EET 2035C – Electrical Circuits

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Experiment 4

Proof of Thevenin, Norton and Maximum Power Transfer Theorems

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OBJECTIVE

The purpose of this lab experiment is to build and analyze series and parallel combination circuits for the purpose of observing the Thevenin and Norton theorems, the Maximum Power Transfer theorem, and practical implementation of adjustable resistances such as with a potentiometer and a rheostat.

LIST OF EQUIPMENT/PARTS/COMPONENTS

• Resistors: $1k\Omega$, $1.5k\Omega$, $2k\Omega$, $3k\Omega$

Multimeter (DMM)

DC Power Supply

Breadboard Trainer

THEORETICAL BACKGROUND

Fundamentals

Much of this section follows information provided in Lab Report 1.

There are many measurements to be taken in this lab, including the resistance of resistors, potential difference, and current through components of a circuit. To measure the voltage across an individual component, a **voltmeter** must be attached to the respective component in parallel. The circuit, however, still functions normally, as the voltmeter has a large resistance value and only allows enough current to make a measurement, as the rest continues through the circuit. Voltage can also be measured regardless of whether there is a current flowing or not [1].

Alternatively, measuring the current through a component in a circuit requires the use of an **ammeter**, attached in series to the respective component. In contrast to a **voltmeter**, an **ammeter** requires current to be flowing through it in order to measure the current. It is important to take precautions to not place an ammeter in parallel to any components in a circuit, as it would receive the entire current of said circuit and blow the fuse of the **ammeter**.

In order to measure resistance of a resistor, an **ohmmeter** is used in either parallel or in series and does not require a current flowing through a component as it creates its own current, and measures using it.

The theoretical **Resistance** of a resistor can be found by using the Color Code procedure, through analyzing **table 1** (read from left to right)

Table 1

Band Color	Digit	Multiplier	Tolerance
Black	0	100	
Brown	1	10 ¹	±1%
Red	2	10^{2}	±2%
Orange	3	10^{3}	
Yellow	4	10^{4}	
Green	5	10 ⁵	
Blue	6	10 ⁶	
Violet	7	10 ⁷	
Gray	8	108	
White	9	109	
No Band			±20%
Silver			±10%
Gold			±5%

Formulas and Laws

There are a few formulas and theories that will assist in finding various calculations throughout the lab. **Ohm's Law** is one such law, and states that the voltage through a component is equal to the current through the component multiplied by the resistance of the component [1].

$$V = IR$$

Also, **Kirchhoff's Voltage Law** states that the sum of each voltage drop through a circuit is zero, while the current is equivalent [1].

$$\sum_{k=1}^{n} V_k = 0$$

n = number of components in loop

Kirchhof's Current Law states that the current into a node is equal to the current out of it [1].

$$\sum_{k=1}^{n} I_k = 0$$

n = number of branches connected to a node

There are a few different methods to finding the **power absorbed** or **delivered** by an element of a circuit. Either the voltage and the current can be multiplied together (1), the current squared can be multiplied by the resistance of the element (2), or the voltage squared divided by the resistance of the element (3).

(1)
$$P = VI$$

(2)
$$P = I^2 R$$

$$(3) P = \frac{V^2}{R}$$

In a **series** circuit, if only the voltage source V_S and all resistor values are known, the voltage of any component can be calculated using the **Voltage Divider** formula, which divides the voltage source by the value of every resistor in the circuit **except** the resistance of the component being measured [1].

$$V_{x} = V_{s}(\frac{R_{x}}{\sum_{k=1}^{n} R_{k}})$$

n = number of resistors

x = component being measured

In a **parallel** circuit, if only the total current I_T and all resistor values are known, the current through any component can be calculated using the **Current Divider** formula, which

divides the total current by the value of every resistor in the circuit **except** the resistance of the component being measured [1].

$$=I_T(\frac{R_\chi}{\sum_{k=1}^n R_k})$$

n = number of resistors

x = component being measured

In a **Series-Parallel** circuit, the equivalent resistance method can be used to calculate measurements for all components of the circuit. In this method, the circuit will be broken down to as few components as possible by combining resistors, and working backwards through the circuit following KVL, KCL, and Ohm's Law.

The **Proportionality Theorem** states that the source in a circuit is equal to its response.

$$V_{out} = KV_{out}$$

Another theorem is the **Superposition** theorem. This theorem is used to find the voltage or current in a branch of a bilateral circuit, that which is equal to the sum of the sources acting on the branch, as well as each independently.

When removing a voltage source, short the circuit by adding a connector wire. When removing a current source, open the circuit completely.

Mesh and Nodal analysis are used throughout the lab in order to calculate voltages and currents throughout loops and branches. Through nodal analysis, voltages are assigned to nodes, and arbitrary currents out of each. Then, KCL is employed. When conducting mesh analysis, loop currents are identified in an arbitrary direction, then KVL is applied to each loop. The Matrix Inversion Method is used to solve the resulting system of equations for each method, determining the unknown voltages and currents respectively.

The **Thevenin Theorem** states that any linear circuit can be simplified to a circuit containing a single voltage source with a series resistance, all connected to a load. Thevenin voltage can be found be finding the open circuit output voltage.

Norton's Theorem states essentially the same as Thevenin, however regarding current in a circuit. The equivalent current can be obtained by short circuiting the two remaining terminals.

The **Maximum Power Theorem** indicates that load resistance must be equal to the Thevenin resistance in order to maximize load power in a circuit.

$$P_{Lmax} = V^2 \left(\frac{R_L}{R_{TH} + R_L} \quad or \quad P_{Lmax} = \frac{V_{OC}^2}{4R_{TH}} \right)$$

The DMM

The labelled DMM diagram is adopted from the following reference:

Ali Notash, "Current, Voltage and Resistance in Series and Parallel Circuits", in *EET 3081C* – *Circuit Analysis 1*, Florida: Valencia College, 2018.

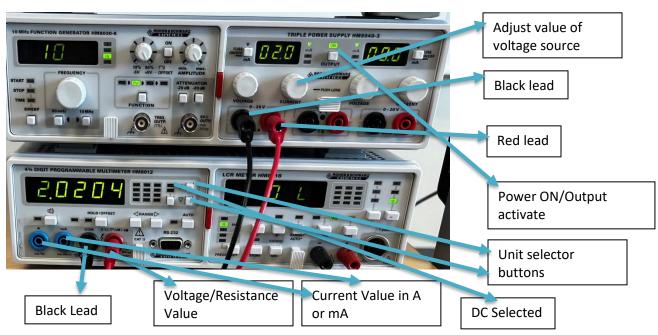


Figure 1

Labelled diagram of the DMM and Power Supply

PROCEDURE / RESULTS / OBSERVATIONS

Procedure is adopted from the following reference:

Prof. Ali Notash, "Current, Voltage, Power, and Resistance in Series and Parallel Circuits," in *Electrical Circuits Laboratory Manual*, 1st Ed. Florida: Valencia College, 2017, pp. 29-33.

Part 1 – Verification of the Thevenin and Norton Theorems.

First, the designated circuit is constructed in Multisim, following the measuring and recording of each resistor in use. Then, the load voltage across R_L and the current through R_L are measured and recorded, shown in **table 2**.

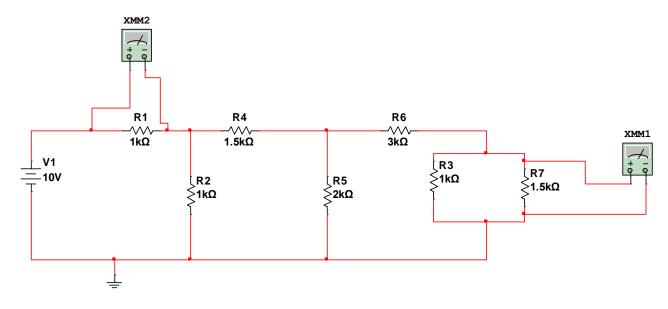


Figure 2

Part 1 Multisim Construction of Designated Circuit

In the following theoretical calculations, the **Nodal Analysis** method was used.

Node 1

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{v_1 - 10v}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} \\ -4v_1 + 9v_2 - 2v_3 = 0$$

$$i_{7} = \frac{v_{3} - v_{2}}{3000}, i_{8} = \frac{v_{3}}{1000}, i_{9} = \frac{v_{3}}{1500}$$

$$-v_{2} + 6v_{3} = 0$$

$$v_{1} = \frac{4.24v}{v_{2} = 1.96v}$$

$$v_{3(V_{RL})} = \frac{0.37v}{1500}$$

$$I_{RL} = \frac{326.09m}{1500}$$

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	$\mathbf{V_{L}}\left(\mathbf{V}\right)$	i _L (A)
Theory	326.09m	217.39u
Simulation	326.09m	217.39u
Measured	326.10m	216.00u

Next, the R_L resistor was removed from the circuit and the open circuit voltage was measured and recorded into **table 3**. The short circuit current is also measured and recorded in the table. Finally, the voltage source is replaced and the circuit is shorted. All data recorded in **table 3**.

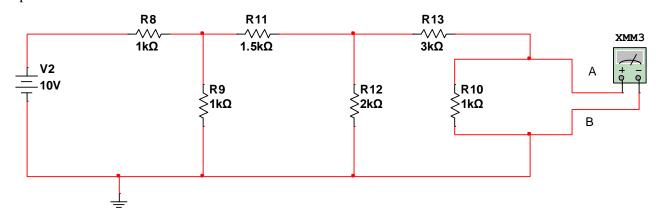


Figure 3

Multisim Construction of Designated Circuit, without RL

For the theory calculations, the **Nodal Analysis** method was again used, alongside **Mesh Analysis** to find the short circuit current.

Node 1

$$i_1 = \frac{i_1 + i_2 + i_3 = 0}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} -4v_1 + 9v_2 - 2v_3 = 0$$

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, \\ -v_2 + 4v_3 = 0$$

$$v_1 = \frac{4.25v}{v_2 = \frac{2.00v}{2.00v}}$$

 $v_{3(V_{RL})} = \frac{0.50v}{2.00v}$

Mesh 1

$$i_1 = \frac{i_1 + i_2 + i_3 = 0}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$
$$2ki_1 - 1ki_2 = 10$$

Mesh 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} -1ki_2 + 4.5ki_2 - 2ki_3 = 0$$

Mesh 3

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{1500} -2ki_2 + 6ki_3 - 1ki_4 = 0$$

Mesh 4

$$i_7 = \frac{v_3 - v_2}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{1500}$$

$$1ki_4 - 1ki_3 = 0$$

$$i_1 = \frac{5.78mA}{1.56mA}$$

$$i_2 = \frac{1.56mA}{1.56mA}$$

$$i_3 = \frac{625uA}{625uA}$$

Table 3

	$\mathbf{V}_{\mathbf{OC}}(V)$	$\mathbf{i}_{\mathrm{sc}}(\mathbf{A})$	$\mathbf{R}_{\mathrm{TH}}\left(\Omega\right)$	$\mathbf{R}_{\mathrm{TH}} = \mathbf{V}_{\mathrm{OC}} / \mathbf{i}_{\mathrm{sc}}$	
Theory	500m	625u 800		800	
Simulation	500m	625u	800	800	
Measured 494.80m		620.00u	799.90	798.06	
% Error 1.04		0.80	0.01	0.24	

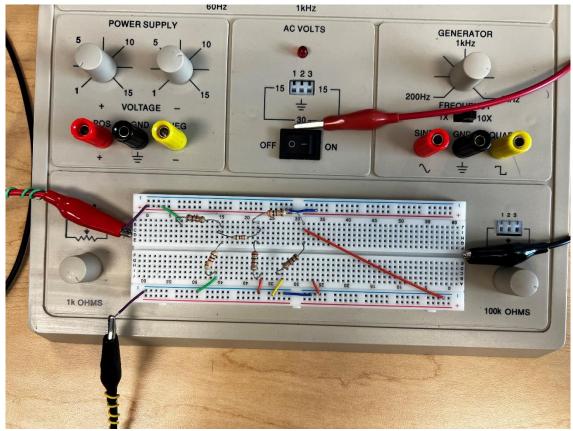
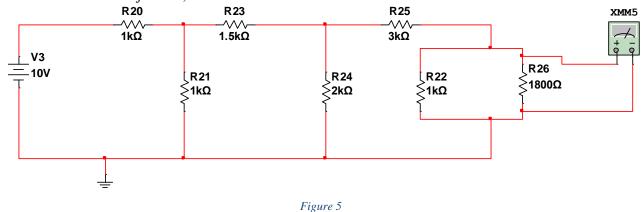


Figure 4

Thevenin Equivalent Circuit on Breadboard

Part 2 – Verification of the Maximum Power Transfer Theorem

Now, the circuit is reconfigured, such that the $1.5k\Omega$ load resistor between terminals A and B, with the breadboard trainer's adjustable, variable resistor.



Reconfigured Multisim Circuit with R26 Representing the Variable Resistor

In increments of 200, increase the variable resistor on the breadboard trainer from 400Ω to $1.8k\Omega$, measuring and recording V_L and i_L throughout.

The theoretical calculations for this process are found below, using **Nodal Analysis**:

 400Ω

Node 1

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{v_1 - 10v}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} - 4v_1 + 9v_2 - 2v_3 = 0$$

Node 3

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{400}$$
$$-2v_2 + 23v_3 = 0$$

$$v_1 = \frac{4.23v}{v_2 = \frac{1.96v}{1.96v}}$$

 $v_{3(V_{RL})} = \frac{0.17v}{1.90}$

 600Ω

Node 1

$$i_1 = \frac{i_1 + i_2 + i_3 = 0}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$
$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} \\ -4v_1 + 9v_2 - 2v_3 = 0$$

Node 3

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{600}$$
$$-v_2 + 9v_2 = 0$$

$$v_1 = \frac{4.23v}{v_2 = 1.93v}$$

 $v_{3(V_{RL})} = \frac{214.29mv}{v_{3(V_{RL})}}$

 800Ω

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{v_1 - 10v}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} \\ -4v_1 + 9v_2 - 2v_3 = 0$$

Node 3

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{800} -4v_2 + 31v_3 = 0$$

$$v_1 = \frac{4.23v}{v_2 = 1.94v}$$
$$v_{3(V_{RL})} = \frac{0.25v}{v_{3(V_{RL})}}$$

 1000Ω

Node 1

$$i_1 = \frac{i_1 + i_2 + i_3 = 0}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000}$$
$$-4v_1 + 9v_2 - 2v_3 = 0$$

Node 3

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{1000} -v_2 + 7v_3 = 0$$

$$v_1 = \frac{4.24v}{v_2 = 1.94v}$$
$$v_{3(V_{RL})} = \frac{0.28v}{v_{3(V_{RL})}}$$

 1200Ω

Node 1

$$i_1 = \frac{i_1 + i_2 + i_3 = 0}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} \\ -4v_1 + 9v_2 - 2v_3 = 0$$

Node 3

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{1200}$$
$$-2v_2 + 13v_3 = 0$$

$$v_1 = \frac{4.24v}{v_2 = 1.95v}$$
$$v_{3(V_{RL})} = 0.30v$$

 1400Ω

Node 1

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{v_1 - 10v}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000} \\ -4v_1 + 9v_2 - 2v_3 = 0$$

Node 3

$$i_7 = \frac{v_3 - v_2}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{1400} \\ -7v_2 + 43v_3 = 0$$

$$v_1 = \frac{4.24v}{v_2 = \frac{1.95v}{1.95v}}$$
$$v_{3(V_{RL})} = \frac{0.32v}{1.95v}$$

 1600Ω

Node 1

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{v_1 - 10v}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000}$$
$$-4v_1 + 9v_2 - 2v_3 = 0$$

$$i_7 = \frac{v_3 - v_2}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{1600} - 8v_2 + 647 = 0$$

$$v_1 = \frac{4.24v}{v_2}$$
 $v_2 = \frac{1.96v}{0.33v}$

 1800Ω

Node 1

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{v_1 - 10v}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1500}$$

$$8v_1 - 2v_2 = 30$$

Node 2

$$i_4 = \frac{v_2 - v_1}{1500}, i_5 = \frac{v_2}{2000}, i_6 = \frac{v_2 - v_3}{3000}$$
$$-4v_1 + 9v_2 - 2v_3 = 0$$

Node 3

$$i_7 = \frac{i_7 + i_8 + i_9 = 0}{3000}, i_8 = \frac{v_3}{1000}, i_9 = \frac{v_3}{1800}$$
$$-3v_2 + 17v_3 = 0$$

$$v_1 = \frac{4.24v}{v_2}$$

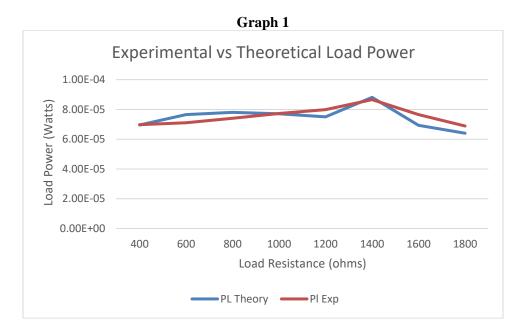
$$v_2 = \frac{1.96v}{0.35v}$$

The power is also measured and calculated, then recorded in the table.

Table 4

	$R_{\rm L}$	400Ω	600Ω	Ω 008	1000Ω	1200Ω	1400Ω	1600Ω	1800Ω
	Theory	166.67m	214.29m	250m	277.78m	300m	318.18m	333.83m	346.15m
VL	Simulation	166.67m	214.29m	250m	277.78m	300m	318.18m	333.83m	346.15m
	Measured	168.5m	213.00m	250.70m	277.90m	301.30m	319.40m	334.80m	347.70m
i _L	Theory	417u	357u	312u	278u	250u	277u	208u	192u
	Simulation	417u	357u	312u	278u	250u	277u	208u	192u
	Measured	414u	333.33u	295.20u	278u	264.90u	271u	228.40u	198u
	Theory	69.5u	76.5u	78u	77u	75u	88.1u	69.3u	64u
PL	Simulation	69.5u	76.5u	78u	77u	75u	88.1u	69.3u	64u
	Measured	6.98E-05	7.10E-05	7.40E-05	7.73E-05	7.98E-05	8.66E-05	7.65E-05	6.88E-05
P _L	(% Error)	3.73E-01	7.19E+00	5.12E+00	3.33E-01	6.42E+00	1.75E+00	1.03E+01	7.57E+00

The graph below depicts the experimental vs the theoretical values for the load power of the circuit. The graphs are largely the same indicating a lack or minimization of error and a confirmation of the goal of the lab, to observe Thevenin and Norton circuits.



Maximum load power occurs at 1400Ω , and is therefore the most optimal resistance rating to use in this circuit. And thus, the **Maximum Power Theorem** is verified experimentally.

Any discrepancies present are likely due to the difference in resistance values that occur in the benchwork, as these values differ slightly, which as displayed in the discussion section of this lab.

RESEARCH QUESTION

Thevenin's and Norton's theorems are very important and useful in applications. One example of this in use is in battery optimization of portable devices. Thevenin's theorem can help to sample internal resistance and open circuit voltage of a battery, allowing for an analysis of the battery's performance under variable conditions.

Another example, this time of Norton's theorem, is found in the way power is distributed in electrical grids. Norton's theorem can be used to predict or test the effects of altering loads in or on a power grid before actually conducting the change. This is essential to allowing for efficient management of these power grids without causing outages or affecting operations.

A third example is found when trying to match output impedance of any device, such as an amplifier to a speaker for maximizing power transfer. Here, Thevenin's theorem would be at play, to find the equivalent voltage source and determine the resistance from an output device.

DISCUSSION

There are many things to keep in mind conducting or following this lab, including maintaining proper bench etiquette and safety practices, ensuring accurate and organized recording of data to minimize loss of information, and foundational understanding of the theorems and laws at practice in the lab. KVL, KCL, and Ohm's law are obvious fundamentals that are necessary for understanding the more complex ones, such as the superposition theorem, the linearity theorem, and both nodal and mesh analysis. Additionally, employing all of these theorems and fundamentals are essential to properly using and understanding both Thevenin's theorem and Norton's theorem.

Superposition analysis in particular must be carefully conducted, as when it is being measured, there is either a short or an open segment of a circuit while current is flowing through it.

Error in the lab could have come from various factors. Aside from the slight difference present from varying resistor values, miscalculations, improper formation of circuits, and faulty equipment could all lead to a higher percentage error. Despite this, the lab largely held little error in terms of consistency between theoretical, simulation, and measured data.

Below are the measured resistor values used for real world testing and circuit construction:

Table 5

1 k Ω (1)	1kΩ (2)	1.5kΩ	$2k\Omega(1)$	$3k\Omega(1)$	1 k Ω (3)	$1.5k\Omega$ (2)
1.006k	983.3	1.48k	1.99k	2.94k	0.99k	1.50k

CONCLUSION

Part 1 of this report allows for the demonstration and the verification of Thevenin's and Norton's theorems by either shorting circuits, or opening them, and finding values such as the Thevenin resistance, the open circuit voltage, and the short circuit current. Mesh analysis and Nodal analysis was used to find help find these values theory-wise, followed by routine benchwork for measured data.

In part 2, Nodal voltage analysis and mesh current analysis were further expanded upon and heavily relied on for theoretical calculations. Again, a circuit was constructed, Similar to the previous circuit, however with the inclusion of a variable resistor serving as the load resistor. This mean many more theoretical calculations, and many more bench measurements, but provided incredibly insightful data allowing for a certain theorem to be tested.

The use of a variable resistor, such as would be found as a potentiometer, is found in this part of the experiment. This allowed us to be exposed to the idea of variable resistance, and test the **Maximum Power Theorem** directly.

REFERENCES

[1] W. H. Hayt, J. E. Kemmerly, J. D. Phillips, and S. M. Durbin, *Engineering Circuit Analysis*, 9th ed. New York, NY: McGraw Hill, LLC, 2019.