

B.Sc. Thesis  
Bachelor of Science in Mathematics and Technology

**DTU Compute**  
Department of Applied Mathematics and Computer Science

# Statistical models for analysis of frequent readings of electricity, water and heat consumption from smart meters

In cooperation with SEAS-NVE

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# Summary

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# Resumé

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# Preface

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This thesis was written at the department of Applied Mathematics and Computer Science at the Technical University of Denmark in fulfillment of the requirements for acquiring a BSc degree in Mathematics and Technology. The project is developed in cooperation with the company SEAS-NVE.

We would like to thank our advisors Lasse Engbo Christiansen and Peder Bacher. Lasse, for always answering emails and having 36 hours a day and Peder, for remembering the sun and to be eternally optimistic. Also a thank you to Anders Buur Hansen for the good cooperation and the opportunity to present our results for SEAS-NVE and Aalborg Forsyning.

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# Contents

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<b>Summary</b>	i
<b>Resumé</b>	iii
<b>Preface</b>	v
<b>Contents</b>	vii
<b>1 Introduction</b>	1
1.1 Introduction to WATTS app . . . . .	1
1.2 Motivation . . . . .	2
<b>2 Data</b>	3
2.1 Original data . . . . .	3
2.2 Cleaning and preparation . . . . .	5
2.2.1 Missing values . . . . .	6
2.2.2 The sun and the wind . . . . .	6
2.2.3 Data checking . . . . .	6
<b>3 Exploratory Analysis</b>	9
3.1 Examination of the Heat Consumption . . . . .	9
3.1.1 Weather data . . . . .	11
3.1.2 BBR data . . . . .	12
3.2 Multicollinearity . . . . .	14
3.3 Data segmentation . . . . .	16
3.3.1 Segmentation by piece-wise optimization . . . . .	17
3.3.2 Segmentation by significant deviations . . . . .	18
<b>4 Models on the Daily Consumption</b>	23
4.1 Linear regression . . . . .	23
4.1.1 Model assumptions . . . . .	24
4.2 Simple linear regression model . . . . .	25
4.2.1 Validation . . . . .	25
4.3 Multiple linear regression model . . . . .	26
4.3.1 Splines . . . . .	28

4.3.2	Multiple linear regression models . . . . .	29
4.3.3	Results . . . . .	30
4.4	Regression model for comparing houses . . . . .	31
4.4.1	Validation . . . . .	31
4.4.2	Results . . . . .	32
4.4.3	Predictions . . . . .	33
4.5	Comparison . . . . .	36
4.6	Visualization of the results . . . . .	37
<b>5</b>	<b>Models on the Hourly Consumption</b>	<b>45</b>
5.1	Description of the Hourly Consumption . . . . .	47
5.2	The ARMA Models and Their Extensions . . . . .	49
5.3	Applying the models . . . . .	51
5.3.1	Modelling with the ARIMA Function . . . . .	52
5.3.2	Predicting with the ARIMA model . . . . .	60
5.3.3	Visualization of the results . . . . .	65
5.3.4	Modelling with the MARIMA Function . . . . .	65
<b>6</b>	<b>Discussion</b>	<b>73</b>
6.1	Future work . . . . .	74
<b>7</b>	<b>Conclusion</b>	<b>77</b>
<b>A</b>	<b>Tables</b>	<b>80</b>
A.1	Estimates and test of the simple linear regression model . . . . .	80
A.2	Estimates and test of the multiple linear regression model . . . . .	82
A.2.1	Significance of parameters for full model . . . . .	82
A.3	Tests of residuals from multiple linear regression model . . . . .	84
A.3.1	Significance of parameters for general regression model . . . . .	86
A.4	Parameters from hourly consumption models . . . . .	87
<b>B</b>	<b>Figures</b>	<b>90</b>
B.1	Scatterplots of data . . . . .	91
B.2	ACF and PACF of time series models . . . . .	93
<b>Bibliography</b>		<b>95</b>
Books and papers . . . . .		95
Websites . . . . .		95
R-packages . . . . .		96

# CHAPTER 1

# Introduction

According to the International Energy Agency [7], heat is the largest energy end-use. Providing heating for homes and industrial purposes accounts for around 50% of the total energy consumption. Renewable heat consumption in the form of bioenergy contribution is expected to grow which will be a better solution for the climate. In relation to the individual consumer, it makes sense to become aware of one's heat consumption, e.g many consumers pay more for their heat consumption than they could. This can be solved by making small adjustments such as replacing radiators with more efficient cooling, replacement of leaky windows, improve insulation of the house etc. Which factors that can influence the heat consumption, are not known to most consumers and thus it can be a challenge to know how to minimize the consumption.

Heat consumption can be described using mathematical models, namely statistical models, and this can lead to an optimization/minimization of the consumption. By examining the influence of different physical phenomena, including the temperature, the wind, the sun, etc. on heat consumption, results can be interpreted and visualized for customers.

## 1.1 Introduction to WATTS app

The app WATTS is designed and created by the danish energy and optical fibre broadband concern, SEAS-NVE. The app provides several features including an overview of the energy consumption to the consumer by showing the actual consumption and predicting the expected consumption. In addition, the app keeps track of the consumers budget and give the user the opportunity to compare their consumption with similar customers. The energy consumption in relation to the expected consumption is visualised with the colours green, yellow and red. The colours are used to indicate whether the consumption is expected to be lower than expected, to exceed the budget by 0-30%, or to exceed the budget by more than 30%. The app



is under expansion such that users are offered the same applications for their heat consumption. In continuation of this, it is possible to add a feature showing the wind dependency on the heat consumption.

## 1.2 Motivation

The aim of this report is to investigate the tap water consumption and thereby provide possible extensions to the app created by SEAS-NVE, WATTS. By illustrating these features in the app, customers can become aware of their heat consumption and at the same time get a sense of what physical phenomena affect their house. Last but not least, a big part of the motivation is driven by the collaboration with a company. The project thus also has a much more meaningful purpose, namely to develop usable models and make any suggestions on how results can be visualized in an app.

Our approach is to develop statistical models in order to analyse which factors influence the heat consumption. First we will explore the data as daily values and use linear regression models for fitting and predicting with the main focus on the estimates for various significant parameters. Subsequently, we will do a more indepth analysis of data by examining hourly values. The aim is to determine whether the estimates of the different parameters can be improved.

# CHAPTER 2

# Data

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The data is provided by SEAS-NVE with measurements from Aalborg Forsyning. The measurements are made with a smart meter from Kamstrup ([13]). The smart meter measures the flow temperature of the water and the return temperature and the total flow in  $m^3/hour$ . The process for the supply of heat from Aalborg Forsyning to houses is described in more detail in [11]. Data are given in three data sets. The house data consists of 71 .csv-files containing 8 attributes for each house which is 513877 data points in all. The second data set includes weather data containing 10,140 observations and predictions of the next 2283 data points, all with 11 attributes. Furthermore, the third data set is from Bygnings- og Boligregistret (BBR) and contain details for each of the houses e.g. total area, year of construction and type of house. The main focus of this section will be how this data is cleaned and prepared for the further analysis.

## 2.1 Original data

The original house and weather data include hourly observations from the period 31-12-2017 23:00 to 7-02-2019 10:00. The time period varies in the house data, i.e there exists data for houses containing observations for approximately six months and there are also houses that contain observations for about 13 months. This will be taken into account when cleaning the data and in the analysis the two groups of houses will be referred to as the long and short houses. Before the data was provided to this project, it had already been slightly altered. When Aalborg Forsyning, who provided the data, received the observations from the hardware installed in the houses of their clients, it is generally not divided into precise one hour intervals. Aalborg Forsyning interprets the time stamps on the observations and changes them to one hour intervals. If observations are missing, an interpolation method is used to simulate the data. The exact procedure is described in [6].

Table 2.1 below shows the attributes from the house data set which is used to define the Heat Consumption as

$$Q = c \cdot m \cdot \Delta T, \quad (2.1)$$

where  $c$  is the specific heat capacity for water which is  $4.186 \text{ kJ/kg} \cdot ^\circ C$ ,  $m$  is the water density in  $kg$  and is calculated from the volume as  $1m^3 = 1000 \text{ kg}$  and  $\Delta T$  is

the cooling degree in  $^{\circ}C$ . In this way, the heat consumption is converted from  $kJ$  to  $kWh$  (since  $1 kWh = 3600 kJ$ ) which is the desired unit for the heat consumption.

Variable	Description
StartTime	Start time and date for measurements.
EndTime	End time and date for measurements.
Energy	Consumption in $kWh$ .
Flow	Amount of water passed through meter in $m^3/hour$ .
Volume	in $m^3$ .
TemperatureIn	Temp. of the water flowing into a house in Degrees/C.
TemperatureOut	Temp. of the water flowing out of a house in Degrees/C.
CoolingDegree	Difference between Temp.In and Temp.Out in Degrees/C.

Table 2.1: Attributes from the original house data.

Table 2.2 shows the attributes in the weather data and Table 2.3 shows which attributes the BBR data consists of.

Variable	Description
StartTime	Start time and date for measurements. Hourly values.
Temperature	Temperature outside in Degrees/C.
WindSpeed	Wind speed in $m/s$
WindDirection	Wind direction in degrees from 0 to 360, 0 being North
SunHour	The level of sunshine in the hour in a scale from 0 to 1
Condition	The weather condition given in numbers described in [10]
UltravioletIndex	The UV index level
MeanSeaLevelPressure	The average atmospheric pressure at mean sea level in $hPa$
DewPoint	The temperature to which air must be cooled to become saturated with water steam (Degrees/C)
Humidity	The amount of water steam present in the air
PrecipitationProbability	The probability that some quantify precipitation will occur (logical factor)
IsHistoricalEstimated	Binary variable, true if the datapoint is a prediction

Table 2.2: Attributes from the original weather data.

Variable	Description
Key	The house ID key
HouseType	Type of house: Apartment, house, industrial etc.
TotalArea	The total area of the house in $m^2$
Floors	The number of floors in the house
Basement	How many $m^2$ basement there is in the house
Attic	How many $m^2$ attic there is in the house
ConstructionYear	The year of construction for the house
Surfaces	The material on the surface of the outdoor walls of the house
ReconstructionYear	The year of the latest reconstruction of the house
AdditionalHeating	If there are any additional heating installed in the house. Fireplace etc.

Table 2.3: Attributes from the BBR data.

## 2.2 Cleaning and preparation

In this section, it is described how the raw data is cleaned and prepared for the statistical analysis.

Due to the fact, that `StartTime` and `EndTime` is always one hour apart, it is redundant to use both of the attributes. The observations of most of the attributes are made at time `EndTime`, and for that reason it is used as `ObsTime` for the observations. For the weather data set, the observations is made at time `StartTime`, and there is no `EndTime` for this data set. When merging these data sets, `ObsTime` is aligned with `StartTime`. The format of these attributes is changed to a `Posixct` value with d-m-Y H:min:sec as the structure.

Every now and then, one or more data points in a row are missing. When this happen, a data point with NA-values for all of the attributes except `ObsTime`, is placed in the data set, which makes the data set easier to use in the modelling process. In some cases, the models that the data set is "complete", i.e. that there are no missing values. This is true for the time series models that will be described in Chapter 5. In the data sets there are no indication of whether or not it is weekend. This attribute is added as well as the school holidays.

Both weather data and the house data are aggregated with mean values for each day in order to convert hourly values into daily values since there are of interest when modelling in chapter 3, two of the attributes is aggregated in a different way, which is explained later.

### 2.2.1 Missing values

In the vast majority of our houses, the data is consistent with data every hour from a given starttime until the end of January 2019. Then for many of the houses, there is a gap of 6 whole days in the start of February, and then 2 more days of data. For this reason, the February data is cut out completely. The rest of the missing readings are handled so that they are based on the values of readings around the gap, i.e. they are interpolated by calculating the average of the readings around the missing reading. The purpose of filling out the missing values with interpolation of data, is to ensure that there exist observations for every hour. This makes it possible to develop time series models to describe data and also predict customers' consumption on an hourly basis.

### 2.2.2 The sun and the wind

A physical factor that could possibly affect the heat consumption is the sun. In raw data, the attributes `Condition`, `SunHour`, and `UltraVioletIndex` can be seen as explanatory variables for the sun. Instead, an attribute, `Radiation`, is added to calculate the solar radiation for a given day. This attribute is determined with use of the R function `calcSol` from the library `solaR`. This value is the radiation of the sun if there is no cloud coverage. That is multiplied by the `SunHour` to weight the radiation with the amount of sun actually penetrating the clouds. The ultraviolet index is a measurement of the strength of ultraviolet radiation and since the attribute `SolarRadiation` is more exact, `UltravioletIndex` is removed from the weather data set. `SunHour` is also removed, since it is now included in `SolarRadiation`.

Another physical factor that might be of importance is the wind. There are data available for both the wind direction in degrees and the wind speed. When the data is aggregated into daily values, it is important to pay special attention to the wind attributes, since it is not logical to take the average of degree values. For example, the average wind direction of 359 degrees and 0 degrees is not 179.5 degrees. Instead the wind direction and wind speed are interpreted as polar coordinates in a coordinate system. They are converted to rectangular coordinates. Then they are aggregated from hourly values into daily values, and returned to polar coordinates. When the wind is aggregated this way, wind directions with high wind speeds are weighted higher than wind directions with low wind speeds. Also the problem with the periodicity of the wind direction is solved.

### 2.2.3 Data checking

As mentioned, there are some measurements missing in the house data and it can therefore be difficult to do modelling for the houses in question. To avoid these difficulties, a so called "Data Checking" function has been made in order to check whether several constraints for the data are fulfilled. There must be a certain number

of observations and the amount of missing data should not exceed a certain fraction of the data observation period.



# CHAPTER 3

# Exploratory Analysis

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First part of the analysis is to explore the different attributes in the data, in order to detect possible patterns or correlations. The exploratory analysis is also used to get an understanding of data and its behaviour. Hence, this chapter is about visualizing the different attributes focusing on their influence on the heat consumption. As the heat in each house is turned off in the summer period, data is segmented such that the summer period is excluded from the data used for modelling.

## 3.1 Examination of the Heat Consumption

To get an overview of the heat consumption for each house, the daily average heat consumption for each house is investigated. Figure 3.1 shows the daily average consumption for all the houses and the daily consumption of two houses - one that follows the trend and one that deviates. These two houses are chosen to be visualised throughout the report and their specifications are presented in Section 3.1.2. It can be seen that the slopes around the summer months are close to 0. As mentioned, the data in focus in this project is where the heat is turned on, hence the period where the heat consumption is close to 0 needs to be removed. Exactly how this is done will be explained and discussed in the data segmentation section. All three plots show some unusual high data points around April 2018. This can be due to the fact that it was snowing in Denmark at that time which is supported by the article found in [5].

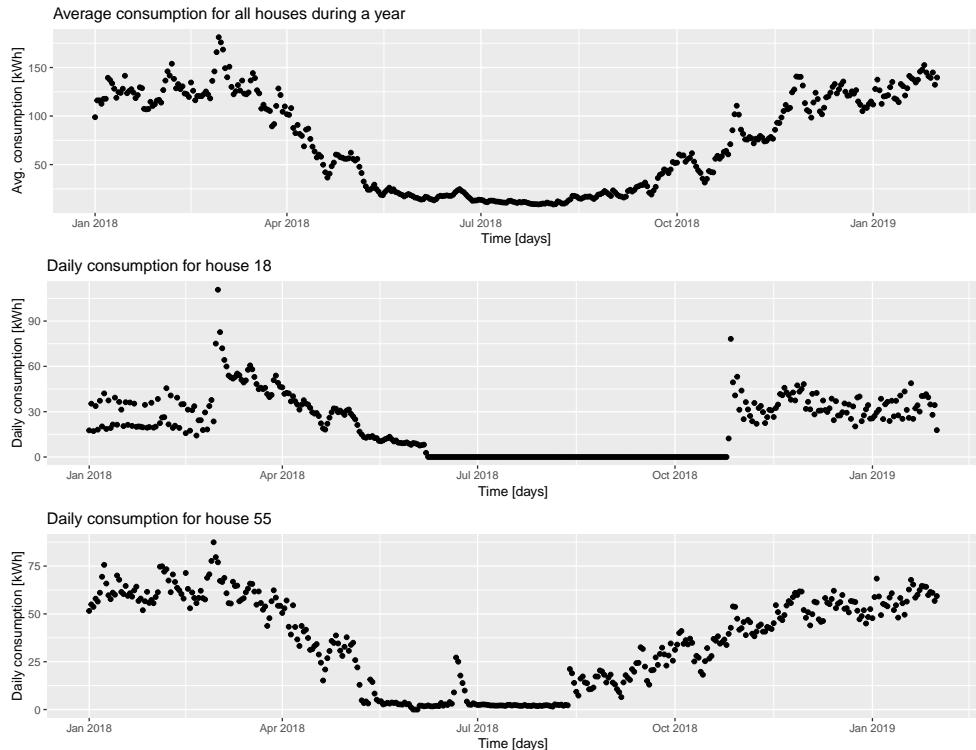


Figure 3.1: Daily consumption during a year (2018). The top plot shows the average consumption for all the houses. The plot in the middle shows an example of a house that deviates from the trend and the last plot shows a house that follows the trend

The remaining attributes from the house data is examined through a scatterplot shown in Figure 3.2 and Figure B.1, in order to find possible linear relationships with the consumption. There are clear linear relationships between the consumption and the flow, the volume, the cooling degree and the temperature going in respectively. It is expected that the consumption depends linearly on the volume and cooling degree, cf. the main equation given in (2.1). The relationships between consumption and the flow and volume are quite similar which is in line with the description of the two attributes given in Table 2.1. It is also seen that the temperature of the water coming out of the system depends on how much is used. So if the return water is quite hot, the house has not fully utilized the heat for the heat consumption.

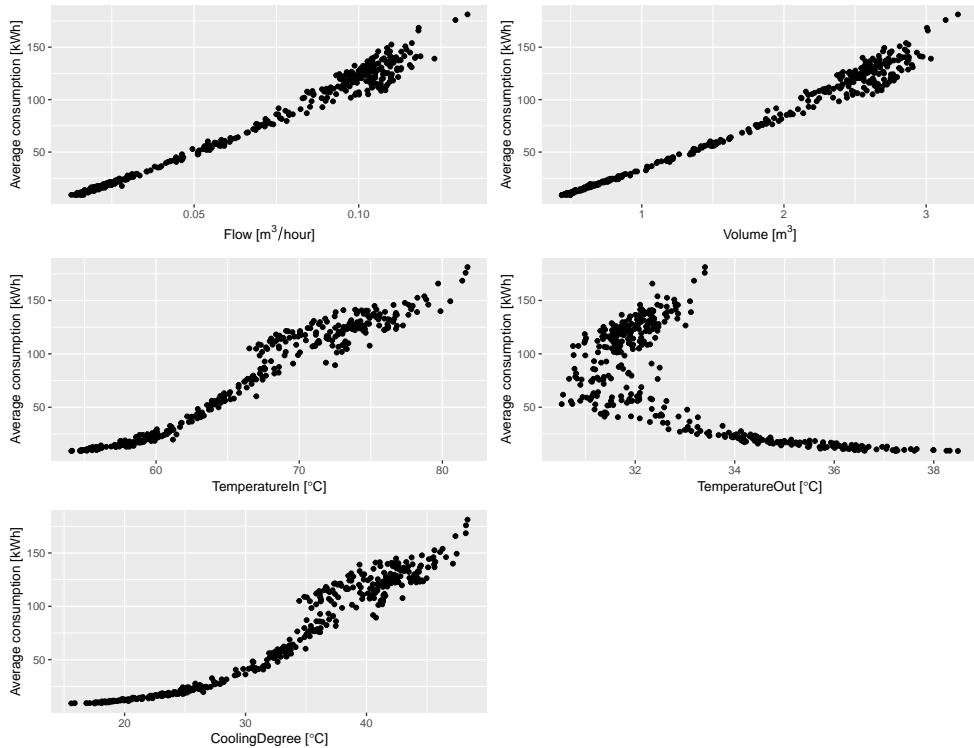


Figure 3.2: Scatterplots of the average consumption for all houses and relevant house attributes. Each point represents a day. There are clear linearly dependencies between Consumption and e.g. Flow

### 3.1.1 Weather data

The weather data is also examined through scatterplots given in Figure 3.3 and Figure B.2 in order to detect dependencies between the average consumption of the houses and the weather attributes. It is already known that the outside temperature has a significant influence on the consumption which is in line with the linearly relationship between the temperature and the consumption in Figure 3.3. Furthermore, consumption is approximately influenced by the dew point which is probably explained by the linear relationship between Temperature and DewPoint illustrated in Figure B.2. The scatterplots show that the consumption overall is independent of the attributes WindSpeed, WindDirection and MeanSeaLevelPressure.

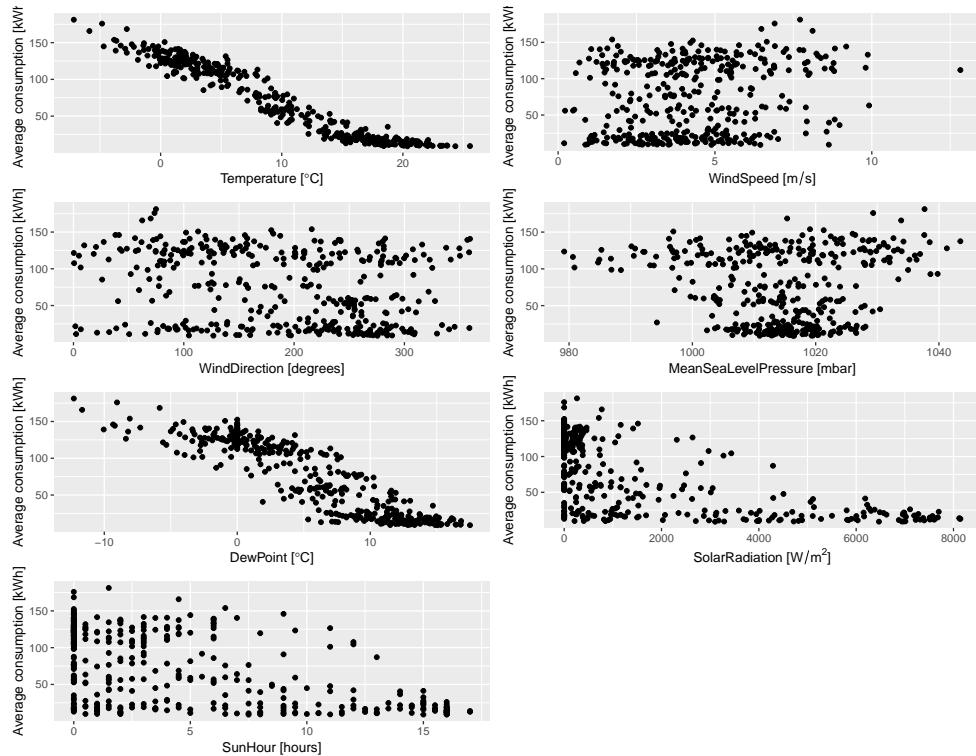


Figure 3.3: Scatterplots of the average consumption for all houses and relevant weather attributes. Each point represents a day. There are clear linearly dependencies between Consumption and Temperature, as was expected

### 3.1.2 BBR data

Presumably, the BBR data has influence on the heat consumption in particular the total area and year of construction. Hence, Figure 3.4 illustrates the house specifications of all houses focusing on the type of house, the total area, the year of construction and reconstruction.

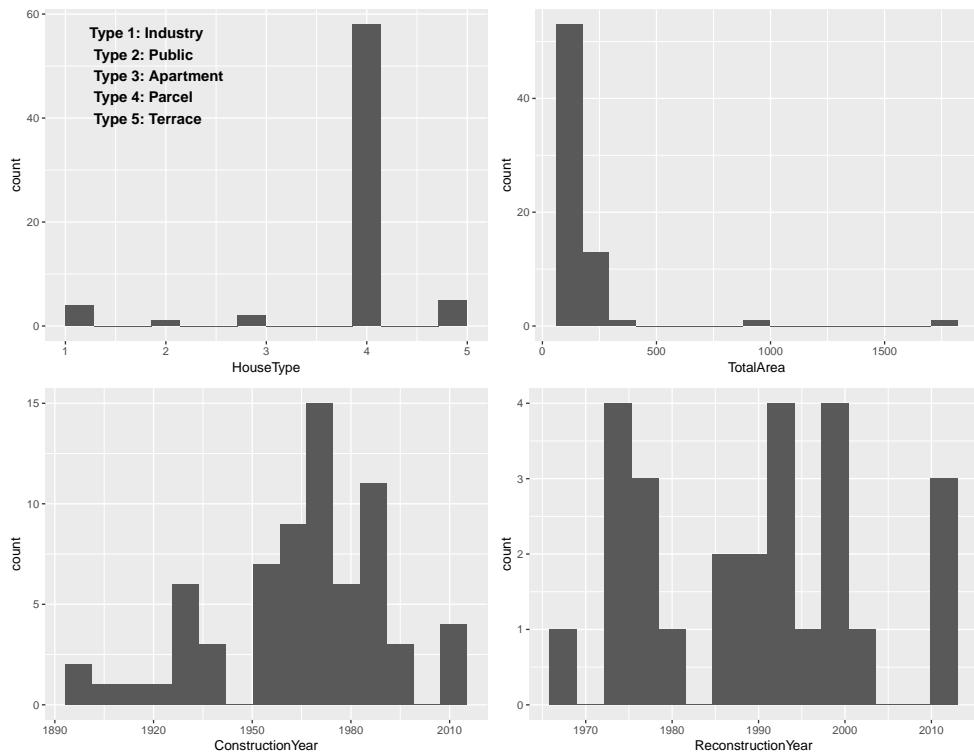


Figure 3.4: Details of the houses from BBR data focusing on the type of house, the total area in  $m^2$ , the year of construction and reconstruction respectively

Since the two houses, 18 and 55, showed in Figure 3.1 will be used throughout the report, it is important to know their specifications:

#### House 18:

- House type: Parcel
- Area:  $128\ m^2$  + attic of  $34\ m^2$
- Year of construction: 1927
- Reconstructed in 1998

#### House 55:

- House type: Parcel
- Area:  $160\ m^2$
- Year of construction: 1971
- Has wood-burning stove

The areas of the houses are somewhat similar, they are of the same house type and their ages are close to each other. However, house 18 has an odd behaviour which will be evident in the later results.

The average of the heat consumption for each house is found/determined for the winter period. By dividing the average consumption with the total area of the house the consumption pr.  $m^2$  is calculated. Figure 3.5 shows the year of construction and the consumption for each of the houses. The year of construction is here determined by either the year of construction or the year of the latest reconstruction of a house.

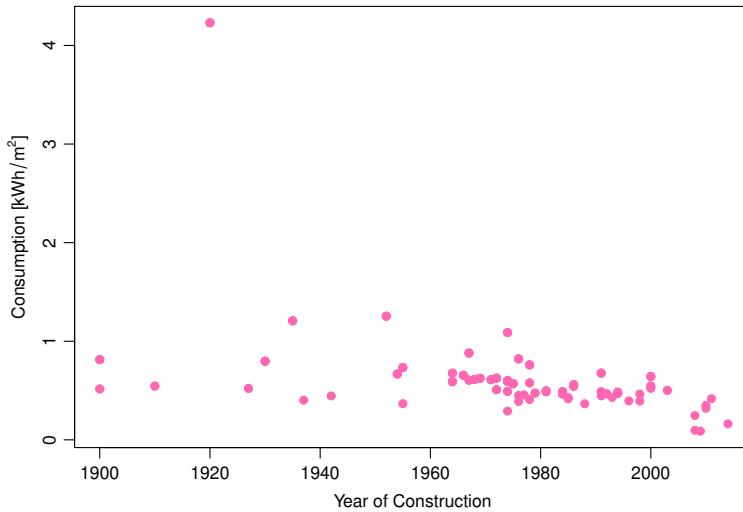


Figure 3.5: Plot showing the year of construction and the average consumption pr.  $m^2$  for each house. It is seen that there is a tendency that the later a house is built or reconstructed, the better is the insulation of the house

Figure 3.5 shows the relation between the year of construction of a house and its average consumption pr.  $m^2$ . A single house stands out with a consumption that is remarkably higher than the rest. The house is a simple apartment of  $61m^2$  build in 1920. Other than that, there seems to be a vague tendency that older buildings have a higher consumption. The buildings from after 1990 all have relatively low consumption, while the older buildings have a higher spread. For houses that have been reconstructed, the reconstruction year has been used instead of the construction year. So some houses in the plot might in fact be older than it is shown here. The nature of the reconstructions is not known, but since they are extensive enough to be registered in BBR, it is assumed to have an effect on the consumption of that house.

## 3.2 Multicollinearity

Multicollinearity occurs when two or more explanatory variables are highly correlated. In linear regression, multicollinearity ... Multicollinearity can be investigated by cal-

culating the correlation using the function `cor()` in R.

Figure B.2 clearly shows that there is a high correlation between `Temperature` and `Dewpoint`. The exact correlation between the two attributes is calculated at 0.936, hence it is decided to remove `Dewpoint`. Furthermore, it is assumed that `Radiation` is a replacement for the attributes describing the sun, namely `Condition` and `SunHour`. This is the basis for expecting a correlation between the radiation and the sun attributes. Figure 3.6 shows a plot of the correlation matrix between the abovementioned attributes.

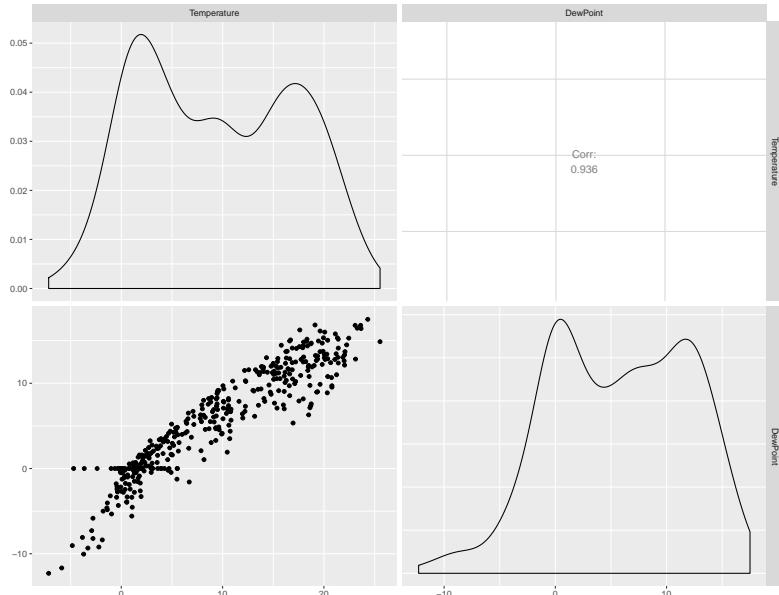


Figure 3.6: Scatterplot showing the correlations between the thwo attributes `Temperature`, `DewPoint`. It is clearly seen that the temperature and dew point are highly correlated

There is a high correlation between `Radiation` and `SunHour` at 0.955, thus `SunHour` is removed from the weather data set.

The complete data set used for modelling in chapter 4 can be seen in table 3.1.

Variable	Description
Date	End time and date for measurements. Hourly values
Temperature	Temperature outside in Degrees/C
WindSpeed	In $m/s$
WindDirection	In degrees
Condition	The condition of the weather given in integers described in [10]
MeanSeaLevelPressure	Avg. atmospheric pressure at mean sea level in mbar.
PrecipitationProbability	Measure of the probability that precipitation will occur.
Observation	The number of observations for each day for each house.
Consumption	CoolingDegree times Volume from House data
Holiday	A categorical attribute with 6 levels: Working day, Weekend, Autumn break, Christmas break, Winter break and Spring break.

Table 3.1: Attributes used for modelling

### 3.3 Data segmentation

All the models used in this project assume that there is a linear relationship between the outside temperature and the heat consumption of a house. It would make sense that the heat consumption of a house increases as the temperature decreases. But this is mostly true when it is actually cold. In the summer time the heat consumption is generally not dependent on the outside temperature. Figure 3.7 shows two examples of the relationship between temperature and consumption. It can be seen that some houses have a very linear relationship between temperature and consumption, and others less so. Most have in common that for high temperatures, the consumption reaches a more or less constant low level. This relationship most likely shows that when the heating is turned on, the consumption rises and falls with the outside temperature. When the heating is not turned on, the only consumption is the tap water consumption. One of the main challenges of this project is that the data does not provide a simple way of distinguishing between tap water consumption and heating consumption. If the inhabitants are not home for a longer period, there will probably be low consumption, even though it might be cold outside. This does not necessarily mean that the house is well isolated. And if there is consumption in warm periods, it is likely to be tap water consumption, and not heating.

To minimize the effect of tap water consumption this section will explore different ways of extracting the period where the consumption is dominated by heating, and not by tap water. This period will be denoted the winter period for obvious reasons. By only applying the regression models described in Chapter 4 to the period dominated by heating, deviations caused by tap water consumption will hopefully be less significant. If a linear model was used on the entire data, the variation of the relation between temperature and consumption would be far from constant during the year, violating

a key assumption of the model. The approach used in this section is to search for a temperature that can be used as a threshold. All days with average temperatures below the threshold will be classified as belonging to the winter period. Two ways of finding this threshold will be described below. Other more sophisticated ways to make the classification of the winter period might be applied, but they will not be considered in this project.

### 3.3.1 Segmentation by piece-wise optimization

The first approach is to make a linear regression on the data with two segments. A breakpoint  $\alpha$  is found, such that the SSE is as small as possible. The second segment is restricted to being constant. This way the breakpoint can be used as the threshold separating the winter period from the rest of the data. This method was tested on every available house, where a new breakpoint was found for each house.

Figure 3.7 shows the regression for two different houses. On house 55 the line fits rather well with the low-temperature data points, but house 18 behaves differently. Neither breakpoints act as a very good threshold. Both houses show very clearly that the assumption that all points below the breakpoint belong to the period without heating is not accurate. Even though this approach can easily take out a lot of data where there is clearly no heating, it will in many cases set the breakpoint too high. The "tail" of the period without heating might still be included, causing a bias in the model, and some variation that is not accounted for. The method is also not very robust. Depending on how the points are spread out, the breakpoint is sometimes as high as 20 degrees, which is not desirable.

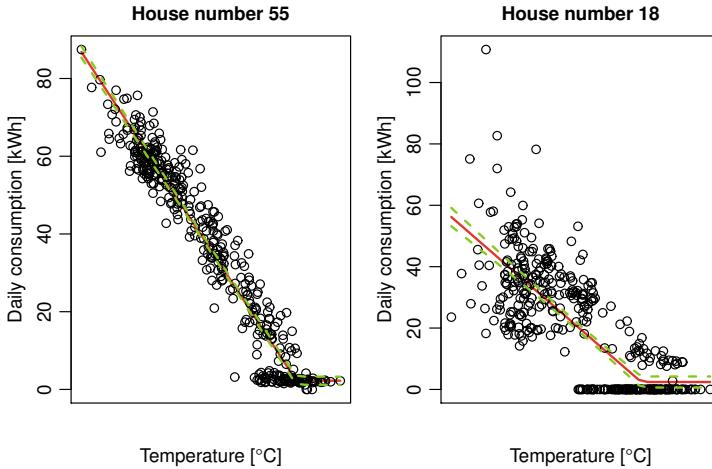


Figure 3.7: Piece-wise optimization of the consumption. The red line is the regression line and the green line is the confidence interval

### 3.3.2 Segmentation by significant deviations

In the second approach, all data points with a temperature above 20 degrees are assumed to belong to the period without heating. Then the mean and standard deviation of a normal distribution containing those points is calculated. For each degree interval below 20 degrees, if 80% of the points in that interval are above the confidence interval of the normal distribution, then that degree interval should be included in the winter period. The threshold is set accordingly. Figure 3.8 shows an example of the procedure on house 55. The left plot shows the confidence interval as a black line. The proportion of the data points above that line is illustrated on the plot on the right. Here the interval between 14 and 15 degrees is the highest where more than 80% is outside the confidence interval. Thus, 15 degrees is chosen as the threshold. The vertical orange line on the left plot shows this threshold at 15 degrees. This threshold captures almost every point in the beforementioned "tail" with low consumption. A few outliers with very low consumption are still included in the winter period. But overall the classification of the winter period seems to work well for this house. Figure 3.9 shows the same procedure for house 18. Here the linear dependency between temperature and consumption is not fulfilled very well for any part of the data, but the threshold cuts out the data where there is clearly no consumption at all.

In general, this model is more robust than the first. It is more selective, and provides

a good way to set the threshold on the correct side of the mentioned "tail" that may occur at temperatures both with and without heating. When comparing Figure 3.8 and Figure 3.9 to Figure 3.7, one can see that this method sets the breakpoint a bit lower, removing more points without heating. If the consumption data behaves badly, and chunks of datapoints are low enough to be within the confidence interval, then a lot of data can potentially be removed, and there might be too little data left.

Until now the focus has been to find a threshold for every individual house. But the goal is to have a general classification of the winter period. Figure 3.10 shows a histogram of the breakpoint values for every house in the data set. The global breakpoint should be in the low end of the scale. It is better to remove data points that could have been used, than to include too many points that belong to a different distribution with a different variation, which could make the assumptions of the model worse. It would not be good to choose the minimum breakpoint, since that would be very vulnerable. A single house with a very low breakpoint might make the model bad for all the other houses. So the breakpoint that is chosen is the first quantile. As it is shown on the figure, this is 12 degrees. All models in the following sections will only be considering data where the temperature less than or equal to 12 degrees. At some point it will also be of interest to look at the summer period. This will be defined as the data with temperatures above 15, as that is the third quantile in Figure 3.10. This way there is a temperature gap between the summer and winter period, that is not included in any of them. Out of the total 396 days covered by the house data, 239 are classified as winter days, 126 as summer days, and 31 days are neither summer nor winter days. Figure 3.11 shows how the days are distributed. It can be seen that the winter days are a continuous period from January to August 2018 and again from November 2019

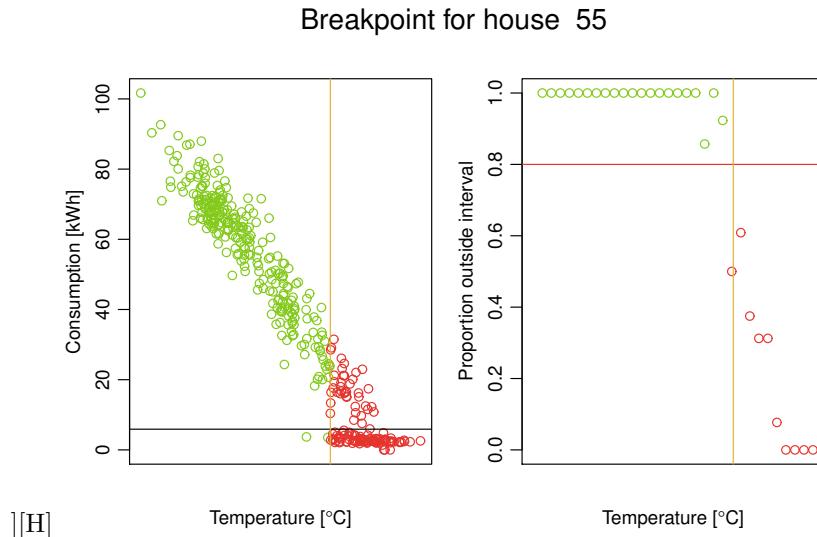


Figure 3.8: An illustration of how the breakpoint is found using segmentation by significant deviations. On the left figure the black line illustrates the confidence interval of the normal distribution containing points above 20 degrees. The right figure shows for each degree interval, the proportion of data points that are outside the confidence interval. The last point below 80% is the chosen breakpoint

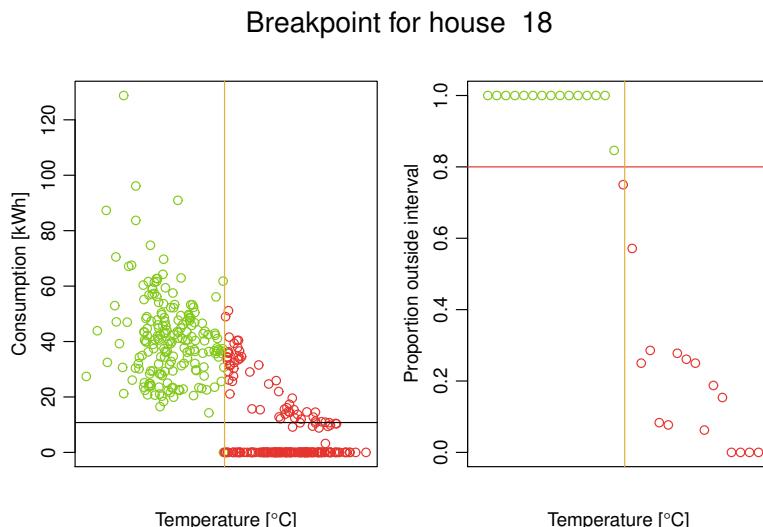


Figure 3.9: This figure shows the same as Figure 3.8, but for house 18 instead. The assumption of a linear relationship between temperature and consumption is worse for this house

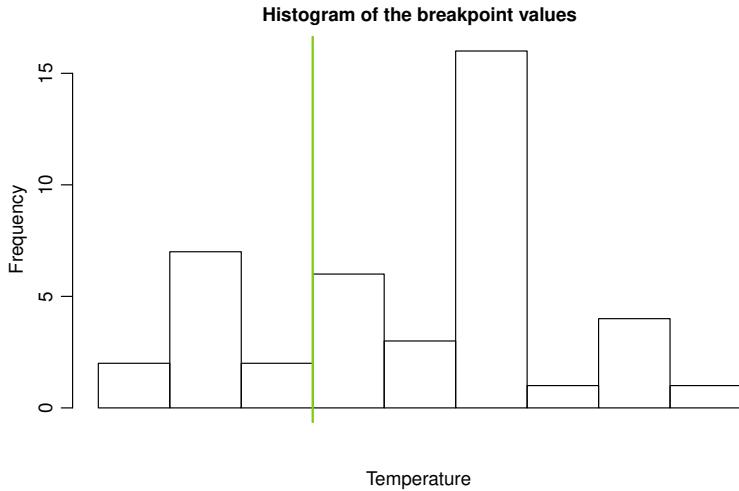


Figure 3.10: A histogram of the alpha values for every house in the third segmentation method. The first quantile is chosen as the overall breakpoint. It is 12 degrees, illustrated by the green line

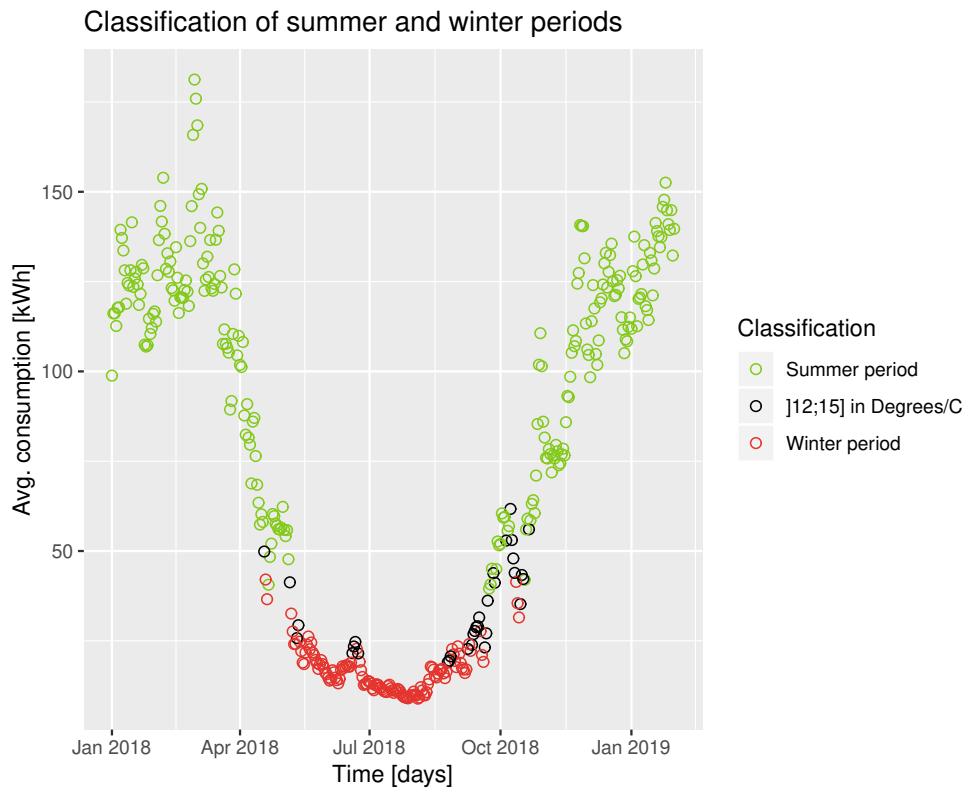


Figure 3.11

# CHAPTER 4

# Models on the Daily Consumption

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Now that data is cleaned and prepared, a statistical analysis consisting of data segmentation and linear regression models can be made. The purpose of the analysis is to detect which attributes affects the performance of a specific house. **Skriv antagelser om konstant indetemperatur og nul måleusikkerheder.**

## 4.1 Linear regression

Linear regression is a method to model the relationship between a dependent variable and one or more independent variables, where the unknown model parameters are estimated from the data. With the dependent variable  $Y$  and the independent variables  $x_1, \dots, x_n$ , the linear regression model is formulated as

$$Y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \dots + \beta_p x_{i,p} + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \sigma^2), \quad i = 1, \dots, n. \quad (4.1)$$

The variables  $\varepsilon_i$  are errors which are assumed to be white noise while also being i.i.d (independent and identically distributed). Equation (4.1) shows a multiple linear regression model as it contains more than one explanatory variable. In this section both a simple linear model and a multiple linear model has been fitted to data given in table 3.1.

As the best linear model  $Y_i$  is desired, the total deviation from the data has to be as small as possible. The least squares method given as

$$\text{SSE} = \sum_{i=1}^n (Y_i - (\beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \dots + \beta_p x_{i,p}))^2 = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (4.2)$$

is chosen for estimating the model. The parameters  $\beta_j$  are optimized to minimize the sum of squared errors of prediction (SSE).

### 4.1.1 Model assumptions

When SSE is minimized the model needs to be validated by checking whether the underlying model assumptions are fulfilled.

- 1 Normality of residuals
- 2 Variance homogeneity
- 3 Variance should be independent of location
- 4 Linear relationship between  $x_j$  and  $Y$

Chapter 7 in [9] explains the model assumptions listed above. To summarize, a model can be checked by looking at diagnostic plots of the residuals. The first plot considered is the Residuals vs Fitted, where the residuals are expected to be randomly scattered. To test whether the residuals are normally distributed a normal quantile–quantile plot is used. Here the residuals are expected to follow a straight line. A Scale-Location plot shows normalized and weighted residuals by sample leverage where the residuals are also expected to be randomly scattered. The last diagnostic plot shows the Residuals vs Leverage which should be a straight line and there should not be any clear patterns. If these assumptions are not met, it can influence the parameter estimates and with it the significance of the parameters. In addition, a Shapiro-Wilk test is performed to check if the residuals of the models are i.i.d with  $\mathcal{N}(0, \sigma^2)$ .

The fitting of the regression models is carried out by using the method stepwise regression which updates the model in each step. In each step it is considered whether a variable is added or subtracted from the set of explanatory variables based on specific criteria. This process is called variable selection and Chapter 7 in [16] explains how this process can be done by using either forward or backward selection. In this project a modified version of backward selection is applied on the multiple regression model. The models are used for comparing which explanatory variables influence each house. Therefore, the models are not reduced using the R function `step`. Instead the significance of the parameters are investigated and then the parameters which are significant for the majority of the houses will be used in an updated linear regression model. Thus, the variable selection is done manually which can be said to be a modified form of backward selection. The level of significance is determined by an F-test where the variables selected have a p-value below a threshold which is chosen at 0.05.

Both a simple linear and a multiple linear regression model will be implemented in order to detect which attributes affect the performance of a specific house. This will be done by interpreting the estimates of the relation between the different explanatory attributes and **Consumption**. As mentioned, the p-value of the estimates of the explanatory variables will be the main focus when investigating which attributes influence the performance. Moreover, transformation of data is not considered since the purpose is to interpret the results and a transformation would make this less intuitive.

## 4.2 Simple linear regression model

A simple linear regression model is fitted to each house with **Consumption** as a function of **Temperature**. Since it is expected that the temperature is the physical phenomenon with the greatest influence on the heat consumption, it is chosen as the independent variable. The temperature is altered based on the threshold for the winter period, which was found to be 12 degrees. The new temperature  $T_{winter}$  is defined as

$$T_{winter} = \begin{cases} \alpha - T & \text{if } T \leq \alpha \\ 0 & \text{Otherwise} \end{cases}. \quad (4.3)$$

This way the temperature input is zero for the data that is not in the winter period. In all models from now on the temperature will be defined by this altered temperature. The simple linear regression model applied to each house is

$$Y_Q = \mu + \beta_T \cdot x_T + \varepsilon \quad (4.4)$$

The models are performed by using the `lm()` function in R. The models will then be validated by examining whether the model assumptions in Chapter 4.1.1 are met and different tests on normality of residuals are performed. The simple model only includes one explanatory variable, thus a variable selection is not performed.

### 4.2.1 Validation

To validate the model, different methods are used. The abovementioned model assumptions are checked and furthermore tests of normally distributed residuals are performed. If the model assumptions are fulfilled and the residuals are i.i.d with  $\mathcal{N}(0, \sigma^2)$ , the model is said to be valid.

Figure 4.1 and Figure 4.2 shows examples of the model applied on two of the houses, where one does not fulfill the assumptions and another model that overall can be said to fulfill the assumptions. The Residuals vs. Fitted plot in Figure 4.1 clearly shows that the residuals are not randomly scattered around mean 0 indicating that the variance are not constant and an odd behaviour appears in the bottom left corner. The QQ-plot shows tails and the residuals do not follow a straight line. The Scale-Location and Residuals vs. Leverage plots also show that the residuals can not be said to be i.i.d. In contrast to the model of house 18, the behavior of the residuals in Figure 4.2 seems more normally distributed. Overall, they are randomly scattered around mean 0 and the QQ-plot shows that the majority of the residuals lie on a straight line. However, the majority of the models do not fulfill the assumptions. In addition, a Shapiro-Wilk test and a sign test is performed. The hypothesis tested in the Shapiro-Wilk test is that the residuals are i.i.d and if the p-value  $> 0.05$  the hypothesis can not be rejected i.e. the residuals are normally distributed. In the sign test the hypothesis is that the number of positive signs are equal to the number of

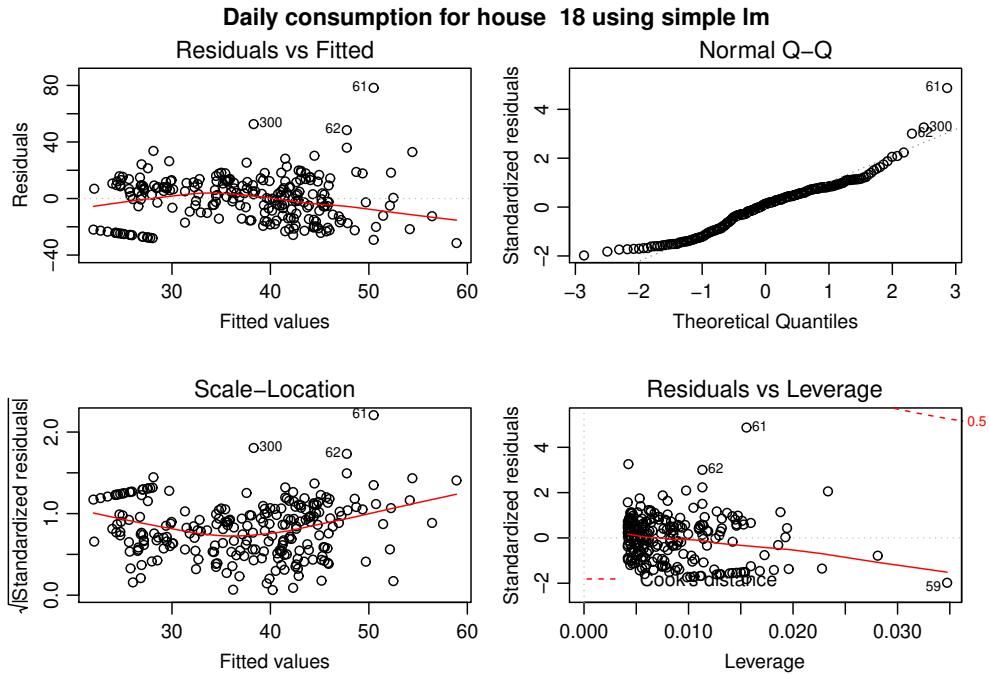


Figure 4.1: Residual plots of house 18 based on the simple linear regression model given in (4.4). The model assumptions of a linear regression model are not fulfilled for this specific house

negative signs, which is one of the requirements when checking for normality. If the model assumptions are also fulfilled, then the model is concluded to be valid. The p-values from both of the tests are found in Table A.2 and it is clearly seen that the majority of the simple linear regression models have residuals with p-value above the significance level at 0.05. Thus, the hypothesis is rejected and it can be concluded that the residuals are not i.i.d.

The estimates from the simple linear regression models can be seen in Table A.1. However, since the simple models do not fulfill the model assumptions, the estimates can not be used for further analysis.

### 4.3 Multiple linear regression model

The linear regression model is extended to a multiple linear regression model as the inclusion of several independent variables is expected to improve the model. The simple model clearly showed that the heat consumption is affected by other physical factors than temperature. Hence, a full multiple linear regression model containing the at-

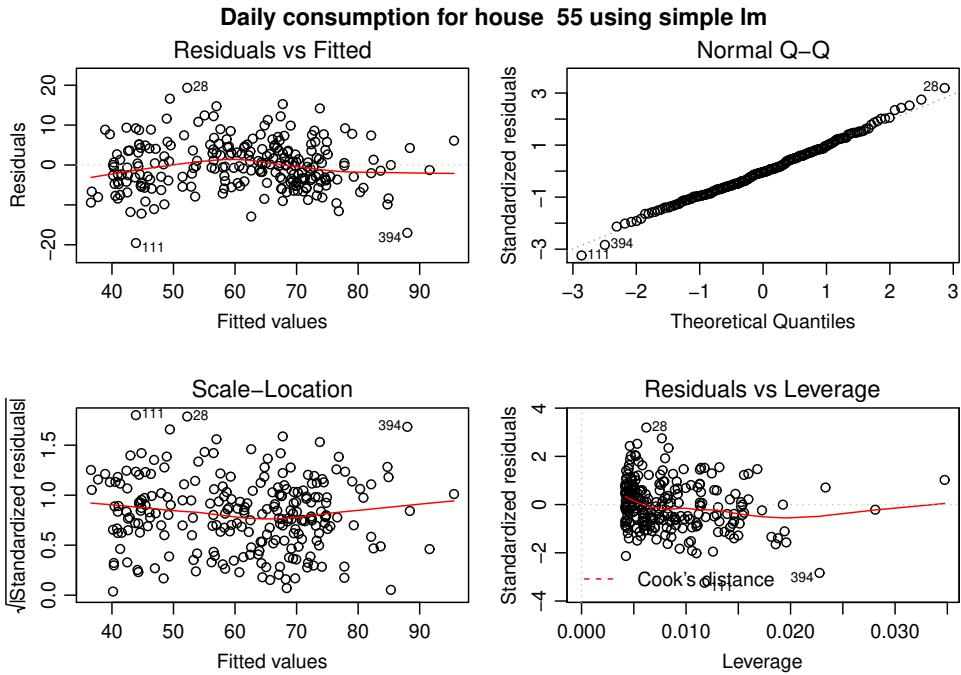


Figure 4.2: Residual plots of house 55 based on the simple linear regression model given in (4.4). The model assumptions of a linear regression model are overall fulfilled

tributes given in Table 3.1 is performed on the model data. The backward selection of the variables are perfomed such that the highly significant parameters from the full multiple regression model are chosen and used to develop an updated model. Since **Condition** and **PrecipitationProbability** are not normalised, they are excluded from the model. In addition, it is mentioned in Chapter 2 that the house data consists of house with observations for approximately a year and house with observations for approximately six months. Thus, the two distinct lengths of observations are modelled slightly different. There do not exist observations for winter break and spring break in the data containing the short houses. The wind directions are not inserted directly into the model. As mentioned in Table 2.2, the wind directions are given as degrees from 0 to 360. But in the model the wind direction is not quantified in this way, rather it should be an effect on each of the four major directions: north, south, east and west. These will in the following be called wind direction categories. One way to classify the wind direction could be to use indicator variables, and assign each possible wind direction to a category. In this project spline functions will be used to determine the effect a certain wind direction has on the categories. As a consequence, a wind direction can have different effects on different categories. For example wind coming directly from north might have a large effect on the north category, but also

a lesser effect on east and west. In real life the wind direction is often distorted close to the ground because of turbulence. Also wind directions on the border between two categories should have an effect on both. For these reasons, splines seem like a good choice for modelling the wind.

### 4.3.1 Splines

Each major direction is described by a basis spline. Together they form a spline basis, that spans the space of the wind direction. The theory on splines introduced here is based on [15]. A spline basis is defined by a knot vector  $\Xi$  and a polynomial degree  $q \in \mathbb{N}$ . The  $i$ 'th basis spline is a function  $\mathcal{N}_{\Xi,i}^q : \mathbb{R} \rightarrow \mathbb{R}$ . If the spline basis should contain  $n$  basic splines, then  $\Xi = \{\xi_1, \xi_2, \dots, \xi_{n+q+1}\}$ . When the knots are equidistant, the spline is uniform. Then the spline is defined by the Cox-de Boor recursion formula:

$$\mathcal{N}_{\Xi,i}^0(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi \leq \xi_{i+1} \\ 0 & \text{otherwise,} \end{cases} \quad (4.5)$$

for the splines of degree 0, and for higher degrees

$$\mathcal{N}_{\Xi,i}^j(\xi) = \frac{\xi - \xi_i}{\xi_{i+j} - \xi_i} \mathcal{N}_{\Xi,i}^{j-1}(\xi) + \frac{\xi_{i+j+1} - \xi}{\xi_{i+j+1} - \xi_{i+1}} \mathcal{N}_{\Xi,i+1}^{j-1}(\xi), \quad (4.6)$$

for  $j = 1, 2, \dots, q$  and  $i = 1, 2, \dots, n + q - j$ . When the splines are uniform, the continuity of the basis splines across a knot is  $q - 1$ . This means that the  $q - 1$ 'th derivative exists and is continuous. In this project splines of degree 2 is used, meaning that the first derivative is continuous at every point on the spline. The splines used here are periodic. That means that after the first  $n$  knots, the knot sequence starts over. For modelling the wind direction, four knots are used for the knot vector, each associated with a wind direction category. Here they are defined as *northeast*, *southeast*, *southwest* and *northwest*. The reason for this will be explained later in this section. For each of these knots there is a basis spline. When the degree of the splines is 2, it means that the spline for a given knot is zero at the opposite knot. For example, when using four knots the spline peaking in the north would always be zero in the south. Higher degrees would not maintain this property. The spline basis can be seen on Figure 4.3. Notice that the sum of the entire spline basis at a given point always adds up to one. The figure shows how the  $i$ 'th spline, associated with the  $i$ 'th knot does not peak at that knot, but in the following interval. This is the reason why the knots are chosen in this way. By choosing the knots to be between the main directions north, east, south and west, this is where the basic splines peek. Now the splines can be used to represent their category in the linear regression model. At a given data point, the wind direction is given as input to each basic spline. The result is then weighted by the windspeed. The result is used as the effect of the effect

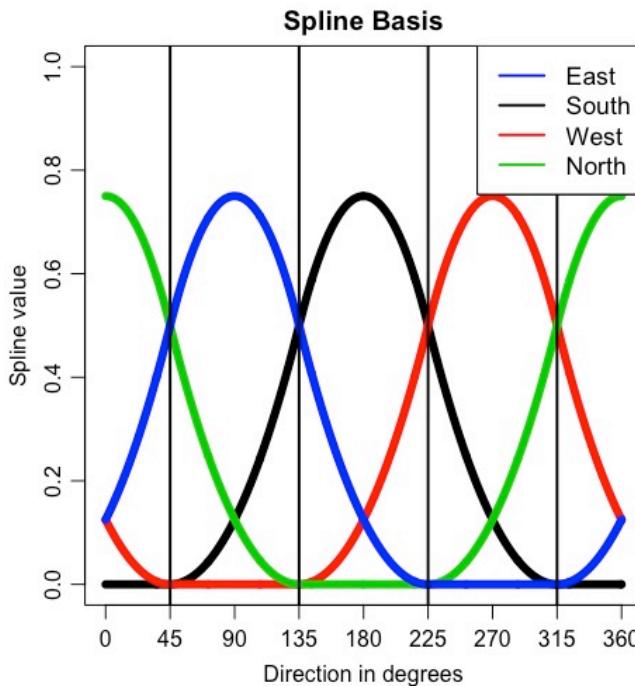


Figure 4.3: The spline basis used to model the wind direction. Each color is a different basis spline, and the vertical lines mark the knots

of the category that spline represents. As an example, a windspeed of 2 from the angle 135 degrees would give the following result for the variables in the regression model:  $x_N = 0$ ,  $x_E = 0.5$ ,  $x_S = 0.5$  and  $x_W = 0$ . These results can be derived from Figure 4.3.

### 4.3.2 Multiple linear regression models

The parameters included in the multiple models will be denoted as follows: Intercept (I), Temperature (T), North (N), East (E), South (S), West (W), Mean Sea Level (MSL), Solar Radiation (SR), Winter Break (WB), Spring Break (SB), Autumn Break (AB), Christmas Break (CB), Weekend (WKND), the interaction between the temperature and the different wind directions (T:N, T:E, T:S, T:W). This lead to the

following two multiple linear regression models:

$$\begin{aligned} Y_{Q,L} = & \mu + \beta_T \cdot x_T + \beta_N \cdot x_N + \beta_E \cdot x_E + \beta_S \cdot x_S + \beta_W \cdot x_W + \beta_{MSL} \cdot x_{MSL} \\ & + \beta_{SR} \cdot x_{SR} + \beta_{WB} \cdot x_{WB} + \beta_{SB} \cdot x_{SB} + \beta_{AB} \cdot x_{AB} + \beta_{CB} \cdot x_{CB} \\ & + \beta_{WKND} \cdot x_{WKND} + (\beta_{T:N} + \beta_{T:E} + \beta_{T:S} + \beta_{T:W}) \cdot x_T + \varepsilon \end{aligned} \quad (4.7)$$

$$\begin{aligned} Y_{Q,S} = & \mu + \beta_T \cdot x_T + \beta_N \cdot x_N + \beta_E \cdot x_E + \beta_S \cdot x_S + \beta_W \cdot x_W + \beta_{MSL} \cdot x_{MSL} \\ & + \beta_{SR} \cdot x_{SR} + \beta_{AB} \cdot x_{AB} + \beta_{CB} \cdot x_{CB} + \beta_{WKND} \cdot x_{WKND} \\ & + (\beta_{T:N} + \beta_{T:E} + \beta_{T:S} + \beta_{T:W}) \cdot x_T + \varepsilon \end{aligned} \quad (4.8)$$

The models include the interaction between the temperature and the wind since it is expected that this interaction has an influence on the heat consumption. That is, it is expected that the influence of the wind is greater when the temperature is lower. In linear regression, it is desired to have as simple a model as possible, and since the models would be too complex when including all the interactions, only the interaction between the temperature and the wind is included. The models also show that the interactions between the attribute *Holiday* and the other attributes are chosen to be excluded. The reason is that *Holiday* is used to investigate how the consumption changes during holiday periods.

The full multiple regression models are steps of the backward selection which is why they are not validated. The purpose of this step is, as mentioned, to determine which parameters are found to be significant in the majority of the models. Hereafter, these parameters are included in a general regression model that can be used for comparing the houses performance. Before performing the models, the wind will be modified which the following section explains.

### 4.3.3 Results

When performing the two models given in (4.7) and (4.8), without reduction, the significance of the parameters are determined and can be found in Table A.3-A.4. In addition, Table 4.1 and Table 4.2 are generated in order to determine which parameters are significant for the majority of the houses. The tables clearly show that the total of significance of the intercept, temperature, east and west as well as the interaction between temperature and west and the solar radiation occurs in more than half of the models. Thus, these are included in the general regression model. But when the influence of the wind on the consumption is examined, it does not make much sense to exclude some of these parameters. Therefore, both north, south, east and west are included as well as their interaction with the temperature. Each parameter from the different holiday attributes is not significant for enough of the models. There might have been some patterns in when the households vacationed, which the tables also indicate, but the impact on consumption is not large enough for the holidays to be included in the general model.

	I	T	N	E	S	W	MSL	SR	WB	SB	AB	CB	WKND	T:N	T:E	T:S	T:W
Sum of ***	5	41	0	18	2	24	6	22	3	3	1	5	4	0	1	7	9
Sum of **	6	1	1	9	5	12	4	10	6	2	0	2	0	1	1	9	9
Sum of *	6	1	5	7	7	2	2	2	3	6	5	3	8	2	3	5	11
Total of 43	17	43	6	34	14	38	12	34	12	11	6	10	12	3	5	21	29

Table 4.1: The distribution of significant parameters from the multiple linear regression model for long houses. There are 43 long houses, thus the total of the significance of each parameter for each house is in relation to the number of long houses

	I	T	N	E	S	W	MSL	SR	AB	CB	WKND	T:N	T:E	T:S	T:W
Sum of ***	0	27	0	4	0	15	0	5	2	0	0	0	0	0	3
Sum of **	2	0	0	6	2	5	2	6	0	0	3	0	0	2	5
Sum of *	2	0	1	8	4	4	4	4	2	5	2	1	1	3	9
Total of 27	4	27	1	18	6	24	6	15	4	5	5	1	1	6	17

Table 4.2: The distribution of significant parameters from the multiple linear regression model for short houses. As for the long houses, the total of the significance is in relation to the number of long houses

## 4.4 Regression model for comparing houses

Based on the tables illustrating the significant parameters for the long and short houses, an updated multiple linear regression model is made. The purpose of this more general model is to compare which parameters influence each house. Furthermore, houses with e.g. same area, construction year etc. can be compared. The comparison model derived from Table 4.1 and Table 4.2, becomes

$$Y_Q = \mu + \beta_T \cdot x_T + \beta_N \cdot x_N + \beta_E \cdot x_E + \beta_S \cdot x_S + \beta_W \cdot x_W \\ + \beta_{SR} \cdot x_{SR} + (\beta_{T:N} + \beta_{T:E} + \beta_{T:S} + \beta_{T:W}) \cdot x_T + \varepsilon. \quad (4.9)$$

Likewise, the models are validated and if the assumptions are fulfilled the models can be used for comparison and predictions. The model in (4.9) is expected to explain the heat consumption of the houses in a more accurate way as it takes several significant parameters into account. As mentioned, the purpose of the general model is to compare the houses for what influences their specific heat consumption and thereby the house's performance. Thus, they are not further reduced.

### 4.4.1 Validation

In order to determine whether the general model is valid, the diagnostic plots in Figure 4.4 and Figure 4.5 are investigated. The residuals from the model for house 18 behave quite odd and are not randomly scattered. More precisely, some of the residuals lie on a straight line, as seen in the residuals vs fitted plot. The reason for this is that the heat consumption for the house is 0 at the end of autumn, as seen in Figure 3.1. This results in that the estimates of the consumption for these

specific points are equal to the corresponding residuals with reversed sign. It seems like this specific house can not be described by a linear regression model, which will be discussed further in Chapter 6. On the other hand, the diagnostic plots of the residuals from house 55 are definitely improved compared to the simple model and the residuals show a randomly behavior. Overall, the diagnostic plots for the general models can be said to be fulfilled. They are not perfect but that is not expected either. There are some houses whose residuals have strange patterns, but the majority of the models can be said to be valid. Furthermore, the Shapiro-Wilk tests and the sign tests in Table A.5 are used to determine if the residuals are normally distributed. The Shapiro-Wilk test shows that the hypothesis of normally distributed residuals for most of the models cannot be accepted. Similarly, the results of the sign tests do not indicate that the residuals can be concluded to be normally distributed. However, this can be said to be contrary to the diagnostic plots, which is quite remarkable. Therefore, this will be discussed in more detail in the comparison of the two regression models. In spite of the abovementioned, the models are concluded to somewhat fulfill the model assumptions and the estimates can be investigated further.

#### 4.4.2 Results

The results of the general regression model performed on each house, are illustrated in Table A.6 and Table A.7 with the significance of each parameter. As expected, the intercept and the temperature are highly significant in all models and with the "correct" sign. That is to say, the expected sign of the temperature is negative since the increase in the temperature will result in a decrease in the heat consumption. The attribute **SolarRadiation** effects the majority of the houses as well. The solar radiation warms the house up through the windows, so it makes sense that this has significance for how large the heat consumption is. In order to compare the houses, the temperature coefficients as well as the solar radiation coefficients are illustrated in Figure 4.6 and Figure 4.7 including a 95% confidence interval. The influence of the temperature on house 18 is minimal but the confidence interval is quite wide compared to the houses lying close to 18. Thus, there is a large uncertainty for this specific model. Likewise, house 55 is not particularly effected by the outdoor temperature. The step response is approximately  $-0.023 \text{ kWh/m}^2$ . The confidence interval is narrow, so the uncertainty for this model is quite small compared to the model for house 18. The model with the largest temperature coefficient is the one for house 63. The step response is around  $-0.273 \text{ kWh/m}^2$ . This might be explained by the construction year of the house - it was build in 1920 and according to Figure 3.4, it is one of the older houses. Contrary, the influence of the temperature is the lowest for house 9. The house was build in 2009, so the expectation that the house performs quite well is somewhat true. When examining the coefficients in Figure 4.7, the model for house 18 shows an odd behavior. The step response is positive ( $0.002 \text{ kWh/m}^2$ ), i.e. the heat consumption increases when the solar radiation is increased. House 55's influence from the sun is small and again the 95% confidence interval is quite narrow.

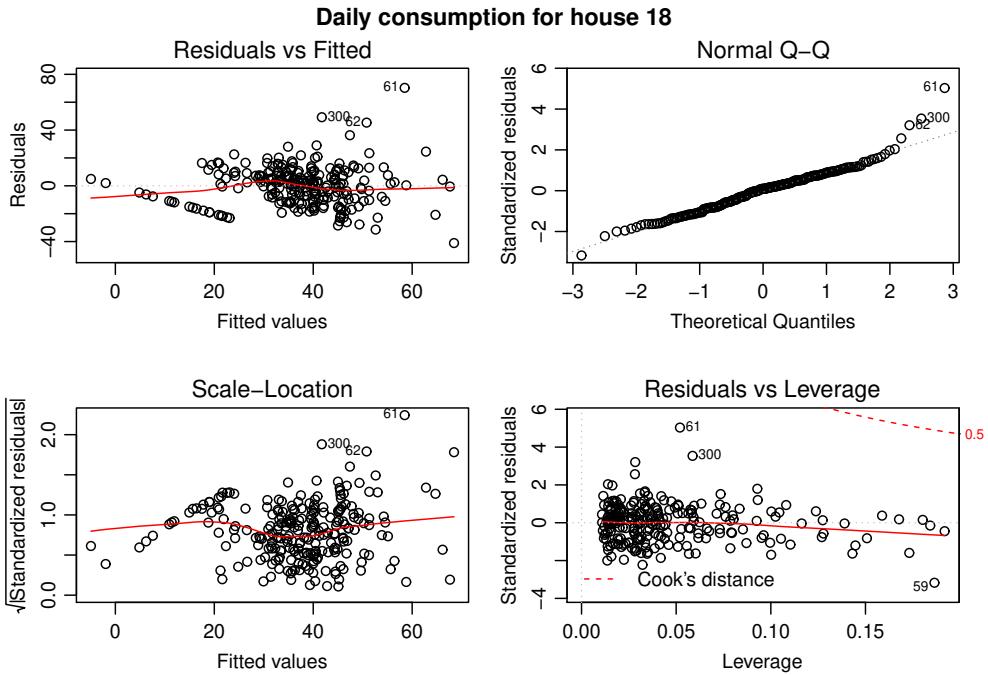


Figure 4.4: Diagnostic plot of the general multiple linear regression model for long house 18.

Based on this and the confidence interval showed in Figure 4.6, the model for house 55 performs well and it can describe the behavior for the specific house. House 10 the most influential in terms of solar radiation. The step response is at  $-0.039 \text{ kWh}$ . However, there does not exist any BBR data on this house apart from the fact that it is a commercial building so whether this behaviour is expected or not is not known.

It can also be seen from Table A.6 and Table A.7 that the wind directed from East and West have great influence on the consumption of most houses. How the exact wind dependency is determined and illustrated is explained in the following about predictions.

#### 4.4.3 Predictions

To see how well the model captures the overall behaviour of the data, it is used to make predictions for unknown data. This is done by dividing the data into a training set and a test set. Here a test set will be used containing January 2019 (31 days). The available weather data for that period is used to make predictions, which are then compared to the actual consumption. These are not one-step predictions, but 31 step

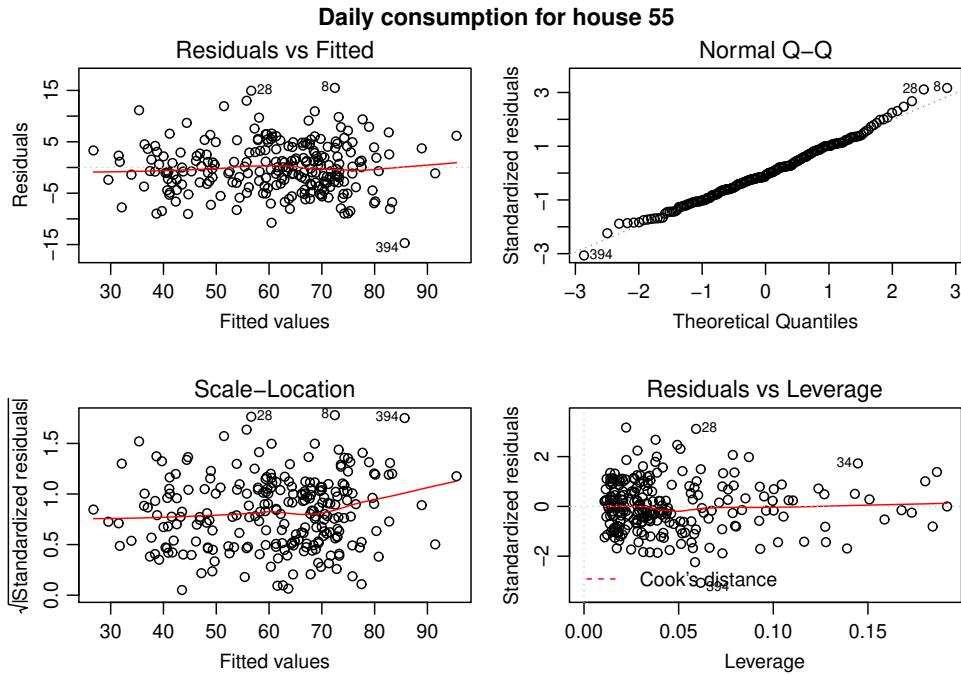


Figure 4.5: Diagnostic plot of the general multiple linear regression model for long house 55

predictions. This means that the model for time  $t$  updates based on the prediction at time  $t - 1$ , not the actual data. In other words, one-step predictions would be the same as predicting one step ahead 31 times and updating the available observations. The predictions are made using the function `predict` in R and Figure 4.8 shows the input values used for the predictions. In consistency with the other results, house 18 and 55 are used for predictions. They are illustrated in Figure 4.9 and Figure 4.10. It can be seen that the predictions for house 55 lie close to the actual data and captures the overall behaviour. The 95% prediction interval is narrow, which indicates that the standard deviations of this model are fairly small. Comparing the predictions to the actual consumption, the actual data is mostly inside the prediction interval. In the last couple of days the model predict spikes in the temperature that are not reflected in the observations. Figure 4.8 indicate why the spikes occur. It seems that there are two days with particularly low temperatures coinciding with the spikes in the predictions. Low temperatures causes the model to expect higher consumption. There can be many reasons why the consumption did not spike as expected, for example the inhabitants of the house might not have been home that day to notice the temperature difference. Such things can be very hard to predict. In general, the model performs well on this house. It should be noted that there is no sun input in

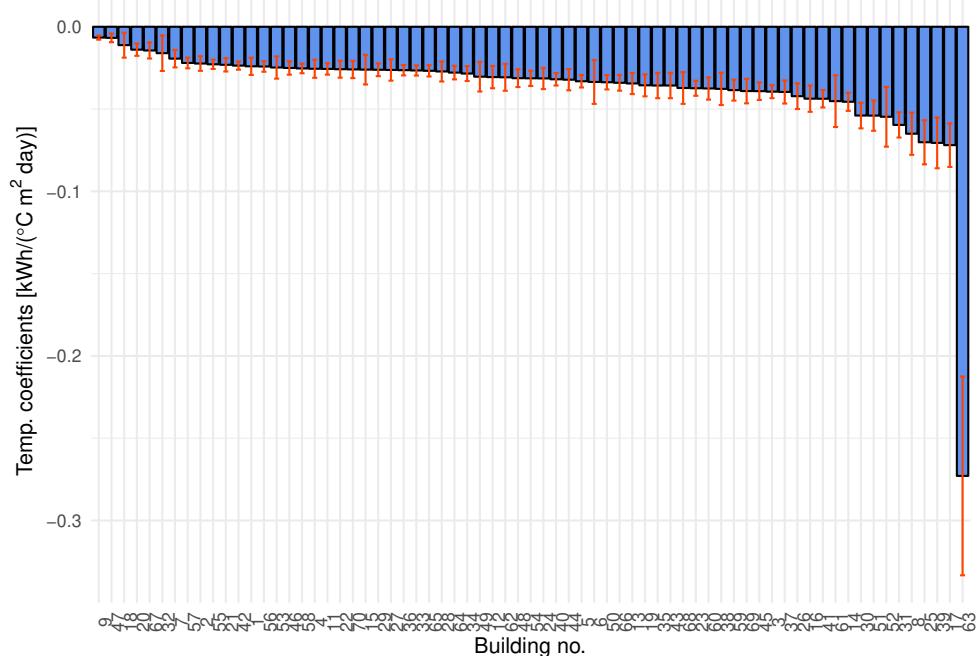


Figure 4.6: The temperature coefficients from the general regression model. The consumption in house 63 is influenced by the temperature in a higher degree than the other houses, since the consumption decreases with  $0.27 \text{ kWh}/\text{m}^2$  when the temperature increases with  $1^\circ\text{C}$

the input weather data for the last half of January. This makes it hard to evaluate how the sun affects the predictions.

House 18, on the other hand, does not behave quite as expected. The model has problems capturing the and the 95% prediction interval much wider than for house 55. There is more unexplained variation in this model. The deviations in the observations cannot be explained by the input from the weather data. So even though all the observations are within the prediction interval, the model is not of much use on this house. There are other factors affecting the consumption that are not captured in the model, which is reflected in the high uncertainty.

The effect of the wind direction on the consumption is investigated by estimating the daily consumption for the house, for a fixed setting, where only the wind direction is changed. The setting used is a day with a temperature of  $0^\circ\text{C}$ , without any solar radiation, and with a wind speed of  $4.27 \text{ m/s}$ , which is the average wind speed throughout the year. A prediction of the daily consumption is made for each wind direction and plotted with a 95% prediction interval. Figure 4.11 shows the wind

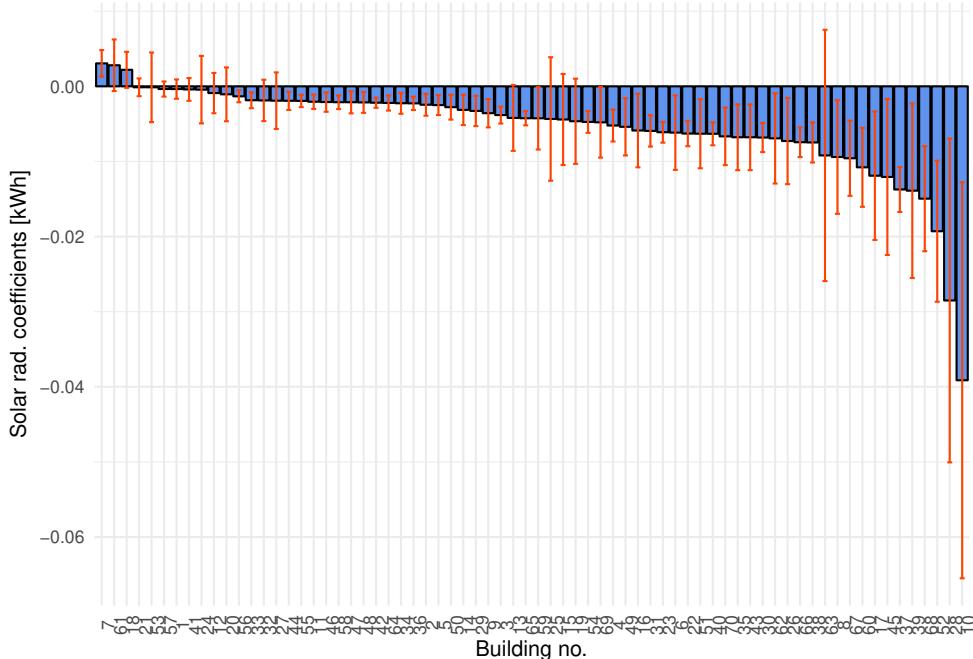


Figure 4.7: The coefficients of the solar radiation from the general regression model. Some of the houses has a step response that increases which is not expected

dependency for house 55. The consumption of house 55 is by far most affected by wind from west. As well as for house 55, the wind dependency is also illustrated for house 18 in Figure 4.12. It shows that the consumption is high when the wind comes from west and east, and low from north and south.

## 4.5 Comparison

When looking at the results of the different tests in Table A.2-A.5, the simple model passes the tests in more cases than the multiple model. This might be explained by the inclusion of the wind in the multiple regression model. The so-called directions are not all significant for all houses. Thus, this can affect the behavior of the residuals. On the other hand, the interpretation of the simple model shows that the temperature has a great influence on the consumption. But from a physical perspective it is known that the heat consumption is influenced by other physical phenomena as well. Hence, the simple regression model is indeed too simple. The general regression model performs as expected. It can say something about which physical phenomena affect the heat consumption of each house. The model can also predict how the expected

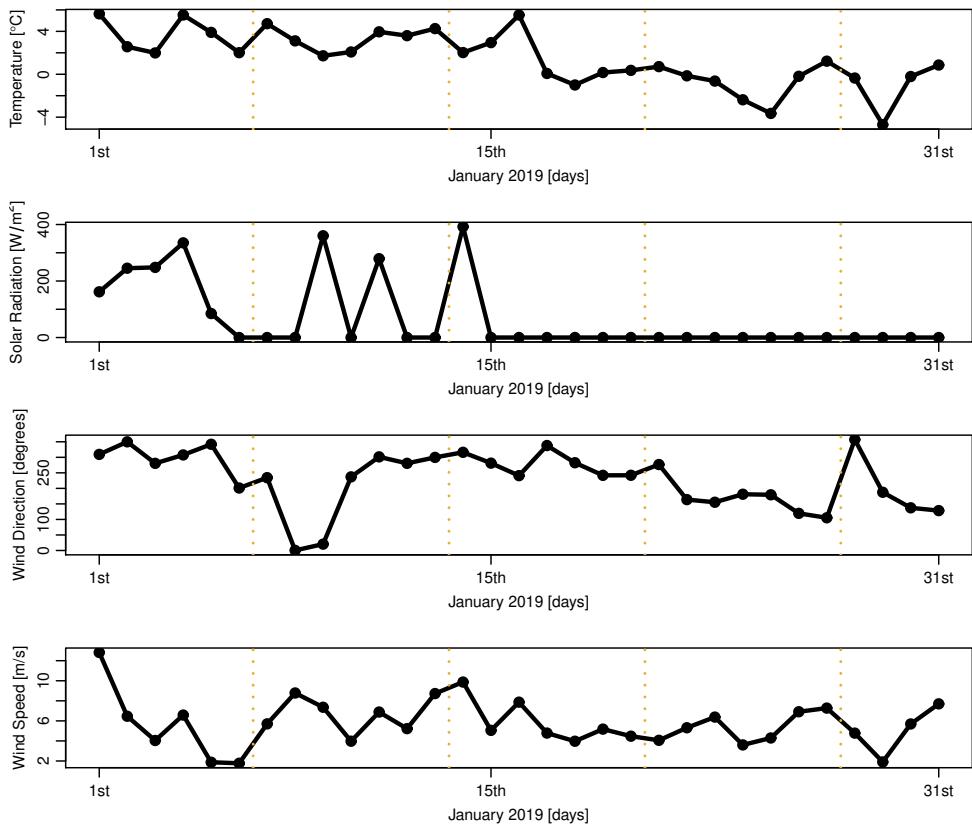


Figure 4.8: The input values used for predictions of the general linear regression model. For the entire last half of January it was never sunny

consumption will look a month ahead. However, this is under the assumption that the weather forecasts are exact.

## 4.6 Visualization of the results

Part of this project is also to give suggestions on how the results of the analyzes can be illustrated to the users of the WATTS app.

The wind dependency showed in Figure 4.11 and Figure 4.12 can be converted to understandable plots in the following way: Each degree get one of the 3 colours, green, yellow and red, based on the 33.3% prediction interval. After subtracting the

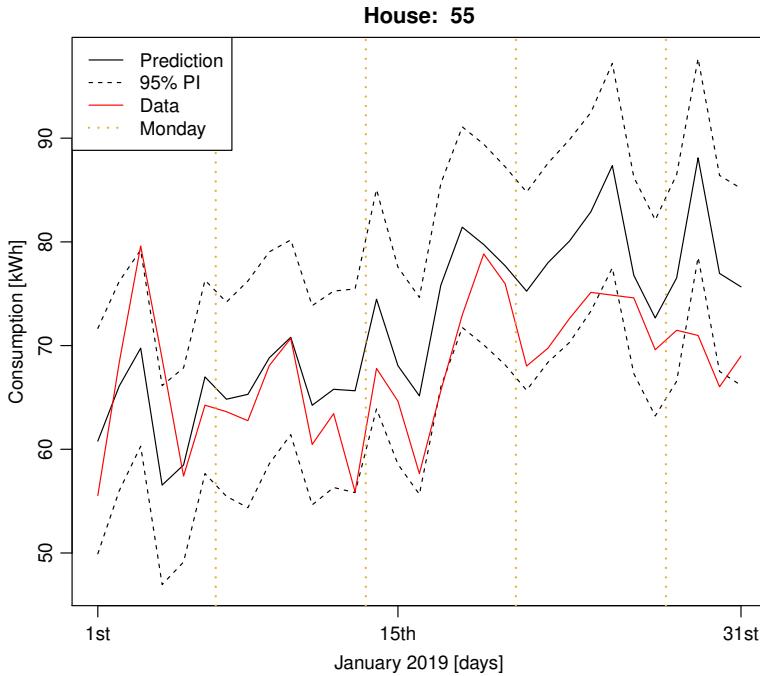


Figure 4.9: Predictions of the last 31 days of the model data for house 55 including a 95% prediction interval. The model captures the behavior of this house quite well

mean of the fit, if 0 is contained in this interval, the degree is coloured yellow. If the entire interval is positive, the degree is coloured red. Likewise if the entire interval is negative, the colour is green. Thus, the coloring is scaled according to the estimate of the wind, and that is relative to the user's own house's performance. For the users of the app, the wind direction is more intuitive as a compass on their phone. Therefore the minimum of the lower bound of the 33.3% prediction interval is subtracted from the fit, and then plotted as a coloured shape. The graph for the wind dependency on the consumption of house 55 and house 18 will look as follows: The red colour indicates where the house is most influential where the consumer must be most attentive in relation to e.g. replacement of windows or better insulation. Figure 4.13 shows that the heat consumption in house 55 increases when the wind is coming from the west. Likewise, Figure 4.14 shows that the heat consumption increases when the wind is blowing from west and also east. These illustrations are relative to the performance of each house. It is difficult to weigh the wind directions precisely when the data does not contain information about the location of the houses which is discussed in Chapter 6. Figure 4.9 and Figure 4.10 are also colorized with red, yellow and green. This coloring is distributed so that there is an equal probability that the heat consumption lies in one of the three areas, ie. the areas are weighted  $\frac{1}{3}$  each. The red

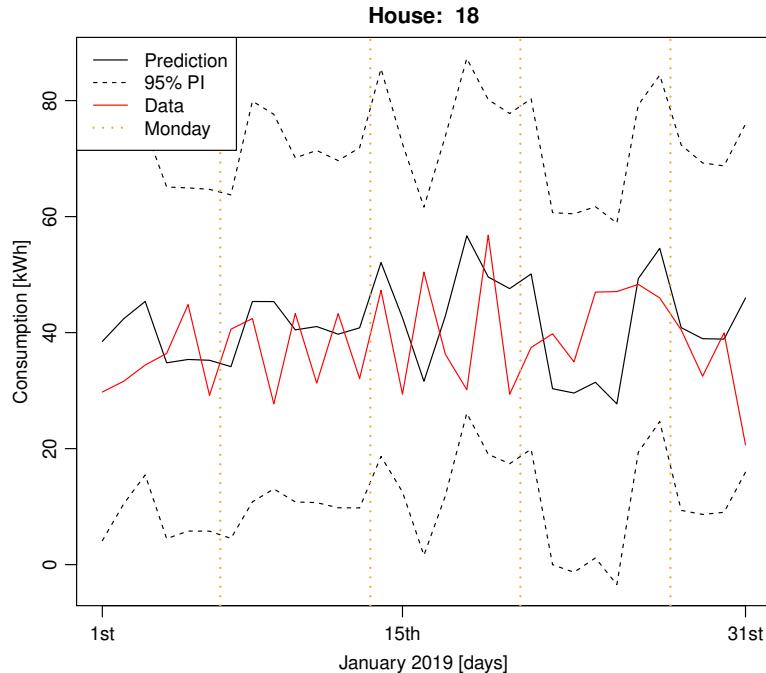


Figure 4.10: Predictions of the last 31 days of the model data for house 18 including a 95% prediction interval. The model captures the behavior of this house, but the prediction interval is quite wide which indicates a large uncertainty about this model

area indicates that the heat consumption exceeds the expected consumption which will result in the consumer having to pay more. If, on the other hand, the consumption is in the green area, the customer consumes less than expected and thus has to pay less than the budget. As can be seen in Figure 4.15 for house 55, it is only in early January that the consumption exceeds the expected. Otherwise, the consumption is around the expected and for most days it is below. To be able to clarify the daily consumption even more, every day in January 2019 can be represented as a pillar, resulting in ???. As mentioned, the consumption for house 55 is most often below the expected consumption, as the bar graph clearly shows with the majority of the pillars colored green. These two suggestions for the WATTS app have also been tried on house 18. The previously mentioned oscillating behavior is clearly seen in Figure 4.16. This shows the consumer that they should probably have an expert investigating the house further. The bar graph in ?? shows that the pillars change colors almost every day. Once again, it can be seen that the heat consumption in house 18 has a very strange behavior.

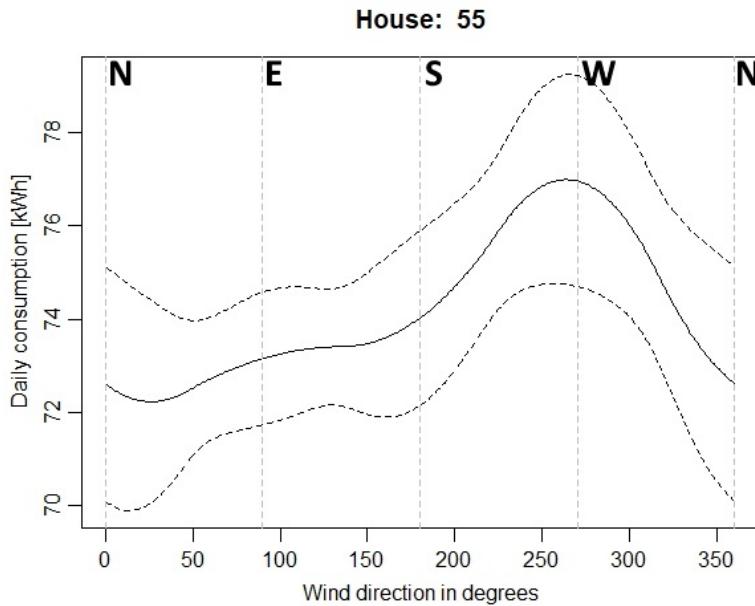


Figure 4.11: Wind direction dependency for the consumption for house 55

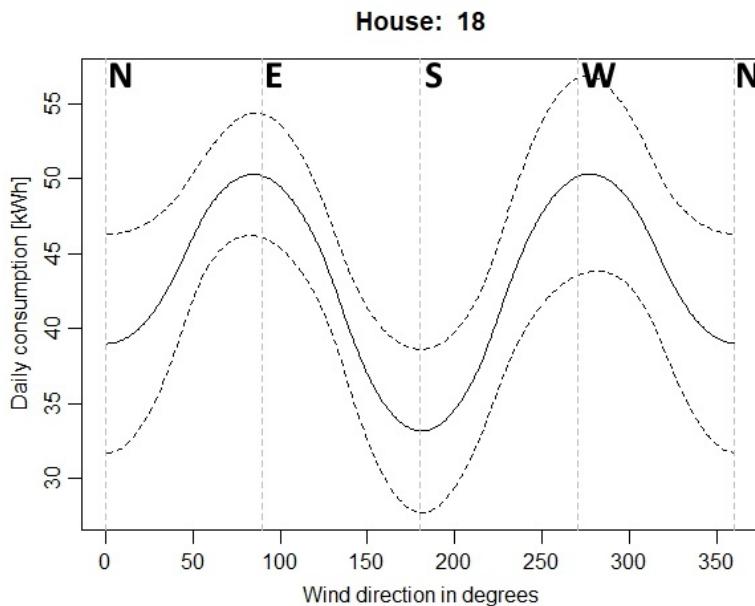


Figure 4.12: Wind direction dependency for the consumption for house 18

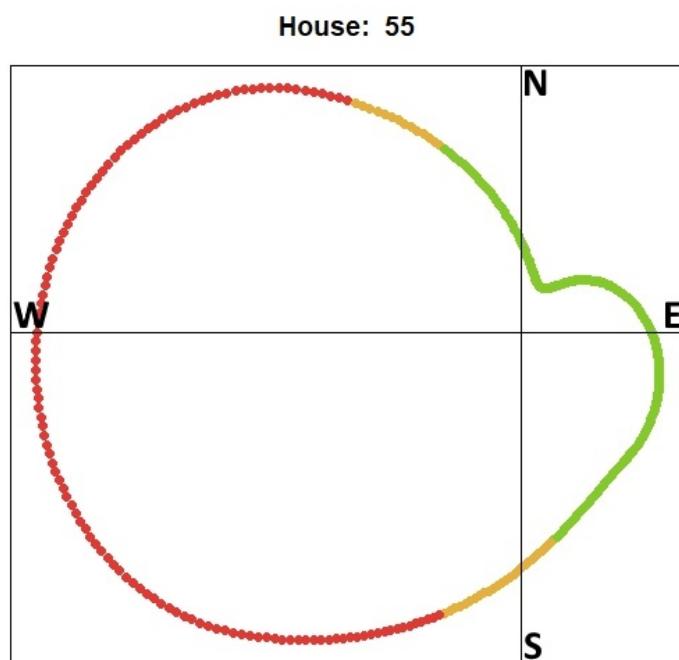


Figure 4.13: Relative wind direction dependency for the consumption for house 55

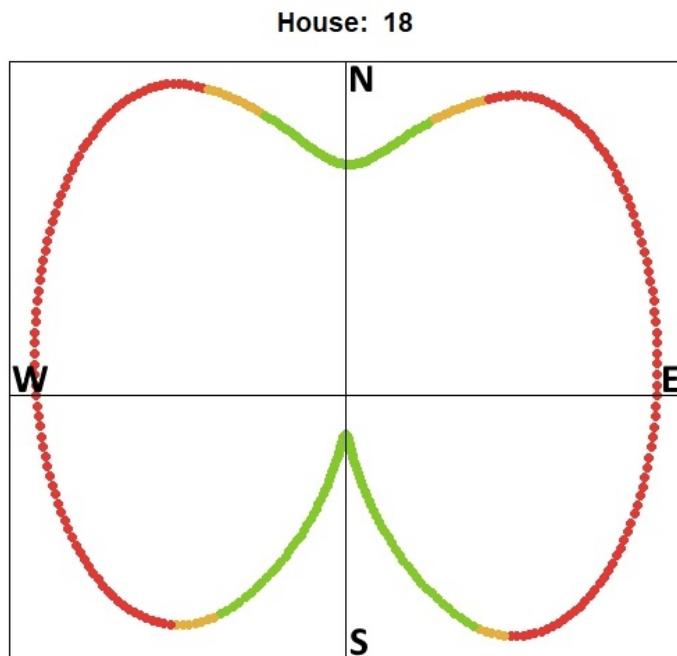


Figure 4.14: Relative wind direction dependency for the consumption for house 18

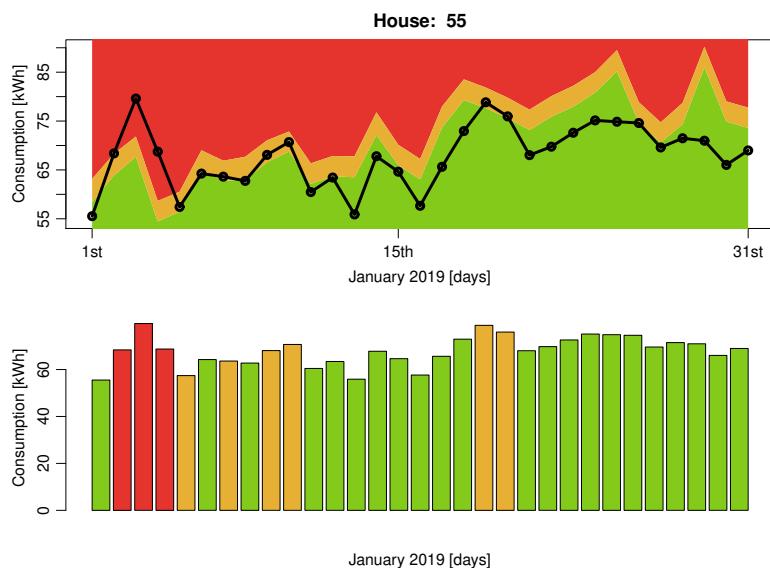


Figure 4.15: This is two other ways of visualizing Figure 4.9. In the first plot, the data is the black line, and the colours show how the house performs compared to the predictions. The lower plot visualizes the consumption of house 55 a bar graph where each day is represented by a pillar.

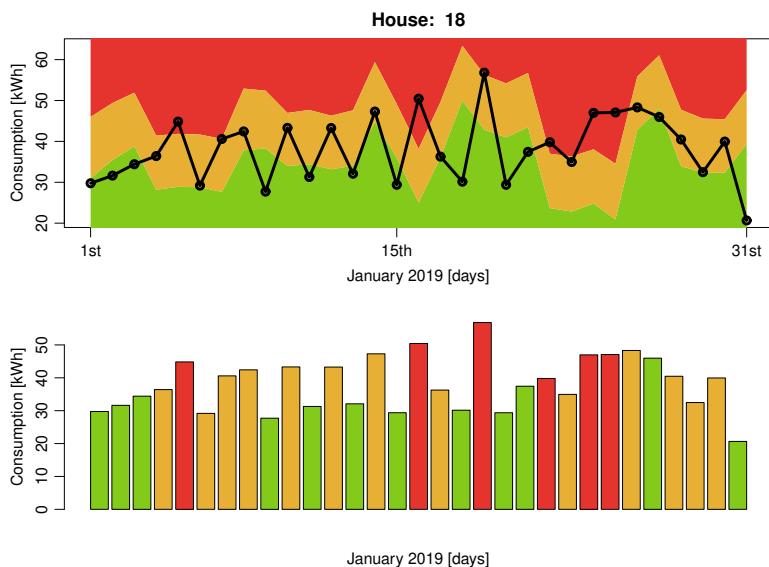


Figure 4.16: This is two other ways of visualizing Figure 4.10. In the first plot, the data is the black line, and the colours show how the house performs compared to the predictions. The lower plot visualizes the consumption of house 18 a bar graph where each day is represented by a pillar.

## CHAPTER 5

# Models on the Hourly Consumption

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In this section a more detailed look at the hourly consumption will be provided. It will be investigated how the consumption is distributed over the course of the day, and what the main differences are between the winter and the summer periods. The main goal is to get an alternative estimate of the effect of the temperature on a house. A model based on hourly data might be able to catch variations that are not present in the hourly data. The type of model that will be applied is an ARIMAX model, that analyzes the data as a time series. Different ARIMAX models will be investigated, it will be tested if the models can make reasonable predictions of a short period with unknown data, and in the step response of the models will be calculated. This step response can be related to the temperature coefficients in the daily models, and the uncertainty of the two will be compared.

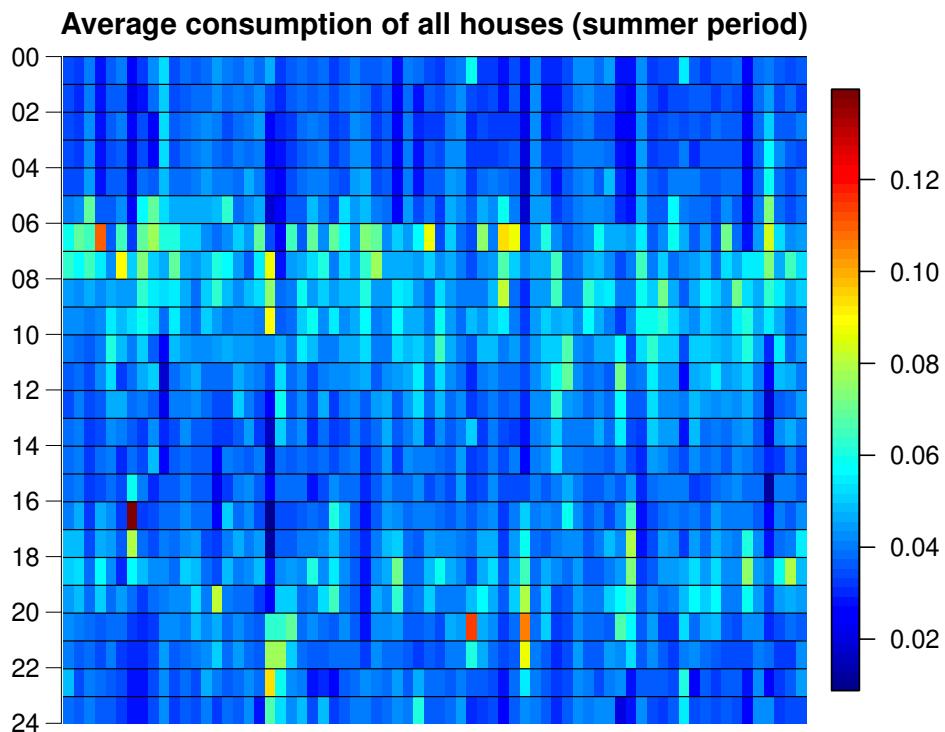


Figure 5.1: The normalized average consumption of every house during the day in the summer period. This is characterized by the days where the average outside temperature is above 15 degrees. The horizontal lines indicate the hours and each vertical strip is a house. The scale indicates the fraction of the total consumption during the day

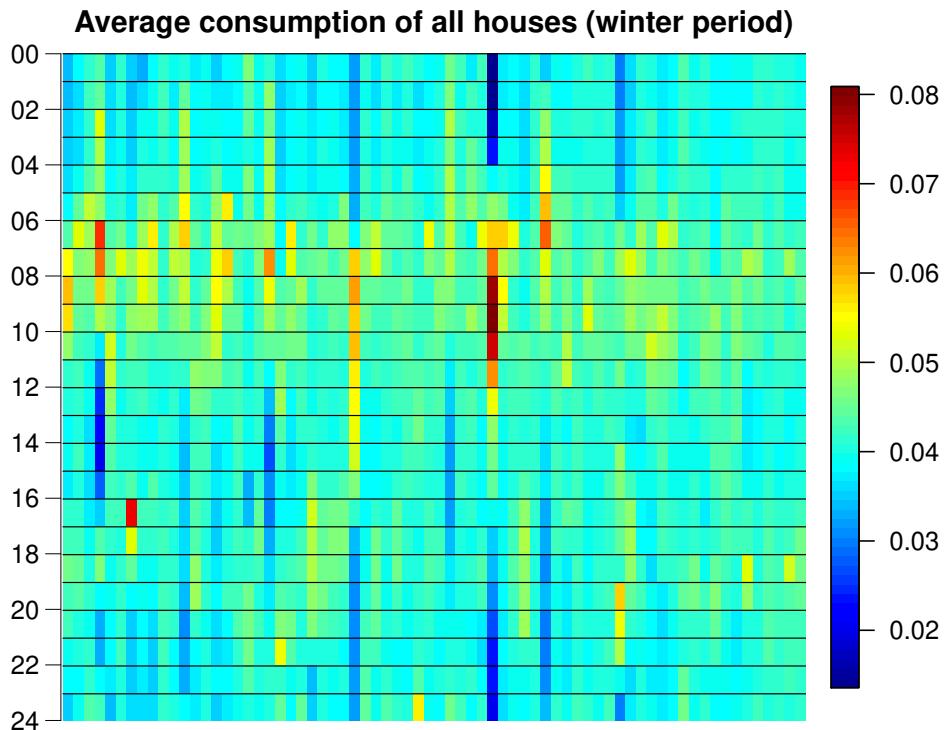


Figure 5.2: This figure shows the same as Figure 3.1, but only in the winter period, characterized by an outside temperature below 12 degrees

## 5.1 Description of the Hourly Consumption

Figure 5.1 and Figure 5.2 show the average consumption of each house during the day for the summer period and the winter period respectively. The different hours can be seen on the y-axis, and the colour coding show the fraction of that house's consumption in that hour interval. Each vertical strip of colours is a single house. Each strip sum up to 1. Looking at the summer period in Figure 5.1, a general trend is apparent: the consumption is usually larger around 7 AM and to some degree around 7 PM. Almost every house peaks in one of these periods, and some peaks go up to 12% of the daily consumption. On the other hand, there is almost no consumption between 11 PM and 5 AM. The same goes for the afternoon between 1 PM and 4 PM. Figure 5.4 shows the average distribution of all houses together with lines indicating the quantiles. On the figure it can be seen that the intervals 06 – 11

and 18 – 19 are in the top quantile. 00 – 05 and 15 – 16 is where the consumption is lowest.

These trends make sense. In the summer period, not much energy is used for heating the house. There is usually a significant amount of tap water consumption in the morning, when people take warm baths and make breakfast. Sometimes a dishwasher might be running as well. Then there is not much consumption while people are at work or school. When they get home in the late afternoon the consumption rises again as they prepare for dinner or use hot water in other ways. During the night time the consumption becomes low again.

The winter period on Figure 5.2 is a bit different. There are still significant peaks in the morning, and to some extent in the evening as well, but in general the consumption is more spread out on the entire day. This is mostly because of the heating consumption in the winter period. While people are not at home or while they are sleeping, the heating is still turned on. The highest peaks only go to 8% of the daily consumption here. One house stands out in this plot. A bit to the left of the middle there is a house where the consumption is several times higher between 8 AM and 12 noon. This house has almost no consumption during the night. But the house is not a commercial building and its area is only  $138 m^2$ . So this house appears to have an efficient night time drop for their thermostat.

These figures illustrate the general trend of the houses, but it is hard to compare them in a meaningful way. But Figure 5.3 shows the average distribution of all houses during the day. Both the winter season and the summer season show the same trends that was discussed above. But this plot also shows how the winter period is more smoothed out than the summer period. Keep in mind that the lines only show the relative distribution, and they do not take into account that the consumption in the winter period is significantly higher. As one can see on the y-axis, the difference between the two curves is very small. A night time period can be defined as the hours 23 – 05. This is the period after the consumption drops in the evening, and before it rises in the morning. In this period, the houses on average use 21,9% of their daily consumption in the summer period, and 23,7% in the winter period. A completely uniform consumption would be 25%. It is not surprising that the consumption in the night hours is lower than the average. Neither is it surprising that the consumption at night in the summer period is relatively smaller than in the winter period. The extra cost of heating the house makes the consumption more spread out on the 24 hours of the day. But it is surprising that the difference between the summer period and the winter period is only 1.8 percentage points. With this in mind, the time series modelling will now be introduced.

In addition, Figure 5.4 is used to detect the significant intervals during a day in relation to the consumption. It is clearly seen that the time interval with highest consumption is 06-11 am and around 6 pm. **Not done**

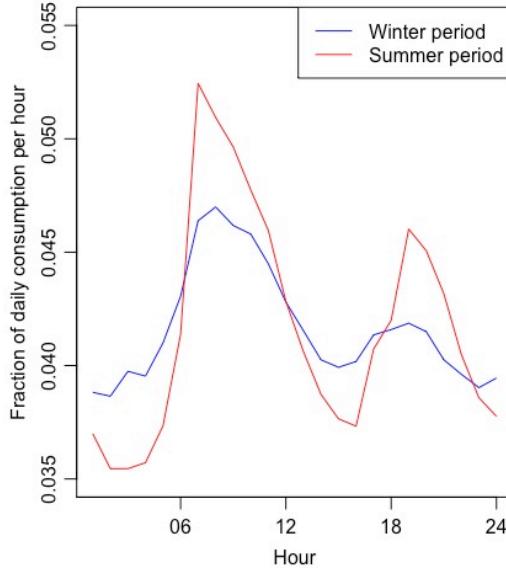


Figure 5.3: The average distribution of the heat consumption during the day for the winter period and the summer period respectively. The winter period is more smoothed out, but they are very similar

## 5.2 The ARMA Models and Their Extensions

The consumption of a house during a certain period with hour intervals is a time series. A time series is a realization of a stochastic process. In this section the ARMA model will be introduced, and an extended ARMA model, the ARIMAX model, will be fitted to the consumption. The theory of the ARMA model is based on chapter 5 from the book "Time Series Analysis" by Henrik Madsen [14]. The ARMA model fits the data to a linear stochastic process, with an autoregression part (AR) and a moving average part (MA). A linear process  $\{Y_t\}$  is a process that can be written as

$$Y_t - \mu = \sum_{i=0}^{\infty} \psi_i \epsilon_{t-i}, \quad (5.1)$$

where  $\mu$  is the mean of the process,  $\{\epsilon_i\}$  is white noise and  $\{\psi_i\}$  is the weights. For now, the mean  $\mu$  is assumed to be zero. To define the ARMA model, the backwards shift operator  $B$  is first introduced as  $B(Y_t) = Y_{t-1}$ . An ARMA process has the form

$$\phi(B)Y_t = \theta(B)\epsilon_t, \quad (5.2)$$

where  $\phi$  and  $\theta$  are polynomials on the shift operator  $B$  with degree  $p$  and  $q$  respectively.  $\theta(B)$  is the autoregressive part and  $\phi(B)$  is the moving average part. The process is

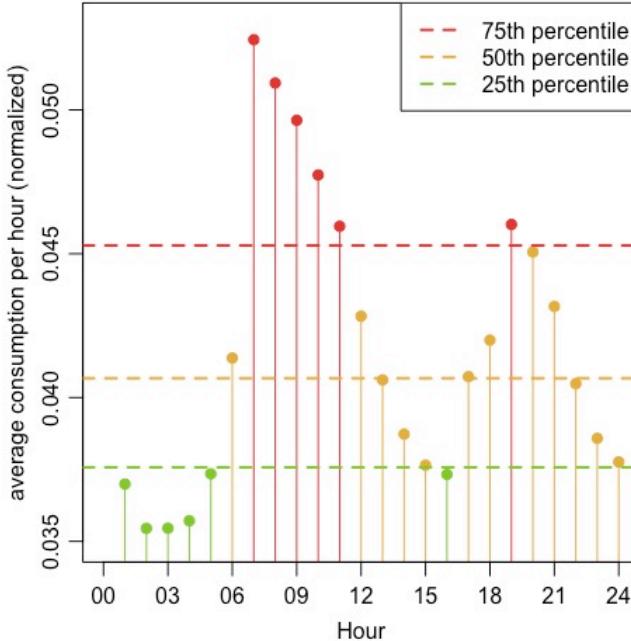


Figure 5.4: The average consumption of all houses during the day. Each point indicate the average consumption in the previous hour interval. The hours in the 75th percentile is 06 – 11 and 18 – 19

denoted as an  $ARMA(p, q)$  process. ARMA processes are linear. If one applies  $\psi(B)$  to  $Y_t$  and substitutes  $Y_{t-1}$ , then  $Y_{t-2}$  and so forth, the form in (5.1) is obtained.

An ARMA process is stationary if all the roots of  $\phi(z^{-1})$  are within the unit circle. Stationarity is a very desirable property. In a stationary process, the mean and variance does not change over time. But often, processes will not be stationary due to long term trends. For example the mean consumption of a house has a periodic trend during the year. This was clearly illustrated on Figure 3.1. But long term trends can be eliminated by introducing differencing. Instead of modelling the process  $\{Y_t\}$ , one can model the process  $\{Y_t - Y_{t-1}\}$ , i.e. the difference between observations. This is formalized with the difference operator  $\Delta = (1 - B)$ . The differenced ARMA model is called the  $ARIMA(p, d, q)$  model, or the autoregressive *integrated* moving average model. It has the form

$$\phi(B)\Delta^d Y_t = \theta(B)\epsilon_t, \quad (5.3)$$

where  $d \in \mathbb{N}$  is the differencing factor. Apart from the long term trends, the model might also have short term periodic trends. In this case a 24 hour periodic trend would be expected. The ARIMA model can be expanded to a seasonal ARIMA with

season  $s$ , such that

$$\phi(B)\Phi(B^s)\Delta^d\Delta_s^D Y_t = \theta(B)\Theta(B^s)\epsilon_t. \quad (5.4)$$

This is a seasonal  $ARIMA(p, d, q) \times (P, D, Q)_s$ .  $\phi$ ,  $B$ ,  $\Delta$  and  $\theta$  are defined as in (5.3). But here  $\Phi$  and  $\Theta$  are also included. They are polynomials in  $B^s$  of degree  $P$  and  $Q$  respectively.  $D$  is the differencing of the seasonal component of the model.

In this particular project we have access to the consumption, but also to the weather data. The exploratory analysis showed that there was a significant correlation between consumption and temperature. The temperature is also a time series, and it can be used as input to the ARIMA model to make a better fit. This is called using an exogenous variable. When an exogenous variable is used, the model is called an ARIMAX model. The following part is based on chapter 8 in [14] and the *R* documentation. The model looks like this

$$\phi(B)\Phi(B^s)\Delta^d\Delta_s^D Y_t = \theta(B)\Theta(B^s)\epsilon_t + \omega(B)X_t, \quad (5.5)$$

where  $X_t$  is the exogenous variable and  $\omega(B)$  is a polynomial in  $B$ . The exogenous part can also be differenced or have seasonal components, but in this project those extensions will not be explored. In fact only  $\omega(B) = \omega_0$  will be used in the modelling process. The software used for the arima processes is of course *R*, but in particular the *arima* function. This function does not estimate the exogenous parameters according to (5.5). It actually starts out by making a regression of the series  $\{Y_t\}$  on the exogenous variable  $\{X_t\}$ . Then this fit is substituted for  $Y_t$  in (5.4). This approach is less precise, since it executes the parameter estimation in two steps, first for the exogenous variable, then for the rest of the variables. An alternative method is to use the *MARIMA* package in *R*. This package computes the estimates in (5.5) by using different approximation methods than the *arima* function. Neither methods should be considered "correct", but they produce different results.

## 5.3 Applying the models

In this section, different ARIMAX models will be applied to the data. First the *arima* function in *R* will be used to test different models. The models will include a season of 24 hours. The *arima* models will then be used to make predictions on data not used for training, to see how well the ARIMAX model performs in general on the data set. Examples will be given on how such predictions could be visualized in WATTS. After that, new models will be generated using the *marima* package in *R*. These models will be used to calculate the impulse response of the temperature, which can be compared to the temperature estimates in the linear regression models. To apply time series models, the data has to be complete without gaps. For this reason the entire data set is used, not only the winter period. Two weeks in january 2019 are left out for testing. The temperature used in the models will be the altered temperature defined

by

$$T' = \begin{cases} \alpha - T & \text{if } T \leq \alpha \\ 0 & \text{Otherwise} \end{cases}, \quad (5.6)$$

just as in the models on the daily data.

### 5.3.1 Modelling with the ARIMA Function

To generate a model, one must decide a model order. It is important that the model is not too complex. With the amount of data available for each house, too many parameters in the model causes the running time to increase drastically. More importantly, sometimes the optimization method used in the *arima* function does not converge. This happens more often when there are more parameters, and the model should be applicable to as many different houses as possible. As mentioned in the section above, stationarity is an important property. If the order of the autoregressive part (both non-seasonal and seasonal) is too high, the model will not be stationary unless differencing is used. Differencing on the other hand makes the model more obscure and harder to interpret. If the model is only differenced once, it means that it models the difference between one hour and the one before that. If the seasonal part is differenced, it models the difference between an hour and the same hour the day before.

These considerations have led to the conclusion that a  $(1, 0, 1) \times (1, 0, 1)$  model is a good starting point. Written in the form of (5.4) it is

$$(1 - \phi_1 B)(1 - \Phi_1 B^{24})X_t = (1 + \theta_1 B)(1 + \Theta_1 B^{24})\epsilon_t. \quad (5.7)$$

Here  $Y_t$  is substituted with the regression fit  $X_t = Y_t - \beta T'_t$ . As mentioned earlier *arima* first estimates the regression variable  $\beta$ , before inserting  $X_t$  into the model. The negative sign of the  $\phi$  values is due to the convention of the *arima* function, where  $Y_t$  is isolated to one side of the equation. The model can also be formulated as

$$\begin{aligned} X_t = & \phi_1 X_{t-1} + \Phi_1 X_{t-24} + \phi_1 \cdot \Phi_1 X_{t-25} \\ & + \theta_1 \epsilon_{t-1} + \Theta_1 \epsilon_{t-24} + \theta_1 \cdot \Theta_1 \cdot \epsilon_{t-25} + \epsilon_t. \end{aligned} \quad (5.8)$$

For most houses this model is not stationary and the optimization does not converge. Figure 5.5 shows the roots of the model when applied to house 55. On the left the AR roots are illustrated, on the right the MA roots are illustrated. When considering stationarity, only the AR roots are of interest. Here it can be seen how every inverse AR root is on the edge of the unit circle. This model introduces 25 roots in  $\phi$ , 1 from the non-seasonal part, and 24 from the seasonal part. Figure 5.6 shows the same for house 18, where every AR root is on the edge of the unit circle except for one. As many of the inverses of these roots as possible should be inside the unit circle. This is not as important if the AR parameters are not very significant. But for this model, the parameters are very significant for both seasonal and nonseasonal AR,

and for both house 55 and 18. The values are listed in Table A.8 and Table A.9 in the appendix. The AR components are significant for most of the houses. For this reason, it makes the most sense to difference the model, and since the seasonal AR produces the most roots, the seasonal part will be differenced. The one root from the non-seasonal part might still be on the edge of the unit circle, but the rest will most likely not be.

Thus, the next model is a  $(1, 0, 1) \times (1, 1, 1)$  model, written as

$$(1 - \phi_1 B)(1 - \Phi_1 B^{24})(1 - B^{24})X_t = (1 + \theta_1 B)(1 + \Theta_1 B^{24})\epsilon_t. \quad (5.9)$$

This is the same model as in (5.7), except for the differencing. When rewritten, the model becomes:

$$\begin{aligned} X_t - X_{t-24} = & \phi_1(X_{t-1} - X_{t-25}) + \Phi_1(X_{t-24} - X_{t-48}) + \phi_1 \cdot \Phi_1(X_{t-25} - X_{t-49}) \\ & + \theta_1 \epsilon_{t-1} + \Theta_1 \epsilon_{t-24} + \theta_1 \cdot \Theta_1 \cdot \epsilon_{t-25} + \epsilon_t. \end{aligned} \quad (5.10)$$

This shows clearly how the model now operates on the difference between a value and the same value a day earlier. Firstly, the stationarity of this model should be evaluated. The inverse roots of house 55 and 18 are shown in Figure 5.7 and Figure 5.8 respectively. For house 55 the 24 seasonal AR roots are within the unit circle. The last root is not. While this is not optimal, it is definitely better than the last model without differencing. For house 18 all the roots are sufficiently within the unit circle. This is true for most of the houses the model was applied to. So in terms of stationarity the model performs well over all. It would be better to introduce non-seasonal differencing as well, but to keep the model simple no further differencing is applied.

After considering the stationarity of the model, the significance of the parameters is examined. The model is applied to 70 houses, making it hard to evaluate based on a few houses. Table 5.1 gives an overview of how many models have significant parameters. It shows how many of the houses have parameters below two and three standard deviations respectively. It can be seen that the seasonal AR1 is the parameter that is most often not significant. For 36 of the houses the parameter value is below two standard deviations, which is about half of the houses. For 50 of the houses, the parameter is below three standard deviations, so most of them are not significant, or barely significant. Looking specifically at house 55, the parameter values and standard deviations are listed in Table 5.2. Here the seasonal AR is only just a little bit bigger than the standard deviation, making it insignificant. If the estimates were inserted into (5.10), one would get the equation

$$\begin{aligned} X_t - X_{t-24} = & 0.9949(X_{t-1} - Y_{t-25}) - 0.0122(X_{t-24} - X_{t-48}) \\ & - 0.0121(X_{t-25} - X_{t-49}) - 0.9124\epsilon_{t-1} \\ & - 0.9572\epsilon_{t-24} + 0.8733\epsilon_{t-25} + \epsilon_t, \end{aligned} \quad (5.11)$$

for  $X_t = Y_t - 0.288T'_T$ . The autocorrelation function (acf) and the partial autocorrelation function (pacf) for this model is visualized in Figure 5.9. There are clearly some

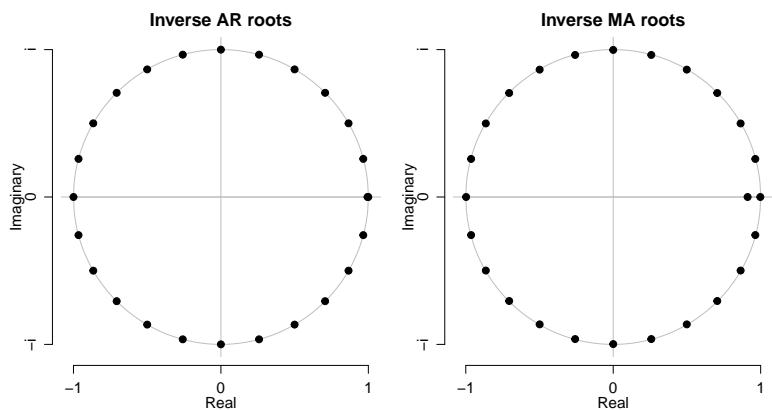


Figure 5.5: The roots of the first model when applied to house 55. All the inverse AR roots are on the edge of the unit circle

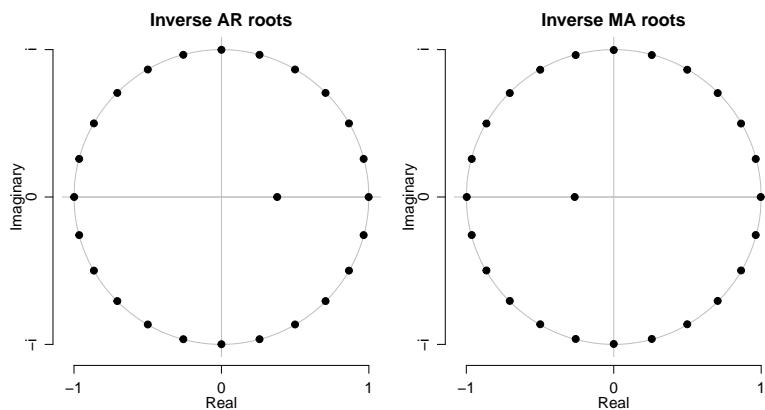


Figure 5.6: The roots of the first model when applied to house 18. All the inverse AR roots are on the edge of the unit circle except for one

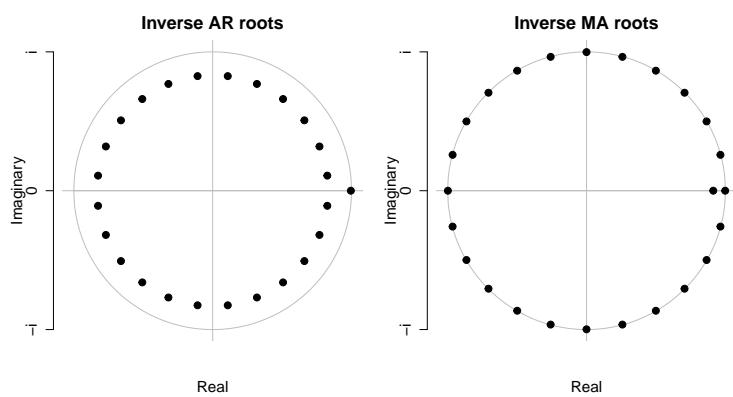


Figure 5.7: The roots of the second model when applied to house 55.

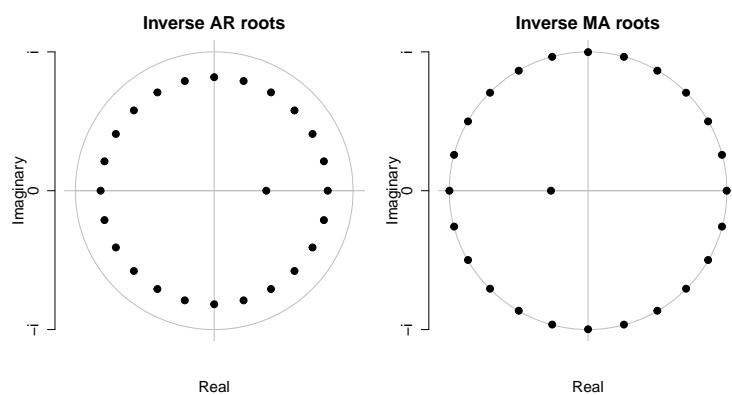


Figure 5.8: The roots of the second model when applied to house 18.

significant spikes on the first couple of lags, both in the acf and in the pacf. There are also a few significant lags scattered out on the rest of the function. It should also be noted that the lags around the seasons, indicated by the red lines, do not stand out. The model clearly has problems in the first couple of lags, where there is autocorrelation between the residuals. On the other hand the seasons seem to be accounted for. The rest of the significant lags are relatively random and not very significant. They could be regarded as white noise. The pacf show signs of some oscillation, but it might just be some distortion in the model. To get a view of the long term effects, Figure B.3 in the appendix show the acf and the pacf for an entire week. Here it is still clear that the model is far from perfect, but the biggest issue is the first couple of lags. After that, none of the lags reach more than about 0.05, which is acceptable. One lag of particular interest is the one at the last red line. This is lag 168 (the seventh season), which is the autocorrelation between weeks. It shows the autocorrelation between the current residual and the same residual one week earlier. A high autocorrelation in lag 168 could be a threat to the performance of the model. Fortunately, no lags around this season seem to stand out for house 55. So for this model the effect between weeks, which is not taken into account, does not seem to have much influence. If one were to try to address the high correlation in the first lags, the non-seasonal part of the model could be expanded with a higher degree of either AR or MA.

Now the model will be evaluated for house 18. The estimates for the model are listed in Table 5.3. Here the seasonal AR1 is even more insignificant than it was for house 55, being less than the standard deviation. The log likelihood of the model for this house ( $-12780$ ) is also much lower than for house 55 ( $-5791$ ), or than the average of the model ( $-6999$ ). After insertion into (5.10) the model for house 18 is

$$\begin{aligned} X_t - X_{t-24} = & 0.3751(X_{t-1} - X_{t-25}) + 0.008(X_{t-24} - X_{t-48}) + 0.003(X_{t-25} - X_{t-49}) \\ & + 0.2672\epsilon_{t-1} - 0.9559\epsilon_{t-24} - 0.2554\epsilon_{t-25} + \epsilon_t, \end{aligned} \quad (5.12)$$

Parameters	AR1	MA1	SAR1	SMA1	Temperature
Below 2 sd	0	3	36	0	6
Below 3 sd	0	3	50	0	14

Table 5.1: The number of houses where each parameter is below two standard deviations and three standard deviations respectively. The  $(1, 0, 1) \times (1, 1, 1)$  model was applied to 70 houses in total. The average log likelihood of the model is  $-6999$

House 55	AR1	MA1	SAR1	SMA1	Temperature
Estimate	0.9949	-0.9124	-0.0122	-0.9572	0.0288
Standard deviation	0.0013	0.0048	0.0111	0.0043	0.0045

Table 5.2: The estimates of the parameters for the  $(1, 0, 1) \times (1, 1, 1)$  model together with their standard deviations for house 55. The log likelihood of the model is  $-5791$

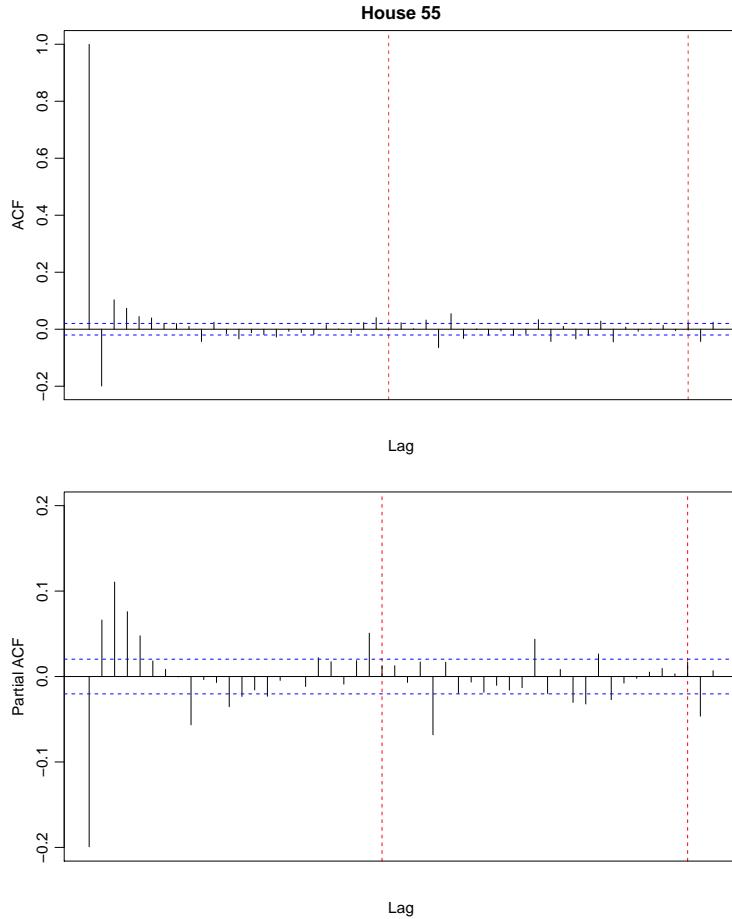


Figure 5.9: The acf and pacf of the second model when applied to house 55. The 24 hour seasons are highlighted with red dashed lines. The blue lines indicate the 95% confidence interval

Where  $X_t = Y_t - 8.15T'_t$ . Looking at the acf and pacf of this house on Figure 5.10, this house does not have the same high autocorrelation in the first lags. Actually both the acf and the pacf look slightly better for house 18 than they did for house 55, even though house 18 overall has shown unpredictable behavior. There are still significant lags in both the acf and the pacf, but as for house 55, they are not very large and can to some extent be regarded white noise. As for the 24 hour seasons, there are no significant outliers. There is no pattern indicating that the residuals for the 24 hour seasons are not sufficiently accounted for in the model. When looking at Figure B.4 in the appendix where an entire week is included, there is a single peak at season four that stands out. But since the other seasons do not show the same behavior, it might as well be a coincidence. Just like for house 55 there is no significant peak at the seventh season. Overall, even though there are many significant peaks in both the acf and the pacf, there is no obvious pattern connected to the seasons or the first lags. And none of the peaks reach a value higher than 0.1.

### 5.3.2 Predicting with the ARIMA model

Now the model found above will be used for predictions. The test data used is two weeks in January 2019. The predictions are made using the R function `predict`. The weather data used as input can be seen in Figure 5.11. Just as in the linear regression model, the input series is assumed to be a perfect prediction without measuring errors. The predictions are not one-step predictions, but 14 step predictions. The results of applying the predictions to house 55 can be seen in figure Figure 5.14. It very quickly becomes clear that the data behaves very differently than the model predicts. There are a lot of very high spikes, each followed by an hour with zero consumption. This tendency is not captured at all in the model, and it is clear that the time series models cannot be used for anything useful for this house unless the data is altered in some way. The temperature input was very low for a few days in the end of the test period, but the effect this had on the predictions is minimal compared to the oscillations in the data. House 18 in figure Figure 5.15 shows the same unexpected behavior. There seems to be no connection between the time of day and the massive spikes for either of the houses. And since the spikes are usually followed by a consumption of precisely zero, the behavior seems to be linked to how the data is collected and pre-processed by Aalborg Forsyning. Given that the anomalies come in pairs of measurements that are two or three times higher than the neighbours and than measurements of

<b>House 18</b>	<b>AR1</b>	<b>MA1</b>	<b>SAR1</b>	<b>SMA1</b>	<b>Temperature</b>
Estimate	0.3751	0.2672	0.008	-0.9559	0.0825
Standard deviation	0.0171	0.0175	0.011	0.0040	0.0078

Table 5.3: The estimates of the parameters for the  $(1, 0, 1) \times (1, 1, 1)$  model together with their standard deviations for house 18. The log likelihood of the model is  $-12780$

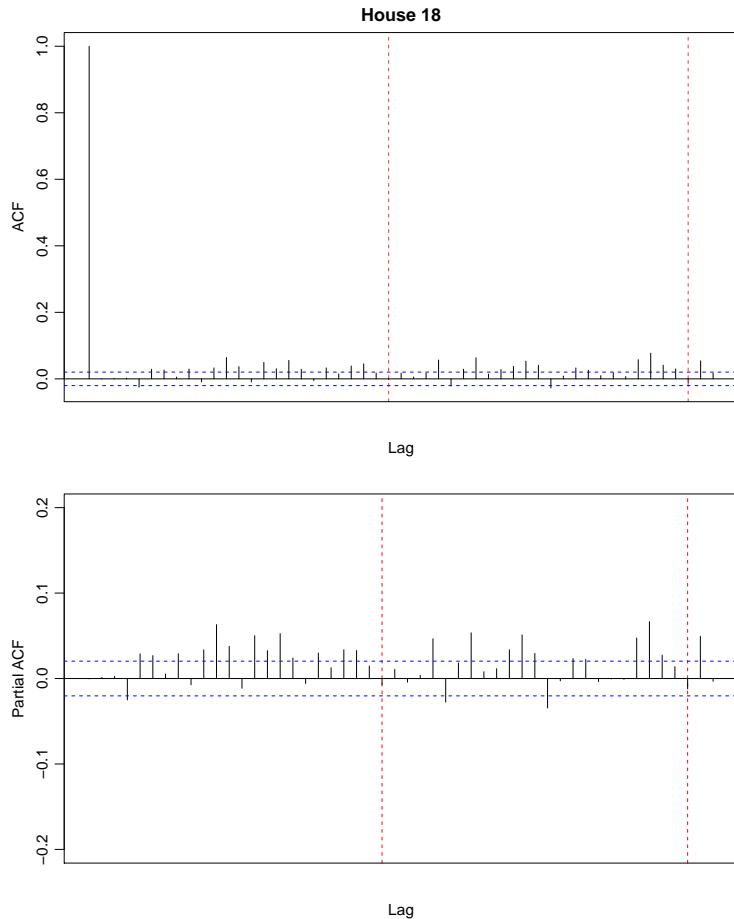


Figure 5.10: The acf and pacf of the second model when applied to house 18. The 24 hour seasons are highlighted with red dashed lines. The blue lines indicate the 95% confidence interval

zero, the most plausible explanation is that several measurements are accidentally grouped into a single hour interval, leaving the next interval with no consumption. The phenomenon with a high spike followed by a zero happens almost every day for most houses, and sometimes several times a day.

To come up with a more fair evaluation of the model, the data is changed slightly to compensate for the random spikes. Every time there is a consumption of precisely zero, that data point and the one before are both changed to the average of the two. This smoothes out the outliers that seem to be caused by erroneous data collection, without changing the rest of the data too much. The model is now trained and tested on the changed data. The results are shown in Figure 5.14 and Figure 5.15. For house 55 the data still have some significant spikes in the first part, but it is more well-behaved in the rest of the test period. Here it is mostly within the prediction interval, but the prediction interval is rather wide. In general the model shows that there is a lot of variance in the data that is not captured by the model. The model could be applied, but it will not perform very well. As for house 18, the data is still way off compared to the predictions. The smoothing did not change the way the data behaved, and there are still a lot of spikes that are so far from the prediction interval, that the model does not say anything useful about this house.

For the houses that only had data between September 2018 and January 2019, it turned out that there were no anomalies of the type described earlier in this section. This could mean that the data from these houses is collected in a more reliable way. Therefore predictions for one of these houses will also be included. The predictions are illustrated in figure Figure 5.16. The figure shows clearly that the hourly data is still much too unstable to apply the models. There is a lot of variance that is not captured at all in the predictions, and the prediction interval is pretty wide.

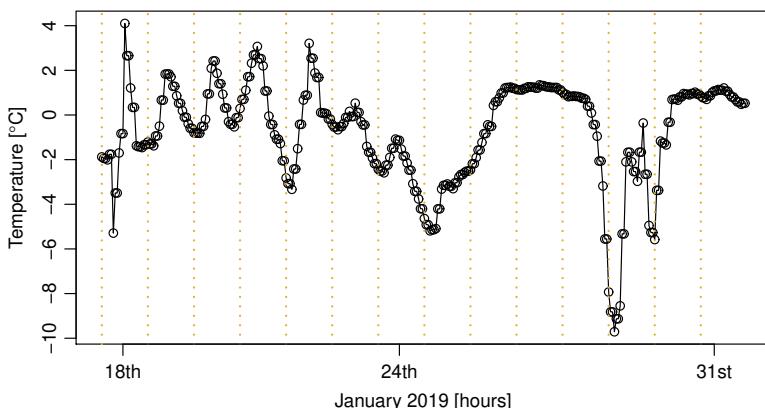


Figure 5.11: The input series of the temperature used for predictions made with the arima model

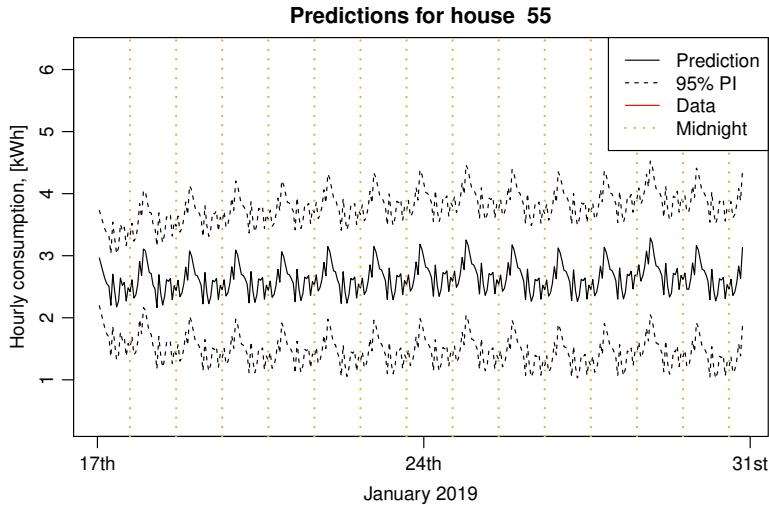


Figure 5.12: Predictions of the arima model for house 55. The data is so far from the predictions and oscillate so much that the model should not be relied upon

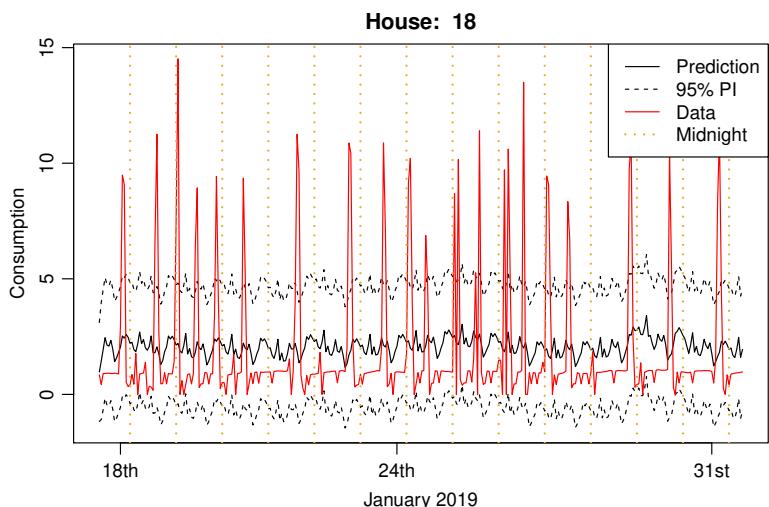


Figure 5.13: Predictions of the arima model for house 18. This is just as bad as for house 18

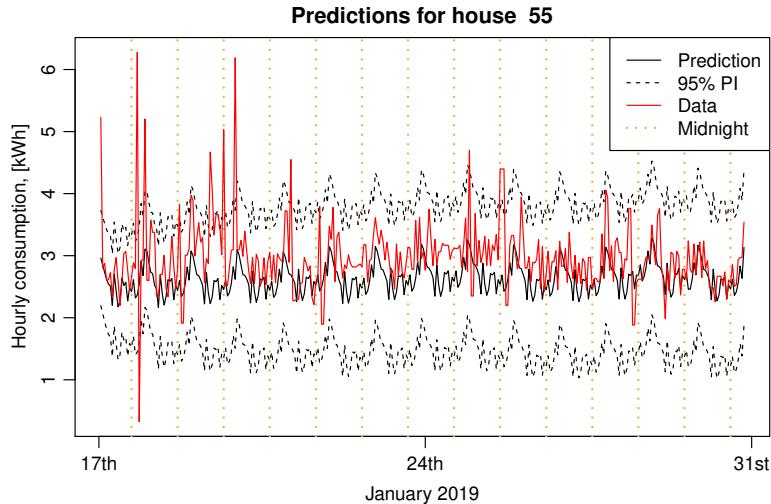


Figure 5.14: Predictions of the arima model for house 55 on the modified data. There are still a few spikes, but the model performs reasonable on this data set

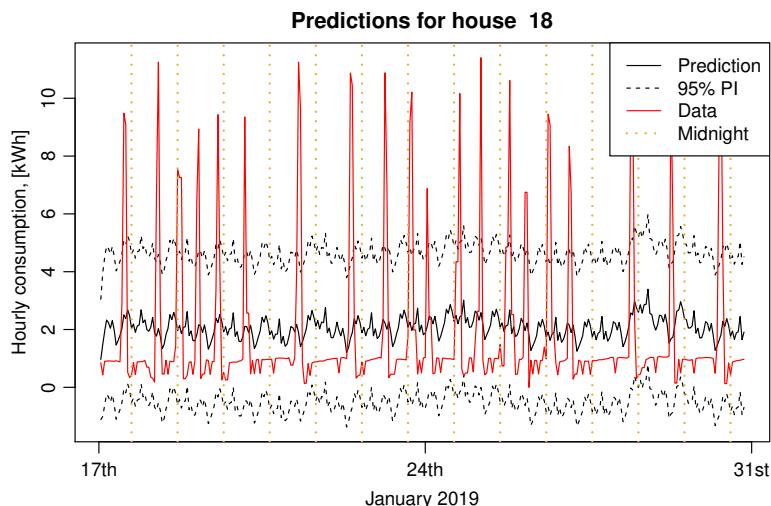


Figure 5.15: Predictions of the arima model for house 18 on the modified data.

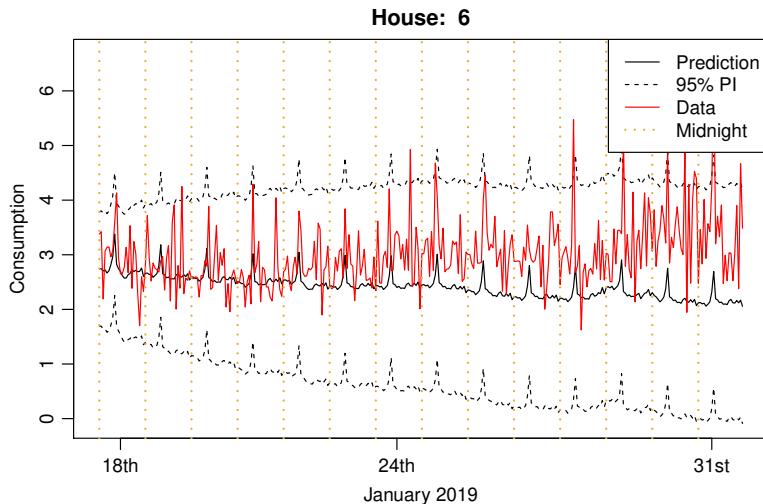


Figure 5.16: Predictions of the arima model for house 6, which is one of the houses with data for a shorter period

### 5.3.3 Visualization of the results

The results received from the time series models so far have not been very reliable, and in general the model should not be used for predictions at this stage. But if one were to visualize the predictions found, it could be done by making a colouring similar to that of the regression model. An example is shown in Figure 5.17 with predictions for just two days. The colouring is made in the same way as for the predictions made by the regression model in Figure 4.15, where there according to the model there should be an equal amount of each colour.

### 5.3.4 Modelling with the MARIMA Function

It has been mentioned earlier that MARIMA works in a fundamentally different way than the ARIMA function. It uses an iterative method to estimate the parameters, including all the regression variables. The exact procedure can be seen in [17]. The goal is to make a model that can be compared to the linear regression model. This model will be more focused on the physical factors affecting the consumption. This means that there will be no differencing and no seasonal component. The model will seek to compute the consumption based on some weather data as exogenous variables, the consumption of the previous couple of hours and a moving average of the residuals. The exogenous variables will be the temperature and the solar radiation, since these are the variables that are the most significant in the linear regression model. Some of the wind directions were not very significant, so the wind is not included in the

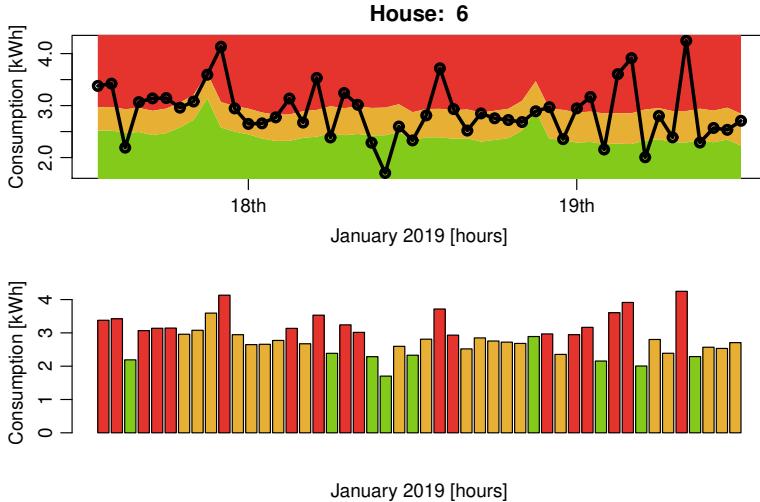


Figure 5.17: Visualization of the results of the arima model applied to house 6 for two days in January 2019. The exact period is from 00 : 00 on the 18th of January to 23 : 00 on the 19th of January

model. The overall formulation of the model that will be estimated is

$$\phi(B)Y_t = \omega_T(B)T_t + \omega_{SR}(B)SR_t + \theta(B)\epsilon_t, \quad (5.13)$$

where  $Y_t$  is the consumption at time  $t$ ,  $T_t$  is the modified temperature (see (4.3)),  $SR_t$  is the solar radiation,  $\epsilon_t$  is the error, and  $\phi(B)$ ,  $\omega_T(B)$ ,  $\omega_{SR}(B)$ , and  $\theta(B)$  are all polynomials of the shift operator  $B$ . The moving average ( $\theta(B)$ ) is included to make the model compensate for measuring errors made along the way. The degree of  $\omega_T(B)$ ,  $\omega_{SR}(B)$ , and  $\theta(B)$  will be one in models considered in this paper. One could argue that the temperature or solar radiation could have an influence on more than one lag. ...The model evaluation will decide which order of  $\phi(B)$  that provides the best estimates. This degree is a measure of how many of the previous hours have an influence on the current consumption. The degrees that will be tested are one, two, three and four. Most of the tested houses give optimal results for these degrees, and higher degrees would result in a model that is too complex.

If the inside temperature had been included in the data, it would have been present in (5.13) together with the outside temperature and the solar radiation [8]. But since it is assumed to be constant, the inside temperature is included in the mean of the model, which has been omitted in the equation. There is a way to allow the inside temperature to vary over the course of the day that will be tested here. That is to make a fourier series with a certain number of terms that can describe the variations. The period of the series is 24 hours. Each sine and cosine function in the series will be assigned a parameter in the model. By taking the sum of the trigonometrical

functions and their weights, a "mean" that is dependent on the hour of the day is obtained. The expanded model is

$$\begin{aligned}\phi(B)Y_t = & \sum_{i=1}^n \left( \omega_i^{sin} \sin \left( 2\pi \cdot t \cdot \frac{24}{i} \right) + \omega_i^{cos} \cos \left( 2\pi \cdot t \cdot \frac{24}{i} \right) \right) \\ & + \omega_T(B)T_t + \omega_{SR}(B)SR_t + \theta(B)\epsilon_t,\end{aligned}\quad (5.14)$$

where  $\omega_i^{sin}$  and  $\omega_i^{cos}$  are the parameters for the sine and cosine function of the  $i'th$  term of the Fourier series. To limit the number of variables in the model, a Fourier series with  $n = 2$  is used for modelling. This gives a model with seven parameters other than the ones from  $\phi(B)$ .

For model selection the Bayesian information criterion (BIC) is used where the model with the lowest BIC is preferred [18]. This criterion is chosen, as the penalty term for resolving overfitting problems is larger for BIC than AIC. The criterion is formulated as

$$BIC = \ln(n)k - 2\ln(\hat{L}), \quad (5.15)$$

where  $\hat{L}$  is the maximized value of the likelihood function,  $n$  is the number of observations and  $k$  is the number of parameters estimated by the model.

The models will be tested for house 55 and 18, and for house 6 as well. House 55 and 18 are modified in the same way as they were in the `arima` section. As described earlier, the orders of the models tested are MARIMA(1, 0, 1), MARIMA(2, 0, 1), MARIMA(3, 0, 1) and MARIMA(4, 0, 1). The BIC values for each of the models can be seen in Table 5.4. These are the models where the Fourier series is not included, i.e. the model described in (5.13). Based on the BIC, the AR orders should be 2, 4 and 2 for house 55, 18 and 6 respectively.

BIC	House 55	House 18	House 6
MARIMA(1, 0, 1)	19431.2	31088.3	7272.7
MARIMA(2, 0, 1)	17907.1	31093.4	7145.1
MARIMA(3, 0, 1)	17928.6	31049.6	7147.1
MARIMA(4, 0, 1)	17939.1	30985.2	7156.8

Table 5.4: The BIC of the different models without Fourier series. The lowest BIC is chosen as the best order for each house. This gives an AR degree of 2, 4 and 2 for house 55, 18 and 6 respectively.

The parameters for the optimal BIC model for house 55 are listed in Table A.10 without the Fourier series and in Table A.11 with the Fourier series in the appendix. For that house every parameter is more or less significant, but for the other houses some of the parameters are not significant. The solar radiation is mostly not significant. It is hard to interpret the parameters of the model in a physical way. There is no obvious connecting between for example the temperature parameter  $\omega_T$  and the

estimate for the temperature in the models for the daily data. But one way to relate the time series models to the daily estimates is to calculate the step response. The step response is defined as the total change in the consumption over time when one of the inputs is changed by one. Figure 5.18 shows an example for house 55. The temperature is increased by one and the rest of the parameters are kept constant. The consumption converges towards a new level as time passes. When the consumption has reached its new level, the difference between the initial consumption and the new consumption is a measure of the step response of the temperature.

The different step responses without Fourier series for the houses are illustrated in Figure 5.19. For each house, the leftmost estimate is the one for the regression model on the daily data. The middle estimate is the one generated by the Marima found based on the BIC criterion. It can be seen that even though the estimate values are similar, the confidence interval is much wider for the Marima model. To get an estimate with a tighter confidence interval, another round of model selection is made, where the model with the smallest variance is chosen (the CL model), instead of basing the model selection on BIC. This is the rightmost estimate on the figure for each house. Compared to the BIC model, the CL model naturally provides more certain estimates, but they are still not as good as the ones for the regression model. For house 18 the difference is very big, but for house 55 and 6 the CL model is just slightly better.

Figure 5.20 shows the same estimates for the model with the Fourier series included. This improves some of the estimates. For house 55 and house 18 the overall perfor-

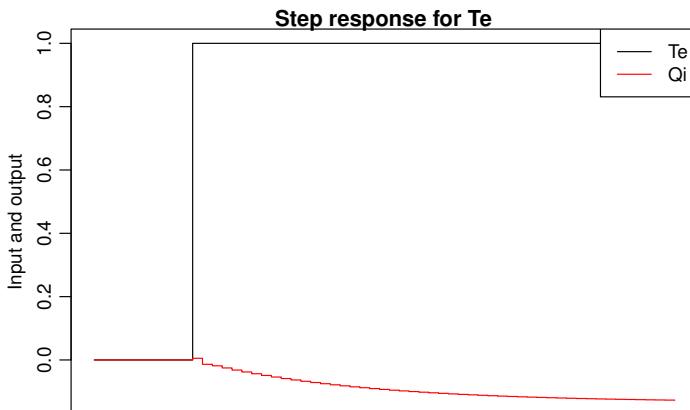


Figure 5.18: The step response for the temperature input for house 55. The red line converges towards a new constant level, which is the step response

mance is improved, but the estimates are still not as good as the regression model. House 55 seems to be the one where the certainty is closest to the regression model. But it still has a larger confidence interval, and the others are even worse. The step response estimates for the temperature with the CL model and with the Fourier series is listed in table... and the step response estimates for the solar radiation in ... The solar radiation will not be compared to the regression model, because the parameters were not nearly as significant as the temperature, making the estimates unreliable. To summarize, the step response temperature estimates for the best model and the best house is still not as good as the regression model estimates for that house. The time series models simply cannot produce results that are as precise as the regression models.

	$\omega_T$	Std. errors	2.5%	97.5%
House 55	0.131	0.007	0.117	0.146
House 18	0.102	0.003	0.096	0.109
House 6	0.160	0.117	-0.070	0.389

Table 5.5

	$\omega_S$	Std. errors	2.5%	97.5%
House 55	0.0006	0.0003	-6.21e-06	0.1318
House 18	-0.0007	0.0001	-0.0009	0.1026
House 6	0.0013	0.0069	-0.0122	0.1732

Table 5.6

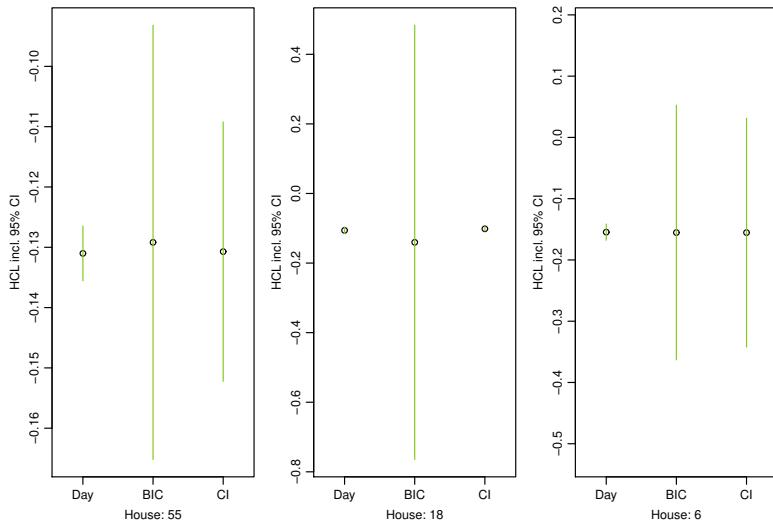


Figure 5.19: The temperature estimates from the regression models compared to the temperature step response for the Marima models without Fourier series. The regression model on the daily data is far superior. The Marima estimates are included for the models found based on BIC and the models with the smallest confidence interval.

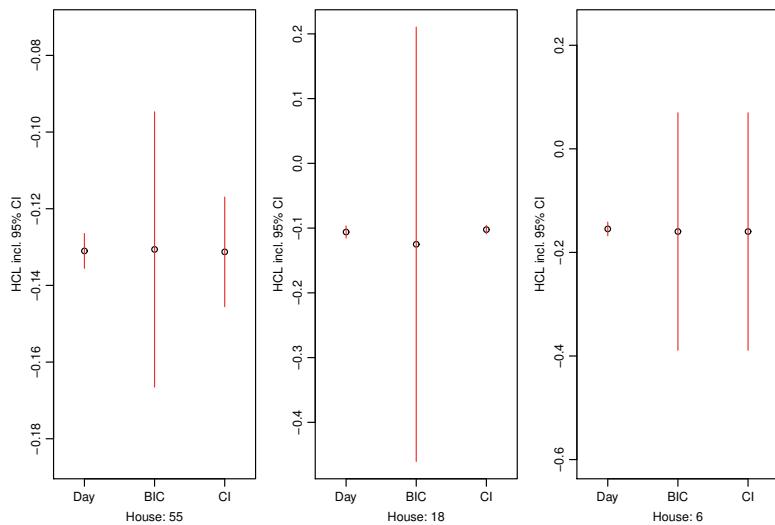


Figure 5.20: The temperature estimates from the regression models compared to the temperature step response for the Marima models with Fourier series. There Fourier series does not improve the step response. The Marima estimates are included for the models found based on BIC and the models with the smallest confidence interval.



# CHAPTER 6

# Discussion

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Now two different types of models have been developed - a linear regression model for describing the heat consumption as daily values and a time series model for the hourly consumption.

For the daily values both, a simple and a multiple regression model have been tested. The simple model only included the temperature as the independent variable. In relation to validation of the model, the diagnostic plots were not met at all. The model did not take several factors into account, which was seen in the residuals behaviour, thus the model was too simple. The multiple model started out as a full model that included all the relevant weather attributes. The purpose was to determine which factors should be included in a general multiple model for comparison of houses. The parameters that were significant for most of the houses were the temperature, solar radiation, the wind directions east and west. In the general regression model, all wind directions and their interaction with temperature were included. The behaviour of the residuals were definitely improved compared to the simple model and the model was concluded to be valid. By investigating the estimates of the temperature from the general model it was seen that the temperature were highly significant with negative sign for all houses and none of the 95% confidence intervals were above zero. Furthermore, predictions for January 2019 using the model were used to support that the estimates indeed were good. The model was good at capturing the overall trend, and therefore the estimates could be concluded to be good enough to describe the influence of the temperature on the heat consumption. However, the predictions were made under the assumption that the weather forecasts were exact for the chosen period. This causes uncertainties for the predictions as they become better than actual data. Overall, the general regression model performs as it should, and it can be used to describe the daily heat consumption despite the uncertainties of using the model for predictions. **Mangler lidt.**

For the hourly values, different time series models was used on the data. First different models were tested and evaluated using the `arima` function. A single model was developed that could be applied to every house in the data set. The model was used to make predictions for unknown data, but it was not able to describe the consumption in a very satisfying way because random variations in the hour data were too dominating. A modification of the data was made to remove some of the

variations from the data, but the model still did not perform well. Then a series of different models was developed using the `Marima` package in R. Different models were applied to a handful of houses. The models were used to compute the step response for each of the houses. But even for the best models modified with Fourier series, the estimates of the step response were much more uncertain than the temperature estimates found with the regression models. The validation of the regression model was much better than the validation of the time series model, so this is clearly the estimate that should be trusted.

As mentioned in Chapter 2 a function has been proposed that can reject houses from being modelled if they do not fulfill certain criteria. The function checks if there is a certain number of observations for each house. One can discuss whether the function could also be used after the model development. Based on the results given in Chapter 4 there are several houses that could probably be omitted because they have too few observations or there are too many interpolations of missing observations in the house measurements. It can be problematic to predict a certain number of days ahead, e.g. 60 days, if there only exists observations for half a year. Furthermore, if there are too many missing values that have been interpolated with the method mentioned in Chapter 2, then a data-checking function should discard the house, as it will have a negative impact on the further modelling of the heat consumption for that specific house.

The lack of information on e.g. the number of residents in the house, the house's location, etc. affects the uncertainties of the models. The tap water consumption can depend on how many people live in the household, and there is therefore a high probability that the hot water consumption is higher than the heat consumption. This creates greater uncertainty as the models do not take the tap water consumption into account. The location of the house affects the estimates of the four different wind directions. If the location of each house was known, it would be easier to interpret the estimates. For example, if a terrace house is located between two other houses, it would make sense that the heat consumption would be less affected when the wind came from the directions where the house adjoins the other two houses.

Overall, there is a major problem with the way data is collected. It is clearly seen on an hourly basis, since data is measured per hour. By converting the measurements into daily values, the strange behavior of the measurements becomes less evident, which results in the regression models being used to describe the daily heat consumption. The biggest challenge then arises when the time series models are used to describe the hourly consumption.

## 6.1 Future work

Although the quality of data on an hourly basis has not been good enough to construct time series models, there is still something to work on in the project. At the

same time, there is also a part that can be improved in relation to data and the way data is measured.

As mentioned, the models for the hourly values do not work particularly well, so one of the things one could primarily work on, is to achieve more stable data. It does not make much sense to continue working on the models, as they cannot produce any useful results at this time. Another way of working on the time series models in the future, could be to investigate the winter period. The heat consumption is low during the summer, so it is the winter period that is of interest, and the models could possibly be improved by this action.

In proportion to data on a daily basis, an immediate idea would be to compare the wind dependency plots presented in Figure 4.14 and Figure 4.13 with the location of the houses and then use Google Earth to visualize the consumers house surrounded by a compass and the coloured shapes. Future work could also be a clustering that compares the houses' consumption with other houses in the same class. The clustering could be made based on the way the houses are heated, the houses' areas, year of construction etc.

The distribution of the tap water consumption is not quite clear, so if it could be identified when houses have some sort of "type" consumption such as wood stove, water pump, etc., it would be possible to remove the tap water consumption from data. It would most likely result in an improvement in the performance of the models. Knowledge of the different heating patterns and methods in the houses, could then be used to classify the houses, and specific models could thus be developed for the different classes.

The method used for choosing the break point explained in Chapter 3 could also be constructed in a more sofisticated way. And last but not least, the Data Checking function could be expanded such that it could determine how much data is needed for one house in order to make the models more robust. For example, one could also consider removing houses where the heat consumption is zero for an extended period, such as house 18.



## CHAPTER 7

# Conclusion

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Den helt overordnede konklusion er, at vi kan lave statistiske modeller til analyse af varmeforbruget på dagsværdier. Men hvis vi gerne vil kigge på det eksakte data, dvs. timeværdierne, så er der desværre for mange usikkerheder ved målingerne **noget om hvordan målerne har målt det** og det bliver for svært at analysere varmeforbruget. Vi har forsøgt os med tidsrækkestørrelser, men de kan ikke opfange den overordnede svingende tendens, der er i data, hvilket gør dem mere upålidelige. Tidsregistreringen skal være ordentlig, så der rent faktisk er en måling pr. time



APPENDIX **A**

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**Tables**

## A.1 Estimates and test of the simple linear regression model

House	Intercept	Temp.	House	Intercept	Temp.
1	66.93507	-2.94519	36	79.613802	-3.423364
2	94.849392	-3.837028	37	175.01976	-10.07022
3	115.650301	-5.686036	38	99.935399	-5.612365
4	80.488183	-5.200701	39	176.52784	-10.82685
5	108.187010	-5.373519	40	120.942228	-6.225172
6	70.482101	-3.106761	41	91.264752	-5.856187
7	60.107374	-3.329154	42	74.110457	-3.148313
8	181.291957	-7.881039	43	96.635504	-5.028666
9	39.385258	-2.171951	44	49.816478	-2.296001
10	468.90208	-21.34012	45	702.95358	-31.44629
11	79.618560	-3.402589	46	79.822266	-4.284348
12	128.062571	-5.867193	47	36.271842	-2.470427
13	120.621917	-4.788174	48	85.677722	-4.535974
14	166.767618	-8.160587	49	111.41920	-5.08165
15	84.352201	-4.469187	50	124.573116	-5.443588
16	112.795215	-6.271305	51	125.613274	-5.697299
17	228.84416	-11.34288	52	138.081716	-9.374576
18	45.110057	-1.924089	53	76.837848	-4.261505
19	126.011426	-5.524056	54	87.162333	-4.655499
20	50.140339	-3.119199	55	73.522690	-3.077972
21	80.400589	-3.458884	56	61.14127	-2.77719
22	72.57428	-4.31138	57	83.940177	-3.589415
23	111.675240	-5.030761	58	72.667315	-3.228788
24	113.672924	-5.108836	59	99.937438	-5.062137
25	211.766719	-8.393969	60	128.735211	-7.009724
26	141.164214	-6.250304	61	103.67781	-4.93304
27	76.54401	-3.38948	62	109.019035	-4.845601
28	847.9319	-45.9522	63	329.14403	-14.03652
29	139.121920	-6.472266	64	64.96314	-3.41983
30	124.229223	-6.362637	65	66.288135	-3.581266
31	135.181578	-7.069286	66	149.708831	-7.375122
32	67.205748	-2.310322	67	103.599478	-4.068565
33	100.039780	-4.430941	68	128.908447	-6.170165
34	105.436649	-4.222979	69	112.26372	-5.04636
35	96.635504	-5.028666	70	101.162713	-4.680857

Table A.1: Estimates of the simple linear regression model

House	Shapiro-Wilk	Sign	House	Shapiro-Wilk	Sign
1	1.269662e-07	1.545711e-01	36	0.0318435	0.6049162
2	0.2120532	0.8971008	37	0.5588932	0.4376862
3	0.06289541	0.89710079	38	0.002469713	0.244235318
4	0.1693807	0.8971008	39	0.1319804	0.8534084
5	0.9020079	0.8971008	40	0.2151643	0.6980220
6	0.5118580	0.5792903	41	0.7165372	0.6980220
7	2.400099e-10	9.597271e-05	42	0.5484956	0.1956672
8	0.3662207	0.5792903	43	0.8539095	0.8534084
9	0.01292524	0.35527968	44	0.09057111	0.79590239
10	0.5039661	0.7043793	45	0.8307638	0.5178173
11	0.2445798	0.6980220	46	0.7925841	1.0000000
12	5.838099e-06	4.376862e-01	47	1.763434e-05	2.847400e-03
13	0.01173622	0.85340845	48	4.364622e-05	8.971008e-01
14	0.2712269	0.5178173	49	0.09749062	0.69802197
15	0.9682545	1.0000000	50	0.6649854	0.6049162
16	0.07409811	1.0000000	51	0.6341959	0.4596914
17	0.9973268	0.5792903	52	4.159959e-05	7.959024e-01
18	1.017533e-06	3.006793e-01	53	0.4897047	1.0000000
19	0.2113022	0.3552797	54	0.008756327	0.195667225
20	0.6677595	0.7117033	55	0.5201494	0.4376862
21	0.1555910	0.5178173	56	0.4827258	1.0000000
22	0.003089513	1.0000000	57	0.005584645	0.795902390
23	0.09877236	0.30067932	58	0.4809962	0.8971008
24	0.4613530	0.5792903	59	0.6763879	0.7117033
25	0.9855876	0.4596914	60	0.6583508	1.0000000
26	0.01226589	0.35527968	61	2.841694e-05	1.968680e-02
27	0.5094042	1.0000000	62	0.009033716	0.355279682
28	0.5752663	0.3006793	63	0.6070834	0.8534084
29	0.1056845	0.3651867	64	0.002157001	0.517817269
30	0.4512082	0.8971008	65	0.09290001	0.19566723
31	0.4481276	1.0000000	66	0.002398244	0.604916209
32	4.134475e-08	1.380207e-02	67	0.735418	1.0000000
33	0.4464175	0.5178173	68	0.2093689	0.5792903
34	5.011367e-06	5.178173e-01	69	0.0001833683	0.5792902502
35	0.8539095	0.8534084	70	0.1393319	0.2671823

Table A.2: P-values from Shapiro-Wilk test and sign test on the simple linear regression model

## A.2 Estimates and test of the multiple linear regression model

### A.2.1 Significance of parameters for full model

Index	I	T	N	E	S	W	MSL	SR	WB	SB	AB	CB	WKND	T:N	T:E	T:S	T:W	
1		-***		+. .		+*** +*			-***	-***				+. .	-***			
2	+**	-***		+**		+***		-*	-*	-*		+*				-***		
3	-***		+***			+*** +*** -***	-***	+. .		-*				+**	-*			
4	-*	-*** +*		+***		+*** +*** -***	-***											
5	-***		+***			+*** +*** -***	-***	+**	+***					+*	+***	-**		
7	+*** -*** -**	+*** -**	+*** -**	+**	+**	-*** -***	+**	+*	+***					-***	+*	-**		
11	+** -***		+***			+***		-***	+***			+*			+**			
12	-***					+*						-*						
14	+*	-***		+***		+***		-**	+***		-*				+**			
18	+*** -*		+*** -*	-*	+**	-**	+.				-.			+**	-***			
21	+** -***		+*			+***					+*				-***			
22	-***			+*** +**		-***		-**			+.			-*				
23	+* -*** -.	+*	+***			-***			-.					+***	-***			
28	-***		+**			+. .	-***		-*		-***	-***						
29	+. -***		+***			+***		-**	-*	+.			+. .	+*	-***			
30	+. -***		+***			+***		-***	+*	-*				+. .	-			
31	-***		+***			+*** +**	-***	+.	-.				+*	+***	-***			
32	-**			+***		-**		-**	-*					+*	-*			
33	-***		+***			-**												
34	+. -***		+***	+. .	+***	-**			-*	+***	-*				*			
36	+** -***		+***			+**	-***				-*				+**			
37	-***		+***			+** +**	-***											
38	-*** -***		+**			+*** +*** -***	+**	-.		-*	+.		+. .	+. .	-***			
40	+* -***		+* +**	+**		-***		-**	-*	-.				+. .	-*			
41	+*** -***		+***			+*** -.		+.	-.					+. .	-			
42	-***		+***			+*** +*** -***		+.			-***			+*	-*			
44	-*** -*					-**					+**				+**			
45	+** -***			+*	+***	-.			+*					+*	+. .	-**		
46	-***		+*	+*	+**	-**					-*					-**		
47	-***		+**	+*	+***	+	-*	-.		+***	+***	-**	-***			-***		
48	+* -***		+**			-**	+**					-***				-**		
49	+* -***					-**												
50	-*** -*	+*** -**	-** +*			-*** +**							+. .		+***			
52	+. -*** -*	+*	-*	-*		-** -*** -***			-***				+. .		+***			
54	-***		+**			+** -***			-.	-*				+**	-*			
55	-***		+*			+*** +*	-***		-.	-*				+***	-*			
56	+** -***		+**			+*** -** -.		-.	-*					+*	-*			
57	+. -***		+***			+**									-*			
58	-*** -.	+. .	+***			-***				+**		+*		+***	-*			
61	+*** -***		+**			+** -*** +.	+**		-.		-.			-*	+**	-**		
64	-*** -*	+***		+**		-*** +.				-*		-*		-*	+**	-**		
65	-***		+*			+*** -***									+*	-**		
66	-***		+***	+*	+***	-***			-.	-.				+*	-**			

Table A.3: Significance of parameters from the full multiple linear regression model performed on 'long' houses. The meaning of the stars is as follows: \* = significant, \*\* = more significant and \*\*\* = highly significant.

Index	I	T	N	E	S	W	MSL	SR	AB	CB	WKND	T:N	T:E	T:S	T:W
6		-***		+*		+**		-*	-.					+	-*
8	++*	-***		+*		+***		-*							-**
9	-***	-.			+*	+**	-***	-*		-*	+		+*	-*	
10	-***		+***		+***	+	-**			-**	+			-*	
13	-***	+*		+*	+*	-*				-.	+		+**	-*	
15	-***	+***			+**		-.		-.					-.	
16	-***	+*		+***	+	-*								-**	
17	-***	+**	+**	+***		-**				-.				-***	
19	-***	+**	-.		+		-.			-.	+		+**		
20	-***			+*											
24	-***	+*		+***		+*								-***	
25	-***	+**		+***								+	-*		
26	-***	+	+	+***	+*	-**	-***							-*	
27	-***	+*	+**	+***						-*				-*	
35	-***	+**	+*	+***		-**				+*				-**	
39	++*	-***		+*	-*	-.				-.					
43	-***	+**	+*	+***		-**			+*					-**	
51	-***	+		+***		-**			-*			+	-*		
53	-***	+*		+*										-.	
59	-***	-.	+***	+*	+***	-.					+	+	-.		
60	-***	-*	+**	+	+***	-***					+*	+	+*	-***	
62	+	-***		+*		-.	-*								
63	+	-***	+*							-**			+		
67	+	-***	+***		+***	-***	+	+*			-*	+	-*	-**	
68	-***			+*		-***		+*	-**			+*	-*		
69	-***			+	+**	-.	-***				+				
70	-***		+	+	+***	+**	-***						-*		

Table A.4: Significance of parameters from the full multiple linear regression model performed on 'short' houses. The meaning of the stars is as follows: \* = significant, \*\* = more significant and \*\*\* = highly significant.

### A.3 Tests of residuals from multiple linear regression model

House	P-value	Sign	House	P-value	Sign
1	7.458816e-09	6.049162e-01	36	0.6551522	0.8971008
2	0.03683475	0.36518675	37	0.00211908	0.89710079
3	0.1522292	0.8971008	38	0.09384231	0.24423532
4	0.1385351	0.8971008	39	0.7203760	0.5792903
5	0.4889633	1.0000000	40	0.0814062	1.0000000
6	0.06605537	1.0000000	41	0.7499187	0.6049162
7	1.772660e-08	6.471864e-03	42	0.5544048	1.0000000
8	0.001297956	0.355279682	43	0.9688807	0.8534084
9	0.002104755	0.195349059	44	0.05668529	0.51781727
10	0.1306790	0.2546051	45	0.6782350	0.8971008
11	0.709406	1.000000	46	0.2546043	1.0000000
12	3.319923e-07	9.239943e-02	47	0.001065575	0.120376700
13	0.0001239323	0.8534084459	48	4.674662e-09	5.178173e-01
14	0.2859703	0.5178173	49	0.0352531	0.5178173
15	0.8776810	0.8534084	50	0.9042102	0.8971008
16	0.03106733	1.0000000	51	0.2314035	1.0000000
17	0.4268815	1.0000000	52	0.02903943	0.43768624
18	0.0000130237	0.3651867453	53	0.4957589	0.8534084
19	0.809942	1.000000	54	0.04954421	0.15457115
20	0.3582973	0.7117033	55	0.2428003	0.5178173
21	0.1858593	0.8971008	56	0.09574403	0.795902392
22	0.07530409	1.0000000	57	0.0213735	0.7959024
23	0.006563705	0.698021972	58	0.7065081	0.6980220
24	0.9487515	0.8534084	59	0.07945885	1.0000000
25	0.2231339	0.4596914	60	0.5896005	0.7117033
26	0.002878431	0.459691387	61	5.313489e-06	5.208653e-02
27	0.5797924	0.8534084	62	0.001550156	0.267182305
28	0.008843725	0.019686796	63	0.5771746	0.3552797
29	0.006802018	0.698021972	64	0.006368894	0.795902390
30	0.08408281	0.51781727	65	0.01968257	0.43768624
31	0.001679153	0.300679317	66	0.0006757623	0.7959023901
32	8.699853e-07	3.823830e-02	67	0.1761617	1.0000000
33	0.9174810	0.7959024	68	0.2359113	0.8534084
34	4.238303e-05	6.049162e-01	69	0.008934653	0.195349059
35	0.9688807	0.8534084	70	0.8963474	1.0000000

Table A.5: P-values from Shapiro-Wilk test for normality on the general multiple linear regression model



### A.3.1 Significance of parameters for general regression model

Index	I	T	N	E	S	W	SR	T:N	T:E	T:S	T:W
1	+***	-***				+**			+. .	-**	
2	+***	-***			+**	+***	-***			-***	
3	+***	-***			+***	+***	-***		+*	-*	
4	+***	-***	+			+***		-***			
5	+***	-***			+***	+***	-***	+*		+***	-**
7	+***	-***	-*		+***	-.	+**	+***	-**	+*	-**
11	+***	-***			+***	+***	-***			+*	
12	+***	-***			+		+*				
14	+***	-***			+***	+***	-**			+**	
18	+***	-**			+***	-**	+**	+		+***	-***
21	+***	-***			+		+***				-***
22	+***	-***				+***	+**	-***			-*
23	+***	-***	-.		+	+***	-***			+***	-***
28	+***	-***			+	+	-**				
29	+***	-***			+***		+***	-**	+*	+**	-***
30	+***	-***			+	+	+***	-***			-*
31	+***	-***			+***	+***	-***	+**		+***	-***
32	+***	-**				+	+**		+		-.
33	+***	-***			+***	+**	+***	-***			-**
34	+***	-***			+***	+	+***	-**		+	-**
36	+***	-***			+***		+***	-***		+**	
37	+***	-***			+***	+**	+**	-***			
38	+***	-***			+		+***	-***			-***
40	+***	-***			+	+**	+**	-***	+. .	+*	-*
41	+***	-***				+.	+***		+. .	+. .	-.
42	+***	-***			+***		+***	-***			-*
44	+***	-***	-*				-**			+*	
45	+***	-***				+	+***	-*	+*	+*	-***
46	+***	-***			+	+	+***	-**			-**
47	+***	-***			+**		+***	-**	-**		-***
48	+***	-***			+**	+	+***	-**			-**
49	+***	-***					-**				
50	+***	-***	-.		+***	-*	+**	-***	+. .	+***	-.
52	+***	-***	-.			-**	-.	-***		+***	
54	+***	-***			+**		+**	-***		+*	-*
55	+***	-***			+	+	+***	-***		+**	-*
56	+***	-***			+**		+***	-***		+**	-*
57	+***	-***			+***		+***				-**
58	+***	-***	-.		+		+***	-***	+*	+***	-**
61	+***	-***			+	+	+		-.		-.
64	+***	-***	-*		+***		+***	-***	-.	+**	-**
65	+***	-***			+		+***	-***		+*	-**
66	+***	-***	+		+***	+**	+***	-***		+**	-**

Table A.6: Significance of parameters for 'long' houses from the general regression model. The meaning of the stars is as follows: \* = significant, \*\* = more significant and \*\*\* = highly significant.

Index	I	T	N	E	S	W	SR	T:N	T:E	T:S	T:W
6	+***	-***		+		+**	-*				
8	+***	-***		+*		+***	-*		+*	-.	
9	+***	-***	-.			+*	-***	+	+*	-**	
10	+***	-***		+***		+**	-**			-.	
13	+***	-***		+		+**	-.	+	+**	-**	
15	+***	-***		+**		+**					
16	+***	-***		+		+***	-*			-**	
17	+***	-***		+*	+***	+***	-**			-***	
19	+***	-***		+*	-.			+		+**	
20	+***	-***			+*						
24	+***	-***		+*		+***				-***	
25	+***	-***		+**		+***			+*	-*	
26	+***	-***				+***	-*	+	+*	-*	
27	+***	-***		+	+**	+**				-*	
35	+***	-***		+**	+	+***	-**			-**	
39	+***	-***				+*	-*		+		
43	+***	-***		+**	+	+***	-**			-**	
51	+***	-***			+	+***	-**		+*	-**	
53	+***	-***		+*		+*				-.	
59	+***	-***	-*	+**	+	+***	-*	+	+*	-.	
60	+***	-***	-*	+**		+***	-***	+	+**	-***	
62	+***	-***				+*	-*				
63	+***	-***		+*					+		
67	+***	-***		+***		+***	-***		-*	+	-**
68	+***	-***				+*	-***		+*	-*	
69	+***	-***				+**	-*		+*		
70	+***	-***		+		+***	-***			-**	

Table A.7: Significance of parameters for 'short' houses from the general regression model. The meaning of the stars is as follows: \* = significant, \*\* = more significant and \*\*\* = highly significant.

## A.4 Parameters from hourly consumption models

House 18	AR1	MA1	SAR1	SMA1	Intercept	Temperature
Estimate	0.9945	-0.9117	0.9924	-0.9497	1.5178	0.0327
Standard deviation	0.0014	0.0048	0.0019	0.0052	0.4818	0.0053

Table A.8: Estimates for the  $(1, 0, 1) \times (1, 0, 1)$  arima model for house 55

House 18	AR1	MA1	SAR1	SMA1	Intercept	Temperature
Estimate	0.3782	0.2647	0.9702	-0.9326	0.4947	0.1103
Standard deviation	0.0170	0.0175	0.053	0.0072	0.0652	0.008

Table A.9: Estimates for the  $(1, 0, 1) \times (1, 0, 1)$  arima model for house 18 without the Fourier series

Coefficient	Estimate	Significance
$\phi_1$	7.299e-01	***
$\mu$	1.546e-01	**
$\omega_T$	-6.590e-03	**
$\omega_{SR}$	-7.989e-05	*
$\phi_2$	2.191e-01	***
$\theta_1$	-8.730e-01	***

Table A.10: MARIMA(3, 0, 1) parameters for house 55.

Coefficient	Estimate	Significance
$\phi_1$	6.386e-01	***
$\mu$	1.503e-01	**
$\omega_T$	-6.535e-03	*
$\omega_{SR}$	-4.165e-05	
$\omega_1^{sin}$	1.566e-02	.
$\omega_2^{sin}$	1.039e-02	
$\omega_3^{sin}$	-2.904e-02	**
$\omega_1^{cos}$	3.206e-02	**
$\omega_2^{cos}$	-7.725e-02	***
$\omega_3^{cos}$	1.715e-02	.
$\phi_2$	2.285e-01	***
$\phi_3$	8.284e-02	***
$\theta_1$	-8.233e-01	***

Table A.11: MARIMA(3, 0, 1) parameters for house 55 with a Fourier series of degree 2.

## APPENDIX **B**

# Figures

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## B.1 Scatterplots of data

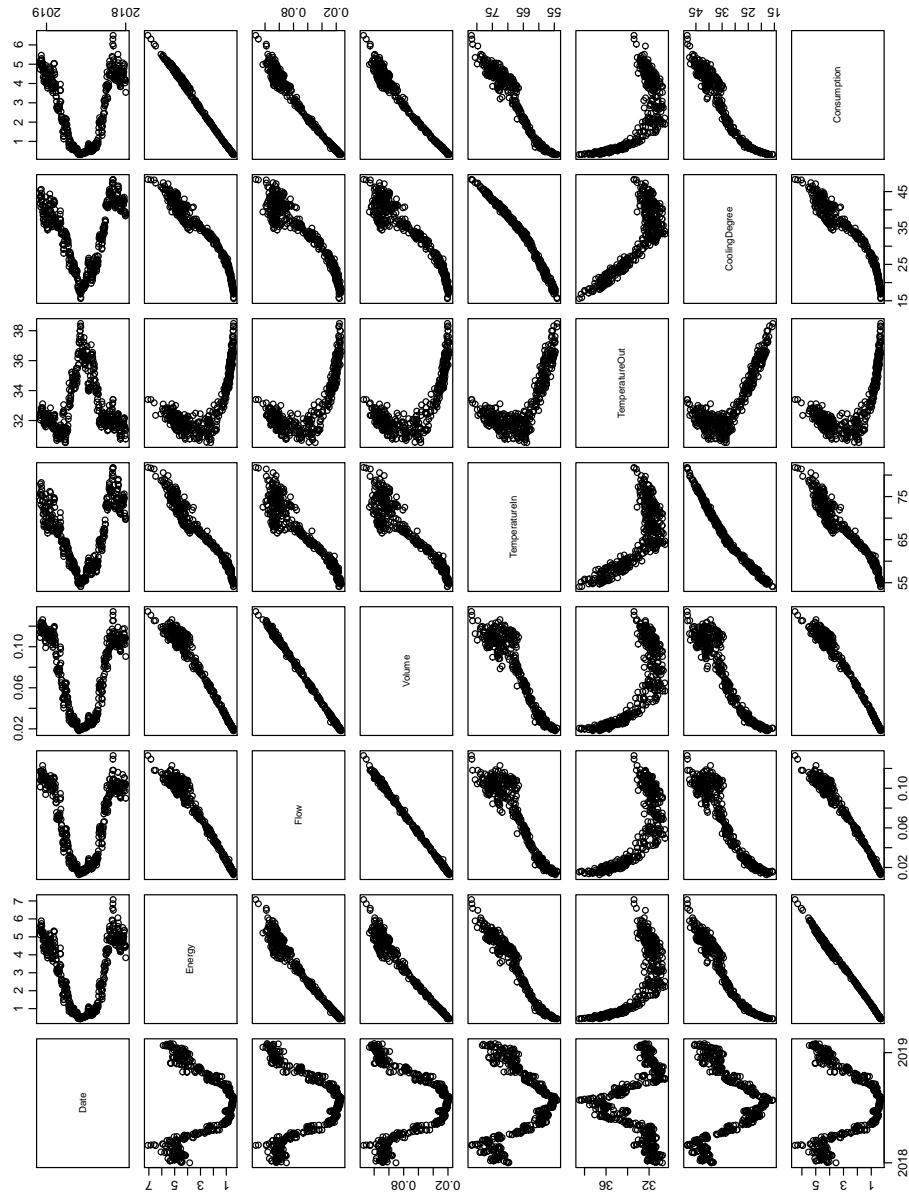


Figure B.1: Scatterplot showing the average of relevant attributes from house data

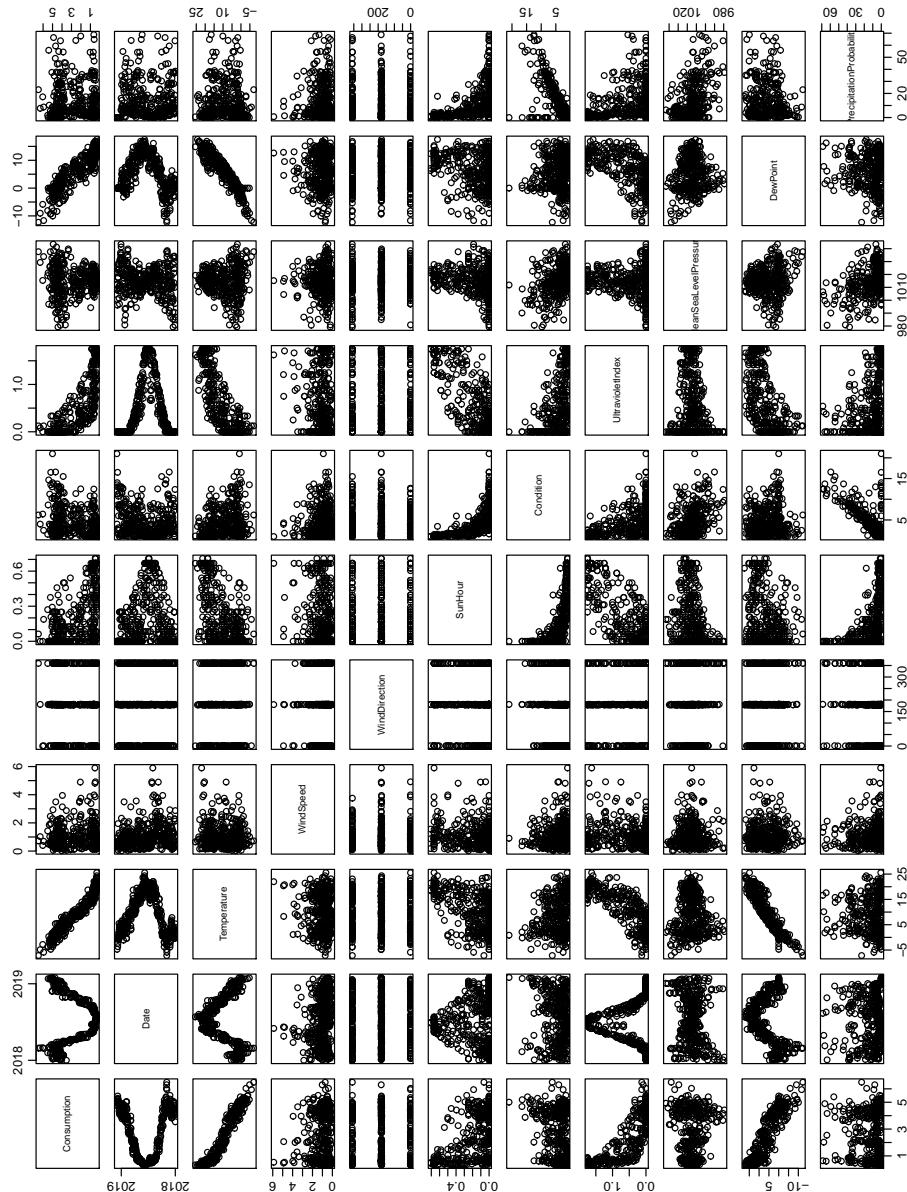


Figure B.2: Scatterplot showing the average of relevant attributes from weather data

## B.2 ACF and PACF of time series models

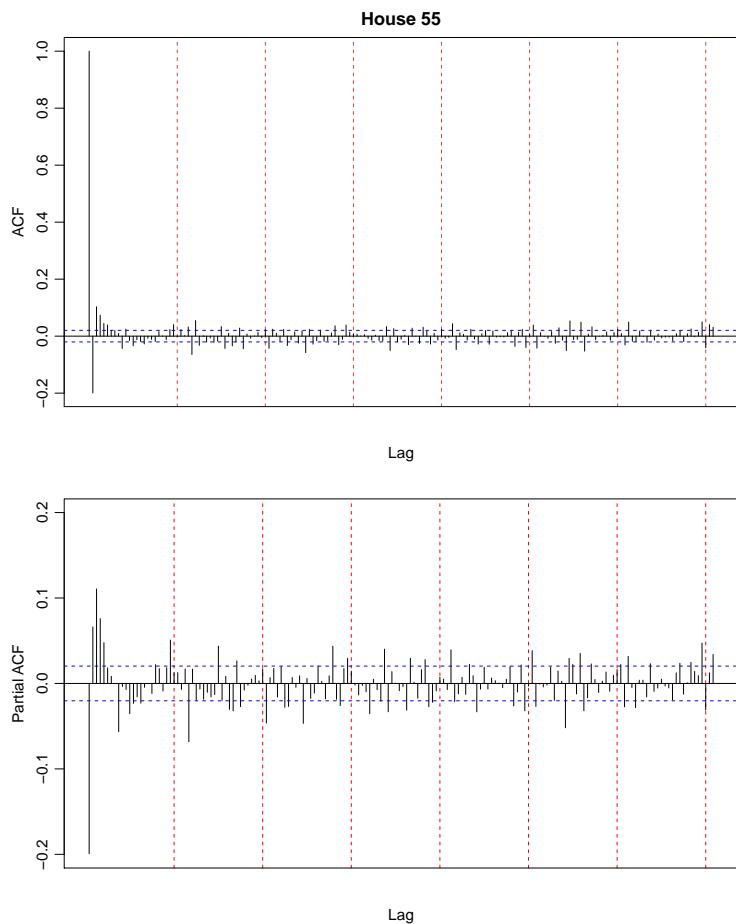


Figure B.3: The acf and pacf of the second model when applied to house 55

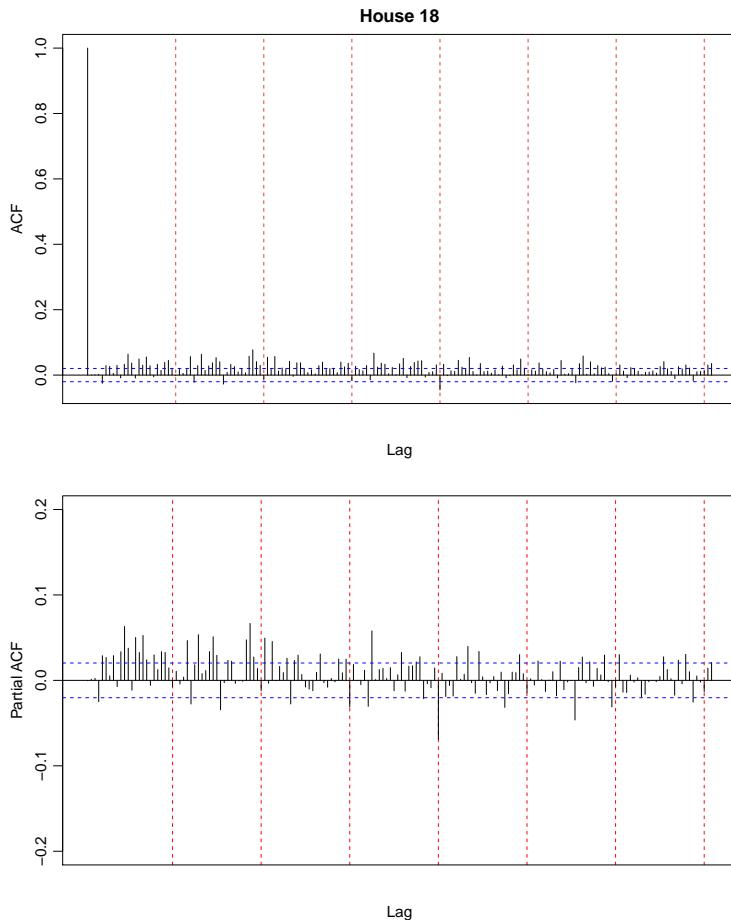


Figure B.4: The acf and pacf of the second model when applied to house 18

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