APPENDIX

A. Proof for ***Corollary*** 1

According to Section V-B, it is known that $\mathcal{P}_4$ is converted from a given feasible offload decision $\mathcal{P}_3$. The $\mathcal{P}_4$ is expressed as

$$\begin{aligned} \mathcal{P}_4 : &\min_{f^t, w^t} \sum_{i=1}^{N} \left[Q_i^t (e_i^t - \varepsilon_i (I^t, f_i^t, w_i^t)) + V \cdot cost_i^t \right] \\ = &\min_{f^t} \sum_{i \in A(t)} \left(-Q_i^t \cdot k(L_i \cdot X_m)(f_i^t)^2 + V \cdot \frac{L_i X_m}{f_i^t} \right) \\ + &\min_{w^t} \sum_{i \in B(t)} \left(\frac{-Q_i^t \cdot p_i \cdot L_i + V \cdot L_i}{r_i^t(w_i^t, h_i^t)} + \frac{V \cdot L_i X_s}{f_s} \right) + \sum_{i \in C(t)} (V \cdot \Psi) + \sum_{i \in U} \left(Q_i^t \cdot e_i^t \right). \end{aligned} \tag{1}$$

The objective functions of $\mathcal{P}_3$ and $\mathcal{P}_4$ are denoted as $\mathcal{J}_3(I^t, f^t, w^t)$ and $\mathcal{J}_4(f^t, w^t)$, respectively. If I_i^t, f_i^t and w_i^t take the optimal value, the objective function of $\mathcal{P}_3$ has the minimum value $\mathcal{J}_3(I^{t*}, f^{t*}, w^{t*})$. Similarly, if f_i^t and w_i^t take the optimal value, the objective function of $\mathcal{P}_4$ has the minimum value $\mathcal{J}_4(f^{t*}, w^{t*})$. For any given feasible offloading decision I^t of $\mathcal{P}_4$, the following equation can be obtained

$$\mathcal{J}_4(f^{t*}, w^{t*}) = \mathcal{J}_3(I^t, f^{t*}, w^{t*}). \tag{2}$$

Therefore, the following inequality can be obtained

$$\begin{aligned} \mathcal{P}_3 : &\min_{I^t, f^t, w^t} \sum_{i=1}^{N} \left[Q_i^t (e_i^t - \varepsilon_i (I_i^t, f_i^t, w_i^t)) + V \cdot cost_i^t \right] = \mathcal{J}_3(I^{t*}, f^{t*}, w^{t*}) \\ \le &\mathcal{J}_4(f^{t*}, w^{t*}) = \mathcal{P}_4 : \min_{f^t, w^t} \sum_{i=1}^{N} \left[Q_i^t (e_i^t - \varepsilon_i (I^t, f_i^t, w_i^t)) + V \cdot cost_i^t \right]. \end{aligned} \tag{3}$$

It can be known that if the given feasible offloading decision of $\mathcal{P}_4$ is optimal, i.e., $I^t = I^{t*}$, $\mathcal{J}_4(f^{t*}, w^{t*}) = \mathcal{J}_3(I^{t*}, f^{t*}, w^{t*})$. Thus, we can say $\mathcal{P}_4$'s solution is the upper bound of $\mathcal{P}_3$'s solution, and when the given offloading decision of $\mathcal{P}_4$ is optimal, $\mathcal{J}_4(f^{t*}, w^{t*})$ is equal to $\mathcal{J}_3(I^{t*}, f^{t*}, w^{t*})$.

B. Proof for ***Theorem*** 1

We first show that $f_{i,L}$ and $f_{i,U}$ are the lower and upper bounds of the mobile devices' feasible CPU-cycle frequency when the computation task is executed locally. Due to delay constraint (18), it can be known that

$$D_i(I_{i,m}^t = 1, f_i^t, w_i^t) = D_{i,m}^t = \frac{L_i \cdot X_m}{f_i^t} \le \tau_d. \tag{4}$$

Thus, the f_i^t should be no less than $\frac{L_i \cdot X_m}{\tau_d}$, i.e., the minimized feasible CPU-cycle frequency of the ith mobile device when $I_{i,m}^t = 1$ is $\frac{L_i \cdot X_m}{\tau_d}$, i.e., $f_{i,L} = \frac{L_i \cdot X_m}{\tau_d}$. Besides, the upper bound of the energy consumed by the ith mobile device in each time slot is limited by $E_{i,max} = \min\left\{\max\{k(L_i \cdot X_m)(f_{i,max})^2, p_i \tau\}, E_{i,\tau}\right\}$. So, for any feasible CPU-cycle frequency f_i^t, the following inequality can be obtained

$$E_{i,m}^t = k(L_i \cdot X_m)(f_i^t)^2 \le E_{i,max}. \tag{5}$$

As can be seen from the above formula, the mobile devices CPU-cycle frequency satisfy $f_i^t \le \sqrt{\frac{E_{i,max}}{k(L_i \cdot X_m)}}$. At the same time, the CPU-cycle frequency of mobile devices is limited to the constraint (20), i.e, $0 \le f_i^t \le f_{i,max}$. Thus, the maximum feasible CPU-cycle frequency $f_{i,U} = \min\{\sqrt{\frac{E_{i,max}}{k(L_i \cdot X_m)}}, f_{i,max}\}$. Noting that if $f_{i,U} < f_{i,L}$, executing tasks locally is not feasible, and the optimal offload decision achieved by the IMOOD Algorithm will be non-local execution computing task, the detailed proof process is in ***Theorem*** *3*.

Next, we will show the optimality of f_i^{t*} from (30) when executing tasks locally is feasible.

For $Q_i^t \geq 0$, we can obtain the first-order derivative

$$\frac{\mathrm{d}\mathcal{J}_{lo,i}^t(f_i^t)}{\mathrm{d}f_i^t} = -2Q_i^t \cdot k(L_i \cdot X_m)(f_i^t) - V \cdot \frac{L_i \cdot X_m}{{f_i^t}^2} < 0. \tag{6}$$

Therefore, $\mathcal{J}_{lo,i}^t(f_i^t)$ decreases with f_i^t, i.e., the minimum value of $\mathcal{J}_{lo,i}^t(f_i^t)$ is achieved by $f_i^t = f_{i,U}$.

For $Q_i^t < 0$, we can obtain the second-order derivative

$$\begin{cases} \frac{d^2}{d{f_i^t}^2}(-Q_i^t \cdot k(L_i \cdot X_m)(f_i^t)^2) = -2Q_i^t \cdot k(L_i \cdot X_m) > 0 \\ \frac{d^2}{d{f_i^t}^2}(V \cdot \frac{L_i \cdot X_m}{f_i^t}) = 2V \cdot \frac{L_i \cdot X_m}{{f_i^t}^3} > 0 \end{cases} \tag{7}$$

Therefore, $\mathcal{J}_{lo,i}^t(f_i^t)$ is convex with respect to f_i^t as both $-Q_i^t \cdot k(L_i \cdot X_m)(f_i^t)^2$ and $V \cdot \frac{L_i \cdot X_m}{f_i^t}$ are convex function of f_i^t. By taking the first-order derivative of $\mathcal{J}_{lo,i}^t(f_i^t)$ and setting it to zero like the follow equation, we can obtain the unique solution $f_{i,0}^t = (\frac{V}{-2Q_i^t k})^{\frac{1}{3}}$ which achieves the minimum value of $\mathcal{J}_{lo,i}^t(f_i^t)$.

$$\mathcal{J}_{lo,i}^t(f_{i,0}^t)' = \frac{\mathrm{d}\mathcal{J}_{lo,i}^t(f_i^t)}{\mathrm{d}f_i^t} = -2Q_i^t \cdot k(L_i \cdot X_m)(f_{i,0}^t) - V \cdot \frac{L_i \cdot X_m}{{f_{i,0}^t}^2} = 0. \tag{8}$$

If $f_{i,0}^t < f_{i,L}^t$, $\mathcal{J}_{lo,i}^t(f_i^t)$ is increasing in $[f_{i,L}^t, f_{i,U}^t]$, and thus $f_i^{t*} = f_{i,L}^t$; if $f_{i,0}^t > f_{i,U}^t$, $\mathcal{J}_{lo,i}^t(f_i^t)$ is decreasing in $[f_{i,L}^t, f_{i,U}^t]$, and thus $f_i^{t*} = f_{i,U}^t$; otherwise, if $f_{i,L}^t \leq f_{i,0}^t \leq f_{i,U}^t$, $\mathcal{J}_{lo,i}^t(f_i^t)$ is decreasing in $[f_{i,L}^t, f_{i,0}^t]$ and increasing in $[f_{i,0}^t, f_{i,U}^t]$, and thus $f_i^{t*} = f_{i,0}^t$, i.e.,

$$f_i^{t*} = \begin{cases} f_{i,L}, & Q_i^t < 0, f_{i,0}^t < f_{i,L} \\ f_{i,0}^t, & Q_i^t < 0, f_{i,L} \leq f_{i,0}^t \leq f_{i,U} \\ f_{i,U}, & Q_i^t \geq 0 \text{ or } Q_i^t < 0, f_{i,0}^t > f_{i,U}, \end{cases} \tag{9}$$

C. *Proof for **Theorem** 2*

According to (35), we can obtain the KKT condition as follows:

$$\begin{cases} \frac{\partial L(w^{t*}, \alpha^{t*}, \beta^{t*}, \gamma^{t*}, \delta^{t*})}{\partial w_i^t} = -\frac{g_i}{{w_i^{t*}}^2} - \frac{\alpha_i^{t*}}{{w_i^{t*}}^2} - \frac{\beta_i^{t*}}{{w_i^{t*}}^2} - \gamma_i^{t*} + \delta^{t*} = 0; \\ \alpha_i^{t*} \geq 0, \beta_i^{t*} \geq 0, \gamma_i^{t*} \geq 0, \delta^{t*} \geq 0; \\ \alpha_i^{t*}(\frac{1}{w_i^{t*}} - \frac{1}{m_i}) = 0, \beta_i^{t*}(\frac{1}{w_i^{t*}} - \frac{1}{n_i}) = 0, \gamma_i^{t*}(w_d - w_i^{t*}) = 0, \delta^{t*}(\Sigma_{i \in B(t)} w_i^{t*} - W) = 0; \\ (\frac{1}{w_i^{t*}} - \frac{1}{m_i}) \leq 0, (\frac{1}{w_i^{t*}} - \frac{1}{n_i}) \leq 0, (w_d - w_i^{t*}) \leq 0, (\Sigma_{i \in B(t)} w_i^{t*} - W) \leq 0. \end{cases} \tag{10}$$

Due to KKT condition, we can know that the optimal solution of $\mathcal{P}_{KKT}$ exists if there are feasible solution to satisfy all constraints of 10. Otherwise, the optimal solution of $\mathcal{P}_{KKT}$ does not exist if there are not any feasible solution to satisfy all constraints of 10. However, noting that the optimal solution of $\mathcal{P}_{KKT}$ does not exist only under the premise that the offloading decision is not feasible, but the IMOOD Algorithm can ensure the optimal offloading decision is feasible. Thus, there is at least one feasible solution to satisfy all constraints of 10 which is the optimal solution of $\mathcal{P}_{KKT}$. Therefore, we only analyze the situation where the solution of $\mathcal{P}_{KKT}$ is feasible.

Firstly, we divide set $B(t)$ into two sets of $F(t)$ and $G(t)$ based on the position of the optimal solution value, $F(t)$ and $G(t)$ are the set of mobile devices whose optimal solutions are on the boundary of constraints and within the constraints, respectively. For the set $F(t)$, the inequality constraint of the Lagrange multiplication function $\mathcal{P}_{KKT}$ have played a binding role, the optimal solutions are on the boundary of constraints. Thus, the optimal solution of $i \in F(t)$ can be obtained as follow:

$$w_i^{t*} = \max\{m_i, n_i, w_{min}\} = z_i, i \in F(t). \tag{11}$$

Secondly, for the set $G(t)$, the inequality constraints of the Lagrange multiplication function $\mathcal{P}_{KKT}$ have not played a binding role, the optimal solutions are within the constraints. Thus, we can obtain

$$\alpha_i^{t*} = \beta_i^{t*} = \gamma_i^{t*} = 0, i \in G(t). \tag{12}$$

According to 10, the optimal solution of $i \in G(t)$ can be obtained as follow:

$$\begin{cases} \frac{g_i}{w_i^{t*2}} = \frac{g_j}{w_j^{t*2}} = \delta^{t*} \\ w_i^{t*} > z_i \end{cases}, i, j \in G(t). \tag{13}$$

Thus, we can obtain the optimal solution of $i \in G(t)$ as follow:

$$w_i^{t*} = \frac{(W - \sum_{j\in F(t)} z_j) \cdot \sqrt{g_i^t}}{\sum_{k\in G(t)} \sqrt{g_k^t}}, i \in G(t), \tag{14}$$

Therefore, combined 11 and 14, the optimal solution of $\mathcal{P}_{KKT}$ can be shown as

$$w_i^{t*} = \begin{cases} z_i, & i \in F(t) \\ \frac{(W - \sum_{j\in F(t)} z_j) \cdot \sqrt{g_i^t}}{\sum_{k\in G(t)} \sqrt{g_k^t}}, & i \in G(t). \end{cases} \tag{15}$$

D. Proof for ***Theorem*** *3*

For $\theta_i = E_{i,max} + V\Psi(E_{i,min})^{-1}, i \in U$, the virtual energy queue of the ith mobile device in tth time slot is obtained to

$$Q_i^t = B_i^t - \theta_i = B_i^t - \theta_i = B_i^t - (E_{i,max} + V\Psi(E_{i,min})^{-1}), i \in U. \tag{16}$$

If the remaining power of the ith mobile device in the tth time slot is sufficient, i.e., $B_i^t \geq E_{i,max}$, the arbitrary solution of $\mathcal{P}_3$ satisfies the constraint (14). Therefore, the optimal solution of $\mathcal{P}_3$ is feasible.

If the remaining power of the ith mobile device in the tth time slice isn't sufficient, i.e., $B_i^t < E_{i,max}$, we can get $Q_i^t < -V\Psi(E_{i,min})^{-1}$. Thus, the follows can be obtained

$$\begin{cases} \mathcal{J}_{lo,i}^t(f_i^{t*}) &= Q_i^t(e_i^t - \varepsilon_i(I_i^t, f_i^t, w_i^t)) + V \cdot cost_i^t > Q_i^t(e_i^t - E_{i,min}) \\ \mathcal{J}_{se,i}^t(w_i^{t*}) &= Q_i^t(e_i^t - \varepsilon_i(I_i^t, f_i^t, w_i^t)) + V \cdot cost_i^t > Q_i^t(e_i^t - E_{i,min}) \\ \mathcal{J}_{dr,i}^t &= Q_i^t(e_i^t - \varepsilon_i(I_i^t, f_i^t, w_i^t)) + V \cdot cost_i^t = Q_i^t \cdot e_i^t + V \cdot \Psi. \end{cases} \tag{17}$$

Due to $Q_i^t < -V\Psi(E_{i,min})^{-1}$, it can be known that

$$\begin{aligned} \min\{\mathcal{J}_{lo,i}^t(f_i^{t*}), \mathcal{J}_{se,i}^t(w_i^{t*})\} \quad &> Q_i^t(e_i^t - E_{i,min}) \\ &= Q_i^t \cdot e_i^t - Q_i^t \cdot E_{i,min} > Q_i^t \cdot e_i^t - (-V\Psi(E_{i,min})^{-1} \cdot E_{i,min}) \\ &= Q_i^t \cdot e_i^t - (-V \cdot \Psi) = Q_i^t \cdot e_i^t + V \cdot \Psi = \mathcal{J}_{dr,i}^t \end{aligned} \tag{18}$$

Thus, $\mathcal{J}_{dr,i}^t < \min\{\mathcal{J}_{lo,i}^t(f_i^{t*}), \mathcal{J}_{se,i}^t(w_i^{t*})\}$ when $B_i^t < E_{i,max}$, according to (37) it is known that the ith mobile device's offloading decision will be that the task dropped, i.e., $I_{i,d}^t = 1$. Therefore, the optimal solution of $\mathcal{P}_3$ is also feasible and satisfies the constraint (14) in this situation.