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The search for the decay of Z boson into two gammas as a test of Bose statistics

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Abstract

We suggest that Bose statistics for photons can be tested by looking for decays of spin-1 bosons into two photons. The experimental upper limit on the decay $Z \rightarrow \gamma\gamma$ is used to establish for the first time the quantitative measure of the validity of Bose symmetry for photons.

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The Standard Model of particle interactions has so far been remarkably consistent with all the experimental data. That adds importance to various ways of looking for possible new physics not described by the Standard Model. Many such possibilities have been considered before; most of them concentrate on some kind of extension of the Standard Model (by adding new particles, symmetries or interactions) and then obtain from experiment various bounds on those extensions. Here, we would like to discuss how recent results in Z measurements can give limits on a more radical departure from standard physics: the possible small violation of Bose statistics.

The problem of small deviations from Fermi or Bose statistics, initially an exotic issue raised in the early 70's, has grown over the last decade into an elaborate area of research leading to new ingenious experiments as well as striking connections with modern mathematical physics [1-34]. Yet most effort, especially in the experimental field, was actually concentrated on discussing small violation of the Pauli exclusion principle rather than violation of Bose statistics. Many dedicated experiments have been performed to give strong bounds on the violation of the Pauli principle. By contrast, nothing similar has been done or suggested with respect to possible deviations from Bose statistics.

For example, there do not exist any direct experimental bounds on Bose symmetry violation in elementary processes (rather than statistical ensembles) involving photons, gluons or gauge bosons. Two discussions of experimental bounds on Bose statistics violation which are known to us are 1) the study of the decay $K_L \rightarrow 2\pi^0$ in Ref. [7] from the point of view of Bose symmetry (rather than CP) violation and 2) the analysis of the upper limit on laser intensities implied by the small deviations from Bose statistics [32] (the latter work however has been criticised in Ref. [34]). In this paper we would like to start filling the gap by deriving the limits on Bose symmetry violations in the system of two photons.

Before going to our main subject, a few words about the spin-statistics theorem are in order. We would like to remind the reader that the spin-statistics theorem proved rigorously in the axiomatic field theory *does not* forbid *small* violations of normal statistics. What this theorem *does* forbid is, so to speak, “*large*” (or better say “100 %”) violations of normal

statistics. More precisely, the theorem says that spin 0 fields cannot be quantized according to Fermi (i.e., with anticommutators) while spin 1/2 fields cannot be quantized according to Bose (i.e., with commutators). Thus the theorem leaves open the question whether *small* violations of statistics exist or not.

There is a clear reason why the system of two photons is especially interesting in testing the degree with which Bose symmetry is exact. It has been known since the early 50's, due to the works of Landau [35] and Yang [36] that a pair of photons cannot be in a state with total angular momentum equal to unity. Therefore, the decay of any spin-1 boson into two photons is absolutely forbidden. Later, Nishijima [37] suggested a more direct way to see the theorem. It is his method that we will use to analyse the consequences of possible Bose symmetry violation for the two-photon system.

Of all (neutral) spin 1 bosons it is natural to concentrate on the heaviest one- the Z-boson- because one can expect that any violations of Bose symmetry, if any, would be better manifested at higher energy scales. The method is to write down the most general form of the decay amplitude of the spin-1 particle into two photons and then apply the conditions of gauge invariance and Bose symmetry to that amplitude. If both conditions are applied, the resulting amplitude is exactly zero. We are going to show that if we impose the condition of gauge invariance but do not require the Bose symmetry, the resulting amplitude is not zero. This left-over amplitude depends on only one parameter which is natural to call "the Bose symmetry violating parameter". We then obtain the two-gamma decay rate of $Z \rightarrow \gamma\gamma$ [38]. In this way we are able for the first time to obtain a direct bound on Bose symmetry violation for photons.

Now, a few remarks about the relation of our method to the existing models of small Bose symmetry violation. The most successful model is "the quon model" [27-34]. Quons are particles described by the commutation relations of the form:

$$a_k a_l^\dagger - q a_l^\dagger a_k = \delta_{kl}, \quad (1)$$

where both quon creation operator, a_l^\dagger , and quon annihilation operator, a_k , are involved. (A particular choice $q = 0$ corresponds to the case of the so-called infinite statistics). Note that there are no commutation relations involving only creation operators or only annihilation operators. Moreover, such commutation relations are not needed for calculation of matrix elements. That means, that the above commutation relations, together with the usual vacuum definition, $a|0\rangle = 0$, form a perfect basis for quon quantum mechanics. If one goes then to quon quantum field theory, then it was shown that such theory has to be non-local, but the full details of such theory are to be developed yet [27-34]. Because of this, we do not try at this stage to relate our phenomenological model of Bose symmetry violation to the quon model. Neither do we attempt to connect our parameter of Bose symmetry violation, see below, to the q parameter. These problems will be considered elsewhere.

Let us turn now to our main purpose: the construction of $Z \rightarrow \gamma\gamma$ decay amplitude. We require that this amplitude satisfies all the standard conditions, such as relativistic invariance and gauge invariance, but we do not require this amplitude to be symmetric under the exchange of photon ends.

The most general Lorentz invariant form of the amplitude S of the decay $Z \rightarrow \gamma\gamma$ is:

$$S(k_1, k_2, \epsilon_1, \epsilon_2) = c_{\lambda\mu\nu}(k_1, k_2) \epsilon_0^\lambda \epsilon_1^\mu \epsilon_2^\nu, \quad (2)$$

where k_1 and k_2 are photon momenta, ϵ_1 and ϵ_2 are photon polarization vectors, ϵ_0 is Z-boson polarization vector.

Next, the condition of Lorentz invariance applied to $c_{\lambda\mu\nu}$ leaves us with 16 possible structures made out of the momenta k_1 and k_2 and tensors $g_{\mu\nu}$ and $\epsilon_{\mu\nu\alpha\beta}$. But recall that the polarization vectors must satisfy the conditions

$$\epsilon_1^\mu k_{1\mu} = 0, \quad \epsilon_2^\nu k_{2\nu} = 0, \quad \epsilon_0^\lambda (k_{1\lambda} + k_{2\lambda}) = 0. \quad (3)$$

Hence terms in $c_{\lambda\mu\nu}$ proportional to $k_{1\mu}$, $k_{2\nu}$ and $k_{1\lambda} + k_{2\lambda}$ do not contribute to S and can therefore be ignored.

Thus we are left with the next most general form of $c_{\lambda\mu\nu}$:

$$\begin{aligned}
c_{\lambda\mu\nu} = & a_1 \epsilon_{\lambda\mu\nu\alpha} k_{1\alpha} + a_2 \epsilon_{\lambda\mu\nu\alpha} k_{2\alpha} + b_1 g_{\lambda\mu} k_{1\nu} \\
& + b_2 g_{\lambda\nu} k_{2\mu} + g g_{\mu\nu} (k_{1\lambda} - k_{2\lambda}) + h (k_{1\lambda} - k_{2\lambda}) k_{1\nu} k_{2\mu}.
\end{aligned} \tag{4}$$

Now, the condition of the electromagnetic gauge invariance reads

$$c_{\lambda\mu\nu} k_1^\mu \epsilon_0^\lambda \epsilon_2^\nu = 0, \tag{5}$$

$$c_{\lambda\mu\nu} k_2^\nu \epsilon_0^\lambda \epsilon_1^\mu = 0, \tag{6}$$

$$c_{\lambda\mu\nu} k_1^\mu k_2^\nu \epsilon_0^\lambda = 0. \tag{7}$$

Going to the rest frame of Z-boson, it can be shown that the necessary and sufficient condition for Eq. (5) and Eq. (6) to hold, are, correspondingly, $a_1 = 0$, $b_2 = 0$ and $a_2 = 0$, $b_1 = 0$.

After putting $a_1 = a_2 = b_1 = b_2 = 0$ we impose the condition Eq. (7) and obtain

$$h = -\frac{2g}{M_Z^2}. \tag{8}$$

Therefore the most general form of the amplitude $c_{\lambda\mu\nu}$ reduces to

$$c_{\lambda\mu\nu} = g(k_1 - k_2)_\lambda (g_{\mu\nu} - \frac{2k_{1\nu}k_{2\mu}}{M_Z^2}). \tag{9}$$

In principle, g could depend on some scalar products of the momenta, but in our case, since all the particles are on mass shell, we have $k_1 k_2 = M_Z^2/2$ (and, of course, $k_1^2 = k_2^2 = 0$), so that g is a pure number. Note that the above amplitude automatically satisfies the condition $(k_1 + k_2)_\lambda c_{\lambda\mu\nu} = 0$.

We see that this amplitude, as expected, violates Bose symmetry because

$$c_{\lambda\mu\nu}(k_1, k_2) = -c_{\lambda\nu\mu}(k_2, k_1), \tag{10}$$

whereas Bose symmetry requires

$$c_{\lambda\mu\nu}(k_1, k_2) = +c_{\lambda\nu\mu}(k_2, k_1). \tag{11}$$

Thus the parameter g can be interpreted as the parameters of Bose statistics violation.

Now, calculating the width of the decay $Z \rightarrow \gamma\gamma$ with the help of the amplitude Eq. (9) we obtain

$$\Gamma = \frac{1}{16\pi M_Z} |S|^2 = \frac{M_Z}{16\pi} g^2. \quad (12)$$

Experimentally, it has recently been measured at LEP [38] that

$$BR(Z \rightarrow \gamma\gamma) < 1.4 \times 10^{-4}. \quad (13)$$

Therefore

$$\frac{\Gamma(Z \rightarrow \gamma\gamma)}{\Gamma_{tot}(Z)} = \frac{g^2 M_Z}{16\pi \Gamma_{tot}(Z)} < 1.4 \times 10^{-4}, \quad (\Gamma_{tot}(Z) \simeq 2.5 GeV). \quad (14)$$

Thus, finally, we can obtain our upper bound on the Bose violating parameter

$$g < 10^{-2}. \quad (15)$$

The same analysis can be made for other spin-1 bosons, too, but since they are lighter than Z , one can expect that the effect of Bose symmetry violation, if it exists at all, would be more strongly suppressed than for the case of Z ; that is why we do not go into details of that.

It is possible to carry out a similar analysis for the case of two gluons, too, but it would be much harder to get any experimental constraints in this case.

To conclude, based on the experimental upper limit on the decay $Z \rightarrow \gamma\gamma$ we have obtained the upper bound on the possible small violation of the Bose symmetry for the system of two photons.

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