# Time Series Analysis. 1. Intro into TS

Alexey Romanenko
alexromsput@gmail.com

FIVT MIPT, September 2017

# Содержание

# Let me introduce: Alexey Romanenko

Education:

- Masters Degree at MIPT, 2011
- PhD thesis: Composition of Time Series Forecasting Algorithm at MIPT, tbd

Work experience:

- SAS Institute (software implementation), 2016 – now
- MIPT (teaching assistant), 2011 – now
- Svyaznoy (one of the largest cellphone retailer in Russia), 2010–2016
- Forecsys (Machine Learning software for business), 2008–2010

View full profile: `https://www.linkedin.com/in/alexromsput/`

## Course Plan

Advanced Machine Learning:

- Time Series Forecasting
- Reinforcement Learning
- Information Retrieval
- Recommendations Systems
- Guest Lectures (1-2)

Страница курса `https://ml-mipt.github.io/`

| Date | Topic |
|---|---|
| 06/09/2017 | Intro in TS forecasting, Exponential Smoothing models |
| 13/09/2017 | ARIMAX and other autoregression models (ARCH, GARCH) |
| 20/09/2017 | Comparing of models, compositions, Aggregating Algorithm |
| 27/09/2017 | TSA in Retail: Hierarchial Forecasting, Demand Forming |

| Date | Topic |
|---|---|
| 06/09/2017 | Intro in TS forecasting, Exponential Smoothing models |
| 13/09/2017 | ARIMAX and other autoregression models (ARCH, GARCH) |
| 20/09/2017 | Comparing of models, compositions, Aggregating Algorithm |
| 27/09/2017 | TSA in Retail: Hierarchial Forecasting, Demand Forming |

You will have to come through:

1. 5 seminars (first you've already heard)
2. 2 HW (for 1-2 hours)
3. 1 Kaggle contest (for 14 days)

| Date | Topic |
| --- | --- |
| 06/09/2017 | Intro in TS forecasting, Exponential Smoothing models |
| 13/09/2017 | ARIMAX and other autoregression models (ARCH, GARCH) |
| 20/09/2017 | Comparing of models, compositions, Aggregating Algorithm |
| 27/09/2017 | TSA in Retail: Hierarchial Forecasting, Demand Forming |

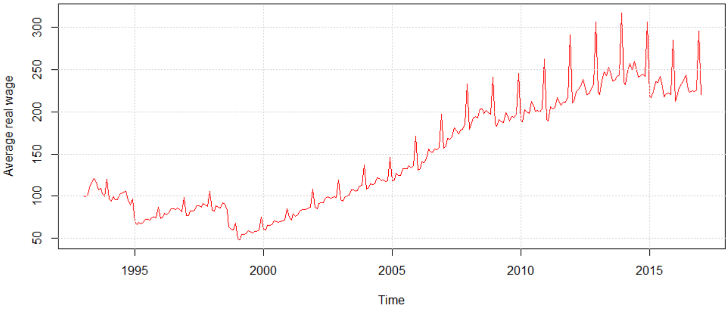
You will have to come through:

1. 5 seminars (first you've already heard)
2. 2 HW (for 1-2 hours)
3. 1 Kaggle contest (for 14 days)

- It is interesting!
- It is difficult!
- We will do it!

# Time Series definition

**Time series**: $y_1, \dots, y_T, \dots, \quad y_t \in \mathbb{R}$, — a sequence of values of some variable, detected in a constant time interval.

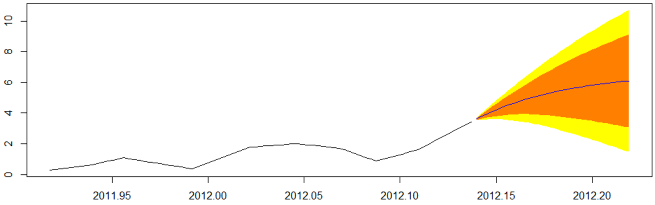



Time series forecasting task — find function $f_T$:

$$y_{T+d} \approx f_T\left(y_T, \dots, y_1, d\right) \equiv \hat{y}_{T+d|T},$$

where $d \in \{1, \dots, D\}$ — delay, $D$ — horizon.

# Forecasting interval, confidence of the forecast



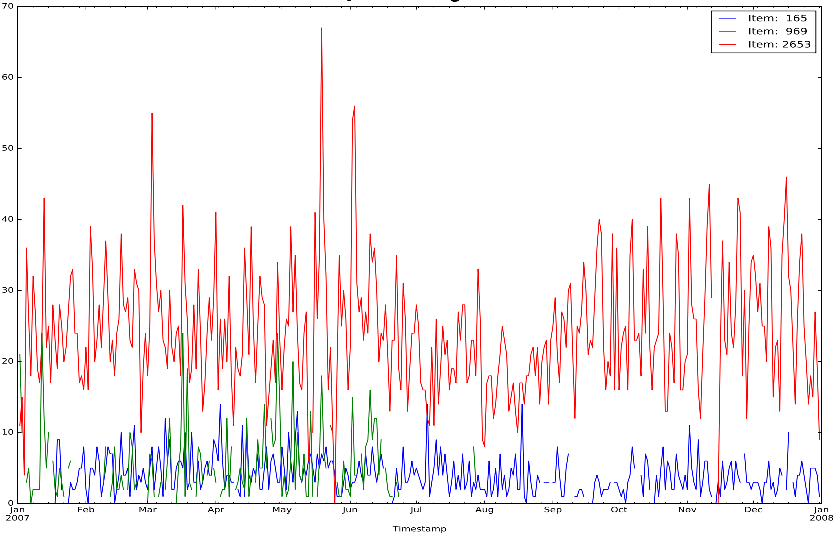

**Example**: April 1997, Grabd-Forks, ND, unexpected flood:
`https://www.youtube.com/watch?v=0iJUgddua-g`
The city was protected by dam of 51 feet;
The National Weather Service (NWS) had forecast that the river would crest at 49 feet
The river crested at 54.35 feet.

50000 citizens were evacuated, 75% buildings were damaged or destroyed,
Property damage $\$3.5$ billion

The forecasting interval was $\pm 9$ feet

# Time series in Retail

Daily sales of goods:



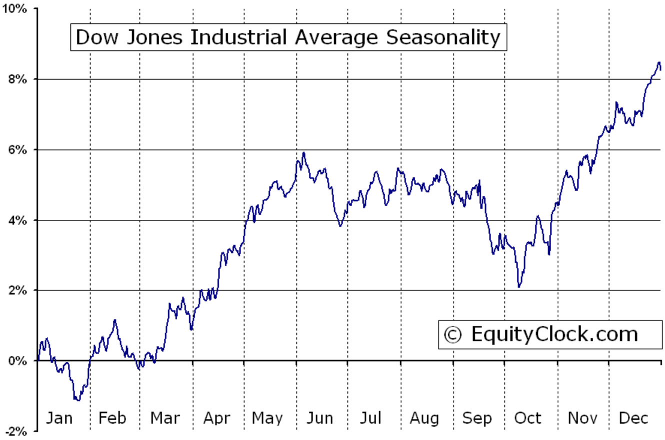
# Time series in Retail

Specification of the TS in Retail


- $10^6 - 10^8$ TS to forecast at once
- missing in data
- out-of-stock (no sales by non-zero demand)
- dependence on promo-events, changes in price
- complex loss-function

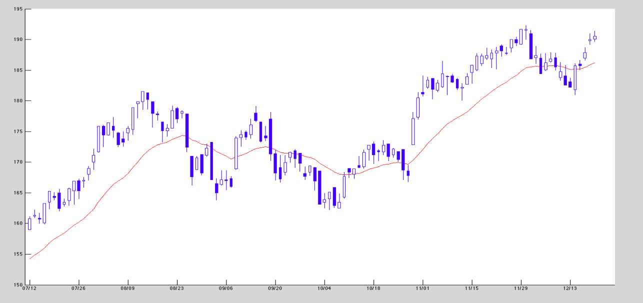
# Time series in Finance

Jndex Daw-Jones:



# Time series in Finance

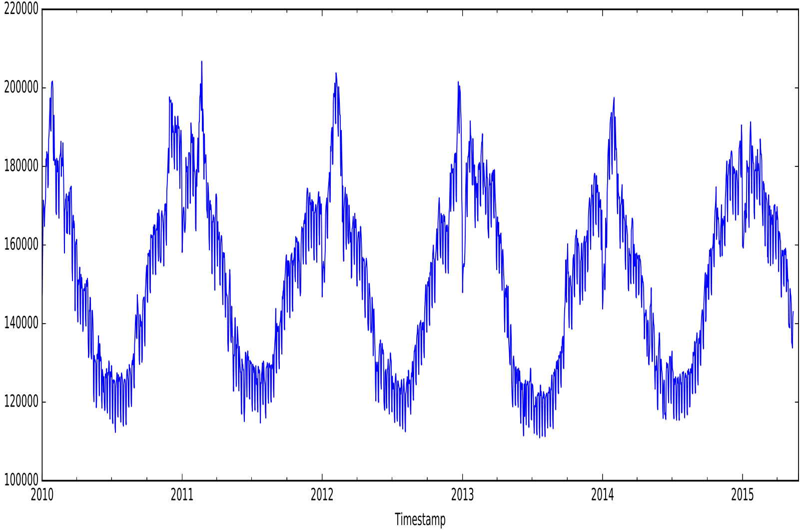
Specification of the TSA in Finance:




- high level of noise
- correlation with other financial index
- highly dependence on external events (big market events, politics)

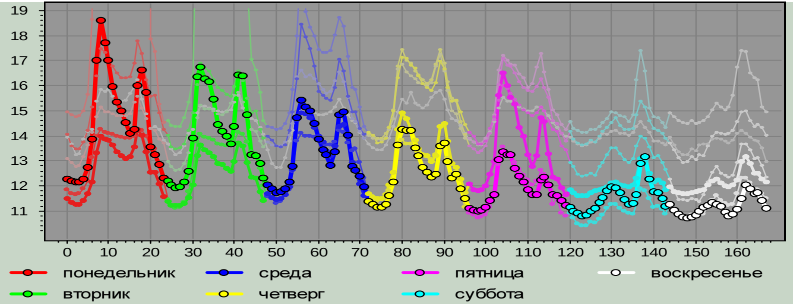
# Time series in Industry: Consumption of Electricity

Volume of Consumption of Electricity:



# Time series in Industry: Consumption of Electricity

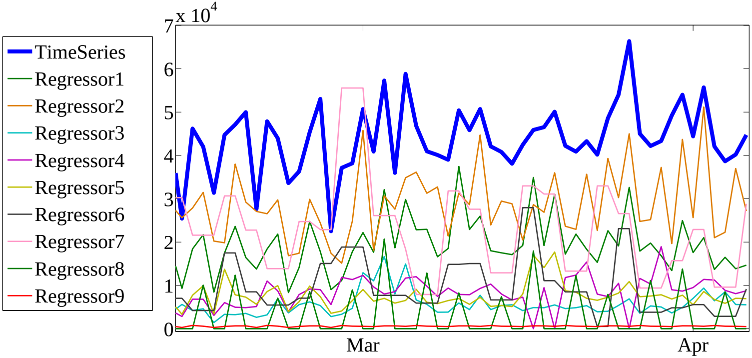
Specification of the TSA




- complex structure (yearly, weekly, daily – seasonality)
- dependence on temperature, price, calendar-events
- nonlinear dependence

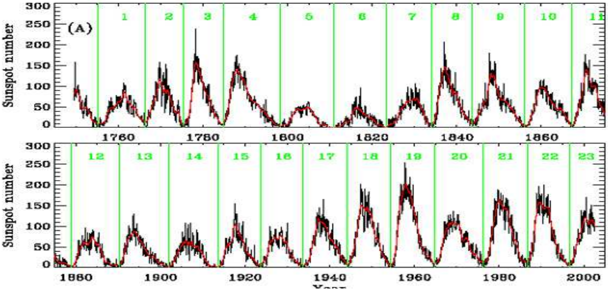
# Time series in Industry: Manufacturing

Total man-hours in Warehouse:




- depends mainly on external factors
- can be described by clear physical model

# Time series in Physics




- a-periodical changes
- complex physical model of dependence (Newton's laws, Kepler's laws, ...)

## Components of Time Series

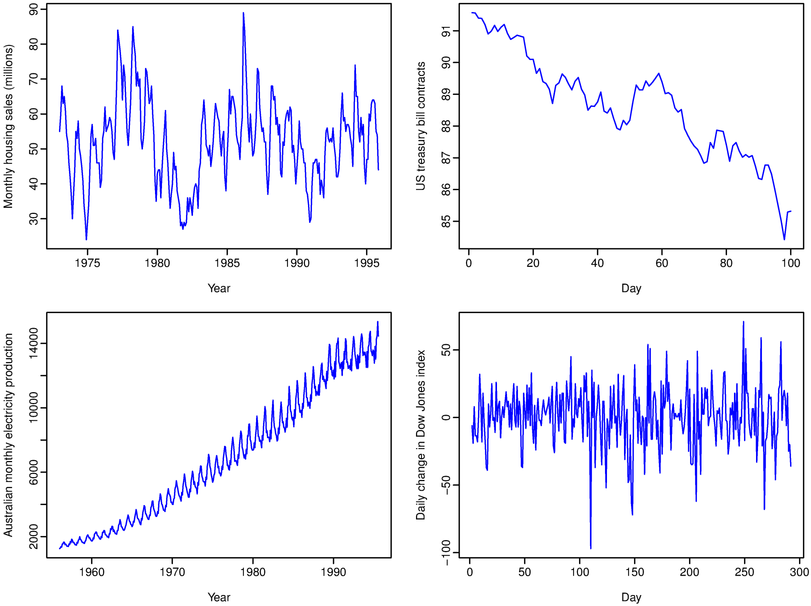
**Level** — average level of values;
**Trend** — monotonic long-term changes of Level;

**Seasonality** — periodical changes of values with constant period;

**Cycle** — changes in time series values (economical cycles, solar activity cycles).

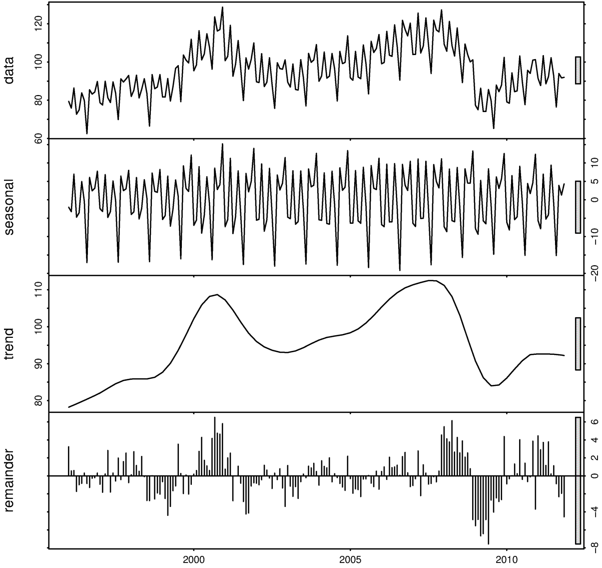
**Error** — random (unbiased) component of time series.

# Components of Time Series



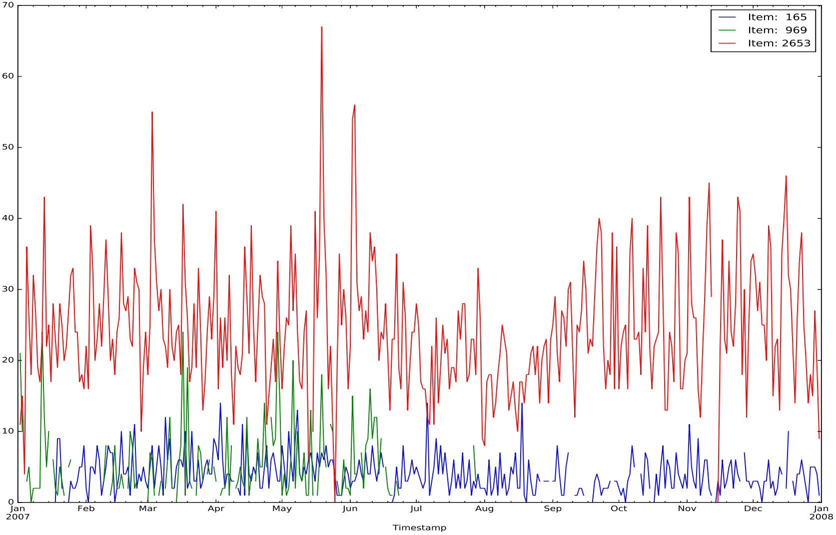
# Components of Time Series

STL-decomposition:



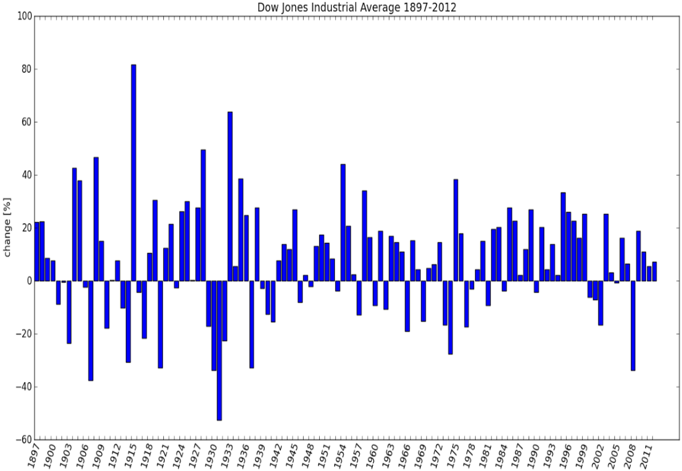
# Time Series Model

Real time series of in Retail Chain:



# Time Series Model

Jndex Dow-Jones:



# Time Series Model

$y_0, y_1, \dots, y_t, \dots$ — is a time series, $y_i \in \mathbb{R}$
The model of time series:

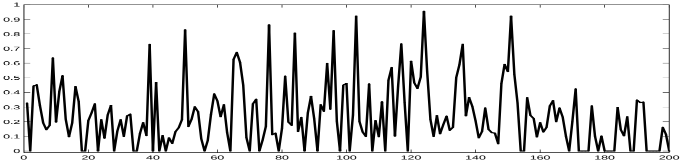
$$y_t = l_t + \varepsilon_t$$

where $l_t$ — level of time series (changing slowly),
$\varepsilon_t$ — (unobserved) error component (noise),

Forecasting model:

$$\hat{y}_{t+d} = \hat{l}_t$$

where $\hat{l}_t$ — an estimation of level,



## Simple Exponential Smoothing

Weighted average with exponentially diminishing weights forecast:

$$\hat{y}_{T+1|T} = \alpha y_T + \alpha(1-\alpha) y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} + \dots$$

$\alpha \uparrow 1 \;\Rightarrow$ greater weight to last points,
$\alpha \downarrow 0 \;\Rightarrow$ greater smoothing.

| Time point | $\alpha = 0.2$ | $\alpha = 0.4$ | $\alpha = 0.6$ | $\alpha = 0.8$ |
|---|---|---|---|---|
| $y_T$ | 0.2 | 0.4 | 0.6 | 0.8 |
| $y_{T-1}$ | 0.16 | 0.24 | 0.24 | 0.16 |
| $y_{T-2}$ | 0.128 | 0.144 | 0.096 | 0.032 |
| $y_{T-3}$ | 0.1024 | 0.0864 | 0.0384 | 0.0064 |
| $y_{T-4}$ | 0.08192 | 0.05184 | 0.01536 | 0.00128 |
| $y_{T-5}$ | 0.065536 | 0.031104 | 0.006144 | 0.000256 |

We find the optimal $\alpha^*$ using moving control:

$$Q(\alpha) = \sum_{t=T_0}^{T_1} \left(\hat{y}_t(\alpha) - y_t\right)^2 \to \min_{\alpha}$$

Empirical rules:
if $\alpha^* \in (0, 0.3)$ the series is stationary, ES works;
if $\alpha^* \in (0.3, 1)$ the series is non-stationary, we need a trend model.

## Simple Exponential Smoothing

- The method suits forecasting of time series without trend and seasonality:

$$\hat{y}_{t+1|t} = l_t,$$
$$l_t = \alpha y_t + (1-\alpha)\, l_{t-1} = \hat{y}_{t|t-1} + \alpha \cdot e_t.$$

where $e_t = y_t - \hat{y}_{t|t-1}$ — forecast error at time point $t$
Proof:

$$\hat{y}_{t+1} := \alpha y_t + (1-\alpha)\hat{y}_t = \hat{y}_t + \alpha \cdot e_t$$

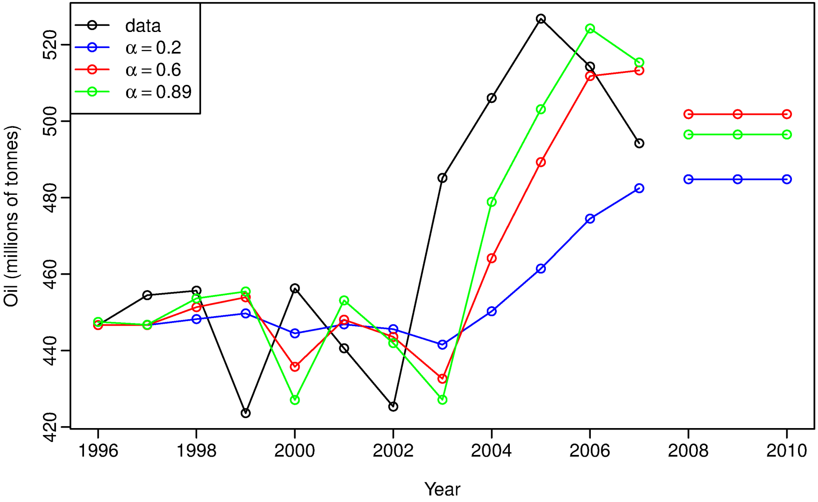
- The forecast depends on $l_0$:

$$\hat{y}_{T+1|T} = \sum_{j=0}^{T-1} \alpha\,(1-\alpha)^j\, y_{T-j} + (1-\alpha)^T\, l_0.$$

We can take $l_0 = y_1$ or optimize it.

- Forecast turns out flat, i.e. $\hat{y}_{t+d|t} = \hat{y}_{t+1|t}$.

## Simple Exponential Smoothing




Simple ES applied to data on oil production in Saudi Arabia (1996–2007).

# Tracking Signal

Tracking signal [Trigg, 1964]

$$K_t = \frac{\hat{e}_t}{\tilde{e}_t} \qquad \begin{aligned} \hat{e}_{t+1} &:= \gamma e_t + (1-\gamma)\hat{e}_t; \\ \tilde{e}_{t+1} &:= \gamma |e_t| + (1-\gamma)\tilde{e}_t. \end{aligned}$$

Recommendation: $\gamma = 0.05 \dots 0.1$

Statistics adequacy test (at $\gamma \geq 0.1,\ \ t \to \infty$):

hypotheses $H_0$: $\mathsf{E}\varepsilon_t = 0,\ \ \mathsf{E}\varepsilon_t\varepsilon_{t+d} = 0$
is accepted at significance level $\delta$ if

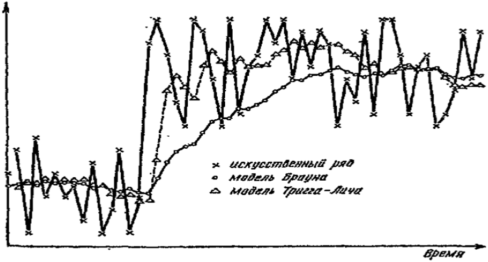
$$|K_t| \leq 1.2\Phi_{1-\delta/2}\sqrt{1-\gamma/(1+\gamma)},$$

$\Phi_{1-\delta/2}$ — normal distribution quantile,
$\Phi_{1-\delta/2} = \Phi_{0.975} = 1.96$ at $\delta = 0.05$

# Trigg-Leach Model [Trigg, Leach, 1967]

**Problem:** adaptive models adjust poorly to sharp changes of structure

**Solution:** $\alpha = |K_t|$

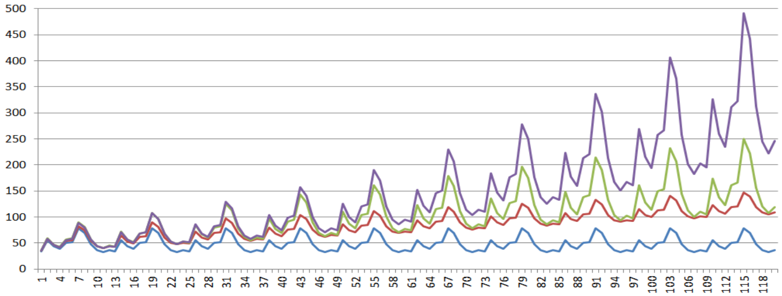



**Drawbacks:**

1) reacts poorly to single outliers; $(\alpha_t = |K_{t-1}|)$
2) requires fitting $\gamma$ given recommended $\gamma = 0.05 \ldots 0.1$.

# Examples of Trend and Seasonality

Example: Combination of trend and seasonality (model data)




Ряд 1 — seasonality and no trend
Ряд 2 — linear trend, additive seasonality
Ряд 3 — linear trend, multiplicative seasonality
Ряд 4 — exponential trend, multiplicative seasonality

## Holt Model = Linear Trend

Linear trend with no seasonality effect:

$$\hat{y}_{t+d} = l_t + b_t d,$$

where $l_t$, $b_t$ — adaptive coefficients of linear trend

Recursive formula:

$$l_t := \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1}) = \hat{y}_t + \alpha e_t;$$
$$b_t := \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} = b_{t-1} + \alpha\beta e_t.$$

Particular case — Brown linear growth model:

$$\alpha = \alpha, \quad \beta = \alpha$$

# Other Methods that Account Trend

Multiplicative linear (exponential) trend:

$$\begin{aligned}
\hat{y}_{t+d|t} &= l_t b_t^d, \\
l_t &= \alpha y_t + (1-\alpha)\left(l_{t-1} b_{t-1}\right), \\
b_t &= \beta \frac{l_t}{l_{t-1}} + (1-\beta)\, b_{t-1}.
\end{aligned}$$

$\alpha, \beta \in [0, 1]$.

## Other Methods that Account Trend

Additive damped trend:

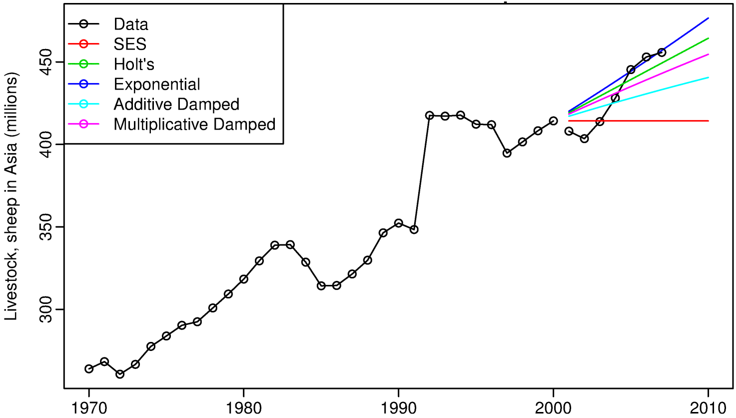
$$\begin{aligned}
\hat{y}_{t+d|t} &= l_t + \left(\phi + \phi^2 + \cdots + \phi^d\right) b_t, \\
l_t &= \alpha y_t + (1-\alpha)\left(l_{t-1} + \phi b_{t-1}\right), \\
b_t &= \beta\left(l_t - l_{t-1}\right) + (1-\beta)\,\phi b_{t-1}.
\end{aligned}$$

Multiplicative damped trend:

$$\begin{aligned}
\hat{y}_{t+d|t} &= l_t b_t^{\left(\phi + \phi^2 + \cdots + \phi^d\right)}, \\
l_t &= \alpha y_t + (1-\alpha)\, l_{t-1} b_{t-1}^{\phi}, \\
b_t &= \beta \frac{l_t}{l_{t-1}} + (1-\beta)\, b_{t-1}^{\phi}.
\end{aligned}$$

$\alpha, \beta \in [0,1]\,,\ \phi \in (0,1)\,.$

# Other Methods that Account Trend




Forecast of sheep population in Asia with regard for trend.

| | SES | Holt's | Exponential | Additive damped | Multiplicative damped |
|---|---|---|---|---|---|
| $\alpha$ | 1 | 0.98 | 0.98 | 0.99 | 0.98 |
| $\beta$ | | 0 | 0 | 0 | 0.00 |
| $\phi$ | | | | 0.98 | 0.98 |

# Winters Model = Multiplicative Seasonality

Multiplicative Seasonality of Period $p$:

$$\hat{y}_{t+d} = l_t \cdot s_{t-p+(d \bmod p)},$$

$s_0, \ldots, s_{p-1}$ — seasonality profile of period $p$.
Recursive formula:

$$l_t := \alpha(y_t/s_{t-p}) + (1-\alpha)l_{t-1} = l_{t-1} + \alpha e_t/s_{t-p};$$
$$s_t := \beta(y_t/l_t) + (1-\beta)s_{t-p} = s_{t-p} + \beta(1-\alpha)e_t/l_t.$$

Proof of the last equation:

$$s_t := s_{t-p} + \beta\left(y_t/l_t - s_{t-p}\right) = s_{t-p} + \beta\left(y_t - s_{t-p}l_t\right)/l_t =$$
$$s_{t-p} + \beta\big(y_t - s_{t-p}(l_{t-1} + \alpha e_t/s_{t-p})\big)/l_t = s_{t-p}+$$
$$+\beta\left(\underbrace{y_t - s_{t-p}l_{t-1}}_{e_t} - \alpha e_t\right)/l_t$$

# Additive Seasonality ES Model

Additive seasonality with period of length $p$:

$$\begin{aligned} \hat{y}_{t+d|t} &= l_t + s_{t-p+(d \mod p)}, \\ l_t &= \alpha\left(y_t - s_{t-p}\right) + (1-\alpha)\left(l_{t-1}\right) = l_{t-1} + \alpha e_t; \\ s_t &= \gamma\left(y_t - l_{t-1}\right) + (1-\gamma)\, s_{t-p} = s_{t-p} + \gamma(1-\alpha)e_t. \end{aligned}$$

# Theil-Wage Model

Linear trend with additive seasonality of period $s$:

$$\hat{y}_{t+d} = (l_t + b_t d) + s_{t+(d \bmod s)-p}.$$

$l_t + b_t d$ — trend cleaned of seasonality,
$s_0, \ldots, s_{p-1}$ — seasonality profile of period $p$.

Recursive formula:

$$\begin{aligned}
l_t &:= \alpha(y_t - s_{t-p}) + (1-\alpha)(l_{t-1} + b_{t-1}) = l_{t-1} + b_{t-1} + \alpha e_t;\\
b_t &:= \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} = b_{t-1} + \alpha\beta e_t;\\
s_t &:= \gamma(y_t - l_t) + (1-\gamma)s_{t-p} = s_{t-p} + \gamma(1-\alpha)e_t.
\end{aligned}$$

# Winters Model with Linear Trend

Multiplicative seasonality of period $s$ with a linear trend:

$$\hat{y}_{t+d} = (l_t + b_t d) \cdot s_{t+(d \bmod p)-p},$$

$l_t + b_t d$ — trend cleaned of seasonality,
$s_0, \ldots, s_{p-1}$ — seasonality profile of period $s$.

Recursive formula:

$$l_t := \alpha(y_t/s_{t-p}) + (1-\alpha)(l_{t-1} + b_{t-1}) = l_{t-1} + b_{t-1} + \alpha e_t/s_{t-p};$$
$$b_t := \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} = b_{t-1} + \alpha\beta e_t/s_{t-p};$$
$$s_t := \gamma(y_t/l_t) + (1-\gamma)s_{t-p} = s_{t-p} + \gamma(1-\alpha)e_t/l_t.$$

## Winters Model with exponential trend

Multiplicative trend model exponential trend:

$$\hat{y}_{t+d} = l_t(r_t)^d \cdot s_{t+(d \bmod p)-p},$$

$l_t(r_t)^d$ — exponential trend without seasonality,
$s_0, \ldots, s_{p-1}$ — seasonal trend $p$.

Recurrent version:

$$l_t := \alpha(y_t/s_{t-p}) + (1-\alpha)l_{t-1}r_{t-1} = l_{t-1}r_{t-1} + \alpha e_t/s_{t-1};$$
$$r_t := \beta(l_t/l_{t-1}) + (1-\beta)r_{t-1} = r_{t-1} + \alpha\beta e_t/st - 1;$$
$$s_t := \gamma(y_t/l_t) + (1-\gamma)s_{t-p} = s_{t-p} + \gamma(1-\alpha)e_t/l_t.$$

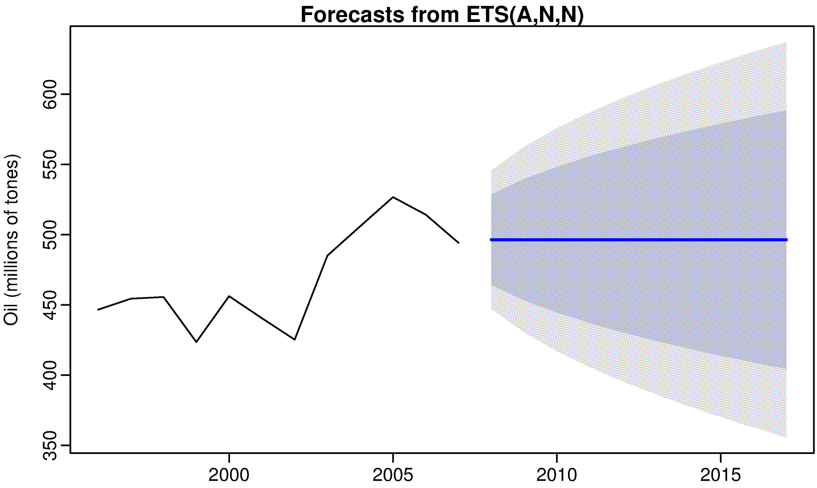
# ES Models

| | Seasonality | | |
|---|---|---|---|
| Trend | N (None) | A (Additive) | M (Multiplicative) |
| N (None) | (N,N) | (N,A) | (N,M) |
| A (Additive) | (A,N) | (A,A) | (A,M) |
| Ad (Additive damped) | (Ad,N) | (Ad,A) | (Ad,M) |
| M (Multiplicative) | (M,N) | (M,A) | (M,M) |
| Md (Multiplicative damped) | (Md,N) | (Md,A) | (Md,M) |

We may additionally suggest an additive (A) or a multiplicative (M) error (the type of error does not influence single-value prediction). Multiplicative error is suitable only for strictly positive time series.

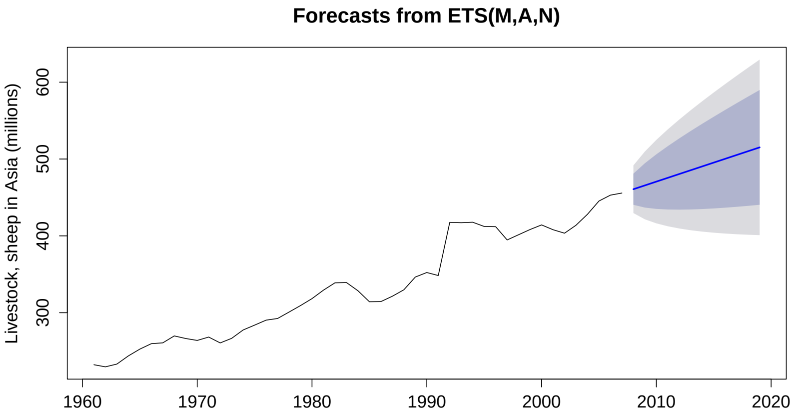
The final model may be written as $ESM(\cdot, \cdot, \cdot)$.

# Examples of Forecast



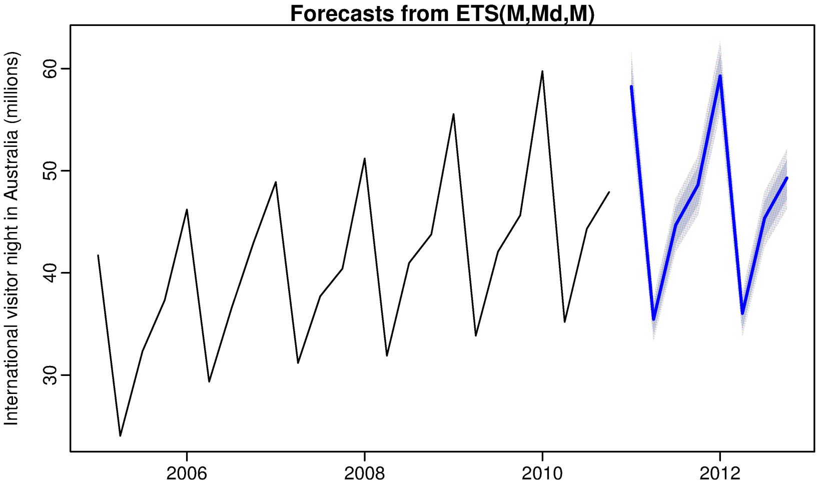

For the data on oil production in Saudi Arabia function ESM selects simple ES.

## Examples of Forecast




For the sheep population in Asia function ESM selects the model with multiplicative error and an additive linear trend.

## Examples of Forecast



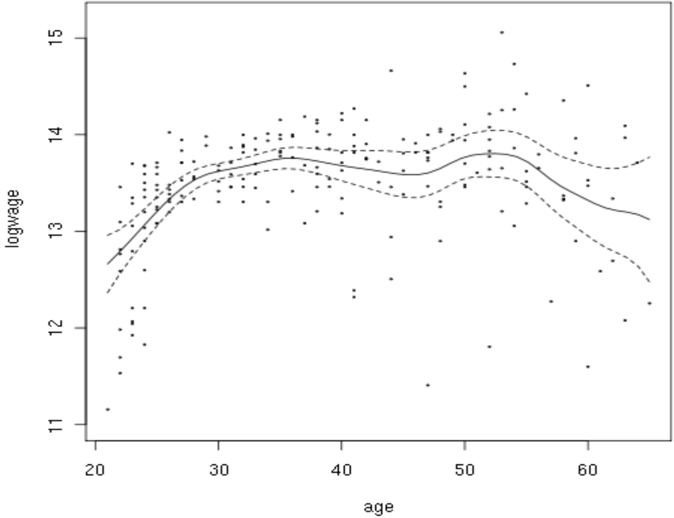

For the quantity of nights spent by tourists in Australia function ESM selects a model with multiplicative error, seasonality and a damped trend.

## Data model

- $X$ — samples ($\mathbb{R}^n$); $Y$ — answers ($\mathbb{R}$);
  $X^\ell = (x_i, y_i)_{i=1}^{\ell}$ — train samples;
  $f(x, y)$ — joint distribution data come from;
- Data model:
  $$y_i = m(x_i) + \varepsilon_i,$$
  $m(x)$ — unknown function;
  
  $\varepsilon_i$ — error (random variable)
  $$\mathsf{E}(\varepsilon_i) = 0, \quad \mathsf{D}(\varepsilon_i) = \sigma_i^2, \quad \mathsf{Cov}(\varepsilon_i, \varepsilon_j) = 0 \forall i \neq j$$
- estimator (according to bayes optimal decision rule)
  $$a(x) = \hat{m}(x) = \mathsf{E}\left[Y | X = x\right] = \int y f(y, x) dy = \frac{\int y f(x, y) dy}{\int f(x, y) dy}$$

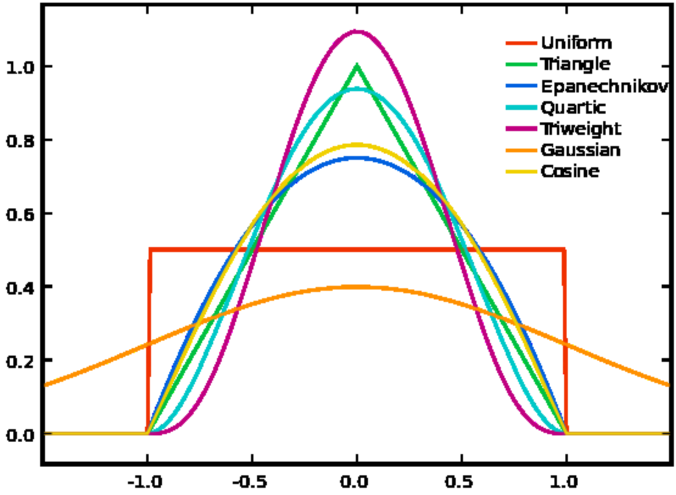
We just need to estimate numerator and denominator.

# Data Examples



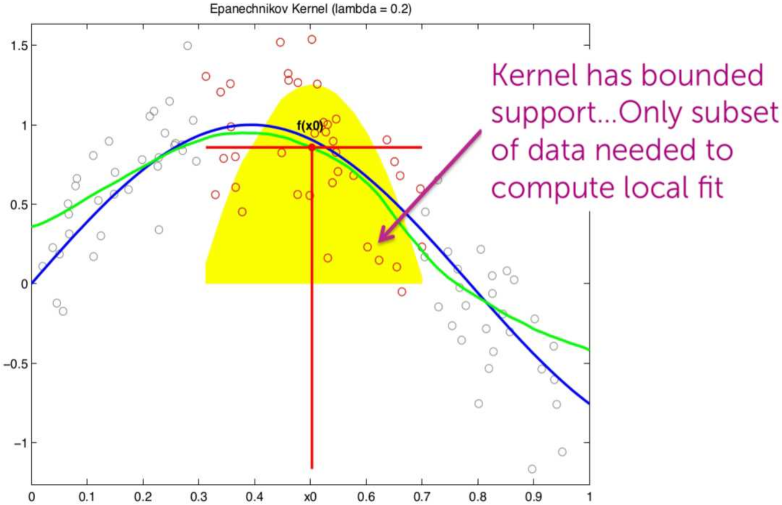
# Kernel estimator

Kernel is non-negative real-valued integrable even (symmetric) function



# Kernel estimator

Kernel Estimator:



## Kernel estimator

Density estimator:

$$\hat{f}(x,y) = \frac{1}{\ell h_x h_y}\sum_{i=1}^{\ell} K\left(\frac{\rho(x,x_i)}{h_x}\right) K\left(\frac{y-y_i}{h_y}\right) = \frac{1}{\ell}\sum_{i=1}^{\ell} K_{h_x}(\rho(x,x_i))K_{h_y}(y-y_i);$$

Estimation of numerator:

$$\int y\hat{f}(x,y)dy = \frac{1}{\ell}\int y\sum_{i=1}^{\ell} K_{h_x}(\rho(x,x_i))K_{h_y}(y-y_i)dy =$$

$$= \frac{1}{\ell}\sum_{i=1}^{\ell} K_{h_x}(\rho(x,x_i)) \underbrace{\int yK_{h_y}(y-y_i)dy}_{y_i} = \frac{1}{\ell}\sum_{i=1}^{\ell} K_{h_x}(\rho(x,x_i))y_i;$$

Estimation of denominator:

$$\int \hat{f}(x,y)dy = \frac{1}{\ell}\int \sum_{i=1}^{\ell} K_{h_x}(\rho(x,x_i))K_{h_y}(y-y_i)dy =$$

$$= \frac{1}{\ell}\sum_{i=1}^{\ell} K_{h_x}(\rho(x,x_i)) \underbrace{\int K_{h_y}(y-y_i)dy}_{1} = \frac{1}{\ell}\sum_{i=1}^{\ell} K_{h_x}(\rho(x,x_i)) = \hat{f}(x);$$

## Formula of Nadaraya–Watson

As consequence:

**Formula (*kernel smoothing*) Nadaraya–Watson:**

$$a_h(x; X^\ell) = \frac{\sum\limits_{i=1}^{\ell} y_i w_i(x)}{\sum\limits_{i=1}^{\ell} w_i(x)} = \frac{\sum\limits_{i=1}^{\ell} y_i K\left(\frac{\rho(x,x_i)}{h_x}\right)}{\sum\limits_{i=1}^{\ell} K\left(\frac{\rho(x,x_i)}{h_x}\right)}.$$

where $w_i(x) = K\left(\frac{\rho(x,x_i)}{h}\right)$ — weights of $x_i$ with respect to $x$;
$K(r)$ — *Kernel*, bounded, smooth; $h$ — window of smoothing.

We could get the same solution as follows: find as constant $a(x) = \alpha$ in neighborhood of $x \in X$:

$$Q(\alpha; X^\ell) = \sum_{i=1}^{\ell} w_i(x)(\alpha - y_i)^2 \to \min_{\alpha \in \mathbb{R}};$$

# Argumentation of formula Nadaraya-Watson

## Theorem

*If next conditions are met:*

*1) sample* $X^\ell = (x_i, y_i)_{i=1}^{\ell}$ *is generated from a joint distribution* $f(x, y)$*;*

*2) Kernel* $K(r)$ *is bounded:* $\int_0^\infty K(r)\,dr < \infty$*,* $\lim\limits_{r\to\infty} rK(r) = 0$*;*

*3)* $\mathsf{E}(y|x)$ *has limited second moment:*
$\mathsf{E}(y^2|x) = \int_Y y^2 p(y|x)\,dy < \infty$ *for all* $x \in X$*;*

*4) sequence* $h_\ell$ *decreases but not fast:*
$\lim\limits_{\ell\to\infty} h_\ell = 0$*,* $\lim\limits_{\ell\to\infty} \ell h_\ell = \infty$*.*

*Then there is convergence in probability:*

$$a_{h_\ell}(x; X^\ell) \xrightarrow{P} \mathsf{E}(y|x) \text{ in all point } x \in X,$$

*and* $\mathsf{E}(y|x)$*,* $f(x)$ *и* $\mathsf{D}(y|x)$ *are continuous functions and* $f(x) > 0$*.*

## Simple Exponential Smoothing

Simple regression model for TS forecasting — constant $\hat{y}_{t+1} = c$,

Kernel function $K(\rho(y_t, y_\tau)) = \alpha^{t-\tau}$

In accordance with formula Nadaraya–Watson:

$$c \equiv \hat{y}_{t+1} = \frac{\sum_{i=0}^{t} \alpha^i y_{t-i}}{\sum_{i=0}^{t} \alpha^i}$$

Note, we can obtain the same result by solving optimization problem:

$$F(c) = \sum_{t=0}^{T} \alpha^{T-t} (y_t - c)^2 \to \min_c, \quad \alpha \in (0, 1).$$

Proof: just find solution of the equation $\frac{\partial F(c)}{\partial c} = 0$.

## Simple Exponential Smoothing

Write the same for $\hat{y}_t$ and use approximation $\sum_{i=0}^{t} \alpha^i \approx \sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha}$, get

$$\hat{y}_{t+1} = \hat{y}_t \alpha + (1-\alpha) y_t$$

Replace $\alpha = 1 - \alpha \Rightarrow$ Simple ES:

$$\hat{y}_{t+1} := \hat{y}_t + \alpha(y_t - \hat{y}_t) \quad = \quad \alpha y_t + (1-\alpha)\hat{y}_t,$$

**Corollary**

*Simple ES gives the optimal solution if loss function looks like*

$$Loss(T) = \sum_{t=1}^{t} \alpha^{T-t} (y_t - \hat{y}_t)^2,$$

*where $\alpha$ is the same as in SES.*

## Other theoretical Properties of Simple ES

The model of time series:

$$y_t = l_t + \varepsilon_t$$

where $l_t$ — level of time series (changing slowly),

$\varepsilon_t$ — (unobserved) error component (noise),

$$\mathsf{E}(\varepsilon_i) = 0, \quad \mathsf{D}(\varepsilon_i) = \sigma^2 < \infty \;\; \forall i, \quad \mathsf{Cov}(\varepsilon_i, \varepsilon_j) = 0 \; \forall \; i \neq j$$

Error of forecast:

$$e_t = y_t - \hat{y}_t$$

Math expectation of squared loss of SES:

$$\mathsf{E}\left((e_t)^2\right) \approx \mathsf{D}(\varepsilon)(1 + (t-1)\alpha^2)$$

# Conclusion

- time series differs in business regions
- time series forecasting problem has features

## Conclusion

- time series differs in business regions
- time series forecasting problem has features

Pro ES:

- ES is very simple model
- very useful for short TS or simple TS
- can be easily modified for different components of TS
- Most of ES models imply quiet a clear interpretation

## Conclusion

- time series differs in business regions
- time series forecasting problem has specific

Pro ES:

- ES is very simple model
- very useful for short TS or simple TS
- can be easily modified for different components of TS
- Most of ES models imply quiet a clear interpretation

Cons ES:

- ES does not take into account independent variables
- heuristic method (there is no theoretical guaranties about it's work )
- Forecast of ES depends on initialization ($l_0$)