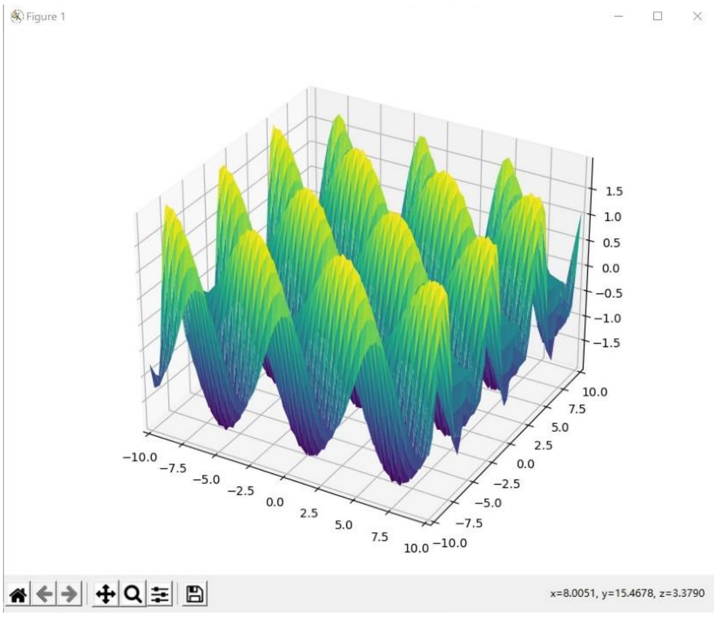
Problem: Search for the minimum of function $g(x,y) = \sin(x+y) + \cos(x+2y)$ in the whole space.

First, we plot the function in the interval $x \in [-10,10], y \in [-10,10]$. From the graph, we can tell that $g(x,y)$ is a periodic function. 2π is a period of the function,



as $g(x,y) = g(x+2\pi, y)$ and $g(x,y) = g(x, y+2\pi)$. In each period, the function has only one local minimum. So, if we have a method to search for the local minimum, then the result we've searched for is the minimum of the function in the whole space.

Then we define a function SD_method(), which means the steepest-descent method. The search process should move along the direction of descending the $g(x,y)$, so the increment $\Delta x, \Delta y$ has the sign opposite to $\nabla g(x,y)$. So, $(\Delta x, \Delta y) = -a(\frac{\partial g(x,y)}{\partial x}, \frac{\partial g(x,y)}{\partial y})$,where a is the step of the iteration. The function takes 1 parameter as input which means the step. The value of the step depends on the problem we faced, and in this problem, 0.01 is small enough. And we also set the original coordinates as $(-1.0, -1.0)$.

Inside the function, we can use a while-loop. Inside the while-loop, we calculate the value of $\Delta x = -(step * \frac{\partial g(x,y)}{\partial x})$ and $\Delta y = -(step * \frac{\partial g(x,y)}{\partial y})$. When $|g(x,y) - g(x+\Delta x, y+\Delta y)| < 1 \times 10^{-20}$, we take $g(x,y)$ as a minimum of the function and break the while-loop. And if $|g(x,y) - g(x+\Delta x, y+\Delta y)| \geq 1 \times 10^{-20}$, set $x + \Delta x$ as a new value of x and $y + \Delta y$ as a new value of y, and then continue the while -loop. As the while-loop continues, we will finally get the minimum of the function.

Input: The function $g(x, y) = \sin(x + y) + \cos(x + 2y)$

Output: The minimum of the function

$x \leftarrow -1.0$, $y \leftarrow -1.0$

$step \leftarrow 0.01$

While True **Do**

    $delta\_x \leftarrow -\left(step * \frac{\partial g(x,y)}{\partial x}\right)$

    $delta\_y \leftarrow -\left(step * \frac{\partial g(x,y)}{\partial y}\right)$

    If $|g(x, y) - g(x + delta\_x, y + delta\_y)| < 1 \times 10^{-20}$ **Then**

        $minimum \leftarrow g(x, y)$

        Break

    Else

        $x \leftarrow x + delta\_x$, $y \leftarrow y + delta\_y$

Return *minimum*

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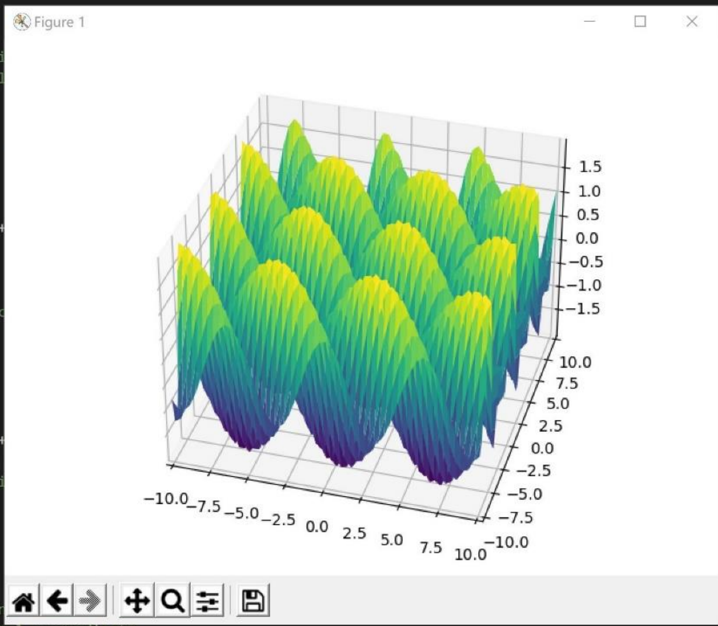
```python
import sympy as sp
from sympy.abc import x,y
from sympy.plotting import plot3d

def main():
    plot(-10,10)
    # First, plot the function i
    # From the graph we can tell

    result = SD_method(0.01)
    print('when x=%.4f, y=%.4f,


def function():
    g = sp.sin(x+y) + sp.cos(x +
    g_x = sp.diff(g,x)
    g_y = sp.diff(g,y)
    list1 = [g,g_x,g_y]
    # We construct a list that c

    return list1


def plot(m,n):
    g = sp.sin(x+y) + sp.cos(x +
    plot3d(g, (x,m,n), (y,m,n))
    # Plot the function in the i


def SD_method(t):
# Steepest-Descent method
    step = t
    x0 = -1.0        # The origin
    y0 = -1.0        # The original y-coordinate.
    fun = function()
    f = fun[0]       # The given function.
    f_x = fun[1]     # The x partial derivative of function.
    f_y = fun[2]     # The y partial derivative of function.
```



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C: > Users > 11765 > Desktop > 学习 > 物理 > 计算物理 > 作业 > homework2 > 2_2_17307110134.py > SD_method

```python
import sympy as sp
from sympy.abc import x,y
from sympy.plotting import plot3d

def main():
    plot(-10,10)
    # First, plot the function in the interval -10<x<10, -10<y<10.
    # From the graph we can tell that g(x,y) is a periodic function.

    result = SD_method(0.01)
    print('when x=%.4f, y=%.4f, the function reaches its minimum.' % (result[0],result[1]))


def function():
    g = sp.sin(x+y) + sp.cos(x + 2*y)
    g_x = sp.diff(g,x)
    g_y = sp.diff(g,y)
    list1 = [g,g_x,g_y]
```

终端 问题 输出 调试控制台

```
f(x)=-2.00000000, the times of iteration is 5756.
f(x)=-2.00000000, the times of iteration is 5757.
f(x)=-2.00000000, the times of iteration is 5758.
f(x)=-2.00000000, the times of iteration is 5759.
f(x)=-2.00000000, the times of iteration is 5760.
f(x)=-2.00000000, the times of iteration is 5761.
f(x)=-2.00000000, the times of iteration is 5762.
f(x)=-2.00000000, the times of iteration is 5763.
f(x)=-2.00000000, the times of iteration is 5764.
f(x)=-2.00000000, the times of iteration is 5765.
f(x)=-2.00000000, the times of iteration is 5766.
f(x)=-2.00000000, the times of iteration is 5767.
f(x)=-2.00000000, the times of iteration is 5768.
f(x)=-2.00000000, the times of iteration is 5769.
f(x)=-2.00000000, the times of iteration is 5770.
f(x)=-2.00000000, the times of iteration is 5771.
f(x)=-2.00000000, the times of iteration is 5772.
f(x)=-2.00000000, the times of iteration is 5773.
f(x)=-2.00000000, the times of iteration is 5774.
f(x)=-2.00000000, the times of iteration is 5775.
f(x)=-2.00000000, the times of iteration is 5776.
f(x)=-2.00000000, the times of iteration is 5777.
f(x)=-2.00000000 is the minimum of the function, the times of iteration are 5778.
when x=-0.0002, y=-1.5707, the function reaches its minimum.
```