R_Warp_design_with_vorticity

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1 R-Warp design with one velocity component (case with vorticity)

[1]:
```
%display latex
```

2 1. Manifold

[2]:
```
M = Manifold(4, 'M', structure="Lorentzian")
N = Manifold(3, 'N', ambient=M, structure="Riemannian")
```

3 2. Chart

[3]:
```
C.<t,x,y,z> = M.chart(r't:(-oo,+oo) x:(-oo,+oo) y:(-oo,+oo) z:(-oo,+oo)')
```

4 3. Metric

[4]:
```
 g=M.metric(name='dS^2')
```

5 3.1. Functions

[5]:
```
V = M.scalar_field(function('V')(t,x,y,z), name='V')
```

6 3.2. Components of the metric

[6]:
```
g[0,0]=-1 + V**2
g[1,1]=1
g[0,1]=-V
g[2,2]=1
g[3,3]=1
```

[7]:
```
g[:]
```

[7]:

$$\begin{pmatrix} V(t,x,y,z)^2-1 & -V(t,x,y,z) & 0 & 0 \\ -V(t,x,y,z) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

7 3.3. Inverse metrical

[8]:
```
ginv=g.inverse()
```

[9]:
```
ginv[:]
```

[9]:
$$\begin{pmatrix} -1 & -V(t,x,y,z) & 0 & 0 \\ -V(t,x,y,z) & -V(t,x,y,z)^2+1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

8 4. Connection

[10]:
```
nab = g.connection()
```

9 5. Christoffel symbols

[11]:
```
nab.display(only_nonredundant=True)
```

[11]:
$$\begin{aligned}
\Gamma^{t}{}_{tt} &= V(t,x,y,z)^2\,\frac{\partial V}{\partial x} \\
\Gamma^{t}{}_{tx} &= -V(t,x,y,z)\,\frac{\partial V}{\partial x} \\
\Gamma^{t}{}_{ty} &= -\tfrac{1}{2}\,V(t,x,y,z)\,\frac{\partial V}{\partial y} \\
\Gamma^{t}{}_{tz} &= -\tfrac{1}{2}\,V(t,x,y,z)\,\frac{\partial V}{\partial z} \\
\Gamma^{t}{}_{xx} &= \frac{\partial V}{\partial x} \\
\Gamma^{t}{}_{xy} &= \tfrac{1}{2}\,\frac{\partial V}{\partial y} \\
\Gamma^{t}{}_{xz} &= \tfrac{1}{2}\,\frac{\partial V}{\partial z} \\
\Gamma^{x}{}_{tt} &= \Big(V(t,x,y,z)^3 - V(t,x,y,z)\Big)\,\frac{\partial V}{\partial x} - \frac{\partial V}{\partial t} \\
\Gamma^{x}{}_{tx} &= -V(t,x,y,z)^2\,\frac{\partial V}{\partial x} \\
\Gamma^{x}{}_{ty} &= -\tfrac{1}{2}\,\Big(V(t,x,y,z)^2+1\Big)\,\frac{\partial V}{\partial y} \\
\Gamma^{x}{}_{tz} &= -\tfrac{1}{2}\,\Big(V(t,x,y,z)^2+1\Big)\,\frac{\partial V}{\partial z} \\
\Gamma^{x}{}_{xx} &= V(t,x,y,z)\,\frac{\partial V}{\partial x} \\
\Gamma^{x}{}_{xy} &= \tfrac{1}{2}\,V(t,x,y,z)\,\frac{\partial V}{\partial y} \\
\Gamma^{x}{}_{xz} &= \tfrac{1}{2}\,V(t,x,y,z)\,\frac{\partial V}{\partial z} \\
\Gamma^{y}{}_{tt} &= -V(t,x,y,z)\,\frac{\partial V}{\partial y} \\
\Gamma^{y}{}_{tx} &= \tfrac{1}{2}\,\frac{\partial V}{\partial y} \\
\Gamma^{z}{}_{tt} &= -V(t,x,y,z)\,\frac{\partial V}{\partial z} \\
\Gamma^{z}{}_{tx} &= \tfrac{1}{2}\,\frac{\partial V}{\partial z}
\end{aligned}$$

10 5.1. Curvature

[12]:
```
Ric=g.ricci()
Scal=Ric['_{ij}']*ginv['^{ij}']
Ein = Ric-(Scal/2)*g
Riem = g.riemann()
```

11 5.2. Riemann and Ricci Tensors

[13]:
```
Riem.display()
```

[13]:

$$\begin{aligned}\mathrm{Riem}\left(dS^2\right) = &\left(-V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial x}\right)^2 - V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x^2} + \frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2 + \frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 - V\right.\\
&\mathrm{d}t\otimes\mathrm{d}t\otimes\mathrm{d}x + \left(-V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x\partial y} - V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y} - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial y}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}t\otimes\mathrm{d}t\otimes\\
&\mathrm{d}y + \left(-V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x\partial z} - V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z} - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial z}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}t\otimes\mathrm{d}t\otimes\mathrm{d}z +\\
&\left(V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial x}\right)^2 + V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x^2} - \frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2 - \frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 + V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial x}\right.\\
&\mathrm{d}t\otimes\mathrm{d}x\otimes\mathrm{d}t + \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial y}\frac{\partial}{\partial t}\otimes\mathrm{d}t\otimes\mathrm{d}x\otimes\mathrm{d}y + \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial z}\frac{\partial}{\partial t}\otimes\\
&\mathrm{d}t\otimes\mathrm{d}x\otimes\mathrm{d}z + \left(V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x\partial y} + V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial y}\right)\frac{\partial}{\partial t}\otimes\\
&\mathrm{d}t\otimes\mathrm{d}y\otimes\mathrm{d}t - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial y}\frac{\partial}{\partial t}\otimes\mathrm{d}t\otimes\mathrm{d}y\otimes\mathrm{d}x +\\
&\left(V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x\partial z} + V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z} + \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial z}\right)\frac{\partial}{\partial t}\otimes\\
&\mathrm{d}t\otimes\mathrm{d}z\otimes\mathrm{d}t - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial z}\frac{\partial}{\partial t}\otimes\mathrm{d}t\otimes\mathrm{d}z\otimes\mathrm{d}x +\\
&\left(\left(\frac{\partial V}{\partial x}\right)^2 + V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x^2} - \frac{1}{4}\left(\frac{\partial V}{\partial y}\right)^2 - \frac{1}{4}\left(\frac{\partial V}{\partial z}\right)^2 + \frac{\partial^2 V}{\partial t\partial x}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\\
&\mathrm{d}t\otimes\mathrm{d}x + \left(V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial y} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{1}{2}\frac{\partial^2 V}{\partial t\partial y}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\mathrm{d}t\otimes\\
&\mathrm{d}y + \left(V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial z} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t\partial z}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\mathrm{d}t\otimes\mathrm{d}z +\\
&\left(-\left(\frac{\partial V}{\partial x}\right)^2 - V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x^2} + \frac{1}{4}\left(\frac{\partial V}{\partial y}\right)^2 + \frac{1}{4}\left(\frac{\partial V}{\partial z}\right)^2 - \frac{\partial^2 V}{\partial t\partial x}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\\
&\mathrm{d}x\otimes\mathrm{d}t - \frac{1}{2}\frac{\partial^2 V}{\partial x\partial y}\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}y - \frac{1}{2}\frac{\partial^2 V}{\partial x\partial z}\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}z +\\
&\left(-V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial y} - \frac{\partial V}{\partial x}\frac{\partial V}{\partial y} - \frac{1}{2}\frac{\partial^2 V}{\partial t\partial y}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\mathrm{d}y\otimes\mathrm{d}t + \frac{1}{2}\frac{\partial^2 V}{\partial x\partial y}\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\\
&\mathrm{d}y\otimes\mathrm{d}x + \left(-V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial z} - \frac{\partial V}{\partial x}\frac{\partial V}{\partial z} - \frac{1}{2}\frac{\partial^2 V}{\partial t\partial z}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}x\otimes\mathrm{d}z\otimes\mathrm{d}t + \frac{1}{2}\frac{\partial^2 V}{\partial x\partial z}\frac{\partial}{\partial t}\otimes\\
&\mathrm{d}x\otimes\mathrm{d}z\otimes\mathrm{d}x + \left(\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial y} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{1}{2}\frac{\partial^2 V}{\partial t\partial y}\right)\frac{\partial}{\partial t}\otimes\mathrm{d}y\otimes\mathrm{d}t\otimes\mathrm{d}x +\end{aligned}$$

$$\left(\frac{3}{4}\left(\frac{\partial V}{\partial y}\right)^2+\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial y^2}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}y\otimes \mathrm{d}t\otimes \mathrm{d}y+\left(\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial y\partial z}+\frac{3}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial t}\otimes$$
$$\mathrm{d}y\otimes \mathrm{d}t\otimes \mathrm{d}z+\left(-\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial x\partial y}-\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}-\frac{1}{2}\frac{\partial^2 V}{\partial t\partial y}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}y\otimes \mathrm{d}x\otimes \mathrm{d}t-\frac{1}{2}\frac{\partial^2 V}{\partial y^2}\frac{\partial}{\partial t}\otimes$$
$$\mathrm{d}y\otimes \mathrm{d}x\otimes \mathrm{d}y-\frac{1}{2}\frac{\partial^2 V}{\partial y\partial z}\frac{\partial}{\partial t}\otimes \mathrm{d}y\otimes \mathrm{d}x\otimes \mathrm{d}z+\left(-\frac{3}{4}\left(\frac{\partial V}{\partial y}\right)^2-\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial y^2}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}y\otimes$$
$$\mathrm{d}y\otimes \mathrm{d}t+\frac{1}{2}\frac{\partial^2 V}{\partial y^2}\frac{\partial}{\partial t}\otimes \mathrm{d}y\otimes \mathrm{d}y\otimes \mathrm{d}x+\left(-\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial y\partial z}-\frac{3}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}y\otimes \mathrm{d}z\otimes \mathrm{d}t+$$
$$\frac{1}{2}\frac{\partial^2 V}{\partial y\partial z}\frac{\partial}{\partial t}\otimes \mathrm{d}y\otimes \mathrm{d}z\otimes \mathrm{d}x+\left(\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial x\partial z}+\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+\frac{1}{2}\frac{\partial^2 V}{\partial t\partial z}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}t\otimes \mathrm{d}x+$$
$$\left(\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial y\partial z}+\frac{3}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}t\otimes \mathrm{d}y+\left(\frac{3}{4}\left(\frac{\partial V}{\partial z}\right)^2+\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial z^2}\right)\frac{\partial}{\partial t}\otimes$$
$$\mathrm{d}z\otimes \mathrm{d}t\otimes \mathrm{d}z+\left(-\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial x\partial z}-\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}-\frac{1}{2}\frac{\partial^2 V}{\partial t\partial z}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}x\otimes \mathrm{d}t-\frac{1}{2}\frac{\partial^2 V}{\partial y\partial z}\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}x\otimes$$
$$\mathrm{d}y-\frac{1}{2}\frac{\partial^2 V}{\partial z^2}\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}x\otimes \mathrm{d}z+\left(-\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial y\partial z}-\frac{3}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}y\otimes \mathrm{d}t+\frac{1}{2}\frac{\partial^2 V}{\partial y\partial z}\frac{\partial}{\partial t}\otimes$$
$$\mathrm{d}z\otimes \mathrm{d}y\otimes \mathrm{d}x+\left(-\frac{3}{4}\left(\frac{\partial V}{\partial z}\right)^2-\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial z^2}\right)\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}z\otimes \mathrm{d}t+\frac{1}{2}\frac{\partial^2 V}{\partial z^2}\frac{\partial}{\partial t}\otimes \mathrm{d}z\otimes \mathrm{d}z\otimes \mathrm{d}x+$$
$$\left(-\left(V(t,x,y,z)^2-1\right)\left(\frac{\partial V}{\partial x}\right)^2+\frac{1}{4}\left(V(t,x,y,z)^2-1\right)\left(\frac{\partial V}{\partial y}\right)^2+\frac{1}{4}\left(V(t,x,y,z)^2-1\right)\left(\frac{\partial V}{\partial z}\right)^2-\left(V(t,x,y,z)\right.\right.$$
$$\mathrm{d}t\otimes \mathrm{d}t\otimes \mathrm{d}x+\left(-\left(V(t,x,y,z)^2-1\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}-\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial t\partial y}-\left(V(t,x,y,z)^3-V(t,x,y,z)\right)\frac{\partial^2 V}{\partial x\partial y}\right)\frac{\partial}{\partial}$$
$$\mathrm{d}t\otimes \mathrm{d}t\otimes \mathrm{d}y+\left(-\left(V(t,x,y,z)^2-1\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}-\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial t\partial z}-\left(V(t,x,y,z)^3-V(t,x,y,z)\right)\frac{\partial^2 V}{\partial x\partial z}\right)\frac{\partial}{\partial}$$
$$\mathrm{d}t\otimes \mathrm{d}t\otimes \mathrm{d}z+\left(\left(V(t,x,y,z)^2-1\right)\left(\frac{\partial V}{\partial x}\right)^2-\frac{1}{4}\left(V(t,x,y,z)^2-1\right)\left(\frac{\partial V}{\partial y}\right)^2-\frac{1}{4}\left(V(t,x,y,z)^2-1\right)\left(\frac{\partial V}{\partial z}\right)^2+\left(V\right.\right.$$
$$\mathrm{d}t\otimes \mathrm{d}x\otimes \mathrm{d}t+\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial x\partial y}\frac{\partial}{\partial x}\otimes \mathrm{d}t\otimes \mathrm{d}x\otimes \mathrm{d}y+\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial x\partial z}\frac{\partial}{\partial x}\otimes \mathrm{d}t\otimes \mathrm{d}x\otimes$$
$$\mathrm{d}z+\left(\left(V(t,x,y,z)^2-1\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial t\partial y}+\left(V(t,x,y,z)^3-V(t,x,y,z)\right)\frac{\partial^2 V}{\partial x\partial y}\right)\frac{\partial}{\partial x}\otimes$$
$$\mathrm{d}t\otimes \mathrm{d}y\otimes \mathrm{d}t-\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial x\partial y}\frac{\partial}{\partial x}\otimes \mathrm{d}t\otimes \mathrm{d}y\otimes \mathrm{d}x+$$
$$\left(\left(V(t,x,y,z)^2-1\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial t\partial z}+\left(V(t,x,y,z)^3-V(t,x,y,z)\right)\frac{\partial^2 V}{\partial x\partial z}\right)\frac{\partial}{\partial x}\otimes$$
$$\mathrm{d}t\otimes \mathrm{d}z\otimes \mathrm{d}t-\frac{1}{2}\left(V(t,x,y,z)^2-1\right)\frac{\partial^2 V}{\partial x\partial z}\frac{\partial}{\partial x}\otimes \mathrm{d}t\otimes \mathrm{d}z\otimes \mathrm{d}x+$$
$$\left(V(t,x,y,z)\left(\frac{\partial V}{\partial x}\right)^2+V(t,x,y,z)^2\frac{\partial^2 V}{\partial x^2}-\frac{1}{4}V(t,x,y,z)\left(\frac{\partial V}{\partial y}\right)^2-\frac{1}{4}V(t,x,y,z)\left(\frac{\partial V}{\partial z}\right)^2+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial x}\right.$$
$$\mathrm{d}x\otimes \mathrm{d}t\otimes \mathrm{d}x+\left(V(t,x,y,z)^2\frac{\partial^2 V}{\partial x\partial y}+V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial y}\right)\frac{\partial}{\partial x}\otimes \mathrm{d}x\otimes$$
$$\mathrm{d}t\otimes \mathrm{d}y+\left(V(t,x,y,z)^2\frac{\partial^2 V}{\partial x\partial z}+V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+\frac{1}{2}V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial z}\right)\frac{\partial}{\partial x}\otimes \mathrm{d}x\otimes \mathrm{d}t\otimes \mathrm{d}z+$$

$$\left(-V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial x}\right)^{2}-V\left(t,x,y,z\right)^{2}\frac{\partial^{2}V}{\partial x^{2}}+\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^{2}+\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^{2}-V\left(t,x,y,z\right)\frac{\partial^{2}}{\partial t\partial}\right.$$
$$\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}t-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial x\partial y}\frac{\partial}{\partial x}\otimes\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}y-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial x\partial z}\frac{\partial}{\partial x}\otimes$$
$$\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}z+\left(-V\left(t,x,y,z\right)^{2}\frac{\partial^{2}V}{\partial x\partial y}-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial t\partial y}\right)\frac{\partial}{\partial x}\otimes$$
$$\mathrm{d}x\otimes\mathrm{d}y\otimes\mathrm{d}t+\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial x\partial y}\frac{\partial}{\partial x}\otimes\mathrm{d}x\otimes\mathrm{d}y\otimes\mathrm{d}x+$$
$$\left(-V\left(t,x,y,z\right)^{2}\frac{\partial^{2}V}{\partial x\partial z}-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial t\partial z}\right)\frac{\partial}{\partial x}\otimes$$
$$\mathrm{d}x\otimes\mathrm{d}z\otimes\mathrm{d}t+\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial x\partial z}\frac{\partial}{\partial x}\otimes\mathrm{d}x\otimes\mathrm{d}z\otimes\mathrm{d}x+$$
$$\left(V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial t\partial y}+\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial x\partial y}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes$$
$$\mathrm{d}t\otimes\mathrm{d}x+\left(V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^{2}+\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes\mathrm{d}t\otimes$$
$$\mathrm{d}y+\left(V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}+\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes\mathrm{d}t\otimes\mathrm{d}z+$$
$$\left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial t\partial y}-\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial x\partial y}\right)\frac{\partial}{\partial x}\otimes$$
$$\mathrm{d}y\otimes\mathrm{d}x\otimes\mathrm{d}t+\left(-\frac{1}{4}\left(\frac{\partial V}{\partial y}\right)^{2}-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes$$
$$\mathrm{d}x\otimes\mathrm{d}y+\left(-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y\partial z}-\frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes\mathrm{d}x\otimes$$
$$\mathrm{d}z+\left(-V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^{2}-\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes$$
$$\mathrm{d}y\otimes\mathrm{d}t+\left(\frac{1}{4}\left(\frac{\partial V}{\partial y}\right)^{2}+\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes\mathrm{d}y\otimes$$
$$\mathrm{d}x+\left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}-\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes$$
$$\mathrm{d}z\otimes\mathrm{d}t+\left(\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y\partial z}+\frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}y\otimes\mathrm{d}z\otimes\mathrm{d}x+$$
$$\left(V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial t\partial z}+\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial x\partial z}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}z\otimes$$
$$\mathrm{d}t\otimes\mathrm{d}x+\left(V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}+\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}z\otimes\mathrm{d}t\otimes$$
$$\mathrm{d}y+\left(V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^{2}+\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial z^{2}}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}z\otimes\mathrm{d}t\otimes\mathrm{d}z+$$
$$\left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial t\partial z}-\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial x\partial z}\right)\frac{\partial}{\partial x}\otimes$$
$$\mathrm{d}z\otimes\mathrm{d}x\otimes\mathrm{d}t+\left(-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y\partial z}-\frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}z\otimes$$
$$\mathrm{d}x\otimes\mathrm{d}y+\left(-\frac{1}{4}\left(\frac{\partial V}{\partial z}\right)^{2}-\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial z^{2}}\right)\frac{\partial}{\partial x}\otimes\mathrm{d}z\otimes\mathrm{d}x\otimes$$

$$
\begin{aligned}
&\mathrm{d}z \quad + \quad \left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}-\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial x} \quad \otimes \quad \mathrm{d}z \quad \otimes \\
&\mathrm{d}y \quad \otimes \quad \mathrm{d}t \quad + \quad \left(\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y\partial z}+\frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\right)\frac{\partial}{\partial x} \quad \otimes \quad \mathrm{d}z \quad \otimes \quad \mathrm{d}y \quad \otimes \\
&\mathrm{d}x \quad + \quad \left(-V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^{2}-\frac{1}{2}\left(V\left(t,x,y,z\right)^{2}+1\right)\frac{\partial^{2}V}{\partial z^{2}}\right)\frac{\partial}{\partial x} \quad \otimes \quad \mathrm{d}z \quad \otimes \\
&\mathrm{d}z \quad \otimes \quad \mathrm{d}t \quad + \quad \left(\frac{1}{4}\left(\frac{\partial V}{\partial z}\right)^{2}+\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial z^{2}}\right)\frac{\partial}{\partial x} \quad \otimes \quad \mathrm{d}z \quad \otimes \quad \mathrm{d}z \quad \otimes \\
&\mathrm{d}x \quad + \quad \left(V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial x\partial y}+\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+\frac{1}{2}\frac{\partial^{2}V}{\partial t\partial y}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}x \quad + \\
&\left(\frac{1}{4}\left(V\left(t,x,y,z\right)^{2}+3\right)\left(\frac{\partial V}{\partial y}\right)^{2}+V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}t \quad \otimes \\
&\mathrm{d}y \quad + \quad \left(\frac{1}{4}\left(V\left(t,x,y,z\right)^{2}+3\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}+V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \\
&\mathrm{d}t \quad \otimes \quad \mathrm{d}z \quad + \quad \left(-V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial x\partial y}-\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}-\frac{1}{2}\frac{\partial^{2}V}{\partial t\partial y}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \\
&\mathrm{d}x \quad \otimes \quad \mathrm{d}t \quad + \quad \left(-\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^{2}-\frac{1}{2}\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}x \quad \otimes \\
&\mathrm{d}y \quad + \quad \left(-\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}-\frac{1}{2}\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}x \quad \otimes \quad \mathrm{d}z \quad + \\
&\left(-\frac{1}{4}\left(V\left(t,x,y,z\right)^{2}+3\right)\left(\frac{\partial V}{\partial y}\right)^{2}-V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}y \quad \otimes \\
&\mathrm{d}t \quad + \quad \left(\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^{2}+\frac{1}{2}\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}y \quad \otimes \quad \mathrm{d}x \quad + \\
&\left(-\frac{1}{4}\left(V\left(t,x,y,z\right)^{2}+3\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}-V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}z \quad \otimes \quad \mathrm{d}t \quad + \\
&\left(\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}+\frac{1}{2}\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}z \quad \otimes \quad \mathrm{d}x \quad - \quad \frac{1}{2}\frac{\partial^{2}V}{\partial x\partial y}\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}x \quad \otimes \\
&\mathrm{d}t \quad \otimes \quad \mathrm{d}x \quad + \quad \left(-\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^{2}-\frac{1}{2}\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial y} \quad \otimes \quad \mathrm{d}x \quad \otimes \quad \mathrm{d}t \quad \otimes \quad \mathrm{d}y \quad + \\
&\left(-\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}-\frac{1}{2}\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial y}\otimes\mathrm{d}x\otimes\mathrm{d}t\otimes\mathrm{d}z+\frac{1}{2}\frac{\partial^{2}V}{\partial x\partial y}\frac{\partial}{\partial y}\otimes\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}t+\frac{1}{4}\left(\frac{\partial V}{\partial y}\right)^{2}\frac{\partial}{\partial y}\otimes \\
&\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}y+\frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\frac{\partial}{\partial y}\otimes\mathrm{d}x\otimes\mathrm{d}x\otimes\mathrm{d}z+\left(\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^{2}+\frac{1}{2}\frac{\partial^{2}V}{\partial y^{2}}\right)\frac{\partial}{\partial y}\otimes\mathrm{d}x\otimes \\
&\mathrm{d}y\otimes\mathrm{d}t-\frac{1}{4}\left(\frac{\partial V}{\partial y}\right)^{2}\frac{\partial}{\partial y}\otimes\mathrm{d}x\otimes\mathrm{d}y\otimes\mathrm{d}x+\left(\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}+\frac{1}{2}\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial y}\otimes\mathrm{d}x\otimes \\
&\mathrm{d}z\otimes\mathrm{d}t-\frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\frac{\partial}{\partial y}\otimes\mathrm{d}x\otimes\mathrm{d}z\otimes\mathrm{d}x+\left(V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial x\partial z}+\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+\frac{1}{2}\frac{\partial^{2}V}{\partial t\partial z}\right)\frac{\partial}{\partial z}\otimes \\
&\mathrm{d}t\otimes\mathrm{d}t\otimes\mathrm{d}x+\left(\frac{1}{4}\left(V\left(t,x,y,z\right)^{2}+3\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}+V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial y\partial z}\right)\frac{\partial}{\partial z}\otimes\mathrm{d}t\otimes \\
&\mathrm{d}t\otimes\mathrm{d}y+\left(\frac{1}{4}\left(V\left(t,x,y,z\right)^{2}+3\right)\left(\frac{\partial V}{\partial z}\right)^{2}+V\left(t,x,y,z\right)\frac{\partial^{2}V}{\partial z^{2}}\right)\frac{\partial}{\partial z}\otimes\mathrm{d}t\otimes
\end{aligned}
$$

$$\mathrm{d}t \otimes \mathrm{d}z + \left(-V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x \partial z} - \frac{\partial V}{\partial x}\frac{\partial V}{\partial z} - \frac{1}{2}\frac{\partial^2 V}{\partial t \partial z}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}t \otimes \mathrm{d}x \otimes \mathrm{d}t +$$
$$\left(-\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z} - \frac{1}{2}\frac{\partial^2 V}{\partial y \partial z}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}t \otimes \mathrm{d}x \otimes \mathrm{d}y + \left(-\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 - \frac{1}{2}\frac{\partial^2 V}{\partial z^2}\right)\frac{\partial}{\partial z} \otimes$$
$$\mathrm{d}t \otimes \mathrm{d}x \otimes \mathrm{d}z + \left(-\frac{1}{4}\left(V\left(t,x,y,z\right)^2 + 3\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z} - V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial y \partial z}\right)\frac{\partial}{\partial z} \otimes$$
$$\mathrm{d}t \otimes \mathrm{d}y \otimes \mathrm{d}t + \left(\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial y \partial z}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}t \otimes \mathrm{d}y \otimes \mathrm{d}x +$$
$$\left(-\frac{1}{4}\left(V\left(t,x,y,z\right)^2 + 3\right)\left(\frac{\partial V}{\partial z}\right)^2 - V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial z^2}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}t \otimes \mathrm{d}z \otimes \mathrm{d}t +$$
$$\left(\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial z^2}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}t \otimes \mathrm{d}z \otimes \mathrm{d}x - \frac{1}{2}\frac{\partial^2 V}{\partial x \partial z}\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}t \otimes \mathrm{d}x +$$
$$\left(-\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z} - \frac{1}{2}\frac{\partial^2 V}{\partial y \partial z}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}t \otimes \mathrm{d}y + \left(-\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 - \frac{1}{2}\frac{\partial^2 V}{\partial z^2}\right)\frac{\partial}{\partial z} \otimes$$
$$\mathrm{d}x \otimes \mathrm{d}t \otimes \mathrm{d}z + \frac{1}{2}\frac{\partial^2 V}{\partial x \partial z}\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}x \otimes \mathrm{d}t + \frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}x \otimes \mathrm{d}y + \frac{1}{4}\left(\frac{\partial V}{\partial z}\right)^2\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes$$
$$\mathrm{d}x \otimes \mathrm{d}z + \left(\frac{1}{4}V\left(t,x,y,z\right)\frac{\partial V}{\partial y}\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial y \partial z}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}y \otimes \mathrm{d}t - \frac{1}{4}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}y \otimes \mathrm{d}x +$$
$$\left(\frac{1}{4}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial z^2}\right)\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}z \otimes \mathrm{d}t - \frac{1}{4}\left(\frac{\partial V}{\partial z}\right)^2\frac{\partial}{\partial z} \otimes \mathrm{d}x \otimes \mathrm{d}z \otimes \mathrm{d}x$$

[14]:
```
Ric.display()
```

[14]:
$$\mathrm{Ric}\left(dS^2\right) = \left(\left(V\left(t,x,y,z\right)^2 - 1\right)\left(\frac{\partial V}{\partial x}\right)^2 - \frac{1}{2}\left(V\left(t,x,y,z\right)^2 + 1\right)\left(\frac{\partial V}{\partial y}\right)^2 - \frac{1}{2}\left(V\left(t,x,y,z\right)^2 + 1\right)\left(\frac{\partial V}{\partial z}\right)^2 + \left(V\right.\right.$$
$$\mathrm{d}t + \left(-V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial x}\right)^2 - V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x^2} + \frac{1}{2}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2 + \frac{1}{2}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 - V\left(t,x,y,z\right)\right.$$
$$\mathrm{d}x + \left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y} - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t \partial y} - \frac{1}{2}\left(V\left(t,x,y,z\right)^2 + 1\right)\frac{\partial^2 V}{\partial x \partial y}\right)\mathrm{d}t \otimes$$
$$\mathrm{d}y + \left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z} - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t \partial z} - \frac{1}{2}\left(V\left(t,x,y,z\right)^2 + 1\right)\frac{\partial^2 V}{\partial x \partial z}\right)\mathrm{d}t \otimes \mathrm{d}z +$$
$$\left(-V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial x}\right)^2 - V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x^2} + \frac{1}{2}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2 + \frac{1}{2}V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2 - V\left(t,x,y,z\right)\frac{\partial^2}{\partial t \partial}\right.$$
$$\mathrm{d}t + \left(\left(\frac{\partial V}{\partial x}\right)^2 + V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x^2} - \frac{1}{2}\left(\frac{\partial V}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial V}{\partial z}\right)^2 + \frac{\partial^2 V}{\partial t \partial x}\right)\mathrm{d}x \otimes \mathrm{d}x +$$
$$\left(\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x \partial y} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{1}{2}\frac{\partial^2 V}{\partial t \partial y}\right)\mathrm{d}x \otimes \mathrm{d}y + \left(\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x \partial z} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t \partial z}\right)\mathrm{d}x \otimes$$
$$\mathrm{d}z + \left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y} - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t \partial y} - \frac{1}{2}\left(V\left(t,x,y,z\right)^2 + 1\right)\frac{\partial^2 V}{\partial x \partial y}\right)\mathrm{d}y \otimes \mathrm{d}t +$$
$$\left(\frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x \partial y} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{1}{2}\frac{\partial^2 V}{\partial t \partial y}\right)\mathrm{d}y \otimes \mathrm{d}x + \frac{1}{2}\left(\frac{\partial V}{\partial y}\right)^2\mathrm{d}y \otimes \mathrm{d}y + \frac{1}{2}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\mathrm{d}y \otimes$$
$$\mathrm{d}z + \left(-V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z} - \frac{1}{2}V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t \partial z} - \frac{1}{2}\left(V\left(t,x,y,z\right)^2 + 1\right)\frac{\partial^2 V}{\partial x \partial z}\right)\mathrm{d}z \otimes \mathrm{d}t +$$

$$\left(\frac{1}{2}\,V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial x\partial z}+\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+\frac{1}{2}\frac{\partial^2 V}{\partial t\partial z}\right)\mathrm{d}z\otimes\mathrm{d}x+\frac{1}{2}\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}\mathrm{d}z\otimes\mathrm{d}y+\frac{1}{2}\left(\frac{\partial V}{\partial z}\right)^2\mathrm{d}z\otimes\mathrm{d}z$$

12 5.3. Components of Ricci

[15]:
```
Ric[:]
```

[15]:
$$\begin{pmatrix} \left(V\left(t,x,y,z\right)^2-1\right)\left(\frac{\partial V}{\partial x}\right)^2-\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\left(\frac{\partial V}{\partial y}\right)^2-\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\left(\frac{\partial V}{\partial z}\right)^2+\left(V\left(t,x,y,z\right)^2-1\right) \\ -V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial x}\right)^2-V\left(t,x,y,z\right)^2\frac{\partial^2 V}{\partial x^2}+\frac{1}{2} \end{pmatrix}$$

13 5.4. Ricci scalar

[16]:
```
Scal.expr()
```

[16]:
$$2\frac{\partial}{\partial x}V\left(t,x,y,z\right)^2+2\,V\left(t,x,y,z\right)\frac{\partial^2}{(\partial x)^2}V\left(t,x,y,z\right)+\frac{1}{2}\frac{\partial}{\partial y}V\left(t,x,y,z\right)^2+\frac{1}{2}\frac{\partial}{\partial z}V\left(t,x,y,z\right)^2+2\frac{\partial^2}{\partial t\partial x}V\left(t,x,y,z\right)$$

14 5.5. Components of Einstein tensor

[17]:
```
Ein[:]
```

[17]:
$$\begin{pmatrix} -\frac{1}{4}\left(3\,V\left(t,x,y,z\right)^2+1\right)\left(\frac{\partial V}{\partial y}\right)^2-\frac{1}{4}\left(3\,V\left(t,x,y,z\right)^2+1\right)\left(\frac{\partial V}{\partial z}\right)^2-V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial y^2}-V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial z^2} & \frac{3}{4}\,V\left(t, \\ \frac{3}{4}\,V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2+\frac{3}{4}\,V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2+\frac{1}{2}\frac{\partial^2 V}{\partial y^2}+\frac{1}{2}\frac{\partial^2 V}{\partial z^2} & \\ -V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}-\frac{1}{2}\,V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial y}-\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\frac{\partial^2 V}{\partial x\partial y} & \\ -V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}-\frac{1}{2}\,V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial z}-\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\frac{\partial^2 V}{\partial x\partial z} & \end{pmatrix}$$

15 6. Energy Momentum Tensor (EMT)

[18]:
```
var('Lambda_')
var('G')
T = 1/(8*pi*G) * ( Ein + g*Lambda_ )
```

[19]:
```
T[:]
```

[19]:

$$\left(\begin{array}{c} \frac{4\,\Lambda V(t,x,y,z)^2-\left(3\,V(t,x,y,z)^2+1\right)\left(\frac{\partial V}{\partial y}\right)^2-\left(3\,V(t,x,y,z)^2+1\right)\left(\frac{\partial V}{\partial z}\right)^2-4\,V(t,x,y,z)\frac{\partial^2 V}{\partial y^2}-4\,V(t,x,y,z)\frac{\partial^2 V}{\partial z^2}-4\,\Lambda}{32\,\pi G} \quad \frac{3\,V(t,x,y,z)\left(\frac{\partial V}{\partial y}\right)^2+3\,V(t,x,\\ \frac{3\,V(t,x,y,z)\left(\frac{\partial V}{\partial y}\right)^2+3\,V(t,x,y,z)\left(\frac{\partial V}{\partial z}\right)^2-4\,\Lambda V(t,x,y,z)+2\,\frac{\partial^2 V}{\partial y^2}+2\,\frac{\partial^2 V}{\partial z^2}}{32\,\pi G} \\ -\frac{2\,V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial y}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial y}}{16\,\pi G} \\ -\frac{2\,V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial z}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial z}}{16\,\pi G} \end{array}\right.$$

16 6.1. Components of EMT

17 6.1.1. Lapse function $N_0 = \sqrt{(|g^{00}|)}$

[20]:
```
N_0 = sqrt(abs(ginv[0,0]))
N_0.display()
```

[20]:
$$(t, x, y, z) \mapsto 1$$

18 6.1.2. Compute shift vector N^i

[21]:
```
N = [ginv[0, i] for i in range(1, 4)]
N[0],N[1], N[2]
```

[21]:
$$\left(-V\left(t, x, y, z\right), 0, 0\right)$$

19 6.1.3. Define the fluid 4-velocity u^μ

[22]:
```
u = M.vector_field(name="u")
u[0] = 1 / N_0 # u_MU = 1
for i in range(1, 4):
    u[i] = - N[i - 1] / N_0  # Spatial components : u_MU[1]=-V(t,x), u_MU[2]=0,␣
 ↪u_MU[3]=0
u.display()
```

[22]:
$$u = \frac{\partial}{\partial t} + V\left(t, x, y, z\right) \frac{\partial}{\partial x}$$

20 6.1.5. Energy density ϵ

[23]:
```
uu= u['^i'] * u['^j']
```

[24]:
```
epsilon=uu['^{ij}'] * T['_{ij}']
epsilon.display()
```

[24]:
$$\begin{array}{lcl} M & \longrightarrow & \mathbb{R} \\ (t, x, y, z) & \longmapsto & -\frac{\left(\frac{\partial V}{\partial y}\right)^2+\left(\frac{\partial V}{\partial z}\right)^2+4\,\Lambda}{32\,\pi G} \end{array}$$

21 6.1.6. Pressure tensor $p_{ij} = pb_{ij} + \pi_{ij}$

[25]:
```
T[1,1], T[1,2], T[1,3]
```

[25]:
$$\left(-\frac{3\left(\frac{\partial V}{\partial y}\right)^2 + 3\left(\frac{\partial V}{\partial z}\right)^2 - 4\Lambda}{32\pi G}, \frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x \partial y} + 2\frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{\partial^2 V}{\partial t \partial y}}{16\pi G}, \frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x \partial z} + 2\frac{\partial V}{\partial x}\frac{\partial V}{\partial z} + \frac{\partial^2 V}{\partial t \partial z}}{16\pi G} \right)$$

[26]:
```
T[2,1], T[2,2], T[2,3]
```

[26]:
$$\left(\frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x \partial y} + 2\frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{\partial^2 V}{\partial t \partial y}}{16\pi G}, -\frac{4\left(\frac{\partial V}{\partial x}\right)^2 + 4V(t,x,y,z)\frac{\partial^2 V}{\partial x^2} - \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 - 4\Lambda + 4\frac{\partial^2 V}{\partial t \partial x}}{32\pi G}, \frac{\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}}{16\pi G} \right)$$

[27]:
```
T[3,1], T[3,2], T[3,3]
```

[27]:
$$\left(\frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x \partial z} + 2\frac{\partial V}{\partial x}\frac{\partial V}{\partial z} + \frac{\partial^2 V}{\partial t \partial z}}{16\pi G}, \frac{\frac{\partial V}{\partial y}\frac{\partial V}{\partial z}}{16\pi G}, -\frac{4\left(\frac{\partial V}{\partial x}\right)^2 + 4V(t,x,y,z)\frac{\partial^2 V}{\partial x^2} + \left(\frac{\partial V}{\partial y}\right)^2 - \left(\frac{\partial V}{\partial z}\right)^2 - 4\Lambda + 4\frac{\partial^2 V}{\partial t \partial x}}{32\pi G} \right)$$

22 6.1.7. Define the spatial projector $b^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu = h^{\mu\nu}$

[28]:
```
b = u['^i'] * u['^j'] + ginv
b[:]
```

[28]:
$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[29]:
```
b1 = u['^i'] * u['^j']
b1[:]
```

[29]:
$$\begin{pmatrix} 1 & V(t,x,y,z) & 0 & 0 \\ V(t,x,y,z) & V(t,x,y,z)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

23 6.1.8. Spatial heat vector $q^k = -b^{ik}u^j T_{ij}$

[30]:
```
bu = -b['^{ik}']*u['^j']
q2 = bu['^{ikj}']*T['_{ij}']
q2[:]
```

[30]:
$$\left[0, -\frac{\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}{16\pi G}, \frac{\frac{\partial^2 V}{\partial x \partial y}}{16\pi G}, \frac{\frac{\partial^2 V}{\partial x \partial z}}{16\pi G} \right]$$

24 $q_i = g_{ik} q^k $

[31]:
```
q_i = g['_{ij}']*q2['^i']
q_i[:]
```

[31]:
$$\left[\frac{V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial y^2}+V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial z^2}}{16\,\pi G},-\frac{\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}}{16\,\pi G},\frac{\frac{\partial^2 V}{\partial x\partial y}}{16\,\pi G},\frac{\frac{\partial^2 V}{\partial x\partial z}}{16\,\pi G}\right]$$

[32]:
```
g['_{ij}'][:]
```

[32]:
$$\begin{pmatrix} V\left(t,x,y,z\right)^2-1 & -V\left(t,x,y,z\right) & 0 & 0\\ -V\left(t,x,y,z\right) & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

[33]:
```
T[0,1],T[1,0]
```

[33]:
$$\left(\frac{3\,V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2+3\,V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2-4\,\Lambda V\left(t,x,y,z\right)+2\,\frac{\partial^2 V}{\partial y^2}+2\,\frac{\partial^2 V}{\partial z^2}}{32\,\pi G},\frac{3\,V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2+3\,V\left(t,x,\right.}{}\right.$$

[34]:
```
p_mixed = b['^{ik}'] * T['_{kj}']
p_mixed[:]
```

[34]:
$$\begin{pmatrix} 0 & 0 & \\ \frac{3\,V(t,x,y,z)\left(\frac{\partial V}{\partial y}\right)^2+3\,V(t,x,y,z)\left(\frac{\partial V}{\partial z}\right)^2-4\,\Lambda V(t,x,y,z)+2\,\frac{\partial^2 V}{\partial y^2}+2\,\frac{\partial^2 V}{\partial z^2}}{32\,\pi G} & -\frac{3\left(\frac{\partial V}{\partial y}\right)^2+3\left(\frac{\partial V}{\partial z}\right)^2-4\,\Lambda}{32\,\pi G} & \frac{V(t}{} \\ -\frac{2\,V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial y}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial y}}{16\,\pi G} & \frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x\partial y}+2\,\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+\frac{\partial^2 V}{\partial t\partial y}}{16\,\pi G} & -\frac{4\left(\frac{\partial V}{\partial x}\right)^2+4\,V(t,x,y,z)\frac{\partial^2 V}{\partial x^2}-(}{32} \\ -\frac{2\,V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial z}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial z}}{16\,\pi G} & \frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x\partial z}+2\,\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+\frac{\partial^2 V}{\partial t\partial z}}{16\,\pi G} & \end{pmatrix}$$

[35]:
```
Ein_mixed = b['^{ik}'] * Ein['_{kj}']
Ein_mixed
```

[35]:
Tensor field of type (1,1) on the 4-dimensional Lorentzian manifold M

[36]:
```
Ein_mixed = ginv['^{ik}'] * Ein['_{kj}']
Ein_mixed[:]
```

[36]:
$$\begin{pmatrix} \frac{1}{4}\left(\frac{\partial V}{\partial y}\right)^2+\frac{1}{2}\,V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial y^2}+\frac{1}{4}\left(\frac{\partial V}{\partial z}\right)^2+\frac{1}{2}\,V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial z^2} & \\ V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial y}\right)^2+V\left(t,x,y,z\right)\left(\frac{\partial V}{\partial z}\right)^2+\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\frac{\partial^2 V}{\partial y^2}+\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\frac{\partial^2 V}{\partial z^2} & -\frac{3}{4}\left(\frac{\partial V}{\partial y}\right)^2- \\ -V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}-\frac{1}{2}\,V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial y}-\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\frac{\partial^2 V}{\partial x\partial y} & \\ -V\left(t,x,y,z\right)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}-\frac{1}{2}\,V\left(t,x,y,z\right)\frac{\partial^2 V}{\partial t\partial z}-\frac{1}{2}\left(V\left(t,x,y,z\right)^2+1\right)\frac{\partial^2 V}{\partial x\partial z} & \end{pmatrix}$$

[37]:
```
T_mixed = ginv['^{ik}'] * T['_{kj}']
T_mixed[:]
```

[37]:
$$\begin{pmatrix} \frac{\left(\frac{\partial V}{\partial y}\right)^2+2V(t,x,y,z)\frac{\partial^2 V}{\partial y^2}+\left(\frac{\partial V}{\partial z}\right)^2+2V(t,x,y,z)\frac{\partial^2 V}{\partial z^2}+4\Lambda}{32\pi G} & \\ \frac{2V(t,x,y,z)\left(\frac{\partial V}{\partial y}\right)^2+2V(t,x,y,z)\left(\frac{\partial V}{\partial z}\right)^2+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial y^2}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial z^2}}{16\pi G} & -\frac{3\left(\frac{\partial V}{\partial y}\right)^2+2V(t,x,y,z)\frac{\partial^2 V}{\partial y^2}+3\left(\frac{\partial V}{\partial z}\right)^2+2V(t,x,y,z} \\ -\frac{2V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial y}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial y}}{16\pi G} & \frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x\partial y}+2\frac{\partial V}{\partial x}} \\ -\frac{2V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial z}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial z}}{16\pi G} & \frac{V(t,x,y,z)\frac{\partial^2 V}{\partial x\partial z}+2\frac{\partial V}{\partial x}} \end{pmatrix}$$

[38]:
```
ginv[:]
```

[38]:
$$\begin{pmatrix} -1 & -V(t,x,y,z) & 0 & 0 \\ -V(t,x,y,z) & -V(t,x,y,z)^2+1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[39]:
```
T[:]
```

[39]:
$$\begin{pmatrix} \frac{4\Lambda V(t,x,y,z)^2-\left(3V(t,x,y,z)^2+1\right)\left(\frac{\partial V}{\partial y}\right)^2-\left(3V(t,x,y,z)^2+1\right)\left(\frac{\partial V}{\partial z}\right)^2-4V(t,x,y,z)\frac{\partial^2 V}{\partial y^2}-4V(t,x,y,z)\frac{\partial^2 V}{\partial z^2}-4\Lambda}{32\pi G} & \frac{3V(t,x,y,z)\left(\frac{\partial V}{\partial y}\right)^2+3V(t,x,} \\ \frac{3V(t,x,y,z)\left(\frac{\partial V}{\partial y}\right)^2+3V(t,x,y,z)\left(\frac{\partial V}{\partial z}\right)^2-4\Lambda V(t,x,y,z)+2\frac{\partial^2 V}{\partial y^2}+2\frac{\partial^2 V}{\partial z^2}}{32\pi G} & \\ -\frac{2V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial y}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial y}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial y}}{16\pi G} & \\ -\frac{2V(t,x,y,z)\frac{\partial V}{\partial x}\frac{\partial V}{\partial z}+V(t,x,y,z)\frac{\partial^2 V}{\partial t\partial z}+\left(V(t,x,y,z)^2+1\right)\frac{\partial^2 V}{\partial x\partial z}}{16\pi G} & \end{pmatrix}$$

[47]:
```
Verif = Ein_mixed - 8*pi*G * T_mixed
Verif[:]
```

[47]:
$$\begin{pmatrix} -\Lambda & 0 & 0 & 0 \\ 0 & -\Lambda & 0 & 0 \\ 0 & 0 & -\Lambda & 0 \\ 0 & 0 & 0 & -\Lambda \end{pmatrix}$$

[ ]:
```

```