Geometry: Introduction

TSS Math Club

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1 Definitions

1.1 General Symbols and Notation

1.1.1 Points and Lines

- A
- $\overline{AB}$
- $\overrightarrow{AB}$
- $\overleftrightarrow{AB}$
- AB
- $A \perp B$
- $A \parallel B$
- Hashmarks, Feathers, Arcs

1.1.2 Angles

- $\angle A$
- $\measuredangle A$
- $\sphericalangle A$
- A°

- A rad
- $A \cong B$

1.2 Angles

- Right
- Acute
- Obtuse
- Straight
- Reflex
- Opposite
- Complementary
- Supplementary

1.3 Shapes and Polygons

1.3.1 Shape

- Form/outline/boundary of an object

1.3.2 Polygon

- Shape formed by line segments connected at vertices
- Simple Polygon: Polygon formed by non-intersecting line segments connected end-to-end
- Complex Polygon: Self-intersecting polygon formed by intersecting line segments, which create multiple interior regions
- Examples:
  - Pentagon: Simple polygon with five edges and five vertices
  - Star: Complex polygon with ten edges and ten vertices

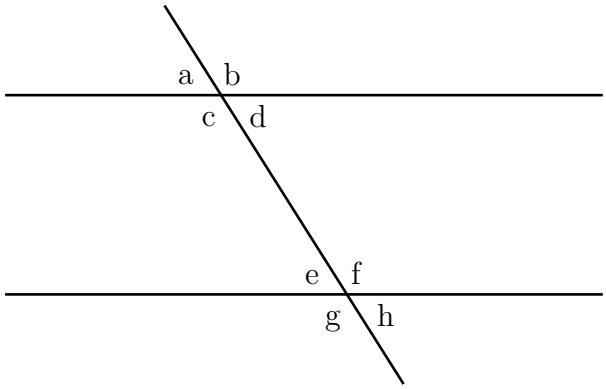
2 Basic Geometry

2.1 Parallelism

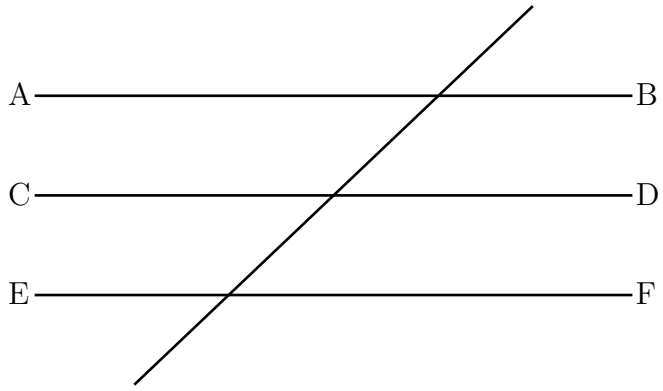
2.1.1 Definition

- Two lines that share an equidistant separation throughout their entire length are parallel.
- Parallel lines do not intersect each other.

2.1.2 Properties

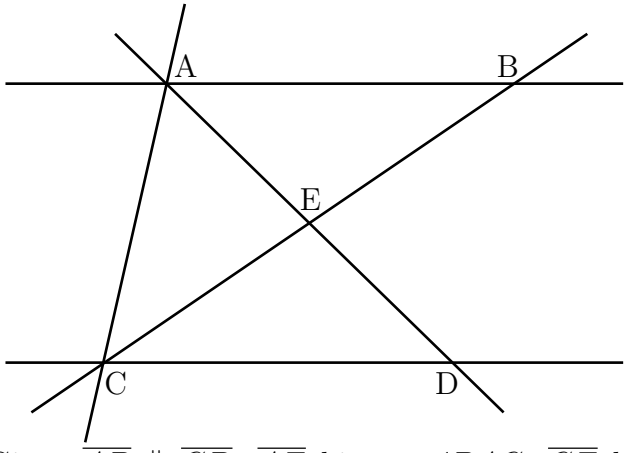


- Corresponding angles are equal (PLT-F): $\angle a = \angle e$
- Alternate interior angles are equal (PLT-Z): $\angle d = \angle e$
- Consecutive interior angles are supplementary (PLT-C): $\angle c + \angle e = 180°$



- Parallelism is a transitive property
- $\overline{AB} \parallel \overline{CD}$ and $\overline{CD} \parallel \overline{EF} \implies \overline{AB} \parallel \overline{EF}$
- All parallel angle theorems will hold true for the angles of the intersections of all parallel lines and a transversal.

2.1.3 Example Problem



Given $\overline{AB} \parallel \overline{CD}$, $\overline{AE}$ bisects $\angle BAC$, $\overline{CE}$ bisects $\angle ACD$
Prove $\overline{AE} \perp \overline{CE}$

2.2 Triangles

2.2.1 Definition

- A simple polygon with three edges and three vertices

2.2.2 Classifications

- Equilateral
- Isosceles
- Scalene
- Acute
- Obtuse

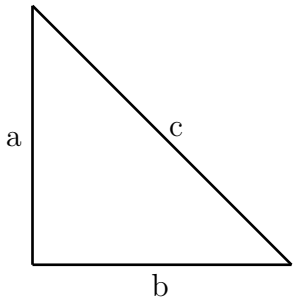
2.2.3 Properties

- Angles: Sum of angles in any triangle is 180°
- Sides: The length of any side is less than the sum and more than the difference of the length of the other two sides

2.2.4 Triangle Inequality Theorem

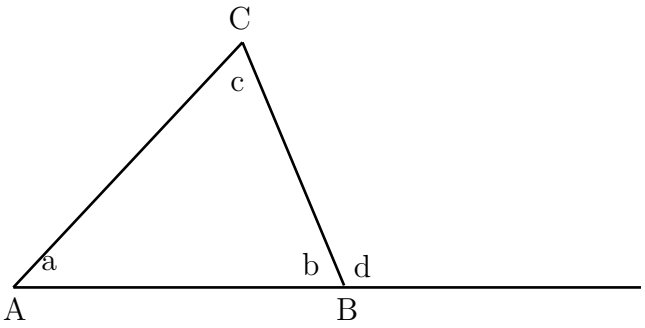
- The sum of the lengths of any two sides in any triangle must be greater than the length of the third side

2.2.5 Pythagorean Theorem



- $a^2 + b^2 = c^2$

2.2.6 Exterior angle theorem



2.2.7 Congruence and Similarity

- Congruent Triangles: Same angles and same size

proven by SSS, SAS, ASA, AAS

ambiguous case SSA

- Similar Triangles: Same angles

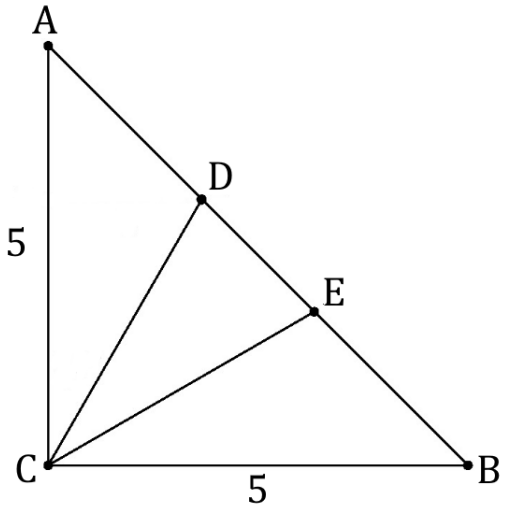
  proven by AA

2.2.8 Perimeter and Area

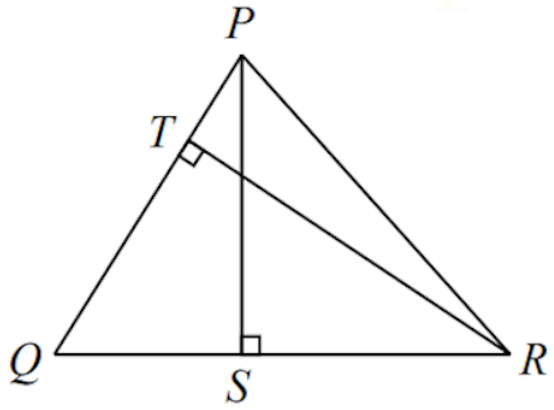
- Perimeter: Sum of lengths of all sides
- Area: For right triangles: $\frac{1}{2} \cdot$ base $\cdot$ height

  For all triangles: Heron's formula

2.2.9 Example Problems



Given $\triangle ABC$ is an isosceles right triangle, $\overline{CD}$ and $\overline{CE}$ trisect $\angle ACB$
Find the area of $\triangle CDE$



Points S and T are on sides QR and PQ of $\triangle PQR$ respectively, such that $\overline{PS} \perp \overline{QR}$, and $\overline{RT} \perp \overline{PQ}$.
Given $PT = 1$, $TQ = 4$, $QS = 3$
Find the length of SR.

2.3 Quadrilaterals

2.3.1 Definition

- A simple polygon with four edges and four vertices

2.3.2 Classifications

- Rectangle

  Square

- Parallelogram

  Rhombus

- Trapezoid

  Acute, Right, Obtuse, Isosceles

- Kite
- Scalene
- Cyclic

2.3.3 Congruence and Similarity

- Congruent Quadrilaterals: Same angles and same size
- Similar Quadrilaterals: Same angles

2.3.4 Perimeter and Area

- Perimeter: Sum of lengths of all sides
- Area: Different formulae for different quadrilaterals

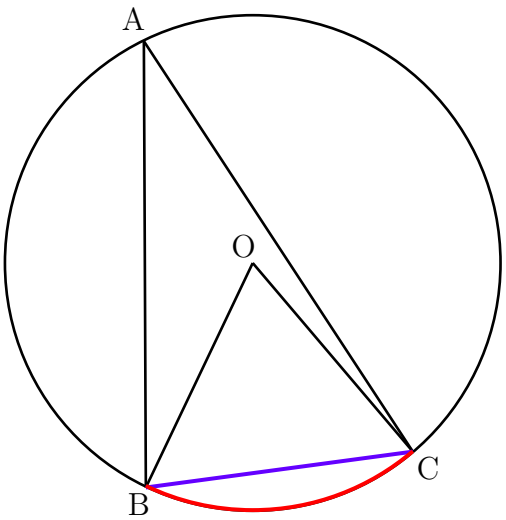
  e.g. length · width for rectangles, base · height for parallelograms

2.4 Circles

2.4.1 Definition

- A closed curve in which all points are equidistant from the center

2.4.2 Geometric objects related to circles



- Center: Center of the circle, Point O
- Radius: Length from center to circumference.
- Arc: A curve that lies on the circumference of a circle.

- Chord: A straight line between two points on a circle.
- Central angle: Angle formed at the center of a circle by two radii
  $\angle BOC$ is a central angle.
- Inscribed angle: Angle formed inside the a circle by two chords
  $\angle BAC$ is an inscribed angle.

2.4.3 Circumference and Area

- Circumference: $2 \cdot \pi \cdot r$
- Area: $\pi \cdot r^2$