Geometry 3 - Miscellaneous

TSS Math Club

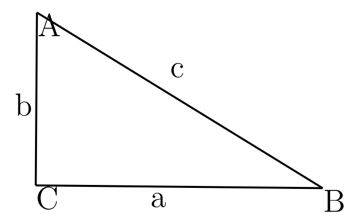
Nov 2022

1 Pythagorean Theorem

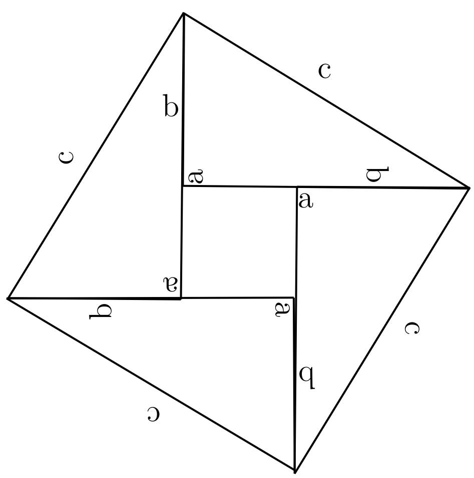
In a right-triangle,

$$a^2 + b^2 = c^2$$

where a and b are two sides and c is the hypotenuse.



1.1 Proof



2 Trigonometry

2.1 Definitions

Sine or $\sin(\theta)$:

Cosine or $\cos(\theta)$:

Tangent or $\tan(\theta)$:

2.2 Pythagorean Theorem

$$\sin(\theta) + \cos(\theta) = 1$$

2.3 Triangle Area Formula with Sine

$$S = \frac{ab \sin C}{2}$$

2.3.1 Proof

2.4 Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = d$$

2.4.1 Proof

2.5 Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

2.5.1 Proof

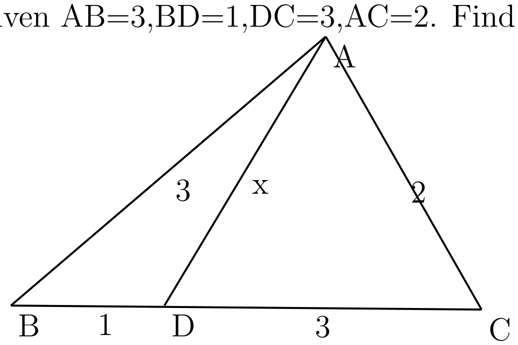
2.6 Problem

2.6.1 Heron's Formula

$$S = \sqrt{s(s-a)(s-b)(s-c)}, \text{where} s = \frac{a+b+c}{2}$$

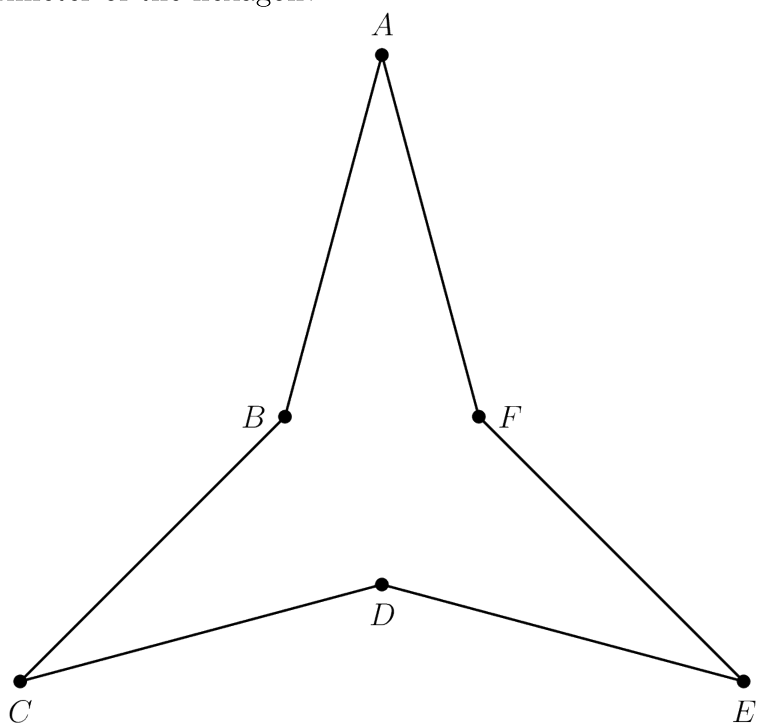
2.6.2 Problem

Given AB=3,BD=1,DC=3,AC=2. Find AD.



2.6.3 Problem

In the figure, equilateral hexagon $ABCDEF$ has three nonadjacent acute interior angles that each measure $30°$. The enclosed area of the hexagon is $6\sqrt{3}$. What is the perimeter of the hexagon?



3 Transversals

3.1 Directed Segments

Definition

3.2 Stewart's Theorem

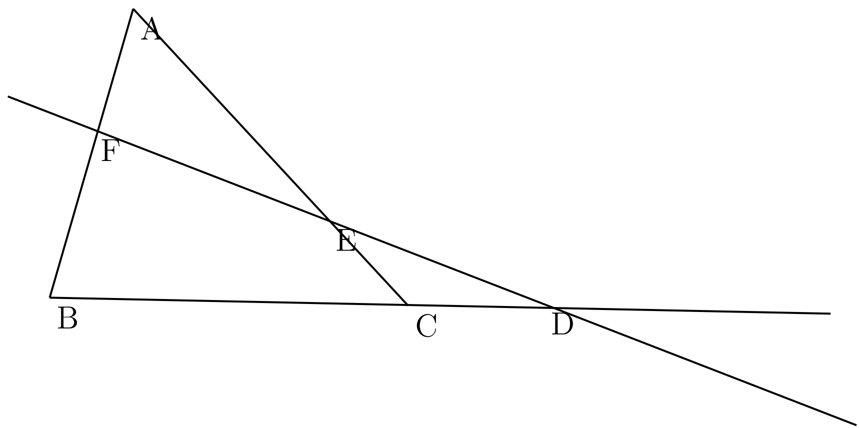
If A,B,C collinear and P is any other point, then

$$PA^2 \cdot BC + PB^2 \cdot CA + PC^2 \cdot AB + BC \cdot CA \cdot AB = 0$$

3.3 Menelaus' Theorem

Suppose we have a triangle ABC, and a transversal line that crosses BC, AC, and AB at points D, E, and F respectively, with D, E, and F distinct from A, B, and C, then

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = -1$$



3.4 Menelaus' Inverse Theorem

Suppose we have a triangle ABC with D on BC, E on AC, F on AB, such that,

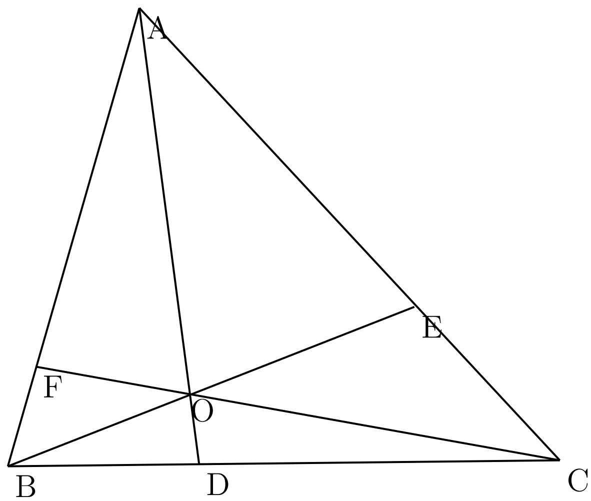
$$PA^2 \cdot BC + PB^2 \cdot CA + PC^2 \cdot AB + BC \cdot CA \cdot AB = 0$$

then D,E, F collinear.

3.5 Ceva's Theorem

Given a triangle ABC, let the lines AO, BO and CO be drawn from the vertices to a common point O (not on one of the sides of ABC), to meet opposite sides at D, E and F respectively, then

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$



3.6 Ceva's Inverse Theorem

Suppose we have a triangle ABC with D on BC, E on AC, F on AB, such that,

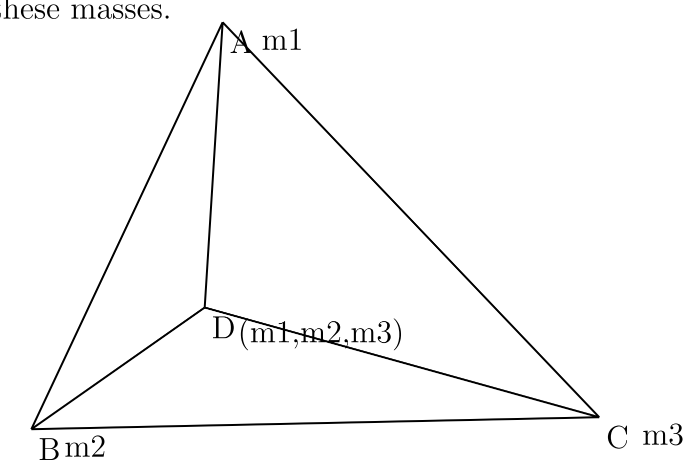
$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

then AD,BE,CF concurrent.

4 Barycentric Coordinate

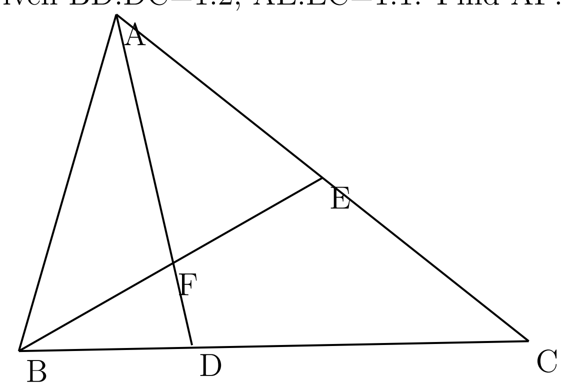
4.1 Definition

The barycentric coordinates of a point can be interpreted as masses placed at the vertices of the simplex, such that the point is the center of mass (or barycenter) of these masses.



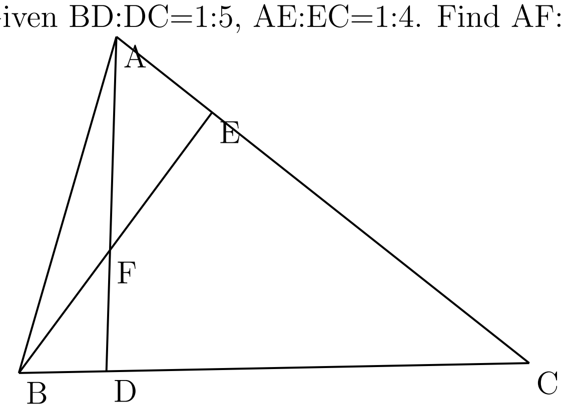
4.2 Example

Given BD:DC=1:2, AE:EC=1:1. Find AF:FD.



4.3 Problem

Given BD:DC=1:5, AE:EC=1:4. Find AF:FD.



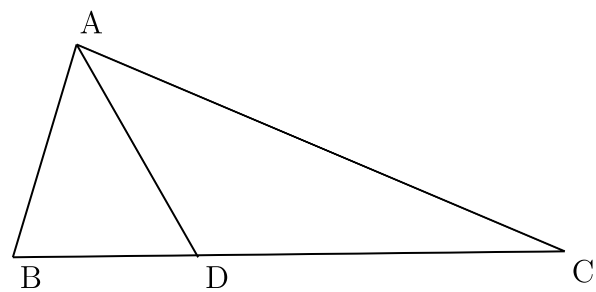
5 Angle Bisector

5.1 Definition

5.2 Angle Bisector Theorem

If AD bisects $\angle A$, then

$$\frac{BD}{CD} = \frac{AB}{AC}$$



5.3 Angle bisectors of a trinagle are concurrent

6 Median