<u>EET 2035C – Electrical Circuits</u>

Summer 2024

Experiment 3

Node-Voltage, Mesh Current, Linearity, and Superposition Theorems

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OBJECTIVE

The purpose of this lab experiment is to build and analyze series and parallel combination circuits for the purpose of observing current, voltage and resistance. These observations will be supported by foundational knowledge of Ohm's Law, KV, and KCL. Experimental measurements will be compared against theoretical and simulation data to ensure proper understanding of each.

LIST OF EQUIPMENT/PARTS/COMPONENTS

- Resistors: 1kΩ, 2.7kΩ, 3.3kΩ, 10kΩ
- DC Power Supply
- Multimeter (DMM)
- Breadboard Trainer

THEORETICAL BACKGROUND

Fundamentals

Much of this section follows information provided in Lab Report 1.

There are many measurements to be taken in this lab, including the resistance of resistors, potential difference, and current through components of a circuit. To measure the voltage across an individual component, a **voltmeter** must be attached to the respective component in parallel. The circuit, however, still functions normally, as the voltmeter has a large resistance value and only allows enough current to make a measurement, as the rest continues through the circuit. Voltage can also be measured regardless of whether there is a current flowing or not [1].

Alternatively, measuring the current through a component in a circuit requires the use of an **ammeter**, attached in series to the respective component. In contrast to a **voltmeter**, an **ammeter** requires current to be flowing through it in order to measure the current. It is important to take precautions to not place an ammeter in parallel to any components in a circuit, as it would receive the entire current of said circuit and blow the fuse of the **ammeter**.

In order to measure resistance of a resistor, an **ohmmeter** is used in either parallel or in series and does not require a current flowing through a component as it creates its own current, and measures using it.

The theoretical **Resistance** of a resistor can be found by using the Color Code procedure, through analyzing **table 1** (read from left to right)

Table 1

Band Color	**Digit**	**Multiplier**	**Tolerance**
Black	0	10^0	
Brown	1	10^1	±1%
Red	2	10^2	±2%
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
No Band			±20%
Silver			±10%
Gold			±5%

Formulas and Laws

There are a few formulas and theories that will assist in finding various calculations throughout the lab. **Ohm's Law** is one such law, and states that the voltage through a component is equal to the current through the component multiplied by the resistance of the component [1].

$$V = IR$$

Also, **Kirchhoff's Voltage Law** states that the sum of each voltage drop through a circuit is zero, while the current is equivalent [1].

$$\sum_{k=1}^{n} V_k = 0$$

$$n = number\ of\ components\ in\ loop$$

Kirchhof's Current Law states that the current into a node is equal to the current out of it [1].

$$\sum_{k=1}^{n} I_k = 0$$

$$n = number\ of\ branches\ connected\ to\ a\ node$$

There are a few different methods to finding the **power absorbed** or **delivered** by an element of a circuit. Either the voltage and the current can be multiplied together (1), the current squared can be multiplied by the resistance of the element (2), or the voltage squared divided by the resistance of the element (3).

$$(1)\ P = VI$$

$$(2)\ P = I^2R$$

$$(3)\ P = \frac{V^2}{R}$$

In a **series** circuit, if only the voltage source V_S and all resistor values are known, the voltage of any component can be calculated using the **Voltage Divider** formula, which divides the voltage source by the value of every resistor in the circuit **except** the resistance of the component being measured [1].

$$V_x = V_s(\frac{R_x}{\sum_{k=1}^{n} R_k})$$

$$n = number\ of\ resistors$$

$$x = component\ being\ measured$$

In a **parallel** circuit, if only the total current I_T and all resistor values are known, the current through any component can be calculated using the **Current Divider** formula, which divides the total current by the value of every resistor in the circuit **except** the resistance of the component being measured [1].

$$= I_T\left(\frac{R_x}{\sum_{k=1}^{n} R_k}\right)$$

$$n = number\ of\ resistors$$

$$x = component\ being\ measured$$

In a **Series-Parallel** circuit, the equivalent resistance method can be used to calculate measurements for all components of the circuit. In this method, the circuit will be broken down to as few components as possible by combining resistors, and working backwards through the circuit following KVL, KCL, and Ohm's Law.

In this lab, some new theorems are introduced, such as the **Proportionality**, or **Linearity** theorem. The proportionality theorem states that the source in a circuit is equal to its response.

$$V_{out} = KV_{out}$$

Another newly introduced theorem for this lab is the **Superposition** theorem. This theorem is used to find the voltage or current in a branch of a bilateral circuit, that which is equal to the sum of the sources acting on the branch, as well as each independently.

When removing a voltage source, short the circuit by adding a connector wire. When removing a current source, open the circuit completely.

Mesh and **Nodal** analysis are used throughout the lab in order to calculate voltages and currents throughout loops and branches. Through nodal analysis, voltages are assigned to nodes, and arbitrary currents out of each. Then, **KCL** is employed. When conducting mesh analysis, loop currents are identified in an arbitrary direction, then KVL is applied to each loop. The **Matrix Inversion Method** is used to solve the resulting system of equations for each method, determining the unknown voltages and currents respectively.

The DMM

The labelled DMM diagram is adopted from the following reference:

Ali Notash, “Current, Voltage and Resistance in Series and Parallel Circuits”, in *EET 3081C – Circuit Analysis 1*, Florida: Valencia College, 2018.

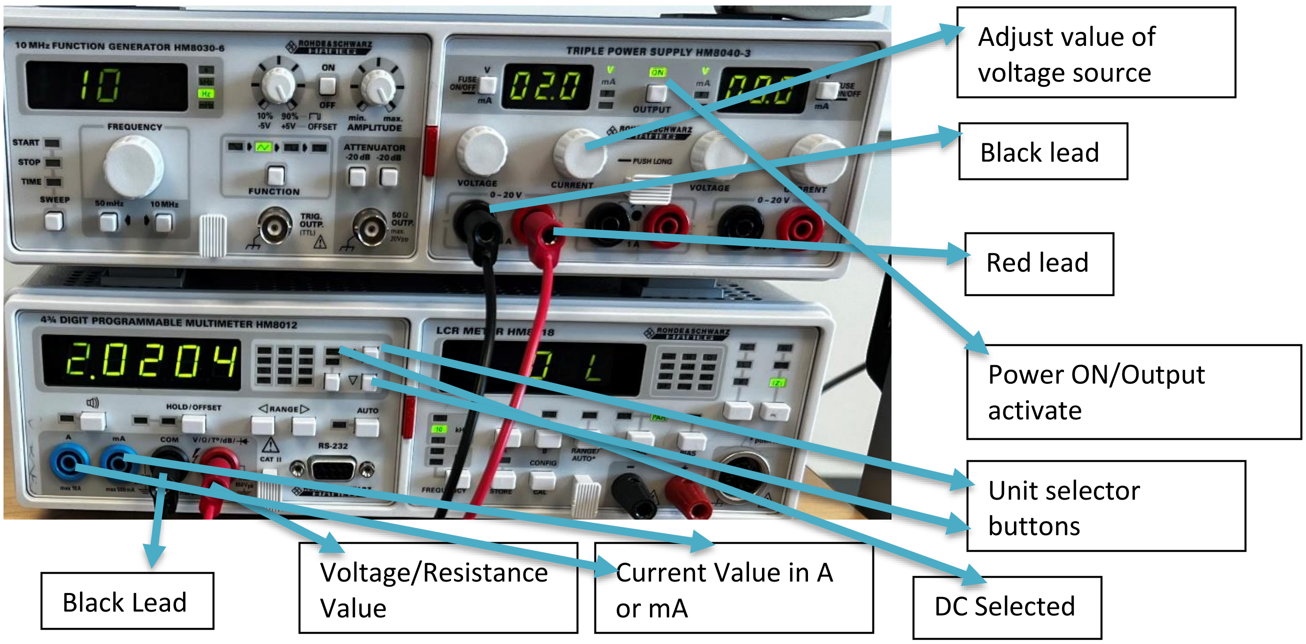


Figure 1

Labelled diagram of the DMM and Power Supply

PROCEDURE / RESULTS / OBSERVATIONS

Procedure is adopted from the following reference:

Prof. Ali Notash, “Current, Voltage, Power, and Resistance in Series and Parallel Circuits,” in *Electrical Circuits Laboratory Manual*, 1st Ed. Florida: Valencia College, 2017, pp. 23-27.

Part 1 – Verifying the Linearity (*proportionality*) principle

First, the designated circuit is constructed in Multisim, and the measurements of five input voltages across V_{out} are taken and compared to the theoretical calculations.

Then, the proportionality constant is calculated by dividing V_{out} by V_{in}, and multiplied by 100 to get the percent value. Then the data is plotted in **graph 1**.

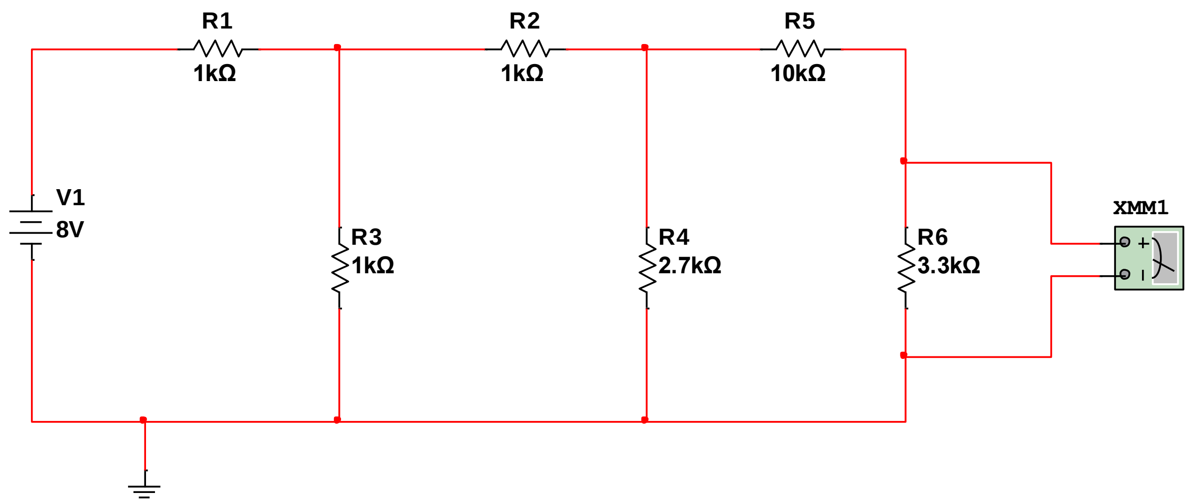


Figure 2

Multisim construction of circuit 1, with multimeter attached to measure V_{out}

The theoretical data is found below.

2V

Node 1	Node 2	Node 3
$2ki_1 - 1ki_2 = 2$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

i_1 = 1.13mA

i_2 = 267.06μA

i_3 = 45.07μA

V_{out} = 45.07μA(3.3k Ω) = 148.73mV

4V

Node 1	Node 2	Node 3
$2ki_1 - 1ki_2 = 4$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

i_1 = 2.27mA

i_2 = 534.14μA

i_3 = 90.14μA

V_{out} = 90.14μA (3.3k Ω) = 297.45mV

6V

Node 1	Node 2	Node 3
$2ki_1 - 1ki_2 = 6$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

i_1 = 3.40mA

i_2 = 801.20μA

i_3 = 135.20μA

V_{out} = 135.20μA(3.3k Ω) = 446.17mV

8V

Node 1	Node 2	Node 3
$2ki_1 - 1ki_2 = 8$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

i_1 = 4.53mA

i_2 = 1.07mA

i_3 = 180.27μA

V_{out} = 180.27μA(3.3k Ω) = 594.89mV

10V

Node 1

$$2ki_1 - 1ki_2 = 10$$

Node 2

$$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$$

Node 3

$$-2.7ki_2 + 16ki_3 = 0$$

i_1 = 5.67mA

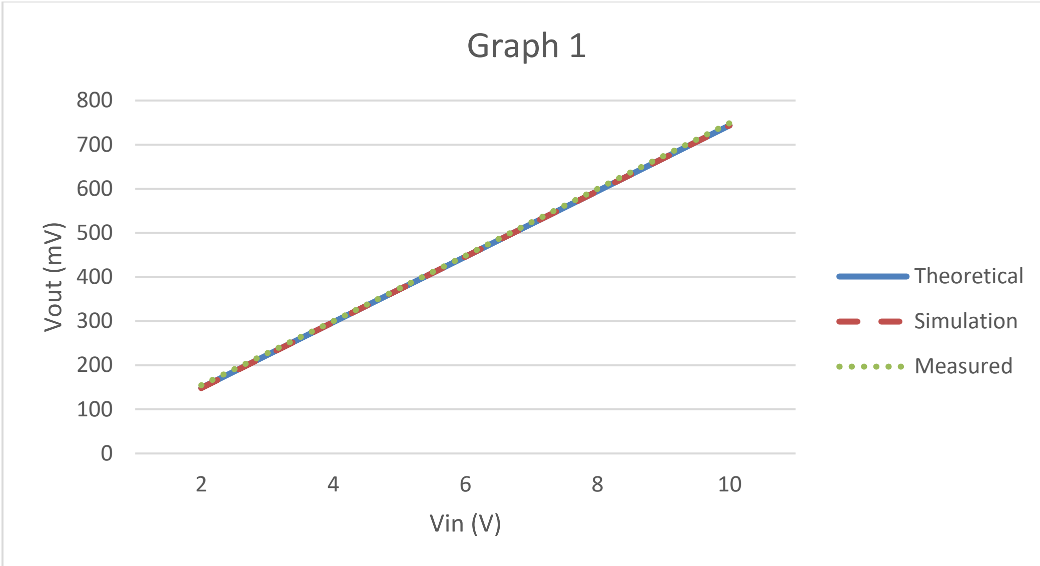
i_2 = 1.34mA

i_3 = 225.34μA

V_{out} = 225.34μA(3.3k Ω) = 743.62mV

Table 2

Input Voltage V_{in}(V)		Output Voltage V_{out}(mV)	Proportionality Constant $K = \frac{V_{out}}{V_{in}}(\%)$
2	**Theory**	148.73	7.44
	Simulation	148.2	7.44
	Measured	154.50	7.73
4	**Theory**	297.45	7.44
	Simulation	297.45	7.44
	Measured	300.14	7.50
6	**Theory**	446.17	7.44
	Simulation	446.18	7.44
	Measured	448.13	7.46
8	**Theory**	594.89	7.44
	Simulation	594.89	7.44
	Measured	599.72	7.49
10	**Theory**	743.62	7.44
	Simulation	743.61	7.44
	Measured	748.72	7.48



This graph depicts input versus output voltage of the given circuit. As shown, the theoretical, simulation, and measured align cohesively. Calculating the slope at any two points yields an almost completely equivalent slope, indicating accuracy between all three measurements. Therefore, result agrees with theory.

Next, changing the input source from a DC source to a $5V_{pp}$, 1kHz sine wave AC source alters not only our output, but our method of measurement as well.
Below are the probes connected to the oscilloscope used to measure AC sine wave signals.

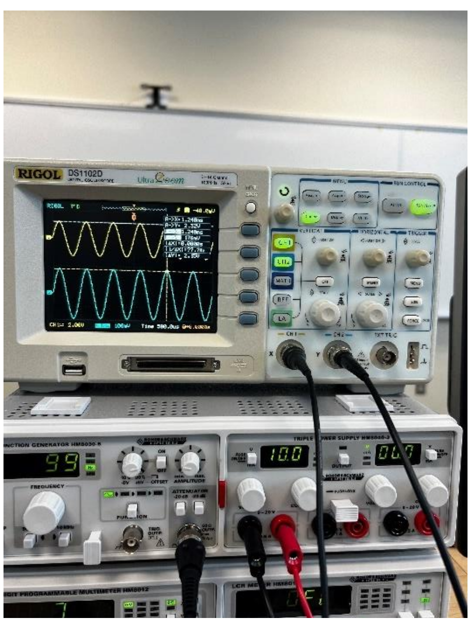


Figure 3

Oscilloscope reading AC signal from constructed circuit

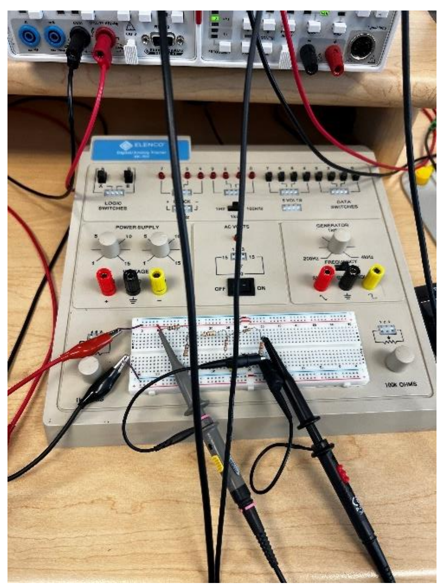


Figure 4

Probes connected to circuit for measurement by oscilloscope

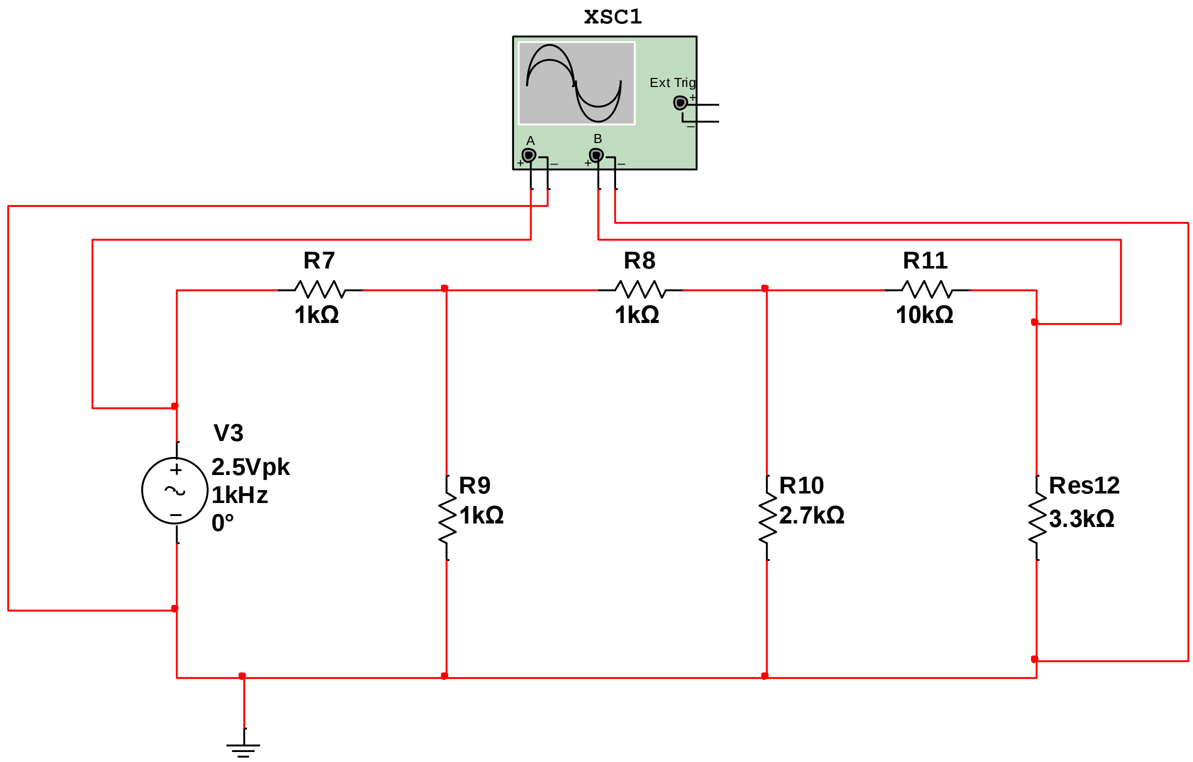


Figure 6

Multisim construction of given AC circuit with attached Oscillioscope

Utilizing the following formula allows for conversion from AC to an input voltage.

$$V(t) = Asin(wt)$$

Below are the calculations for the theoretical data of the AC Circuit input voltage.

0.25ms || $V(t) = 2.5\sin\,(2\pi(1k)0.25e^{-3} = 2.5V$
0.50ms || $V(t) = 2.5\sin\,(2\pi(1k)0.50e^{-3} = 0V$
1.00ms || $V(t) = 2.5\sin\,(2\pi(1k)1.00e^{-3} = 0V$
1.25ms || $V(t) = 2.5\sin\,(2\pi(1k)1.25e^{-3} = 2.5V$

Now, conducting mesh analysis on the circuit yields the following data.

0.25s

Mesh 1	Mesh 2	Mesh 3
$2ki_1 - 1ki_2 = 2.5$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

i_1 = 1.42mA

i_2 = 333.83μA

i_3 = 56.33μA

V_{out} = 56.33μA (3.3k Ω) = 185.90mV

0.50s

Mesh 1	Mesh 2	Mesh 3
$2ki_1 - 1ki_2 = 0$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

$i_1 = 0$

$i_2 = 0$

$i_3 = 0$

$V_{out} = 90.14\mu A\ (3.3k\ \Omega) = 0$

1.00s

Mesh 1	Mesh 2	Mesh 3
$2ki_1 - 1ki_2 = 0$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

$i_1 = 3.40mA$

$i_2 = 801.20\mu A$

$i_3 = 135.20\mu A$

$V_{out} = 135.20\mu A(3.3k\ \Omega) = 0$

1.25s

Mesh 1	Mesh 2	Mesh 3
$2ki_1 - 1ki_2 = 2.5$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 16ki_3 = 0$

$i_1 = 1.42mA$

$i_2 = 333.83\mu A$

$i_3 = 56.33\mu A$

$V_{out} = 56.33\mu A\ (3.3k\Omega) = 185.90mV$

The data is then plotted into a graph for analysis

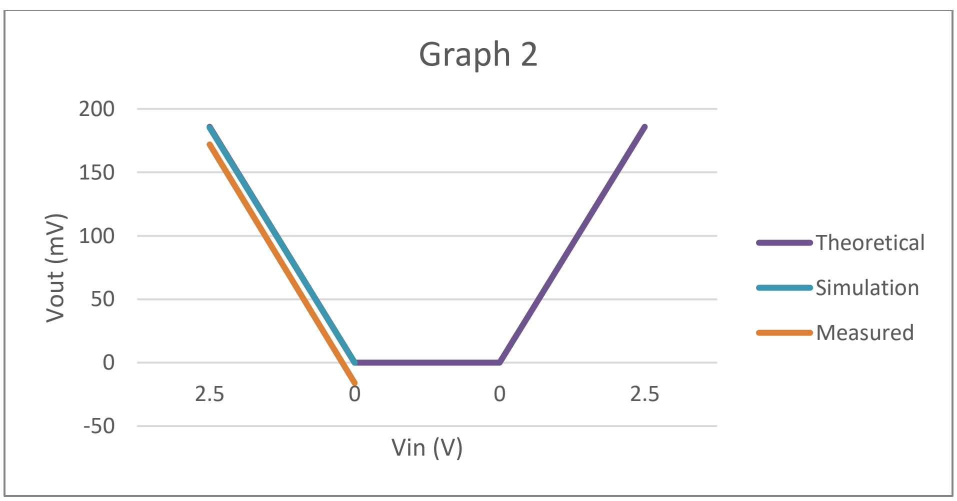


Figure 7

Graph depicting V_{in} against V_{out}

Table 3

Time Instance (ms)		Input Voltage V_{in}(V)	Output Voltage V_{out}(mV)	Proportionality Constant $K = \frac{V_{out}}{V_{in}}$ (%)
0.25	**Theory**	2.5	185.90	7.44
	Simulation	2.5	185.18	7.44
	Measured	2.5	172	7.717
0.5	**Theory**	0	0	0
	Simulation	-279µ	-207µ	7.42
	Measured	-120m	-16	13.33
1	**Theory**	0	0	0
	Simulation	279µ	20.70µ	7.42
	Measured	0	0	0
1.25	**Theory**	2.5	185.90	7.44
	Simulation	2.5	185.18	7.44
	Measured	2.5	176	7.33

There seems to continue to be a linear relationship between V_{in} and V_{out}.

Part 2 – Node-Voltage and Mesh Current Analysis

The following circuit is first created in Multisim, then reconstructed later on the breadboard. As with the previous part of the lab, theoretical, simulation, and measured values are observed and recorded.

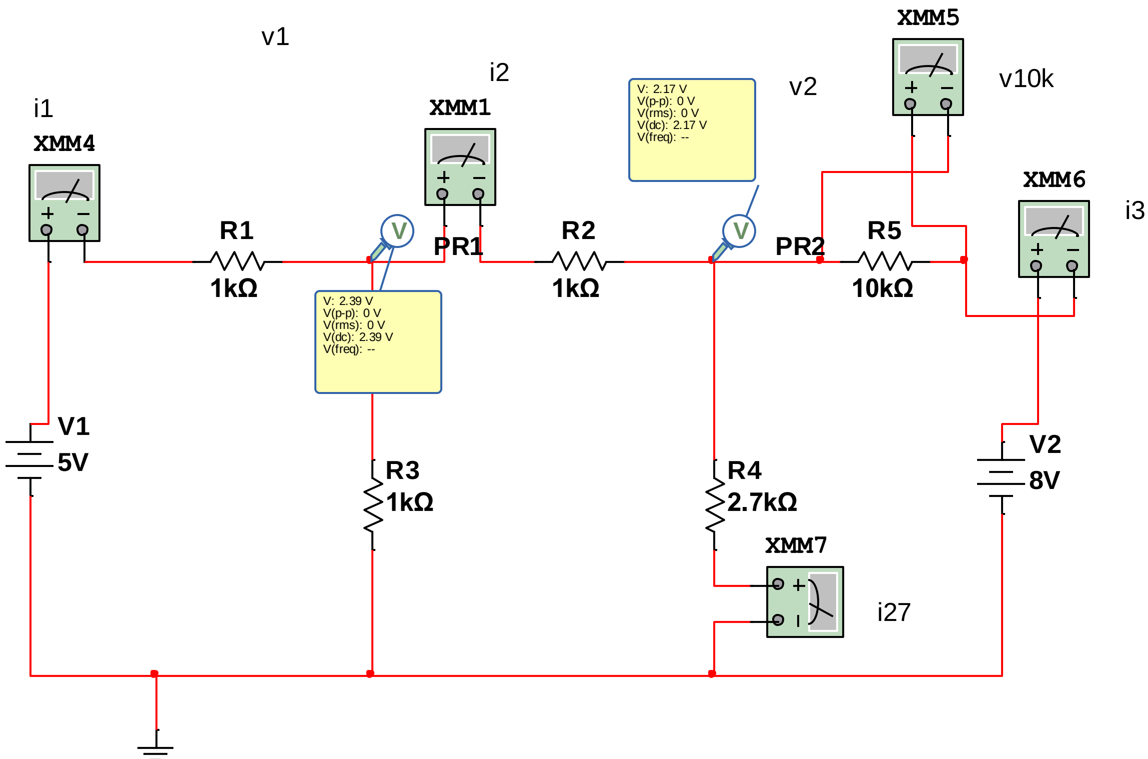


Figure 8

Multisim constructed given circuit, featuring both probes and multimeters

The voltages of nodes 1 and 2 are measured, followed by the voltage drop across the 10kΩ resistor

It can be observed that the difference in voltage between node 2 and VS2 is equivalent to the voltage drop across the 10kΩ resistor:

$$V_2 = 2.17V$$
$$V_{s2} = 8V$$
$$V_{10k\Omega} = 5.83$$
$$V_{s2} - V_2 = 5.83V$$

Nodal analysis yields the following data:

Node 1

$i_1 + i_2 + i_3 = 0$

$$i_1 = \frac{v_1 - 5}{1000}, i_2 = \frac{v_1}{1000}, i_3 = \frac{v_1 - v_2}{1000}$$

$$3v_1 - v_2 = 5$$

$$v_1 = 2.39V$$

Node 2

$I_4 + i_5 + i_6 = 0$

$$i_4 = \frac{v_2 - v_1}{1000}, i_5 = \frac{v_2}{2700}, i_6 = \frac{v_2 - 8}{10000}$$

$$27v_1 - 39.7v_2 = 21.6$$

$$v_2 = 2.17V$$

Mesh current analysis yields the following data:

Mesh 1

$$2ki_1 - 1ki_2 = 5$$

Mesh 2

$$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$$

Mesh 3

$$-2.7ki_2 + 12.7ki_3 = 8$$

i_1 = 2.61mA
i_2 = 220.41μA
i_3 = 583.06μA

$i_{2.7k}$ = 583.06μ + 220.41μ = 803.48μ
$V_{10k\Omega}$ = 10000(583.06μ) = 5.83V

Table 4 below depicts the theoretical, simulation, and measured values for each requested component as per the lab manual experiment.

Table 4

Component	**V_1** (v)	**V_2** (v)	**$V_{10k\Omega}$** (v)	**i_1** (A)	**i_2** (A)	**i_3** (A)	**$i_{2.7k\Omega}$** (A)
Theoretical	2.39	2.17	5.83	2.61m	220.41μ	583.06μ	803.48μ
Simulation	2.39	2.17	5.83	2.61m	220.41μ	583.06μ	803.48μ
Measured	2.23	2.19	5.85	2.51m	223.70 μ	582.50 μ	799.50 μ

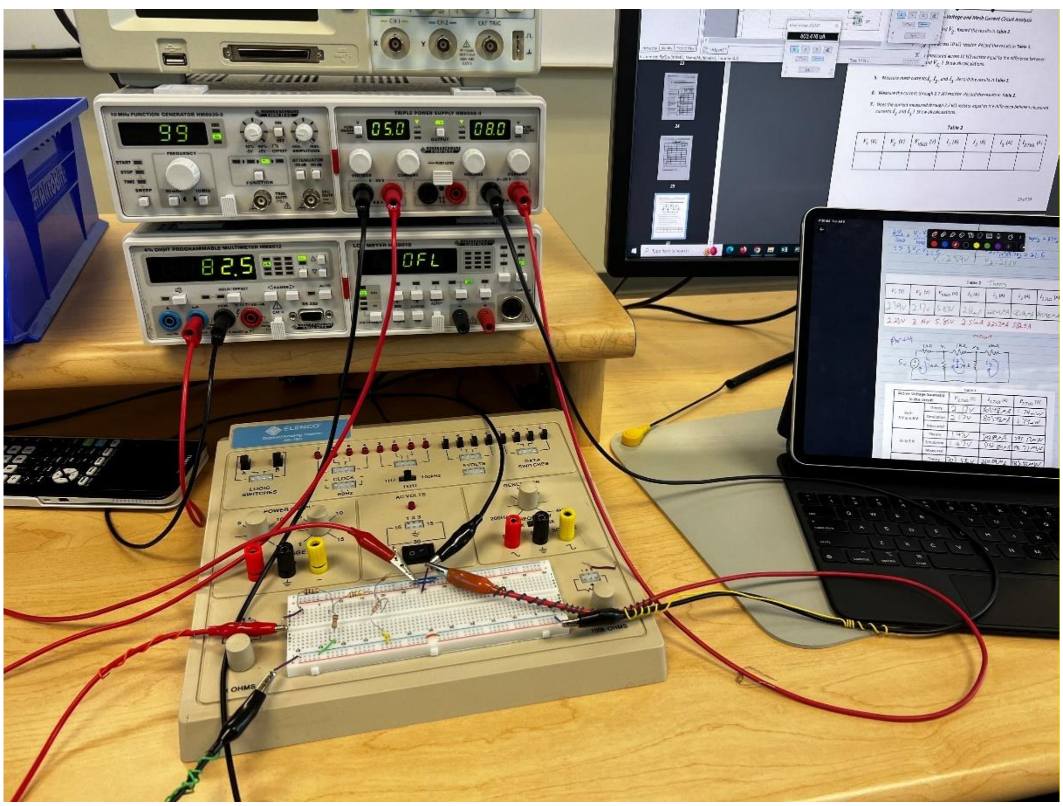


Figure 9

The given circuit constructed on the breadboard trainer, connected to the DMM for measurement

Similarly to the previous observation wherein the difference between V_2 and V_{s2} was equivalent to the voltage drop across the 10kΩ resistor, the sum between the currents i_3 and i_2 is equivalent to the 2.7kΩ resistor:

$$582.50\mu + (223.70\mu) = 799.50\mu$$

In terms of how the theoretical, simulation, and measured data compared, they were all accurate with very marginal error,

Part 3 – Verifying the Superposition Theorem

In this section of the lab, a circuit will be analyzed that has both a 5 volt and an 8 volt DC independent source, similar to the previous circuit. However, this time, the circuit will be analyzed while both sources are active, as well one being deactivated while the other is activated for each.

As previously stated, when removing a voltage source for the purpose of analyzing using the superposition theorem, the circuit must be shorted in lieu of the source, using a piece of wire to bridge the newfound gap.

Below are the Multisim simulation constructed circuits.

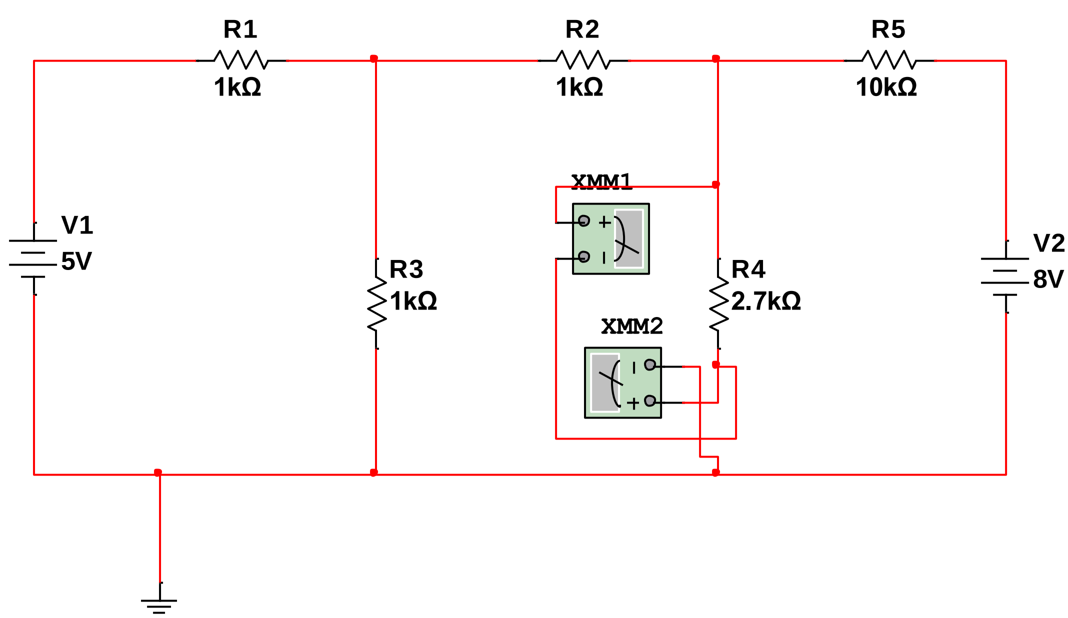


Figure 10

Depicting the designated circuit with both sources activated, as well as multimeters being present

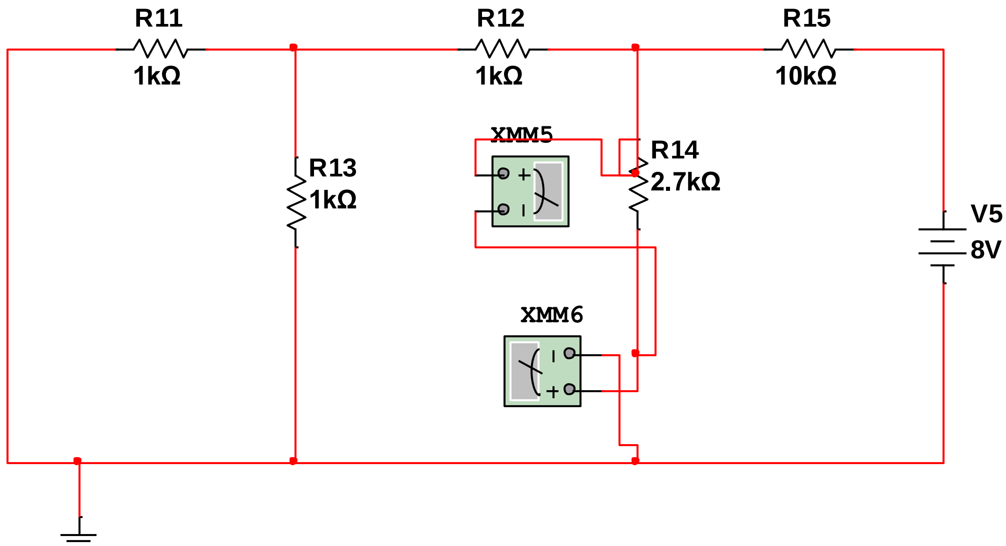


Figure 11

Depicting the designated circuit with only the 8V source activated, and the 5V source deactivated

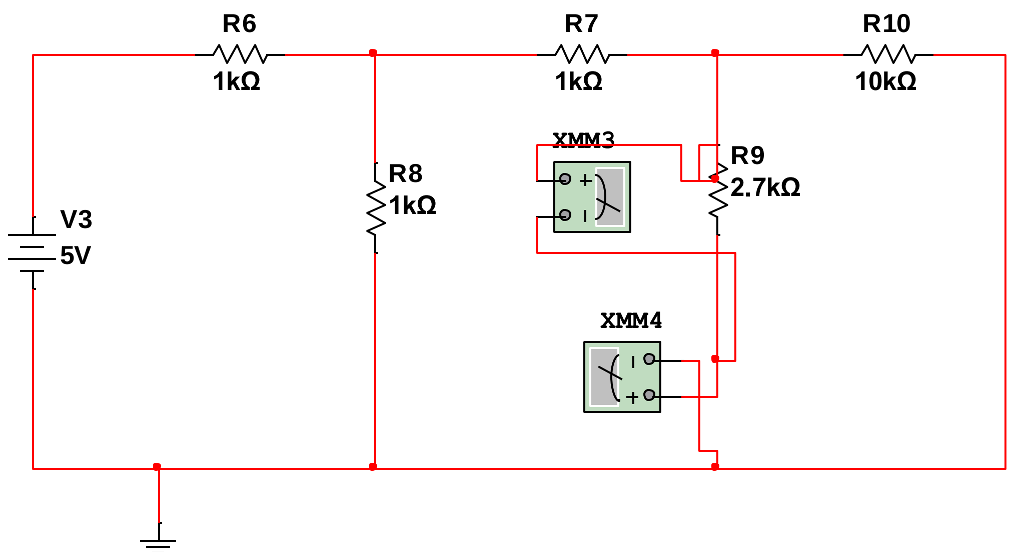


Figure 12

Depicting the designated circuit with only the 5V source activated, and the 8V source deactivated

Conducting a mesh analysis yields the following results:

Only 5V source activated

Mesh 1	**Mesh 2**	**Mesh 3**
$2ki_1 - 1ki_2 = 5$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 12.7ki_3 = 0$

$i_1 = 2.84\text{mA}$
$i_2 = 689.47\mu\text{A}$
$i_3 = 146.58\mu\text{A}$

$i_{2.7k\Omega} = i_3 - i_2 = 542.89\mu\text{A}(2700\Omega) = 1.47\text{V}$

Only 8V source activated

Mesh 1	Mesh 2	Mesh 3
$2ki_1 - 1ki_2 = 0$	$-1ki_1 + 4.7ki_2 - 2.7ki_3 = 0$	$-2.7ki_2 + 12.7ki_3 = 5$

$i_1 = 234.53\mu\text{A}$
$i_2 = 469.06\mu\text{A}$
$i_3 = 729.64\mu\text{A}$

$i_{2.7k\Omega} = i_3 - i_2 = 260.59\mu\text{A}(2700\Omega) = 703.58\text{mV}$

Below is the organized data for part 3 of the lab, including the algebraic sum of the two individual cases.

Table 5

Active Voltage Source(s) in the circuit		$\mathbf{V_{2.7k\Omega}}$ (V)	$\mathbf{I_{2.7k\Omega}}$ (A)	$\mathbf{P_{2.7k\Omega}}$ (W)
Both 5V and 8V	**Theory**	2.17	803.48 µ	1.74m
	Simulation	2.17	803.48 µ	1.74m
	Measured	2.19	799.30 µ	1.75m
Only 5V	**Theory**	1.47	542.89 µ	795.77 µ
	Simulation	1.47	542.89 µ	795.77 µ
	Measured	1.49	444.14 µ	661.77 µ
Only 8V	**Theory**	703.58m	260.59 µ	183.35 µ
	Simulation	703.58m	260.59 µ	183.35 µ
	Measured	824.70m	301.90 µ	248.98 µ
Algebraic sum of the separate 5V and 8V	**Theory**	2.17	803.45 µ	1.74m
	Simulation	2.17	803.45 µ	1.74m
	Measured	2.31	746.04 µ	910.75 µ

For each of the three circuits, the theoretical, simulated, and measured measurements were frequently the same, with some exceptions. Errors may have come from improper measurements on the bench and/or improperly building the circuits either on the breadboard or in Multisim. Also, calculation error are possible in the theoretical aspects of the observations.

Superposition applies to voltage, current, and power in the sense that, while current and voltage are linear values, power is not, as it is reliant on both current and voltage to be measured or calculated. Therefore superposition as a theorem deals primarily with current and voltage more than it does power.

DISCUSSION

There are many things to keep in mind conducting or following this lab, including maintaining proper bench etiquette and safety practices, ensuring accurate and organized recording of data to minimize loss of information, and foundational understanding of the theorems and laws at practice in the lab. KVL, KCL, and Ohm’s law are obvious fundamentals that are necessary for understanding the more complex ones, such as the superposition theorem, the linearity theorem, and both nodal and mesh analysis.

Superposition analysis in particular must be carefully conducted, as when it is being measured, there is either a short or an open segment of a circuit while current is flowing through it.

Error in the lab could have come from various factors. Aside from the slight difference present from varying resistor values, miscalculations, improper formation of circuits, and faulty equipment could all lead to a higher percentage error. Despite this, the lab largely held little error in terms of consistency between theoretical, simulation, and measured data.

Below are the measured resistor values used for real world testing and circuit construction:

Table 6

1kΩ (1)	**1kΩ** (2)	**1kΩ** (3)	**2.7kΩ** (1)	**3.3kΩ** (1)	**10kΩ** (1)
995.60Ω	985.10Ω	995.40Ω	2.67kΩ	3.29kΩ	9.89kΩ

CONCLUSION

Part 1 of this report allows for the demonstration and the verification of the linearity principle by creating a circuit and measuring the input voltage against the output voltage across a resistor, and consequently finding the proportionality constant which stayed the same throughout various input voltage values. Mesh analysis was used to find these output voltage values theory-wise, followed by routine benchwork for measured data.

Also, the use of an oscilloscope was used for the second half of this part, for the purpose of measuring AC source sinusoidal waves.

In part 2, Nodal voltage analysis and mesh current analysis were further expanded upon and heavily relied on for theoretical calculations. Again, a circuit was constructed, this time with two independent voltage sources, and both Nodal and mesh analysis were used sequentially to find all required component measurement values. The linearity theorem was observed here in the difference between V_2 and V_{s2} and the voltage across the 10k resistor, as well as with i_2 and i_3 and the current through the 2.7k resistor.

In part 3, the verification of the superposition theorem took place. In order to properly observe this theorem in action, in turn, each of the two voltage sources must be turned off and turned on and shorted, as well as measured with both together. Here, it is apparent that the algebraic sum of the separate values for the components is equivalent to those same values in the circuit where both are activated.

REFERENCES

[1] W. H. Hayt, J. E. Kemmerly, J. D. Phillips, and S. M. Durbin, *Engineering Circuit Analysis*, 9th ed. New York, NY: McGraw Hill, LLC, 2019.