Combinatorics

TSS Math Club

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1 Introduction to Counting

1.1 Rule of Product

1.1.1 Definition

1.1.2 Tree Diagram

1.1.3 Example 1

How many 4 digit numbers are there with no repeated digits?

1.1.4 Example 2

How many 5 digit odd numbers are there with no repeated digits?

1.1.5 Example 3

How many ways are there for 5 people to stand in a row?

1.2 Rule of Sum

1.2.1 Definition

1.2.2 Example 1

Calvin wants to go to Milwaukee. He can choose from 3 3 bus services or 2 2 train services to head from home to downtown Chicago. From there, he can choose from 2 bus services or 3 train services to head to Milwaukee. How many ways are there for Calvin to get to Milwaukee?

1.3 Case Working

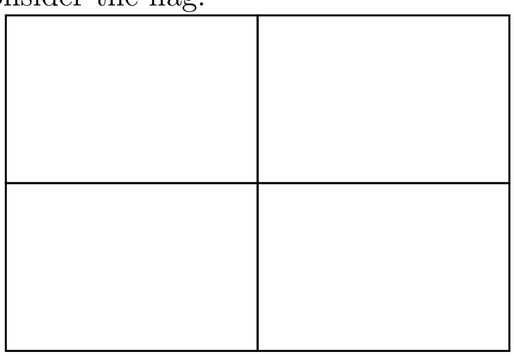
1.3.1 Definition

1.3.2 Example 1

How many numbers less than 10,000 are there with no repeated digits?

1.3.3 Example 2

Consider the flag:



If 10 colours are provided, how many ways are there to colour the flag so that no 2 adjacent regions have the same colour.

1.4 Indirect/Complementary Counting

1.4.1 Example 1

How many positive integers less than 100 are not a multiple of five?

1.4.2 Example 2

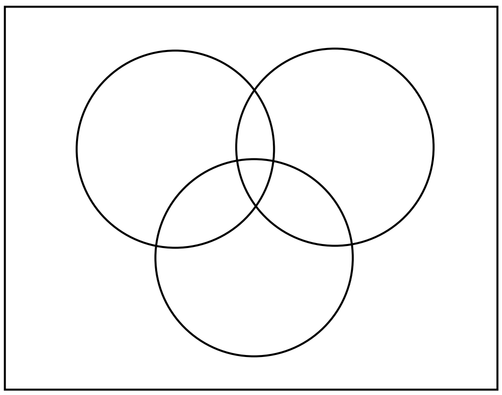
How many four-digit positive integers have at least one digit that is a 2 or a 3?

1.5 Venn Diagram and Inclusion-Exclusion Principle

1.5.1 Venn Diagram

Example: 100 students were interviewed. 28 took PE, 31 took BIO, 42 took ENG, 9 took PE and BIO, 10 took PE and ENG, 6 took BIO and ENG, 4 took all three subjects.

- How many students took none of the three subjects?
- How many students took PE but not BIO or ENG?
- How many students took BIO and PE but not ENG?



1.5.2 Inclusion-Exclusion Principle

Two sets: $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$.
Three sets: $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_3 \cap A_1| + |A_1 \cap A_2 \cap A_3|$.

Example:
There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

2 Permutation and Combination

2.1 Factorial

2.1.1 Notation/Definition

$n! =$

2.1.2 Examples

- $3! =$
- $5! =$
- $(n^2 + 5n + 6)(n + 1)! =$

2.2 Permutation

2.2.1 Definition

2.2.2 Notation

$mPn = P(m, n) =$

2.2.3 Examples

- $5P3 =$
- $P(7, 2) =$
- $P(n, r)P(n - r, m) =$

2.3 With Repetition

2.3.1 No repetiotion

How many arrangements are there for the word MATHS?

2.3.2 With repetiotion

How many arrangements are there for the word TORONTO?

2.3.3 Problem

How many ways can 12 basketball players be assigned to four triple rooms?

2.4 Grouping

2.4.1 Example

Math Club is taking yearbook photo one day with 20 members and Mr.Fraschetti and Mr.Gatti lining in a row. If Mr.Fraschetti and Mr.Gatti insist to stand together, how many arrangements are there?

2.4.2 Problem

At a recent conference of the 11 premiers (including the Prime Minister), find the number of different group photos possible if Lucien Bouchard and Jean Chretien refused to stand next to each other and if the premiers arranged themselves in a line.

2.5 Circular

2.5.1 Example

How many arrangements are there if 8 people sitting around a circular table?

2.5.2 Problem

Grace is making a bracelet with 8 beads that are all different colours. How many bracelets can she make if the bracelet has no visible clasp?

2.6 Combination

2.6.1 Definition

2.6.2 Notation

$mCn = \binom{m}{n} =$

2.6.3 Examples

- $5C3 =$
- $\binom{7}{2} =$
- Prove $\binom{n}{r} = \binom{n}{n-r}$

2.7 Combination Problems

2.8 Stars and Bars

3 Pascal Triangle

3.1 Pascal Triangle

3.2 Binomial Theorem and Multi-nomial Theorem

3.3 Pascal Routes

4 Probability

4.1 Discrete

4.2 Continuous

4.3 Conditional Probability

5 Problems