Algebra 4 - Miscellaneous

TSS Math Club

Feb 2023

1 Euqation

1.1 Definition

Eequation is a statement that the values of two mathematical expressions are equal. To solve a equation means to find the value of the unknow(s) so that the values of two mathematical expressions are equal.

1.2 Tricks in solving equation

1.2.1 Factoring

1.2.2 Quadratic Formula

1.2.3 Subsitution

1.2.4 Rational Roots Theorem

1.2.5 Long divition

1.3 Problems

1.3.1 Solve

$$x^4 + x^2 + 1 = 0$$

1.3.2 Solve

$$(x + 1)(x + 2)(x + 3)(x + 4) = 3$$

1.3.3 Solve

$$x^3 + 5x^2 + 3x - 4 = 0$$

2 Vieta's Theorem and Symetrical Polynomials

2.1 Vieta's Theorem

In algebra, Vieta's formulas are a set of results that relate the coefficients of a polynomial to its roots.

2.2 Symetrical Polynomials

Definition:

2.3 Problems

2.3.1 Problem

Solve x, y, given $x + y = 2, xy = -3$.

2.3.2 2022 AMC 12B Q4

For how many values of the constant k will the polynomial $x^2 + kx + 36$ have two distinct integer roots?

2.3.3 2022 AMC 12A Q15

The roots of the polynomial $10x^3 - 39x^2 + 29x - 6$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

3 Inequality and Estimation

3.1 Solve Inequality

3.1.1 Example: Solve the following inequality

$$\frac{x^4 - x^2}{(x^4 - 13x^2 + 36)^2} \geq 0$$

3.2 Use Inequality to Estimate

3.2.1 Example

a, b, c are postive real numbers, prove

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} > \frac{1}{3}$$

3.2.2 Problem: 2010 AIME I Q5

Positive integers a, b, c, and d satisfy $a > b > c > d$, $a + b + c + d = 2010$, and $a^2 - b^2 + c^2 - d^2 = 2010$. Find the number of possible values of a.

3.2.3 Problem: 2022 CSMC Part B Q2

- Determine all real numbers $a > 0$ for which $\sqrt{a^2 + a} = \frac{2}{3}$
- For each positive integer m, determine the difference between $(m+\frac{1}{2})^2+(m+\frac{1}{2})$ and the nearest perfect square.
- For every positive integer n, prove that the number of positive integers c with $n < \sqrt{c+\sqrt{c}} < n+1$ is even.

3.2.4 Problem

p, q are positive integers, prove at least one of the following is not a perfect square.

- $p^2 + q$
- $q^2 + 4p$