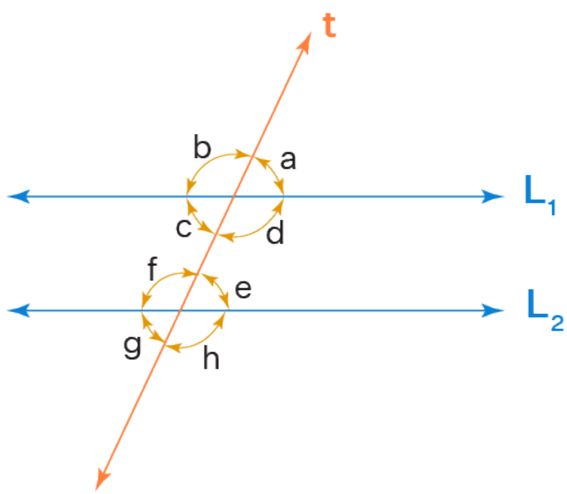
Geometry 1

1. Parallelism and basic geometry

1.1.Parallel lines:

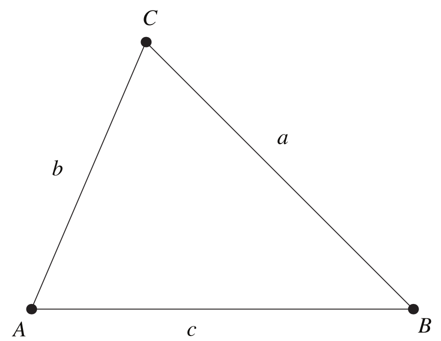
- Parallel postulate:
    - If a line segment intersects two straight lines forming two interior angles on the same side that are less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.
- Definition:

- Parallel Lines and Transversal



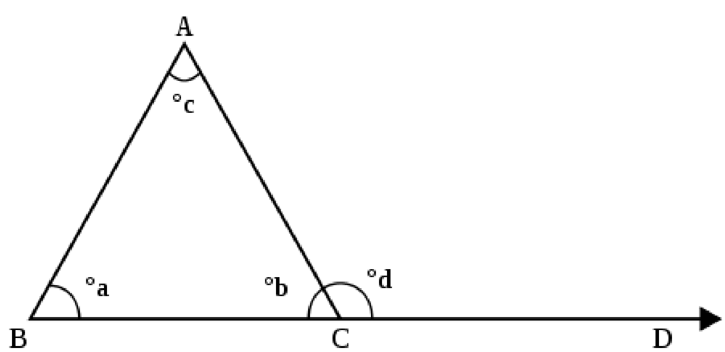
1.2.Parallel is transitive (i.e. l1//l2,l2//l3 -> l1//l3)

1.3.Sum of interior angles of a triangle is 180°



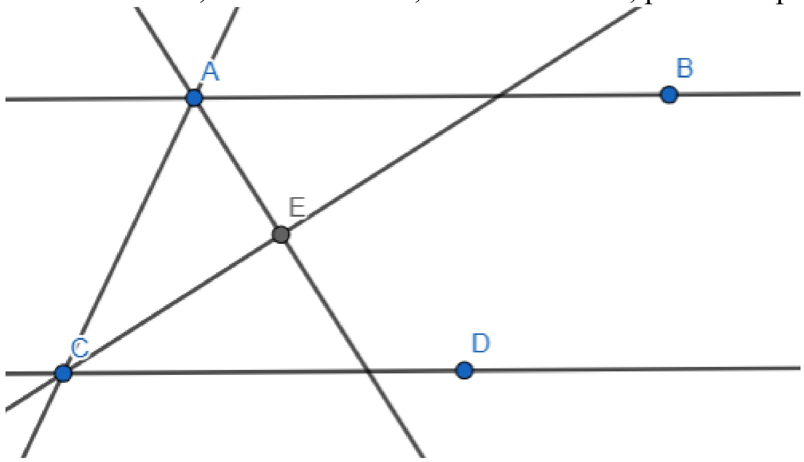
1.4.Sum of interior angles of a n-gon is (n-2)x180°

1.5.Exterior angle theorem



1.6.Another problem

Given AB//CD, AE bisect BAC, CE bisect ACD, prove AE perpendicular to CE.

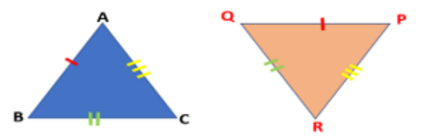
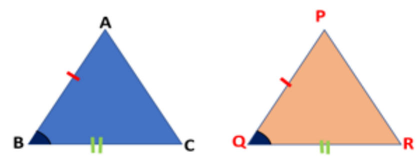
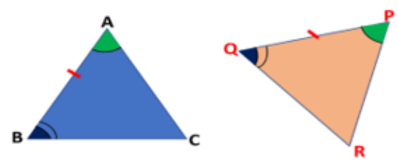
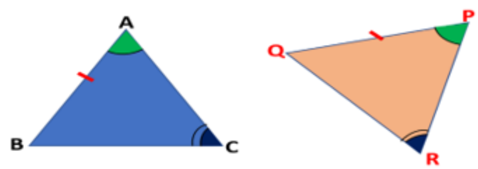
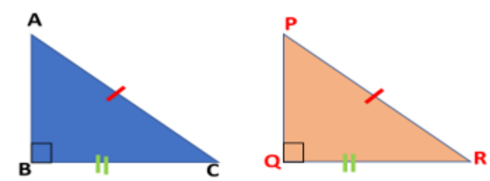


2. Congruence

2.1.Definition:

2.2.Method to prove congruency

Congruent Triangles

SSS (Side – Side – Side) Congruency		*When the three sides of a triangles are equal to the other three sides of another triangle.* $\Delta ABC \cong \Delta PQR$
SAS (Side – Angle – Side) Congruency		*When two sides and the included angle on one triangle is equal to the two sides and included angle of another triangle.* $\Delta ABC \cong \Delta PQR$
ASA (Angle – Side – Angle) Congruency		*When two angles and the included side of one triangle are equal to two angles and the included side of another triangle.* $\Delta ABC \cong \Delta PQR$
AAS (Angle – Angle – Side) Congruency		*When two angles and the non-included side of one triangle are equal to two angles and the non-included side of another triangle.* $\Delta ABC \cong \Delta PQR$
RHS (Right Angle – Hypotenuse – Side) Congruency		*When hypotenuse and one side of a right triangle is equal to the hypotenuse and other side of another right triangle.* $\Delta ABC \cong \Delta PQR$

How to write in a contest:

2.2.1. Side-Side-Angle (SSA), the ambiguous case.

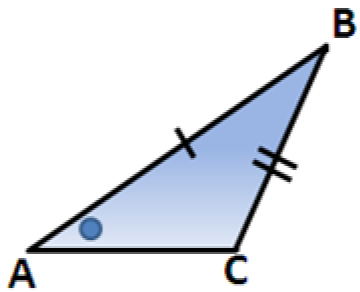


Figure 3-A

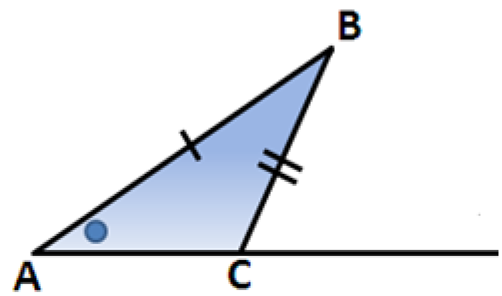


Figure 3-B

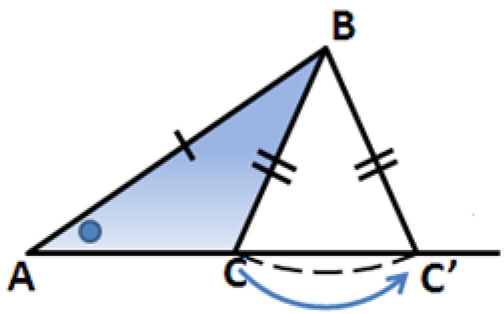


Figure 3-C

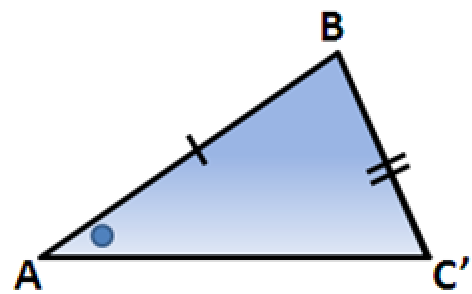
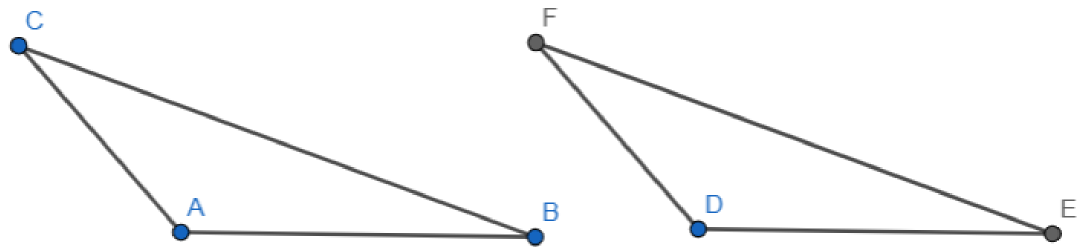


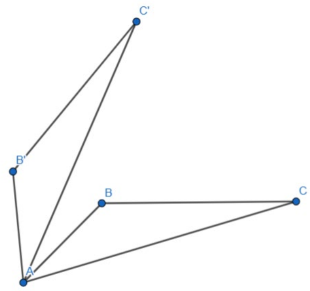
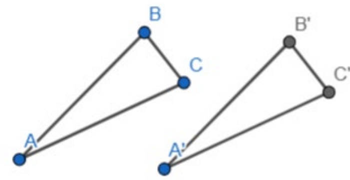
Figure 3-D

2.2.2. Proof related to the ambiguous case:

Given AB=DE, BC=EF, angle A =angle D>90°, prove triangle ABC congruent to triangle DEF.



2.3.Useful congruencies

Rotation	Translation
	

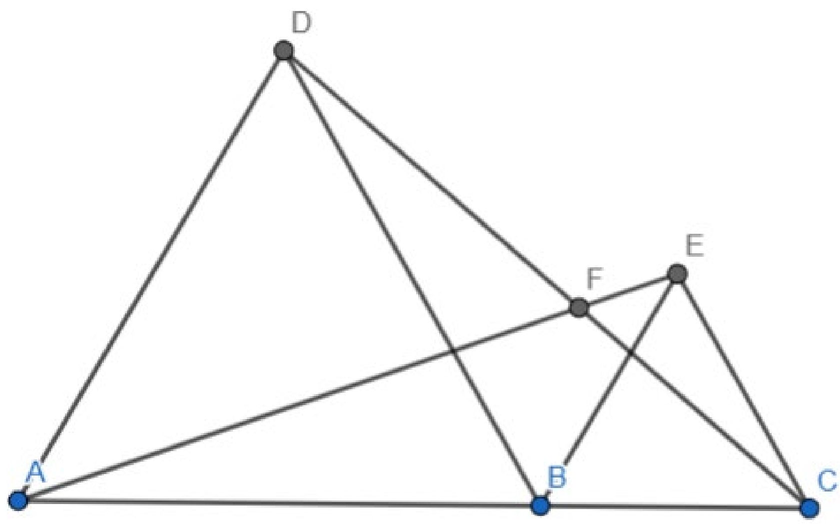
2.4.Properties of congruency

2.4.1. Equal side length

2.4.2. Equal angles

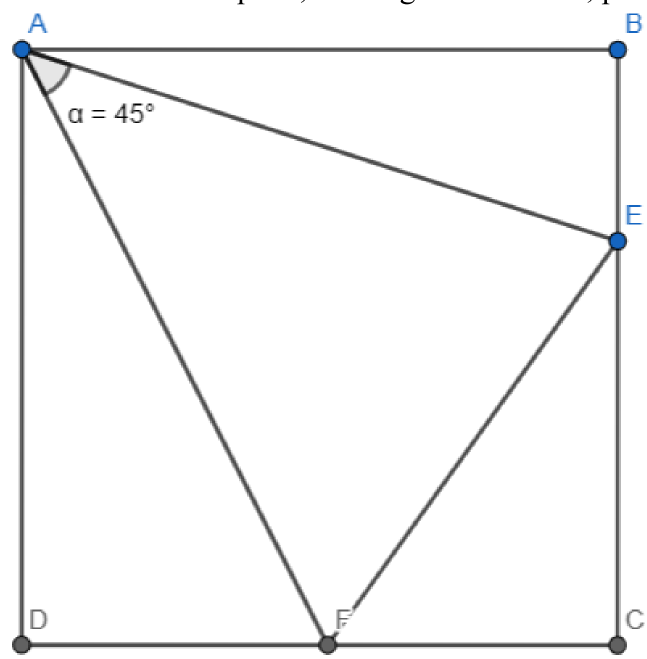
2.5.Problems

2.5.1. Assume ABD and BCE are equilateral triangles, find angle DFA



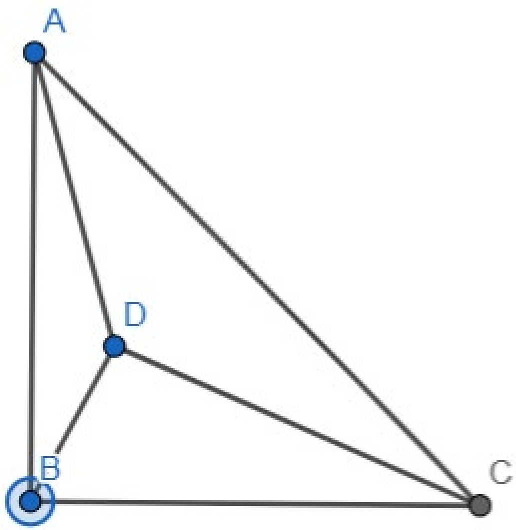
2.5.2.

Given ABCD is a square, and angle EAF = 45°, prove AE bisect BEF.



2.5.3.

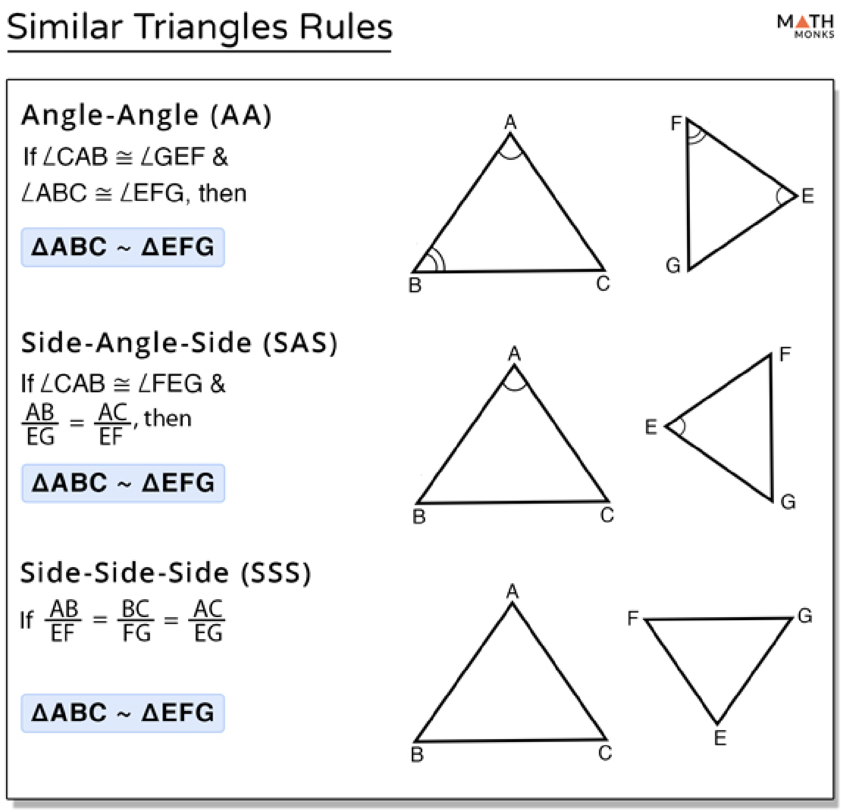
Given AB=BC and angle ABC = 90°, AD=$\sqrt{5}$, BD=$\sqrt{2}$, DC=3, find angle ADB.



3. Similarity

3.1.Definition:

3.2.Method to Prove similar triangles



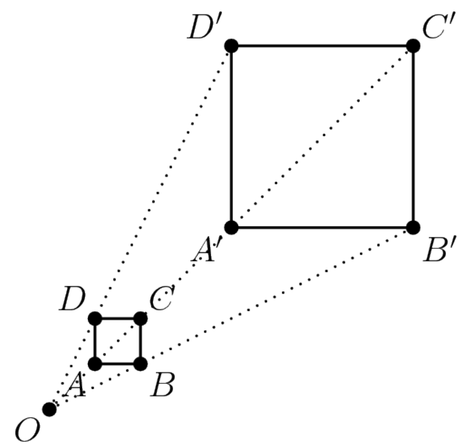
3.3.Properties of similar triangles

3.3.1. Ratio of corresponding sides

3.3.2. Equal angles

4. Homothety

4.1.Definition:



4.2.Properties:

5. Quadrilaterals

5.1.Parallelogram

5.1.1. Definition:

5.1.2. Properties:

5.2. Rhombus

5.2.1. Definition:

5.2.2. Properties:

5.3. Rectangle

5.3.1. Definition:

5.3.2. Properties:

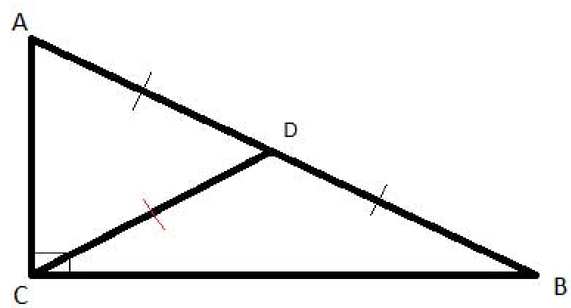
5.4. Square

5.4.1. Definition:

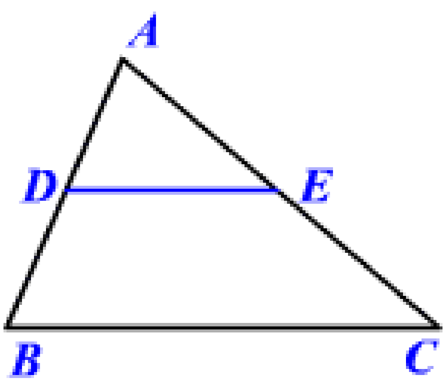
5.4.2. Properties:

6. Midpoint

6.1. Midpoint of a right triangle

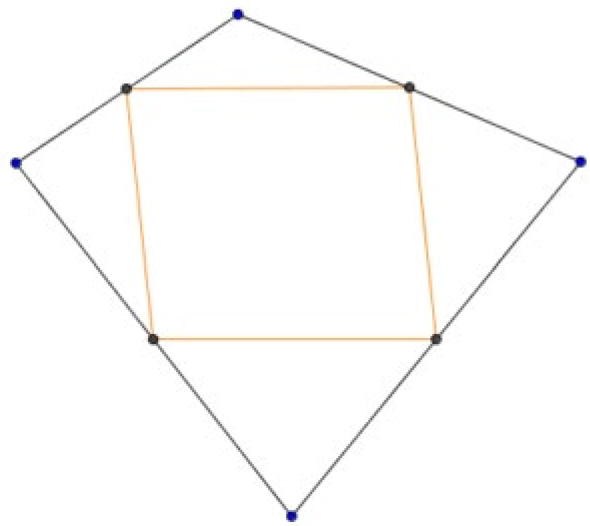


6.2. Midsegment

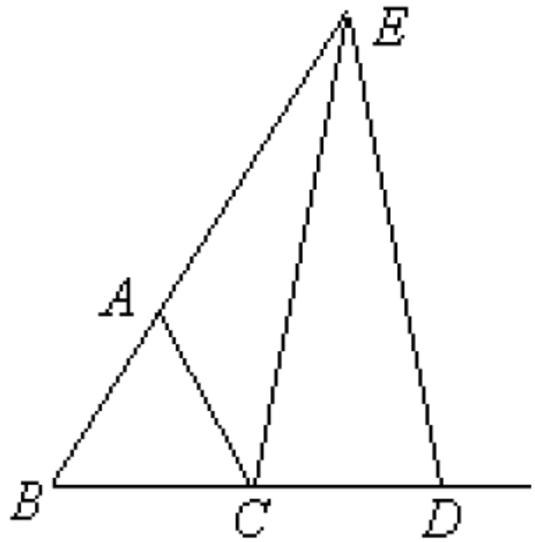


7. Problems

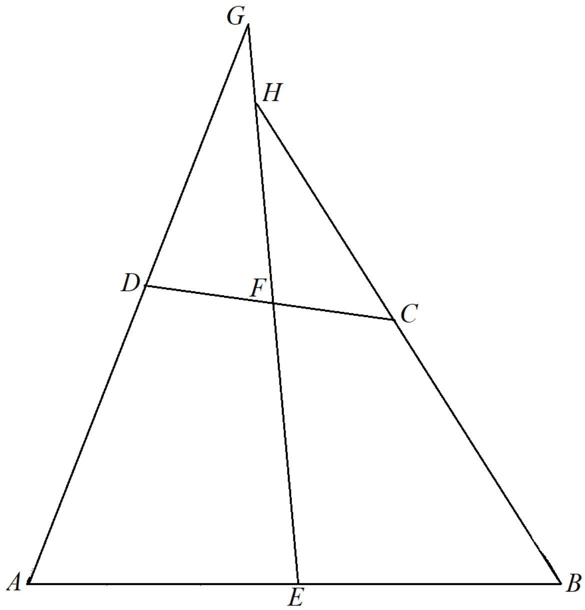
7.1. The quadrilateral formed by joining midpoint of a quadrilateral is a parallelogram



7.2. Given equilateral ΔABC, extend BC to D and BA to E to let AE = BD. Join CE and DE. Prove ∠ECD =∠EDC.



7.3. In quadrilateral ABCD with AD = BC, E and F are midpoints on AB and CD, respectively. EF meets AD at G, meets BC at H. Prove ∠DGF = ∠CHF.



7.4. Given BD bisects angle ABC, CE bisects angle ACB, AF perpendicular to BD, AG perpendicular to CE, prove FG//BC.