Algebra 3 - Sequence and Series

TSS Math Club

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1 Sequence

1.1 Definition

A sequence can be thought of as a list of numbers written in a definite order:

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

The number a_1 is called the first term, a_2 is the second term, and in general a_n is the n-th term.

1.2 Notation

The sequence $\{a_1, a_2, a_3, \ldots\}$ is also denoted by $\{a_n\}$ or $\{a_n\}_{n\geq 1}$

1.3 Examples

1.3.1 Sequence Definited by General Term

$$a_n = \frac{1}{n}$$

1.3.2 Arithmetic Sequence

Definition: An arithmetic sequence is a sequence where each term increases by adding/subtracting some constant d (the common difference).

$$a_{n+1} = a_n + d$$

Useful Formulae:

- $a_n = a_1 + (n-1)d$
- $2a_n = a_{n-1} + a_{n+1}$

1.3.3 Geometric Sequence

Definition: A geometric sequence is a sequence in which each term is found by multiplying the preceding term by the same value r (common ratio).

$$a_{n+1} = ra_n$$

Useful Formulae:

- $a_n = a_1 r^{n-1}$
- $a_n^2 = a_{n-1} a_{n+1}$

1.3.4 Fibonacci Sequence

$a_1 = 1, a_2 = 1, a_{n+2} = a_{n+1} + a_n$

1.3.5 Recursive Sequence

A recursive sequence is a sequence in which terms are defined using one or more previous terms which are given. Example: $a_1 = x, a_2 = y, a_n = \frac{a_{n-1}+1}{a_{n-2}}$
$a_1 = x, a_2 = y, a_3 = \frac{y+1}{x}, a_4 = \frac{x+y+1}{xy}, a_5 = \frac{x+1}{y}, a_6 = x, a_7 = y, \ldots$

2 Limit of a Sequence