# Geometry 3 - Miscellaneous

TSS Math Club

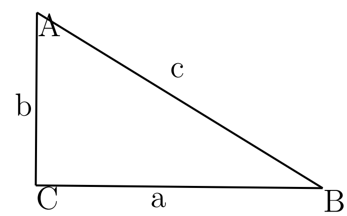
Nov 2022

## 1 Pythagorean Theorem

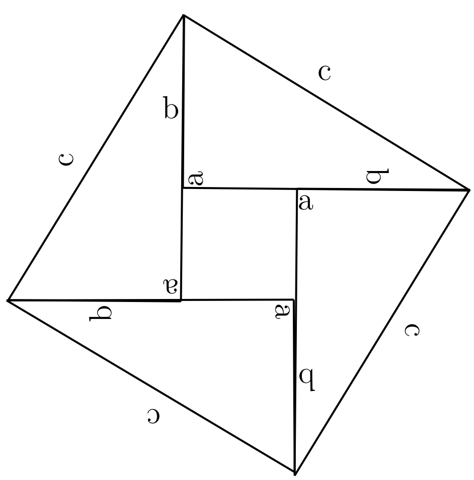
In a right-triangle,

$$a^2 + b^2 = c^2$$

where a and b are two sides and c is the hypotenuse.




### 1.1 Proof



# 2 Trigonometry

## 2.1 Definitions

Sine or $\sin(\theta)$ :

Cosine or $\cos(\theta)$ :

Tangent or $\tan(\theta)$ :

## 2.2 Pythagorean Theorem

$$\sin(\theta) + \cos(\theta) = 1$$

## 2.3 Triangle Area Formula with Sine

$$S = \frac{ab \sin C}{2}$$

### 2.3.1 Proof

## 2.4 Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = d$$

### 2.4.1 Proof

## 2.5 Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

### 2.5.1 Proof

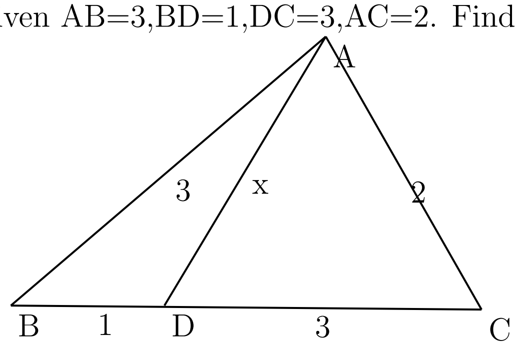
## 2.6 Problem

### 2.6.1 Heron's Formula

$$S = \sqrt{s(s-a)(s-b)(s-c)}, \text{where} s = \frac{a+b+c}{2}$$

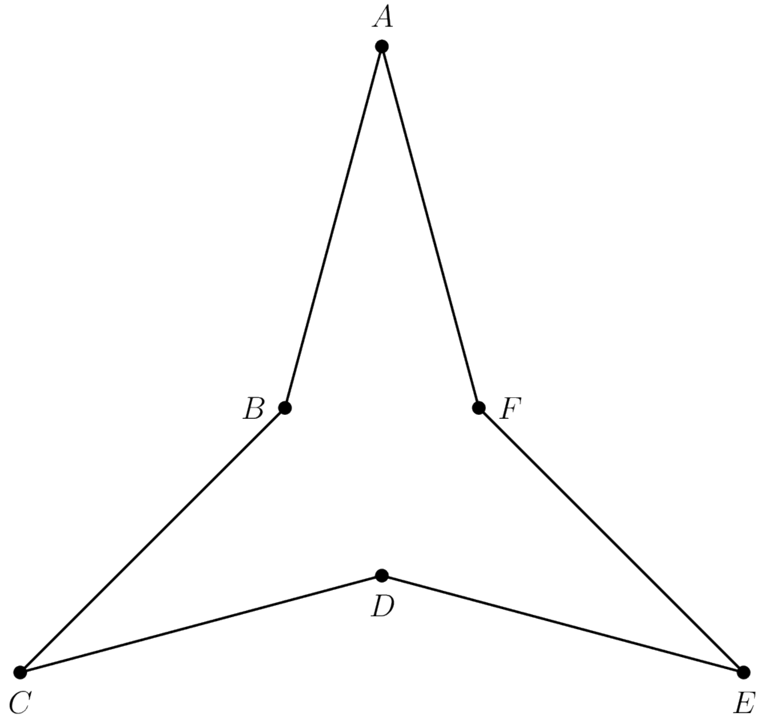
### 2.6.2 Problem

Given AB=3,BD=1,DC=3,AC=2. Find AD.



### 2.6.3 Problem

In the figure, equilateral hexagon $ABCDEF$ has three nonadjacent acute interior angles that each measure $30^\circ$. The enclosed area of the hexagon is $6\sqrt{3}$. What is the perimeter of the hexagon?



# 3 Transversals

## 3.1 Directed Segments

Definition

## 3.2 Stewart's Theorem

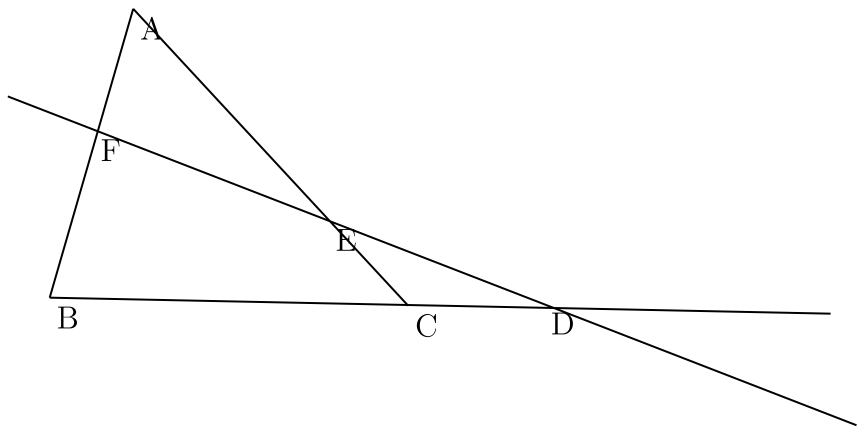
If A,B,C collinear and P is any other point, then

$$PA^2 \cdot BC + PB^2 \cdot CA + PC^2 \cdot AB + BC \cdot CA \cdot AB = 0$$

### 3.3 Menelaus' Theorem

Suppose we have a triangle ABC, and a transversal line that crosses BC, AC, and AB at points D, E, and F respectively, with D, E, and F distinct from A, B, and C, then

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = -1$$




### 3.4 Menelaus' Inverse Theorem

Suppose we have a triangle ABC with D on BC, E on AC, F on AB, such that,

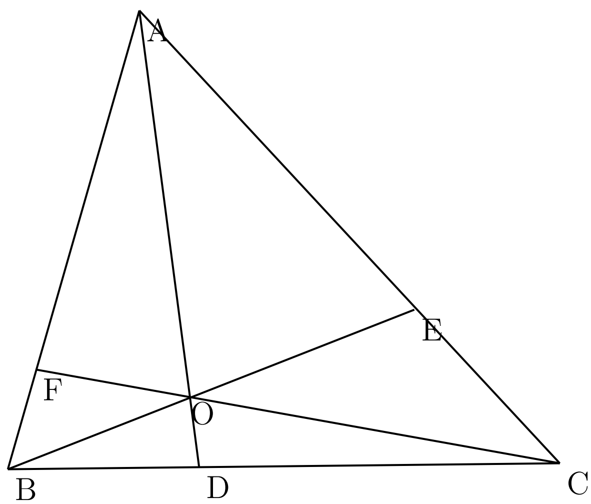
$$PA^2 \cdot BC + PB^2 \cdot CA + PC^2 \cdot AB + BC \cdot CA \cdot AB = 0$$

then D,E, F collinear.

### 3.5 Ceva's Theorem

Given a triangle ABC, let the lines AO, BO and CO be drawn from the vertices to a common point O (not on one of the sides of ABC), to meet opposite sides at D, E and F respectively, then

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$




### 3.6 Ceva's Inverse Theorem

Suppose we have a triangle ABC with D on BC, E on AC, F on AB, such that,

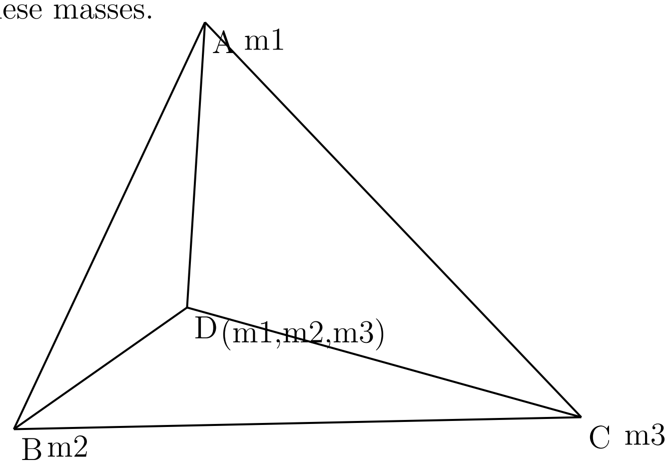
$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

then AD,BE,CF concurrent.

# 4 Barycentric Coordinate

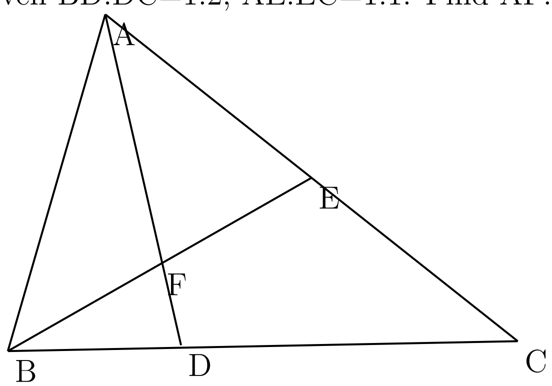
## 4.1 Definition

The barycentric coordinates of a point can be interpreted as masses placed at the vertices of the simplex, such that the point is the center of mass (or barycenter) of these masses.




## 4.2 Example

Given BD:DC=1:2, AE:EC=1:1. Find AF:FD.




## 4.3 Problem

Given BD:DC=1:5, AE:EC=1:4. Find AF:FD.