

Geometry 2 - Circles

TSS Math Club

Oct 2022

1 Basic property of Circles

1.1 Definition of Circles

1.2 Terms to describe geometric object related to circles

Center:

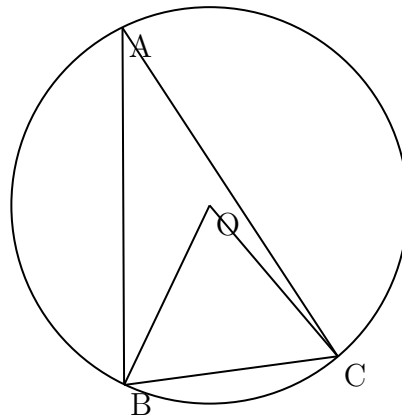
Radius:

Arc:

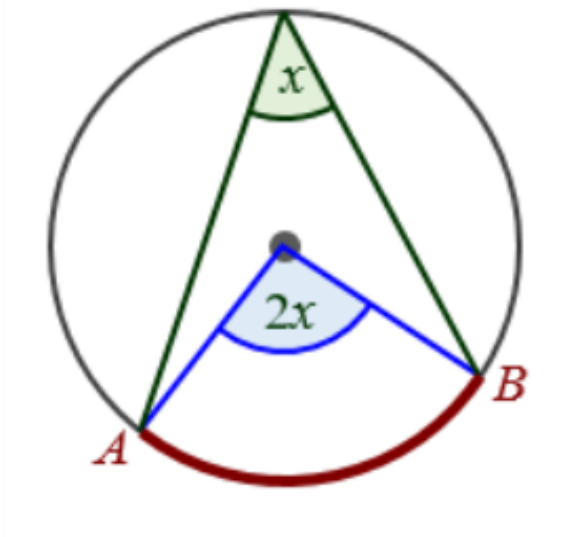
Chord:

Central angle:

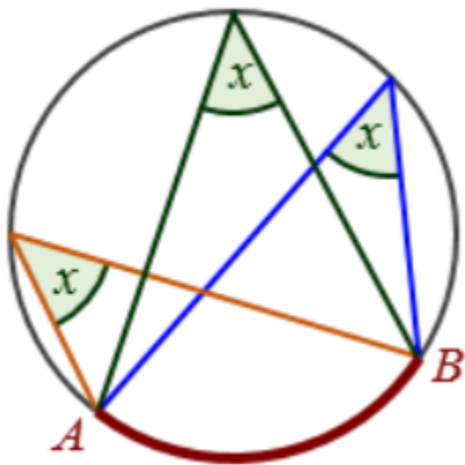
Inscribed angle:



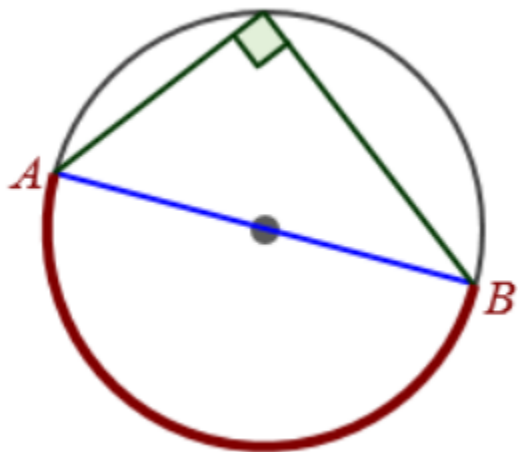
1.3 Central angle is twice any inscribed angle



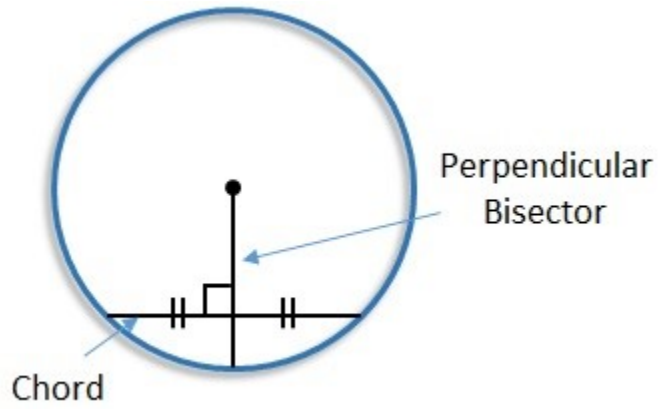
1.4 Inscribed angles subtended by the same arc are equal



1.5 Angle subtended by a diameter is 90°



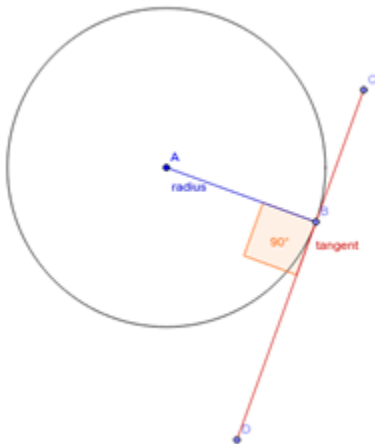
1.6 Perpendicular chord theorem



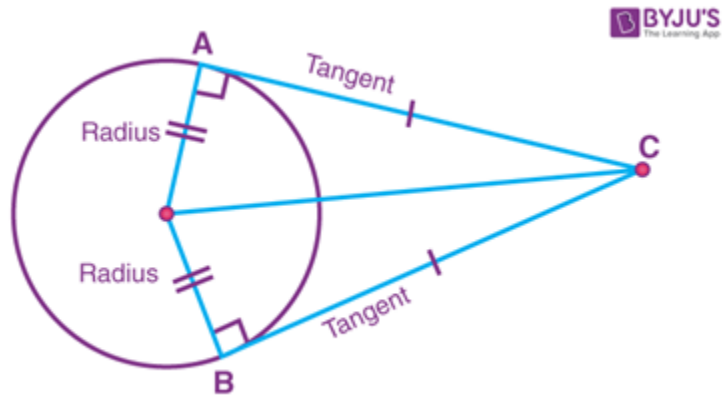
1.7 Tangent to a circle

1.7.1 Definition:

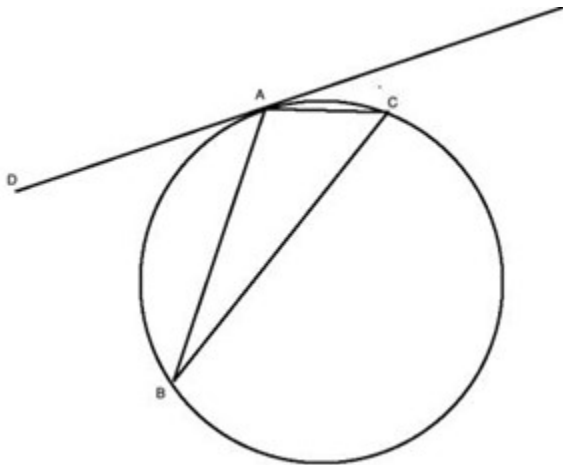
1.7.2 The radius from the center of the circle to the point of tangency is perpendicular to the tangent line



1.7.3 The length of tangents from a point to a circle are equal



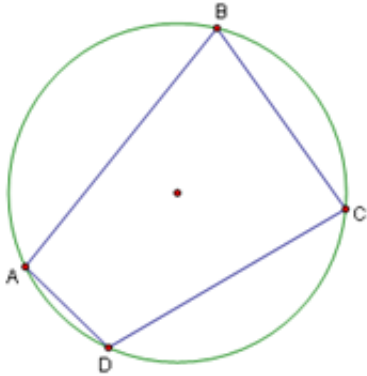
1.7.4 Tangent-Chord Theorem: the angle formed between a chord and a tangent line to a circle is equal to the inscribed angle on the other side of the chord



2 Cyclic Quadrilateral (Four points cyclic)

2.1 Definition

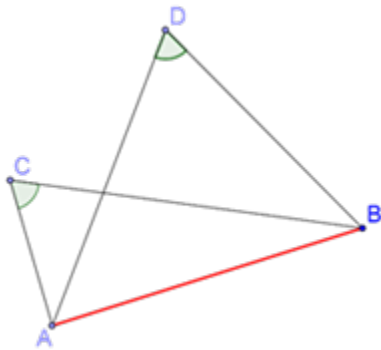
2.2 Opposite angles are added up to 180°



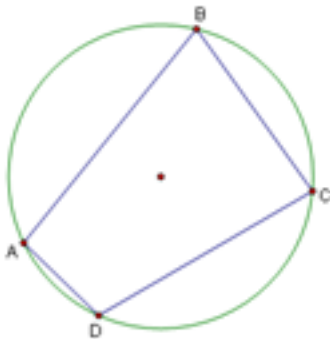
2.3 How to prove four points cyclic

2.3.1 Prove these four points lies equally distance to another point — the center of the circle

2.3.2 Two equal angles subtend a segment (chord in the circle)

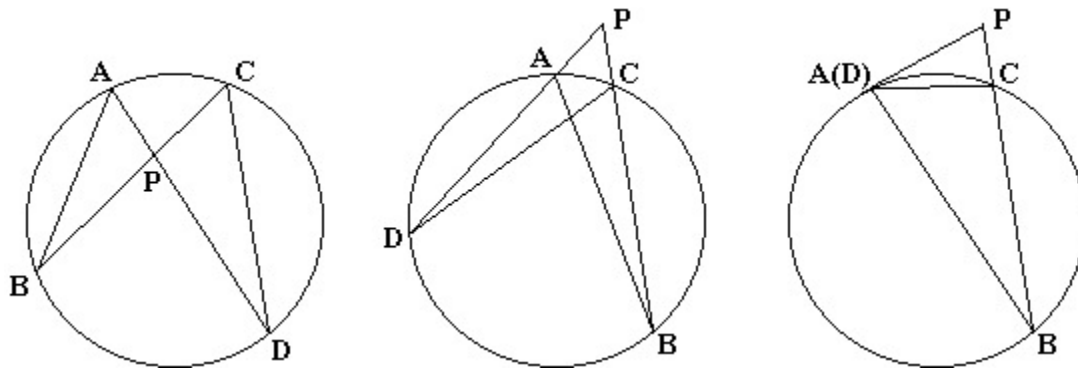


2.3.3 Opposite angles are added up to 180°



3 Similar triangles involving a circle

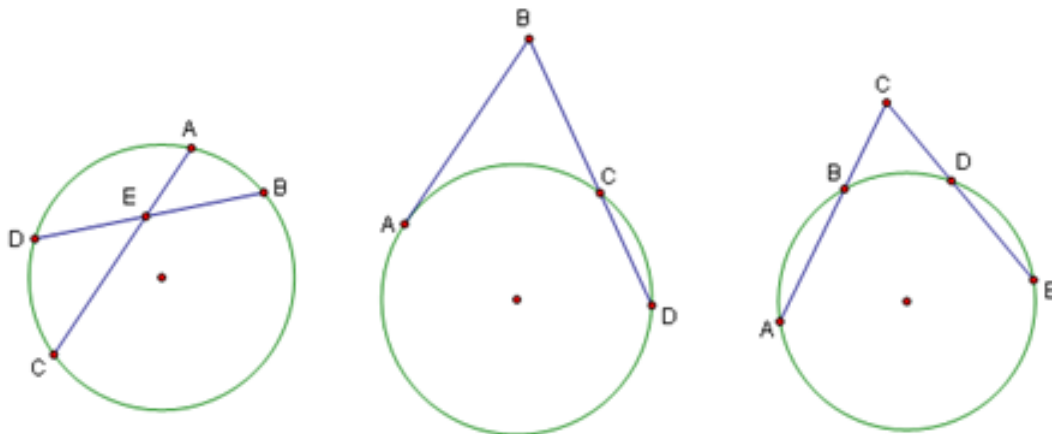
3.1 Identify as many similar triangles as possible



3.2 Power of a point

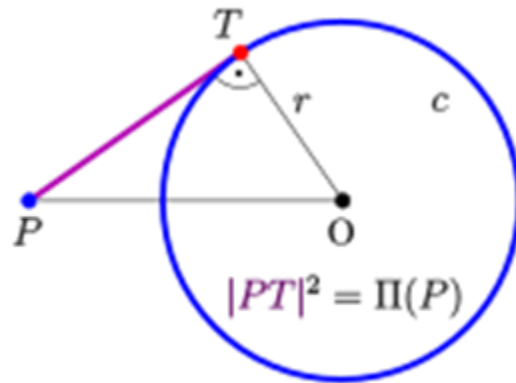
3.2.1 Definition:

3.2.2 Power of point is fixed regardless the choice of chord



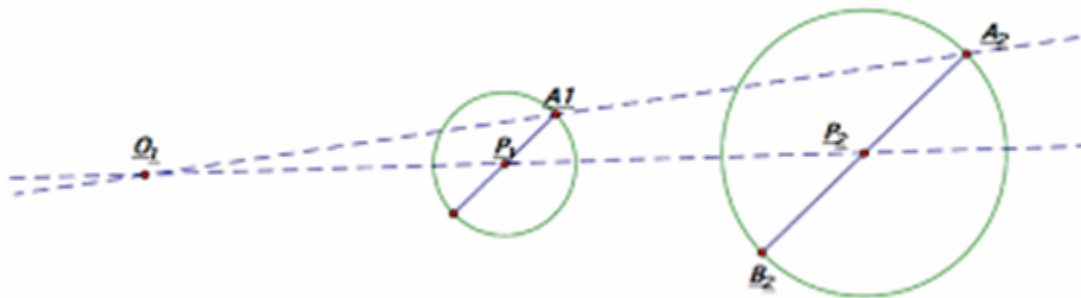
3.2.3 Power of a point formula

$$PoP = PO^2 + r^2$$



3.3 Homothety involving circles

3.3.1 Homothety of a circle is a circle

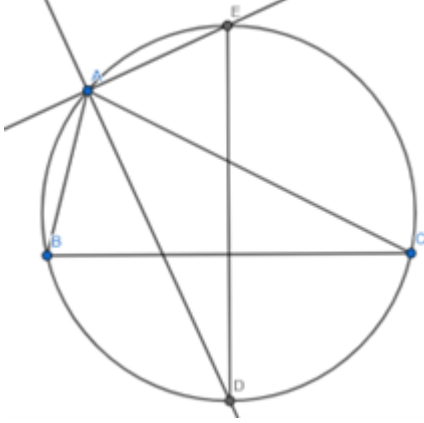


3.3.2 Ratios in the homothety

4 Problems

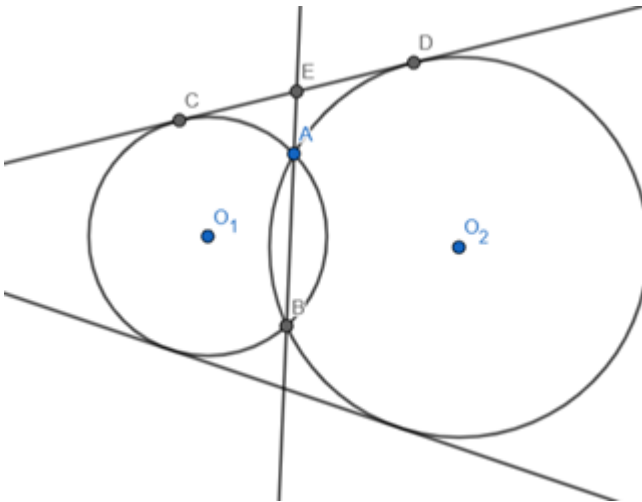
4.1 Problem

Given AD AE are the internal, external angle bisector of angle A , such that D, E are the intersection of the angle bisectors with the circumcircle. Prove DE is a diameter of the circle.



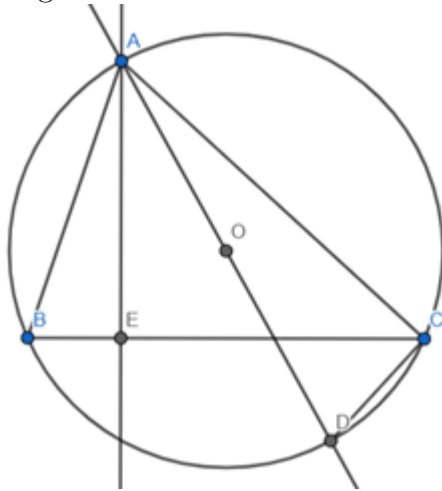
4.2 Problem

Given Circle C_1 , C_2 intersect at A , B , CD is the common tangent to both circles, E is the intersection of AB and CD . Prove E is the midpoint of CD .



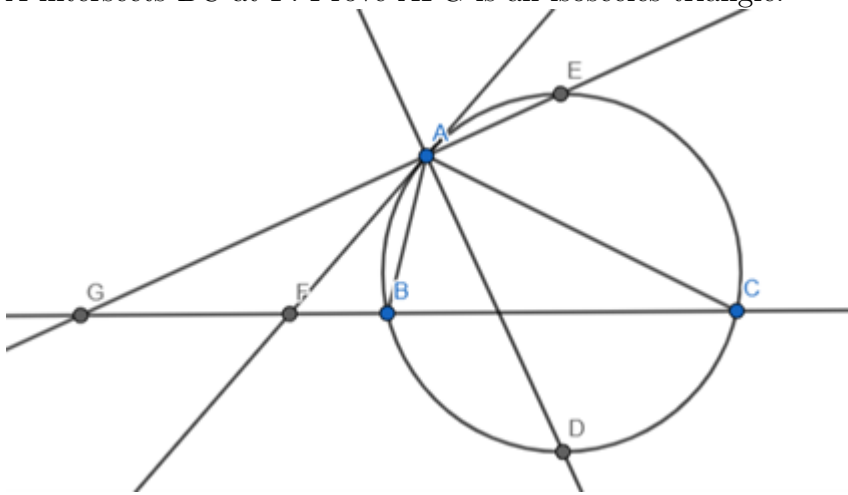
4.3 Theorem

In a triangle $abc=4RS$



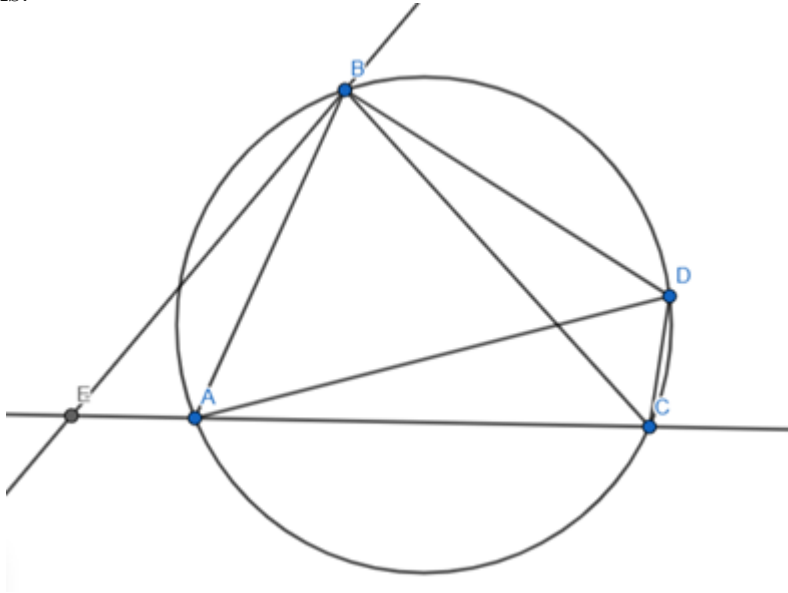
4.4 Problem

Given AE is the external angle bisector of angle A , AE intersects BC at G , the tangent at A intersects BC at F . Prove AFG is an isosceles triangle.



4.5 Ptolemy's theorem

If a quadrilateral is inscribed in a circle then the product of the lengths of its diagonals is equal to the sum of the products of the lengths of the pairs of opposite sides. Or $ab+cd=xy$ where a,b,c,d are the sides of the quadrilateral and x,y are the diagonals.



4.6 Problem

In $\triangle ABC$, point D is inside of ABC such that $\angle DAC = \angle DCA = 30^\circ$ and $\angle DBA = 60^\circ$. E is the midpoint on BC and F is a trisect point on AC such that $CF = \frac{CA}{3}$. Prove $DE \perp EF$.

