the response to temperature and solar radiation. Using such dynamical methods enables energy optimization by load management related to integration of renewable energy and peak-shaving. An example of applying dynamical methods as basis for energy optimization is for district heating where it can lead up to 20% savings of the heat loss in the distribution system Nielsen and Madsen [2002].

The data used in this section consists of heat and electricity consumption data for the period from ultimo September 2008 to primo December 2009 from 56 households connected to the district heating system in Sønderborg, Denmark. Also climate data obtained at a local weather station within a few kilometers from the buildings. The energy consumption data is described in detail in the Report [ENFOR, 2010a]. For 26 of the 56 households the electricity data is available, they are considered in this article. In the Report [ENFOR, 2010b] it is shown that the thermal characteristics of the building can often be well estimated based on measurements of the heat consumption alone. This is the case when the electricity consumption is not too large as it would be if for example electrical floor heating is used.

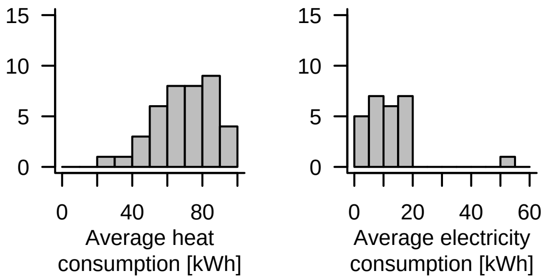


Figure 2: Distribution of daily average heat and electricity consumption in winter 2008/09.

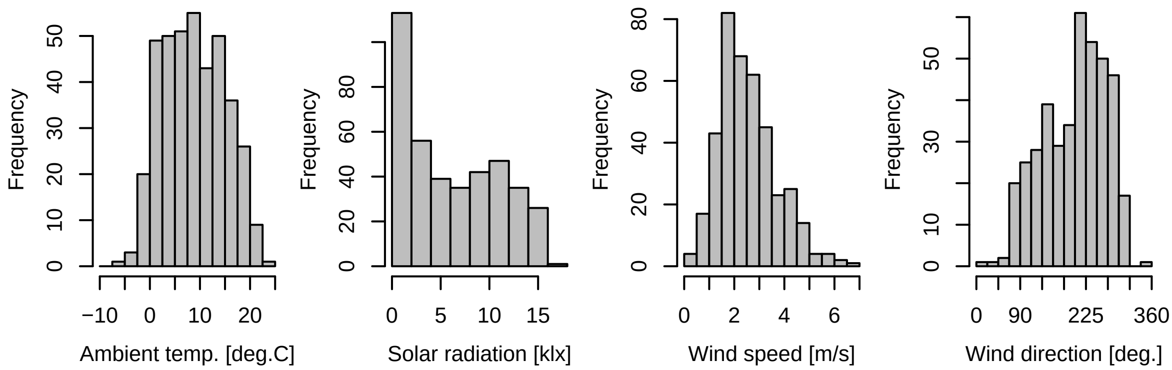


Figure 3: Daily averages of climate variables. The unit of solar radiation is kLux.

4.1 Daily sampling

The analysis is carried out using daily power consumption values, i.e. using a sampling period of 1 day. With a unit resolution of 0.01 GJ this gives the daily consumption a unit resolution of 2.78 kWh/day, which is found to be sufficient for the analysis of daily values. Heat is estimated based on the difference in the accumulated consumption from midnight to midnight. The electricity consumption is treated in the same way. Distribution of daily averages are shown in Figure 2. Climate data are available in the period from 2008-10-06 to 2009-11-18 with a 10 minute sampling interval. The available variables are ambient air temperature T_a in °C, solar radiation R_0 in lux, wind speed w in m/s and wind direction θ in degrees. All climate data are down sampled to diurnal averages. The resulting distribution of the variables for a sampling period of 1 day are shown in Figure 3. The measurement of solar radiation is assumed to be dominated by direct sunlight and thus to be proportional with the effect of the direct sunlight.

The stationary heat transfer for a building is for the main part assumed to be comprised by three ways of heat transfer, namely through walls, windows, and by ventilation. Here heat transfer through the roof is assumed to be included as part of the model for the walls. By considering stationary models for heat transfer trough walls and windows and via ventilation a model with the following characteristics is derived:

- Responses on the temperature are collected into one term for which the coefficient is the UA-value.

- Responses on the solar radiation are collected into one term for which the coefficient is the gA-value.
- Responses on the product of the temperature and the wind speed are collected into one term for which the coefficient is the wA-value.

The model can only be used during the time period where the building is heated to maintain a constant indoor temperature, such that the heat transfer from the building can be measured based on the amount of energy supplied to the household. In the following it is shown how this period can be estimated.

4.1.1 Analysis of daily values of power consumption

The estimation of UA and gA values and wind dependence is based on the assumptions outlined in the previous section. Unknown parameters in the model are UA, gA, $v(\theta)$. The function $v(\theta)$ is modelled either as a constant $v(\theta) = c_w$ or as piecewise constant for the major wind directions. There are only three days with an average wind direction from the northern quarter, and it is chosen to keep only three major wind segments, namely east (E) 0-135 deg., south (S) 135-225 deg., and west (W) 225-360 deg. The piecewise constant approximation to $v(\theta)$ is given as

$$v(\theta) = \sum_{j=E,S,W} I(\theta \in j) c_{wj} \tag{1}$$

where I is an indicator function equal to 1 when the argument is true and otherwise 0. The three coefficients c_{wj} gives wind dependence in the model and is interpreted as 'wA' values such that $\text{wA}_j = c_{wj}$.

4.1.2 Time varying estimates

Initial investigation of the energy consumption data is done by estimating the time variations of the coefficients in a linearized and simplified version of the model outlined above. To reduce the number of parameters to be estimated the interaction between wind speed and air temperature, is not included giving the model

$$Q_t = b_0 - \text{UA} \cdot T_{a,t} - \text{gA} \cdot R_{0,t} + b_1 w_t + e_t \tag{2}$$

where b_0 and b_1 are constants, $T_{a,t}$ is the ambient air temperature, $R_{0,t}$ is the solar radiation, and w_t is the wind speed. The coefficient b_1 cannot be interpreted in relation to the physical model, but it still gives an indication of wind speed dependence in the energy consumption.

The time variations are estimated using locally weighted estimation of the linear model. The method is described by Nielsen [1997] and gives local estimates in time of the model coefficients by only considering observations within a limited time window. This makes it possible to see if they are constant over time, e.g. to look for variations during the heating season and how they change during the summer period. Figure 4 shows these time varying estimates for two households. For the most of the households the UA, gA and b_1 values are relatively stable during the winter period which is also seen for these two households.

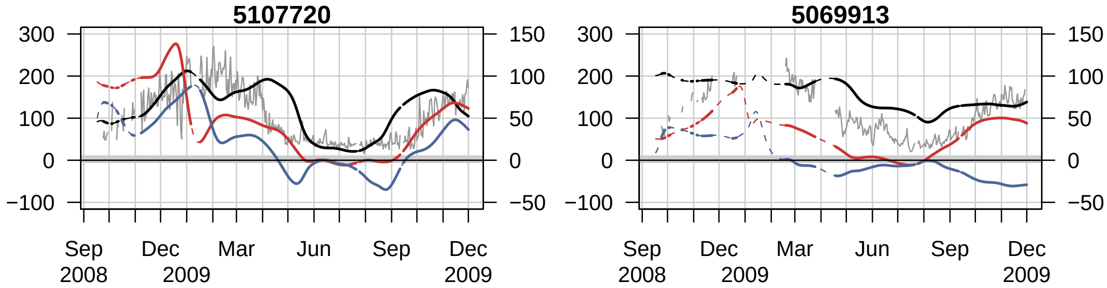


Figure 4: Time varying estimates of coefficients in Eq. (2). Black is UA [°C], red is gA [W/kLux] and blue is b1 [W/m/s] all measured on the left side axis. The underlying gray curve is daily total energy consumption in kWh measured on the right side axis.

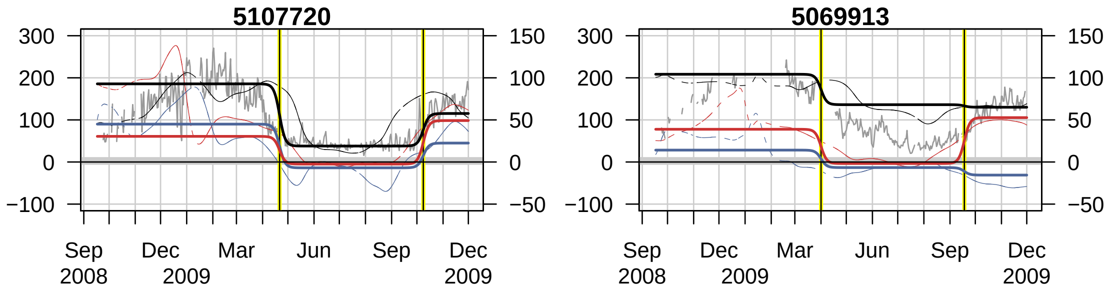


Figure 5: Thick lines are estimates of parametric models for time varying coefficients in Eq. (2). Black is UA [W/ °C], red is gA [W/klux] and blue is b1 [W/m/s] all measured on the left side axis. Thin lines are estimates based on local regression from Figure 7. Vertical black-yellow lines are estimated time point of change. The underlying gray curve is daily total energy consumption in kWh measured on the right side axis.

4.1.3 Parametric modelling of time-variations

Based on the estimates of time variations of the coefficients in the model in Eq. (2) it seems reasonable to assume that the coefficients can be modelled with a constant level for each of the two winter and one summer periods, giving three levels in total. Estimating when the changes in level occur will indicate the exact extent of the heating season for each individual building and this information can then be used to select the longest possible period of the actual heating season for further analysis. The model is estimated by means of partial linear estimation techniques and results are shown for the two selected households in Figure 5.

In previous work for estimation of UA-values alone based on daily averages of energy consumption [Nielsen, 2008] it has been found that there is significant dependence on the ambient temperature one day back for the heat consumption. This dynamic effect is also included here. In order to be able to get a good estimate of the effect of the solar radiation the heating season must comprise into the spring, where there is a significant contribution from the sun.

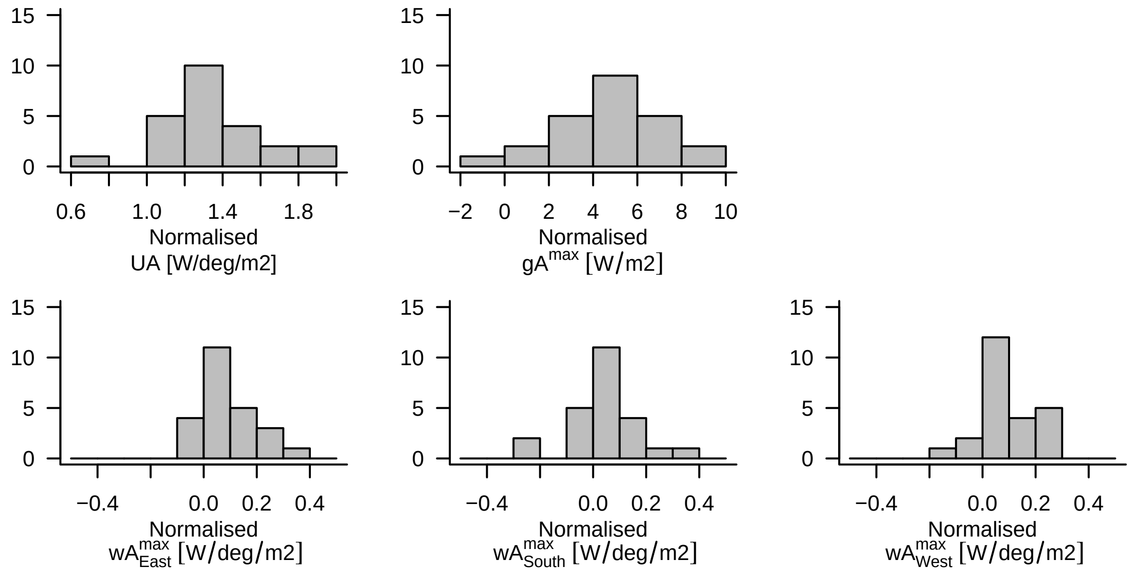


Figure 6: Histograms of estimates of estimates normalized by floor area.

4.1.4 Results

Applying the model, which include sensitivity to wind direction gives the parameter estimates for the winter period shown in Figure 6. The estimated values are within realistic physical values. The estimates of the UA and gA^{max} values are all positive as they are expected to be except for one case of gA^{max}, which is considered an outlier. The estimates of the wA_*^{max} values are mostly positive, although there are some negative estimates indicating a reduced UA value for these wind directions. However, overall the estimates of the wA_*^{max} values gives a picture of the wind dependence for each house, and it is seen that some are clearly more wind sensitive than others. It is also noted that the uncertainties of the parameters are estimated, which provide important information. All details and a thorough analysis of the results can be found in the Report [ENFOR, 2010b].

4.1.5 Discussion on the estimation of energy performance of single family houses

A fundamental assumption of the presented method is that measurements with a time resolution of one day are available. During the recent years such measurements has started to appear, both for electricity, district heating, and natural gas consumption. Obviously, measurements performed with a high frequency imply higher demands for bandwidth and data storage. Maybe less obviously high frequency measurements also require better resolution of the basic measurement equipment. The resolution of the measuring equipment compared with the time resolution can have an important effect on information embedded in the data. In the present study a unit resolution of 2.78 kWh/day has prooved to be sufficient for estimation of the energy performance of single family houses. For energy optimization by load-shifting using dynamical methods a higher time resolution is needed.

4.2 Conclusion

The presented method for estimation of the thermal characteristics of single family houses gives an objective description of the energy performance of individual buildings, based on measurements of energy consumption and climate. It is possible to determine how well a building is insulated, combined with its wind sensitivity for the prevailing wind directions and its ability to passively use solar heating. This implies estimation of the coefficients characterizing the response of the building to differences in temperature (UA-value), solar radiation (gA-value), and wind (wA-value). Such methods based on measurements from smart meters, can enable ICT-facilitated improvements of building energy efficiency by means such as: providing objective methodologies to calculate the thermal performance of buildings for implementation of policy and in measuring its effectiveness, providing advises on the best ways of improving the energy performance of a building, and enhancing the energy awareness of users via interactive web sites or smart phones.

5 Grey-box modelling of the heat dynamics for a building

In the this section a study in which a grey-box model is identified for the heat dynamics of a building is presented. The building is FlexHouse, which is part of the experimental energy system Syslab, at Risø DTU in Denmark. FlexHouse has a controllable electrical heat system and many sensors for data collection. Measurements consisting of five minute values over a period of six days are used, for further details see the report [Bacher and Madsen, 2010], in which a thorough description of the experiments and data is given.

5.1 Data

The following time series consisting of five minute average values are used: Y (°C) a single signal representing the indoor air temperature, T_a (°C) observed ambient temperature at the climate station, Φ_h (kW) total heat input from the electrical heaters in the building, Φ_s (kW/m^2) the global irradiance measured at a climate station located a few meters from the building. Plots of the time series can be found in the uppermost plot of Figure 10. The controlled heat input is formed by two pseudo-random binary sequences (PRBS), which has white noise properties and no correlation with the climate variables.

5.2 Applied models

The models are named from their state vector and where needed a few parameter names. See Bacher and Madsen [2011] for a list of RC-networks corresponding to all applied models.

5.2.1 The full model *TiTmTeThTsAeRia*

The RC-network of the full model, which is the most complex model applied, is illustrated in Figure 7.

This model includes all the individual parts of the building, which it is found feasible to include in linear models based the available data. The individual model parts are indicated on the figure. The state variables are

Sensor The temperature sensors are modelled with a heat capacity and a thermal resistance to the interior.

Interior The interior is in the full model considered to be the indoor air and it is modelled as a heat capacity connected to other parts by thermal resistances.

Medium A thermal medium inside the building is the interior walls and furniture, which is modelled with a heat capacity and a thermal resistance to the interior.

Heater The heaters are modelled by a heat capacity and a thermal resistance to the interior.

Solar The heat input from solar radiation is modelled by the global irradiance times a coefficient.

Envelope The building envelope is modelled with a heat capacity and thermal resistances to both the interior and the ambient environment. A thermal resistance directly coupled to the ambient is also included.

Ambient The ambient environment is represented by the observed ambient temperature.

The full model includes five state variables, that each represents the temperature in a part of the building, and they are: T_s the temperature of the sensor, which for the full model is used as the model output, i.e. y in the measurement equation, T_i the temperature of the interior, i.e. the indoor air, T_m the temperature of an interior thermal medium, i.e. interior walls and furniture, T_h the temperature of the heater. T_e the temperature of the building envelope. The parameters of the model represent different thermal properties of the building. This includes thermal resistances: R_is between the interior and the sensor, R_im between the interior and the interior thermal medium, R_ih between the heaters and the interior, R_ia between the interior and the ambient environment, R_ie between from the interior and the building envelope, R_ea between the building envelope and the ambient environment. The heat capacities of different parts of the building are represented by: C_s for the temperature sensor, C_i for the interior, C_m for the interior walls and furniture, C_h for the electrical heaters, C_e for the building envelope. Finally two coefficients are included, each representing an estimate of an effective area, in which the energy from solar radiation enters the building. They are: A_w the effective window area of the building and A_e the effective area in which the solar radiation enters the building envelope.

5.2.2 The simplest model *Ti*

The simplest model considered is illustrated by the RC-network in Figure 8. The model has one state variable T_i and the following parameters: R_ia the thermal resistance from the interior to the ambient environment, C_i the heat capacity of the entire building, including the indoor air, interior walls, furniture etc., and the building envelope, and A_w the effective window area of the building.

5.3 Model selection

A forward selection procedure is applied to find a sufficient model in the set of models ranging from *Ti* to *TiTmTeThTsAeRia*. The log-likelihood of each model, which is fitted, is listed in Table 1 ordered by the iterations of the model selection. In Table 2 the result of likelihood-ratio tests for model expansion in each iteration are listed. As clearly seen by the very low p-values, the expansions carried out in the first three iterations indicate very significant improvements of the model. In iteration four, the improvement is still below 5%, whereas no significant improvement is found in iteration five. During each iteration the current selected model is evaluated, see Section 5.4.

The procedure thus ends with *TiTeThTsAe* as a the selected model, which is called a sufficient model since it is a sufficient model to describe the data while not being too complex i.e. over-fitted. It is illustrated by the RC-network in Figure 9.

Iteration	Models			
Start	*Ti*			
log-likelihood	2482.6			
m	6			
1	*TiTe*	*TiTm*	*TiTs*	*TiTh*
	3628.0	3639.4	3884.4	3911.1
	10	10	10	10
2	*TiThTs*	*TiTmTh*	*TiTeTh*	
	4017.0	5513.1	5517.1	
	14	14	14	
3	*TiTeThRia*	*TiTeThAe*	*TiTmTeTh*	*TiTeThTs*
	5517.3	5520.5	5534.5	5612.4
	15	15	18	18
4	*TiTeThTsRia*	*TiTmTeThTs*	*TiTeThTsAe*	
	5612.5	5612.9	5614.6	
	19	22	19	
5	*TiTmTeThTsAe*	*TiTeThTsAeRia*		
	5614.6	5614.7		
	23	20		

Table 1: Log-likelihood for the fitted models ordered by iterations of the model selection procedure and in each row by log-likelihood. In each iteration the extended model with highest log-likelihood is selected, which is the rightmost models in the table. The number of estimated parameters for each model is indicated by m.

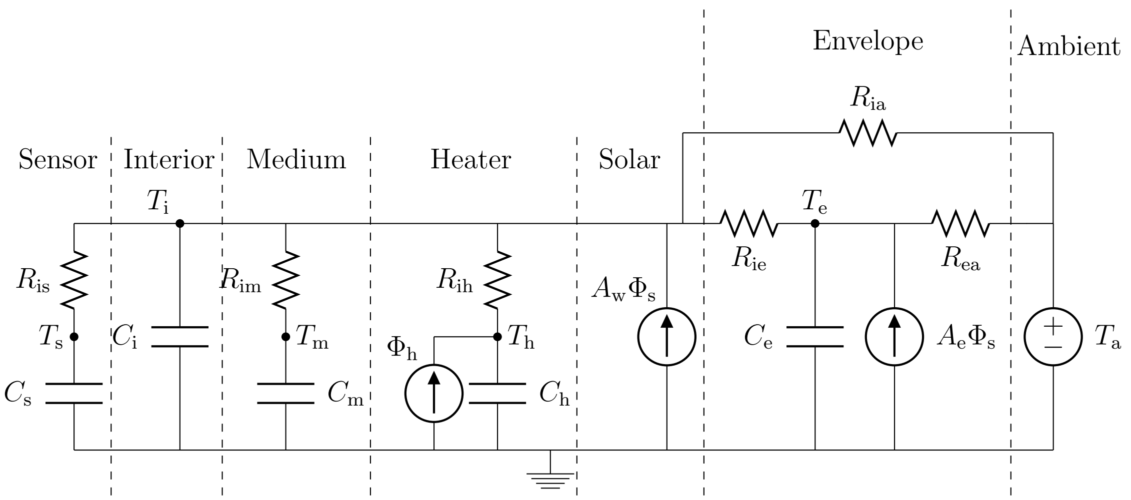


Figure 7: The full model *TiTmTeThTsAeRia* with the individual model parts indicated. This model includes all parts which is included in any of the applied models.

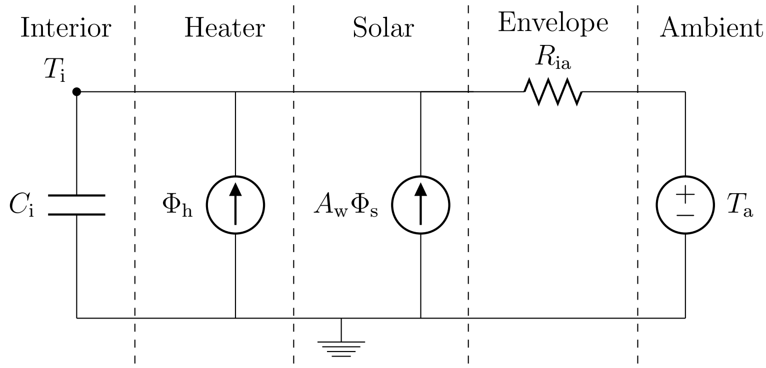


Figure 8: RC-network of the model *Ti*, which is the simplest feasible model.

Iteration	**Sub-model**	**Model**	$m-r$	**p-value**
1	*Ti*	*TiTh*	4	$< 10^{-16}$
2	*TiTh*	*TiTeTh*	4	$< 10^{-16}$
3	*TiTeTh*	*TiTeThTs*	4	$< 10^{-16}$
4	*TiTeThTs*	*TiTeThTsAe*	1	0.011
5	*TiTeThTsAe*	*TiTeThTsAeRia*	1	0.68

Table 2: Tests carried out in the model selection procedure. The difference in number of parameters is indicated by $m-r$.

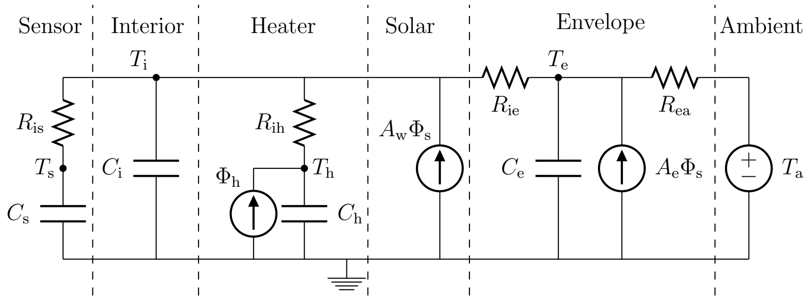


Figure 9: The finally selected model *TiTeThTsAe* with the individual model parts indicated.

5.4 Model evaluation

The selected models are evaluated following the approach described in Bacher and Madsen [2011].

5.4.1 Residuals

Plots of output, inputs, and residuals for each model can be seen in Figure 10. For each model the auto-correlation function (ACF) of the residuals is plotted in Figure 11(a) and the cumulated periodogram (CP) in Figure 11(b). It is seen directly from the plot of the residuals from the simplest model *Ti*, that they are not white noise. The ACF of the residuals also clearly show a high lag dependency, and the CP reveals that the model is not detailed enough to describe the dynamics. Proceeding to *TiTeTh*, it is seen that the level of the residuals is reduced dramatically (see plots of the residuals in Figure 10). The ACF reveals that the characteristics of the residuals are much closer to white noise, which is also seen from the CP, indicating that the model now describe the heat dynamics of the building quite well. The plot of the residuals, ACF, and CP for the selected model *TiTeThTsAe*, show that almost no differences can be observed from the previous model. The highest level of error can be observed where the solar irradiance is high, hence it is found that further improvement of the model should be focused on the part in which the solar radiation enters the building.

5.4.2 Parameter estimates

The parameter estimates are presented in Table 3 together with the time constants calculated for each of the selected models. The total heat capacity and thermal resistance of the building envelope estimated by the selected models are presented in Table 4. As found by evaluating the residuals, see previous section, the models *Ti* and *TiTh* do not describe the dynamics of the system very well, which implies that the estimates of the heat capacities are not reliable. Estimates of the heat capacities found by the tree larger models are more credible, especially it is seen that the time constants are almost equal, indicating that the model comprise the same dynamics. The exact physical interpretation of the smaller heat capacities $C_\mathrm{i}, C_\mathrm{h}$, and C_s cannot be given, but it is noted that their sum, for each of the three larger models, is quite close ranging from 1.03 to 1.08 [kWh/°C].

The total thermal resistance of the building envelope and thereby the UA-values is quite similarly estimated for all models, as seen in Table 4.

5.5 Perspectives of applied dynamical modelling

Identification of a suitable model of the heat dynamics of a building based on frequent readings of heat consumption, indoor temperature, ambient air temperature, and climate variables, will be very useful for different purposes. Important fields of applications are:

Accurate description of energy performance of the building A detailed description of the thermal performance and dynamics of buildings can provide important information for energy- and cost effective improvements of the building. The most

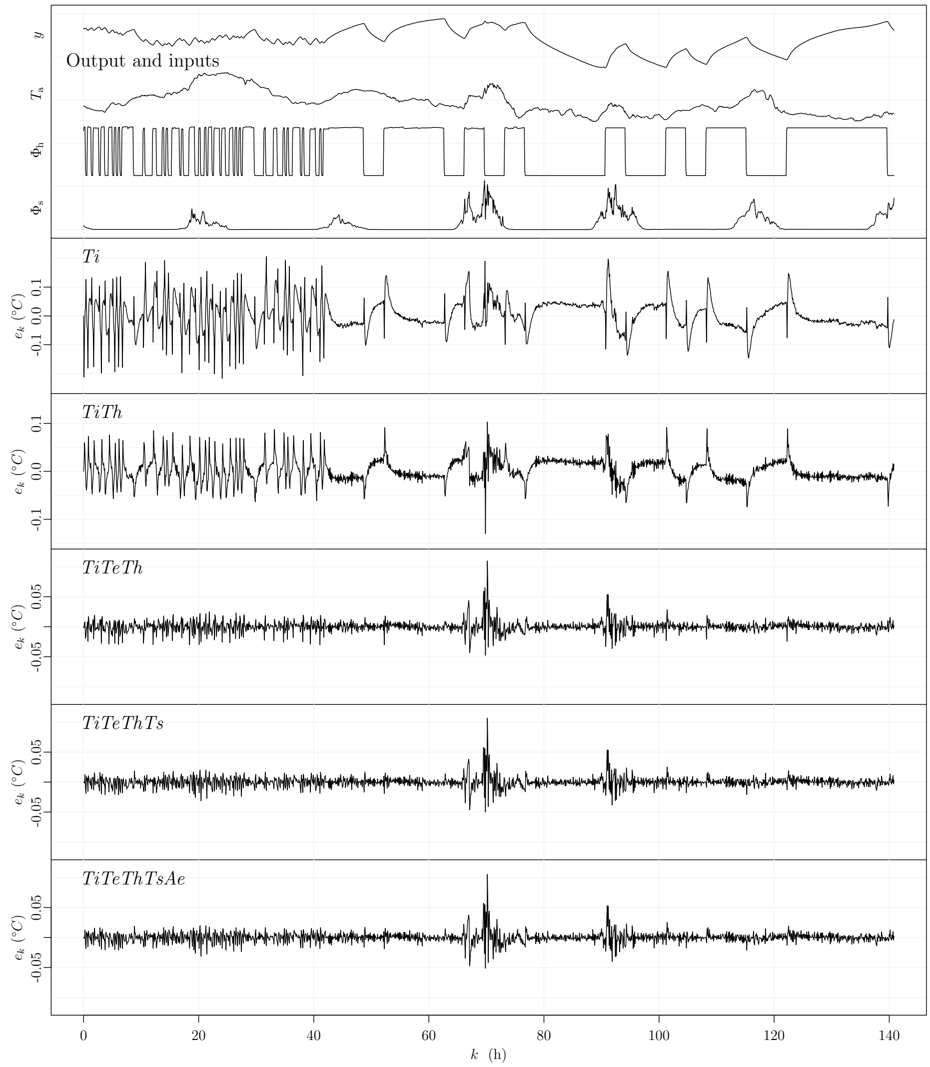
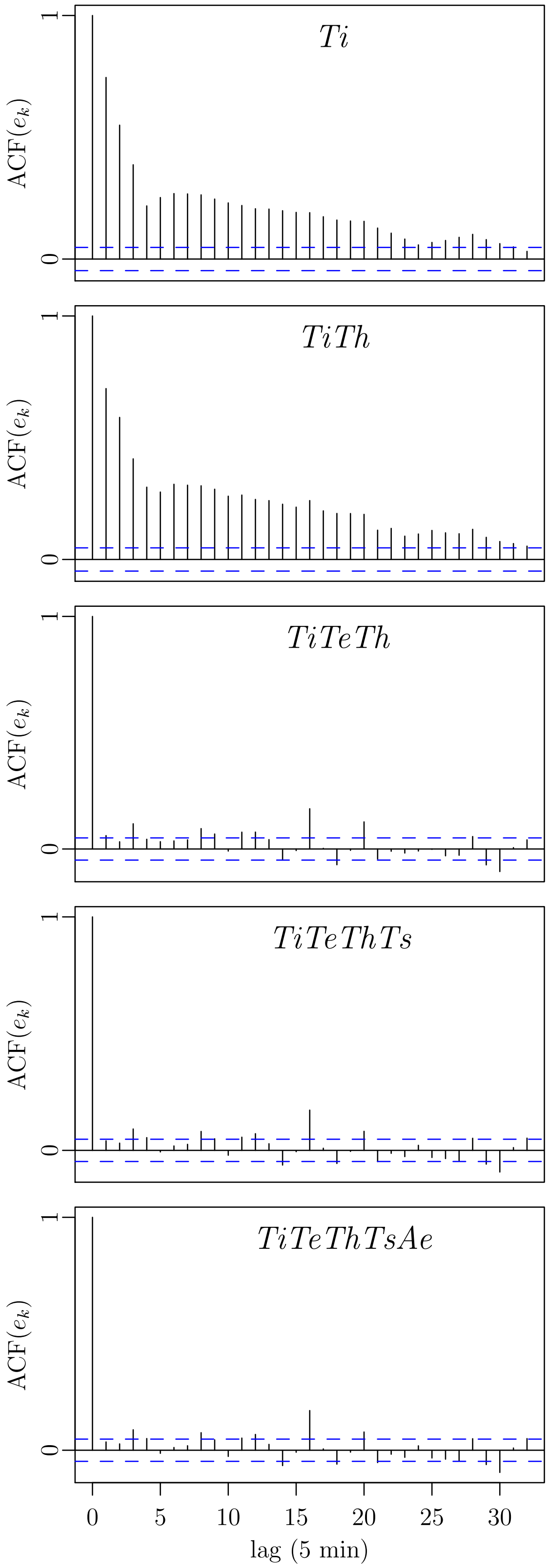
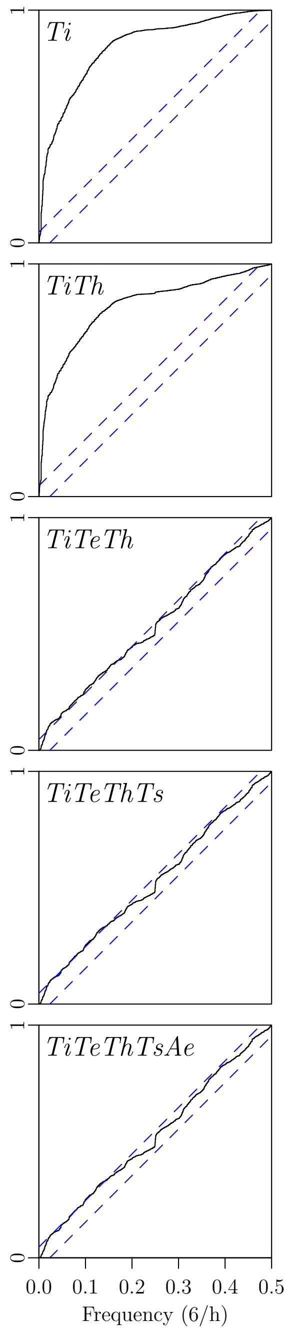


Figure 10: The upper plot is of the output and inputs, and the following plots are of the residuals for each of the models selected in each iteration.



(a) The auto-correlation function of the residuals for each of the models selected in each iteration.



(b) The cumulated periodogram of the residuals for each of the models selected in each iteration.

effective actions to be taken for an individual building can be identified. Furthermore the heat consumption due to physical effects, such as a poor isolated building envelope, can be separated from behavioral effects, e.g. a high indoor temperature.

Forecasting of energy consumption for heating Forecasting of energy consumption for heating can be used for integration of large amounts of renewable energy, such as wind- and solar energy. Implementation of electrical heating with hot water tanks for heat storage in individual houses can be profitable in the near future. Knowledge of the heat dynamics of buildings is essential to forecasting and control of such systems.

Indoor climate control Control of the indoor temperature, ventilation etc. to provide a good indoor climate conditions can be carried out with methods which include models of the heat dynamics. The models can also be extended to include the effect of wind and thereby provide information of the air tightness of buildings.

5.6 Discussion and conclusion

It is clear that the improvement in log-likelihood from model *TiTeThTs* selected in iteration 3 and *TiTeThTsAe* selected in iteration 4 is not very high, and as also discussed the improvement of the model could have focused on the part of the model in which solar radiation enters the building, since the level of the residuals clearly increase

Model	*Ti*	*TiTh*	*TiTeTh*	*TiTeThTs*	*TiTeThTsWithAe*
C_{i}	2.07	1.36	1.07	0.143	0.0928
C_{e}	-	-	2.92	3.24	3.32
C_{h}	-	0.309	0.00139	0.321	0.889
C_{s}	-	-	-	0.619	0.0549
R_{ia}	5.29	5.31	-	-	-
R_{ie}	-	-	0.863	0.909	0.897
R_{ea}	-	-	4.54	4.47	4.38
R_{ih}	-	0.639	93.4	0.383	0.146
R_{is}	-	-	-	0.115	1.89
A_{w}	7.89	6.22	5.64	6.03	5.75
A_{e}	-	-	-	-	3.87
τ_1	10.9	0.16	0.129	0.0102	0.0102
τ_2	-	8.9	0.668	0.105	0.105
τ_3	-	-	18.4	0.786	0.788
τ_4	-	-	-	19.6	19.3

Table 3: The estimated parameters. The heat capacities, C_{x}, are in [kWh/°C]. The thermal resistances, R_{xx}, are in [°C/kW]. The areas, A_{x}, are in [m^2]. The time constants, τ_x, are in hours. Note that the physical interpretation most of the parameters is different for each model.

Model	*Ti*	*TiTh*	*TiTeTh*	*TiTeThTs*	*TiTeThTsAe*
C (total)	2.07	1.67	3.99	4.32	4.36
R (envelope)	5.29	5.31	5.40	5.38	5.28
α_{UA}	1.55	1.55	1.52	1.53	1.55

Table 4: The total heat capacity [kWh/°C] and thermal resistance [°C/kW] of the building envelope found by the selected models. The UA-values α_{UA} normalized with floor area [W/°C/m2].

in the periods with higher solar radiation.

In the presented study a grey-box model is identified for the heat dynamics of a building. A forward model selection procedure based on log-likelihood ratio tests is applied and the model is thoroughly evaluated from both a statistical and physical point of view.

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