Geometry 2 - Circles

TSS Math Club

Oct 2022

1 Basic property of Circles

1.1 Definition of Circles

1.2 Terms to describe geometric object related to circles

Center:

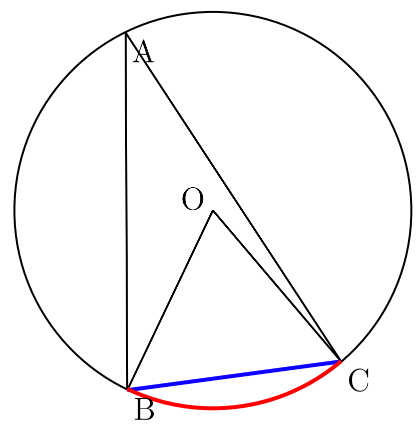
Radius:

Arc:

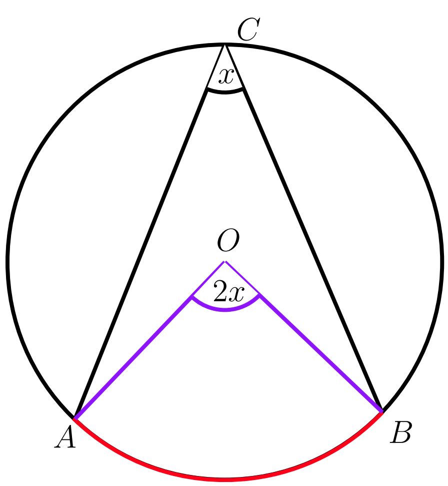
Chord:

Central angle:

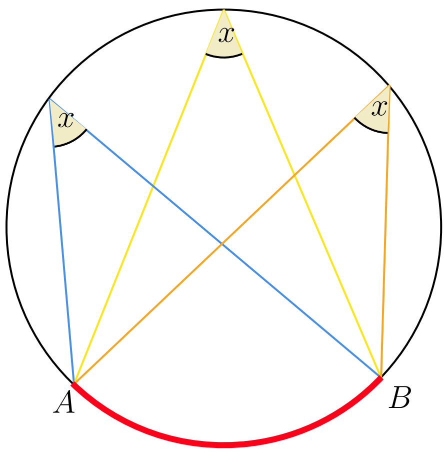
Inscribed angle:



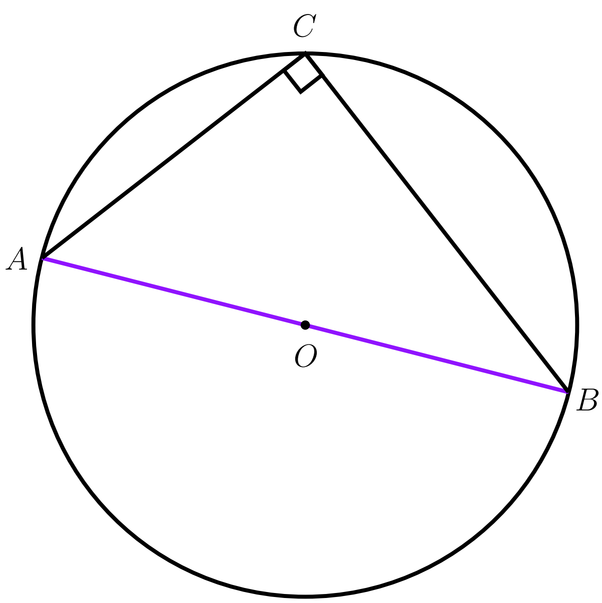
1.3 Central angle is twice any inscribed angle



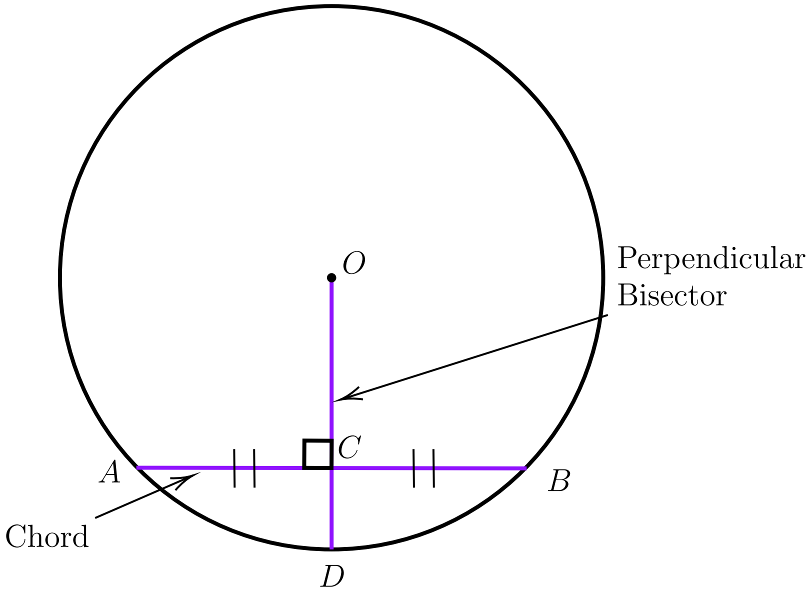
1.4 Inscribed angles subtended by the same arc are equal



1.5 Angle subtended by a diameter is 90°



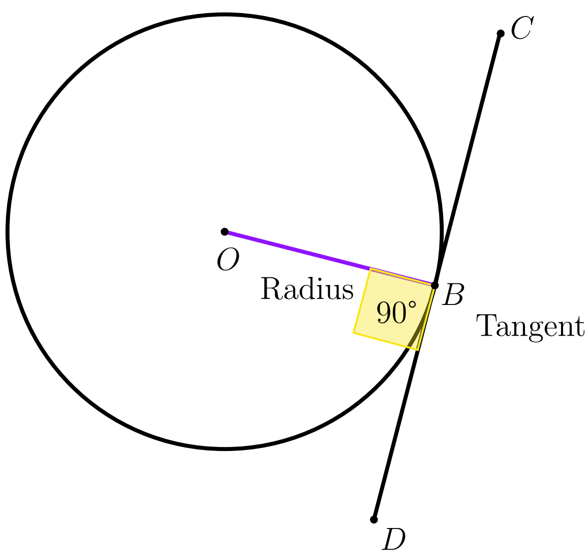
1.6 Perpendicular chord theorem



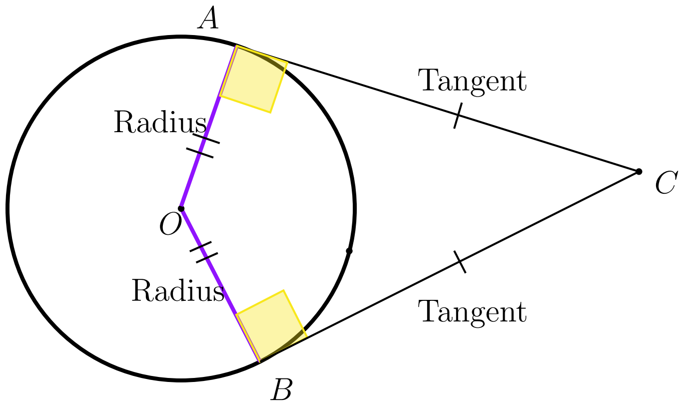
1.7 Tangent to a circle

1.7.1 Definition:

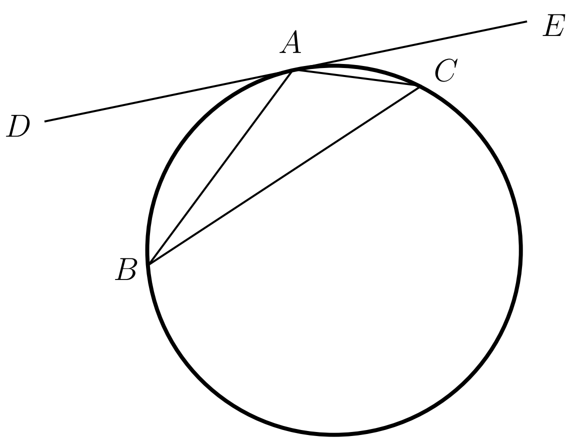
1.7.2 The radius from the center of the circle to the point of tangency is perpendicular to the tangent line



1.7.3 The length of tangents from a point to a circle are equal



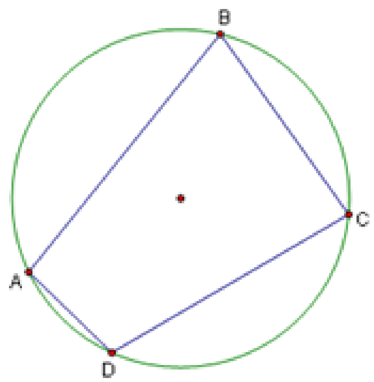
1.7.4 Tangent-Chord Theorem: the angle formed between a chord and a tangent line to a circle is equal to the inscribed angle on the other side of the chord



2 Cyclic Quadrilateral (Four points cyclic)

2.1 Definition

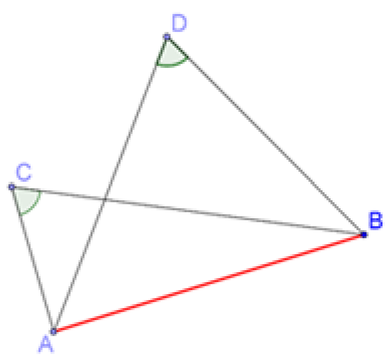
2.2 Opposite angles are added up to $180°$



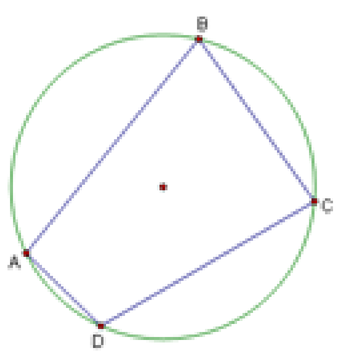
2.3 How to prove four points cyclic

2.3.1 Prove these four points lies equally distance to another point — the center of the circle

2.3.2 Two equal angles subtend a segment (chord in the circle)

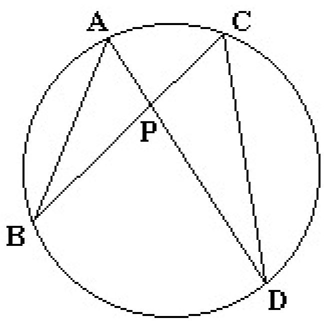


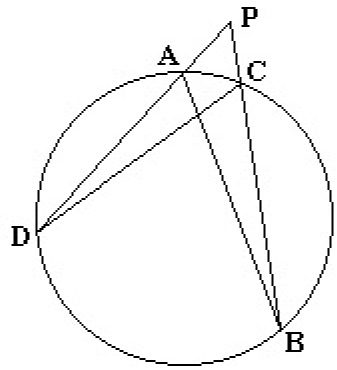
2.3.3 Opposite angles are added up to $180°$

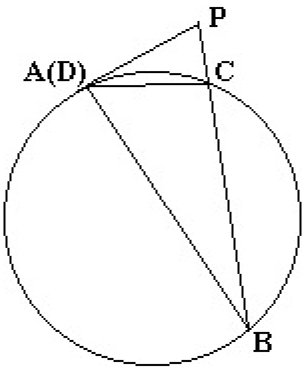


3 Similar triangles involving a circle

3.1 Identify as many similar triangles as possible



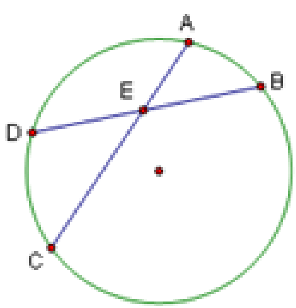


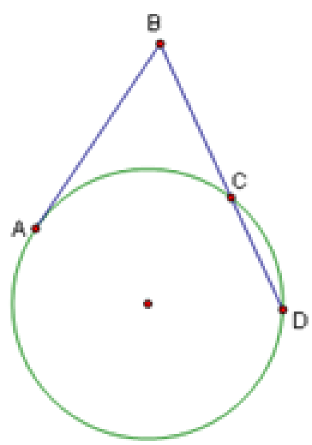


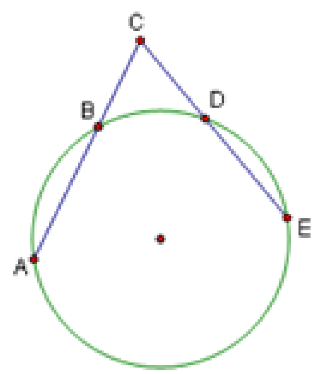
3.2 Power of a point

3.2.1 Definition:

3.2.2 Power of point is fixed regardless the choice of chord

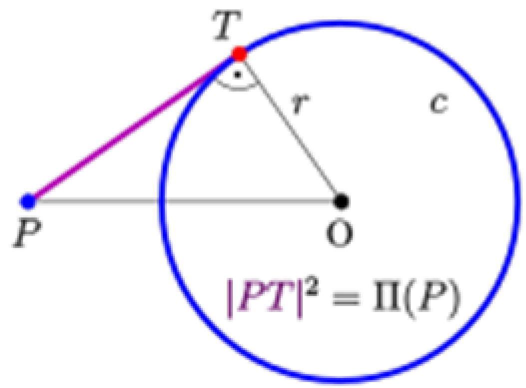






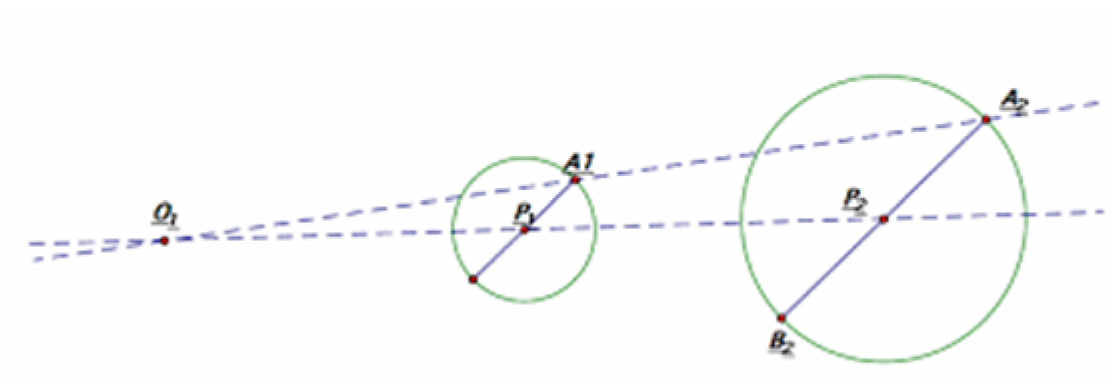
3.2.3 Power of a point formula

$$PoP = PO^2 + r^2$$



3.3 Homothety involving circles

3.3.1 Homothety of a circle is a circle

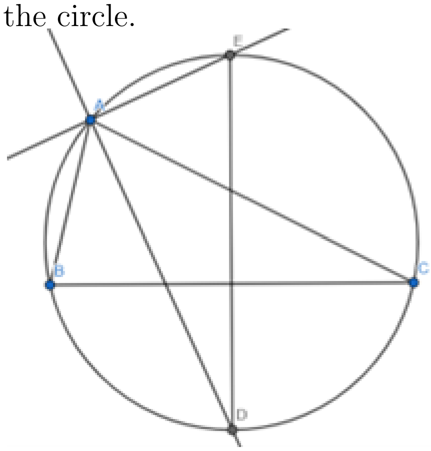


3.3.2 Ratios in the homothety

4 Problems

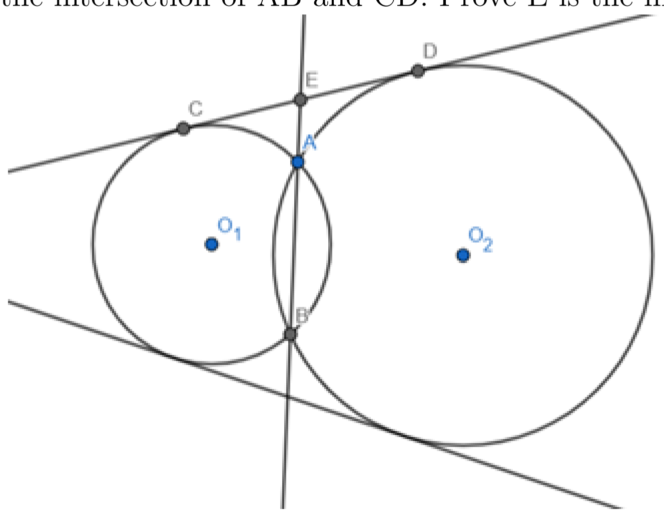
4.1 Problem

Given AD AE are the internal, external angle bisector of angle A, such that D,E are the intersection of the angle bisectors with the circumcircle. Prove DE is a diameter of the circle.



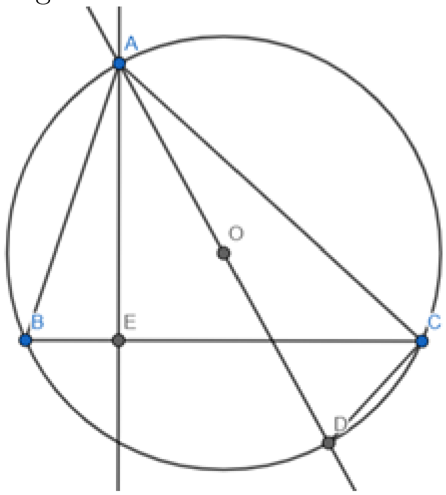
4.2 Problem

Given Circle C1, C2 intersect at A, B, CD is the common tangent to both circles, E is the intersection of AB and CD. Prove E is the midpoint of CD.



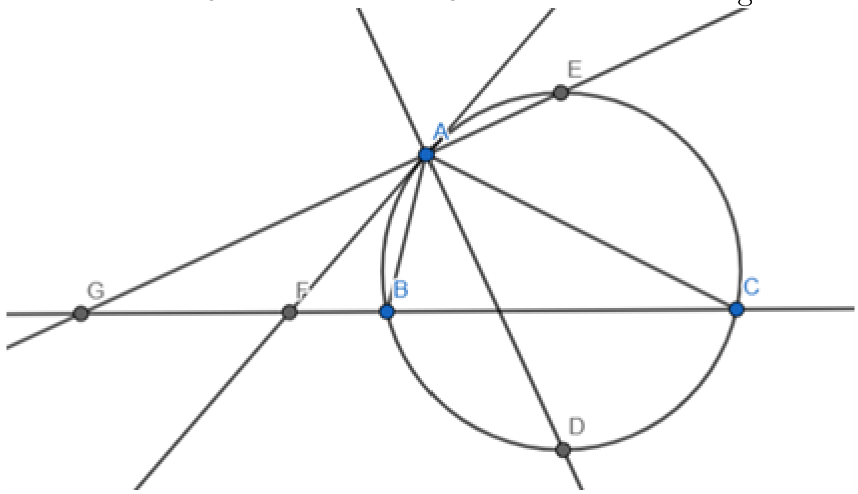
4.3 Theorem

In a triangle abc=4RS



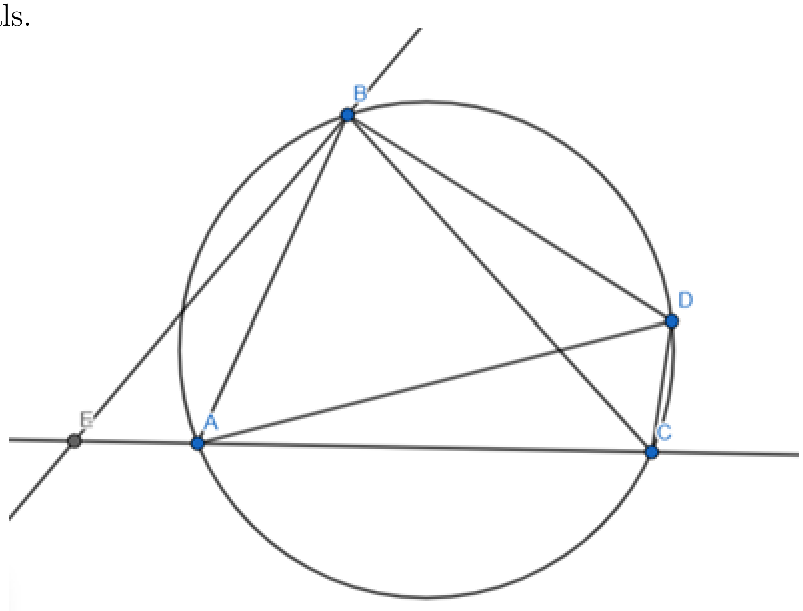
4.4 Problem

Given AE is the external angle bisector of angle A, AE intersects BC at G, the tangent at A intersects BC at F. Prove AFG is an isosceles triangle.



4.5 Ptolemy's theorem

If a quadrilateral is inscribed in a circle then the product of the lengths of its diagonals is equal to the sum of the products of the lengths of the pairs of opposite sides. Or ab+cd=xy where a,b,c,d are the sides of the quadrilateral and x,y are the diagonals.



4.6 Problem

In $\triangle ABC$, point D is inside of ABC such that $\angle DAC = \angle DCA = 30°$ and $\angle DBA = 60°$. E is the midpoint on BC and F is a trisect point on AC such that $CF = \frac{CA}{3}$. Prove $DE \perp EF$.