

# Geometry 2 - Circles

TSS Math CLub

Oct 2022

## 1 Basic property of Circles

### 1.1 Definition of Circles

### 1.2 Terms to describe geometric object related to circles

Center:

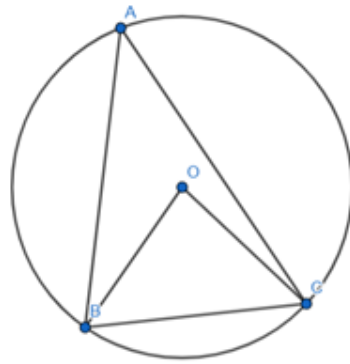
Radius:

Arc:

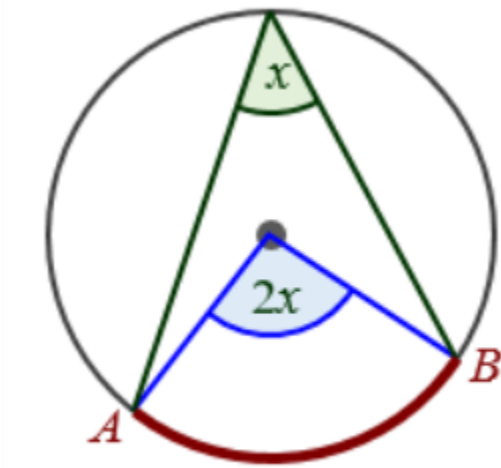
Chord:

Central angle:

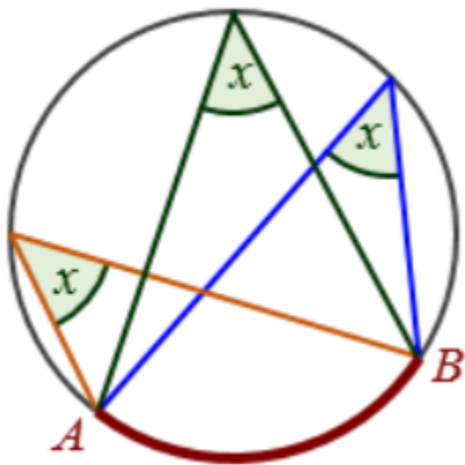
Inscribed angle:



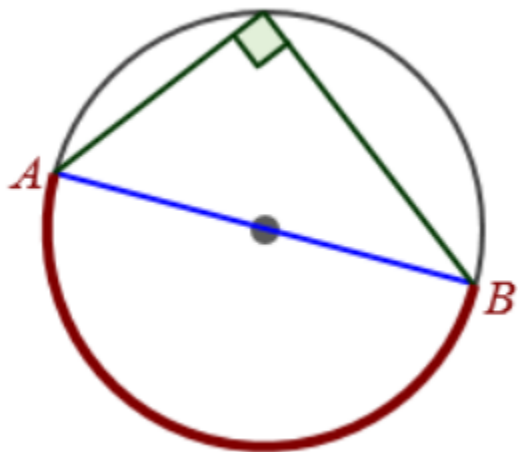
### 1.3 Central angle is twice any inscribed angle



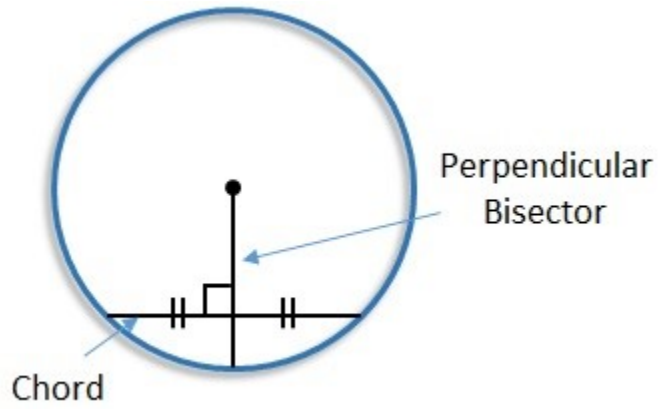
1.4 Inscribed angles subtended by the same arc are equal



1.5 Angle subtended by a diameter is  $90^\circ$



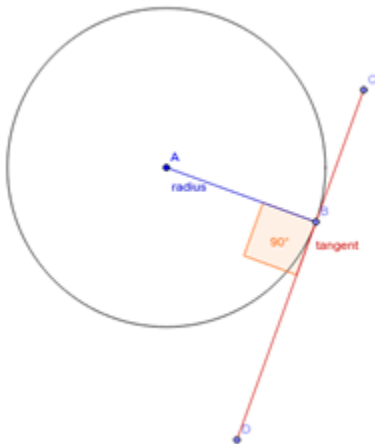
## 1.6 Perpendicular chord theorem



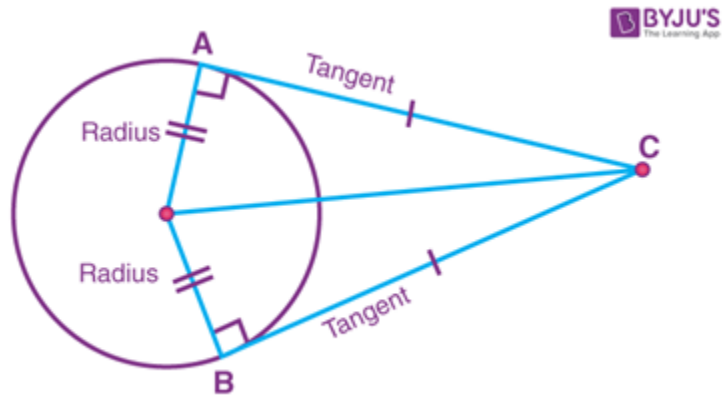
## 1.7 Tangent to a circle

### 1.7.1 Definition:

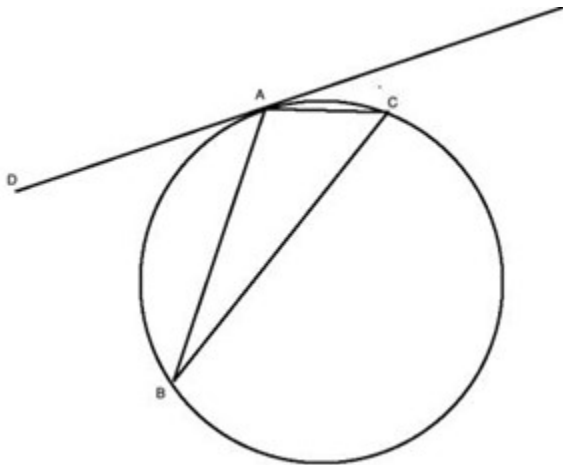
1.7.2 The radius from the center of the circle to the point of tangency is perpendicular to the tangent line



1.7.3 The length of tangents from a point to a circle are equal



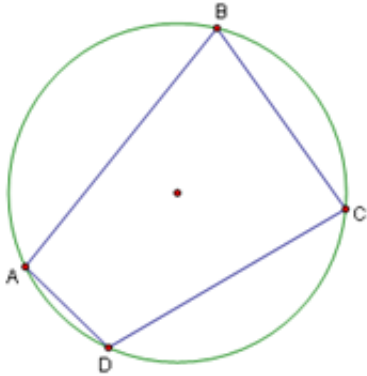
1.7.4 Tangent-Chord Theorem: the angle formed between a chord and a tangent line to a circle is equal to the inscribed angle on the other side of the chord



## 2 Cyclic Quadrilateral (Four points cyclic)

### 2.1 Definition

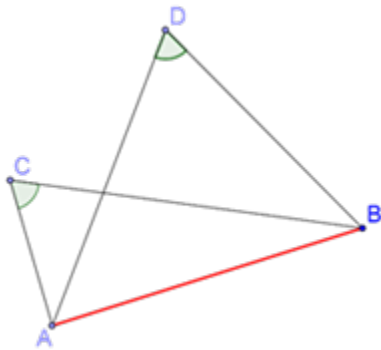
### 2.2 Opposite angles are added up to $180^\circ$



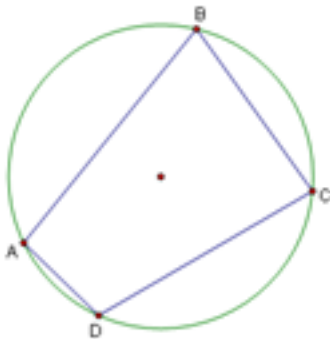
### 2.3 How to prove four points cyclic

2.3.1 Prove these four points lies equally distance to another point — the center of the circle

2.3.2 Two equal angles subtend a segment (chord in the circle)

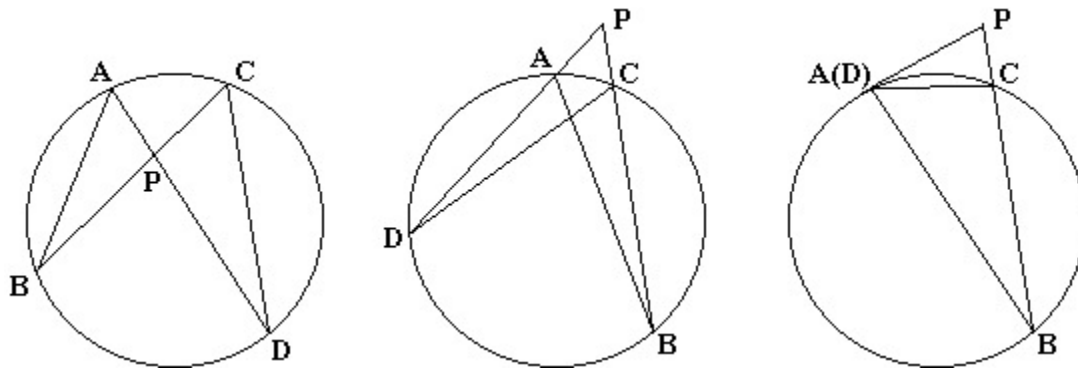


2.3.3 Opposite angles are added up to  $180^\circ$



### 3 Similar triangles involving a circle

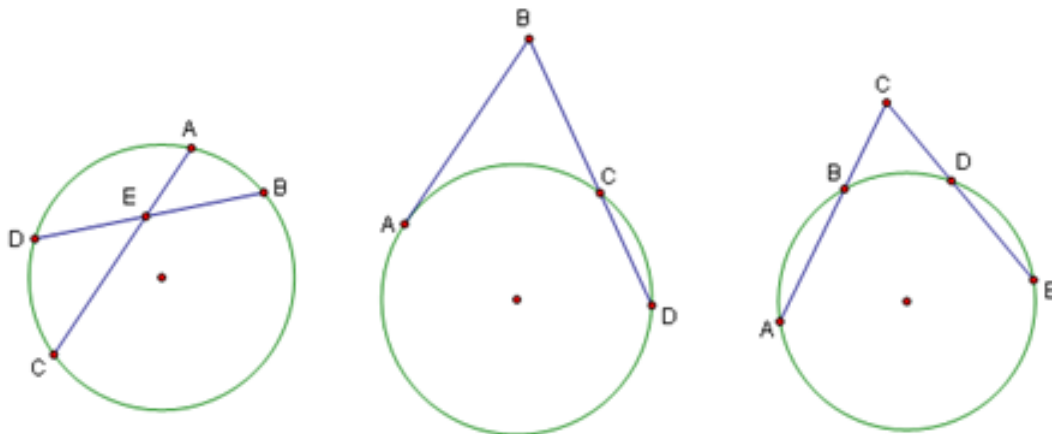
#### 3.1 Identify as many similar triangles as possible



#### 3.2 Power of a point

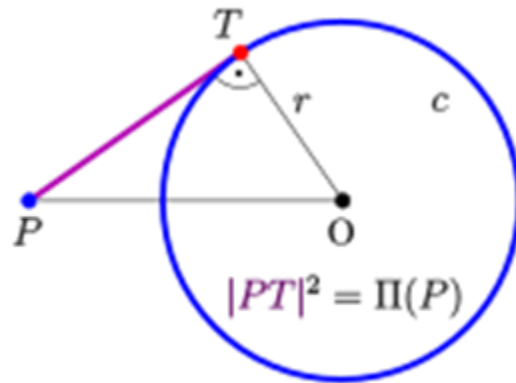
##### 3.2.1 Definition:

##### 3.2.2 Power of point is fixed regardless the choice of chord



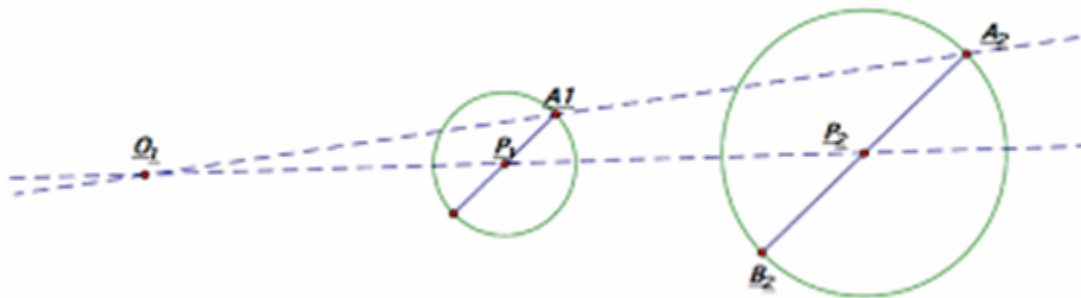
### 3.2.3 Power of a point formula

$$PoP = PO^2 + r^2$$



## 3.3 Homothety involving circles

### 3.3.1 Homothety of a circle is a circle

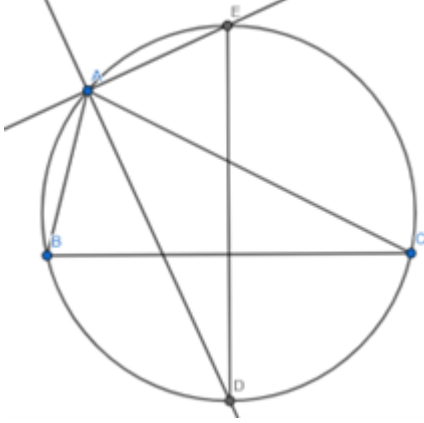


### 3.3.2 Ratios in the homothety

## 4 Problems

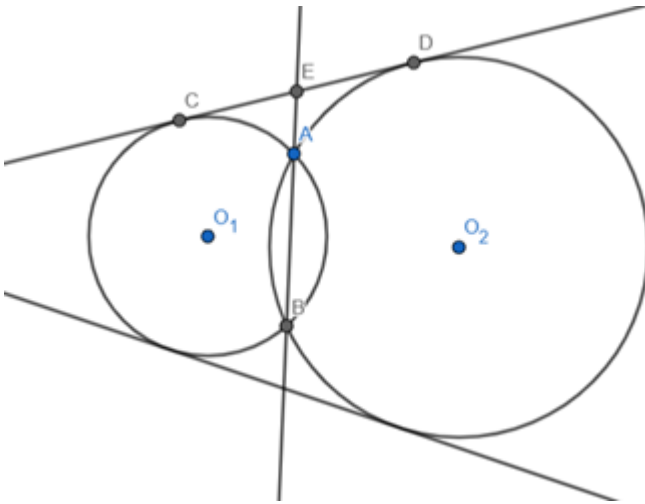
### 4.1 Problem

Given  $AD$   $AE$  are the internal, external angle bisector of angle  $A$ , such that  $D, E$  are the intersection of the angle bisectors with the circumcircle. Prove  $DE$  is a diameter of the circle.



### 4.2 Problem

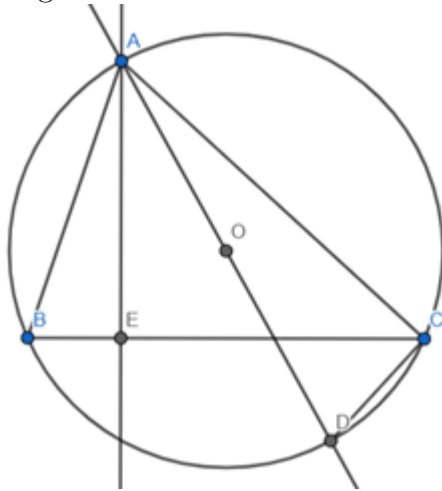
Given Circle  $C_1$ ,  $C_2$  intersect at  $A$ ,  $B$ ,  $CD$  is the common tangent to both circles,  $E$  is the intersection of  $AB$  and  $CD$ . Prove  $E$  is the midpoint of  $CD$ .





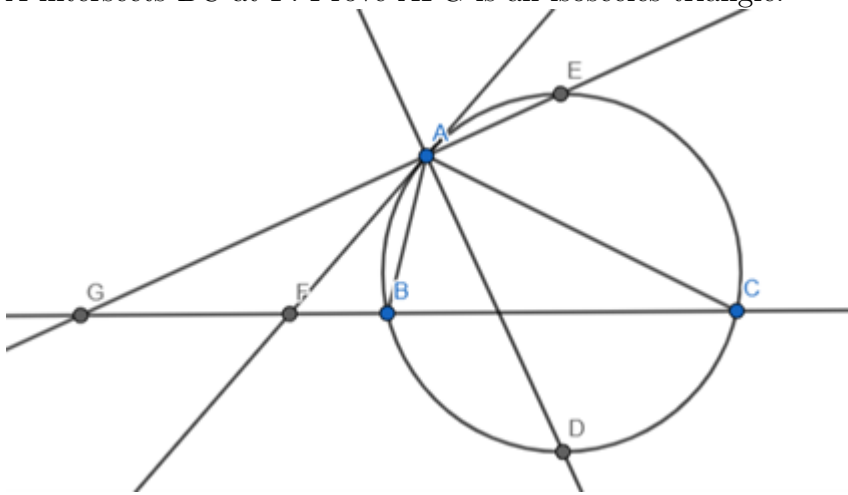
### 4.3 Theorem

In a triangle  $abc=4RS$



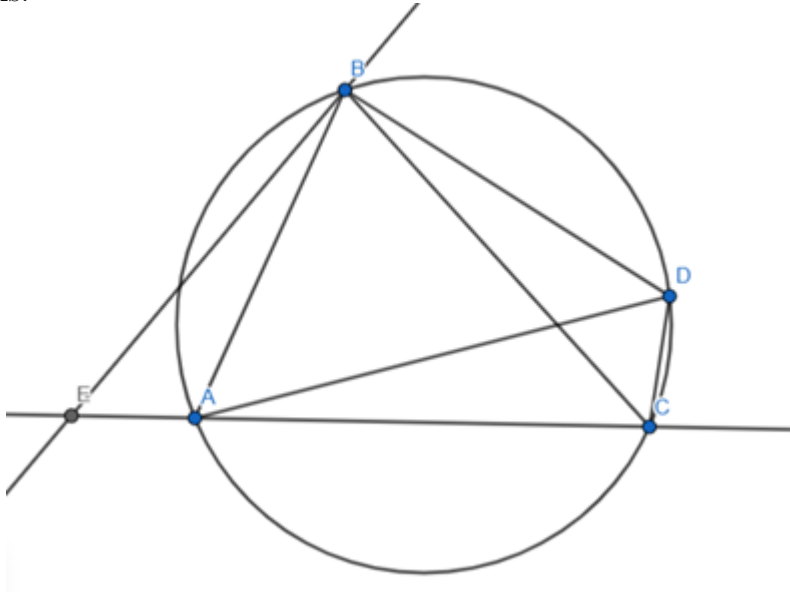
#### 4.4 Problem

Given  $AE$  is the external angle bisector of angle  $A$ ,  $AE$  intersects  $BC$  at  $G$ , the tangent at  $A$  intersects  $BC$  at  $F$ . Prove  $AFG$  is an isosceles triangle.



#### 4.5 Ptolemy's theorem

If a quadrilateral is inscribable in a circle then the product of the lengths of its diagonals is equal to the sum of the products of the lengths of the pairs of opposite sides. Or  $ab+cd=xy$  where  $a,b,c,d$  are the sides of the quadrilateral and  $x,y$  are the diagonals.



#### 4.6 Problem

In  $\triangle ABC$ , point  $D$  is inside of  $ABC$  such that  $\angle DAC = \angle DCA = 30^\circ$  and  $\angle DBA = 60^\circ$ .  $E$  is the midpoint on  $BC$  and  $F$  is a trisect point on  $AC$  such that  $CF = \frac{CA}{3}$ . Prove  $DE \perp EF$ .

