# Senior Thesis Notes

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1. Clustering with Multiple Graphs (Tang et al.)

   - Linked Matrix Factorization (LMF)

     Given tensor $A$, each layer being a separate graph $A^{(m)}$, construct a common factor matrix $P \in \mathbf{R}^{n \times k}$ and layer-specific factor matrices $\Lambda^{(m)} \in \mathbf{R}^{k \times k}$ s.t.

     $$[\hat{P}, \hat{\Lambda}^{(1 \ldots m)}] = \text{arg min} \frac{1}{2} \sum_{m=1}^{M} \left\| A^{(m)} - P\Lambda^{(m)} P^T \right\|_F^2 + \frac{\alpha}{2}$$

     where $\alpha$ is a Frobenius norm regularization term. Here, $\Lambda^{(m)}$ are not necessarily diagonal.

2. Consistency of Community Detection in Multi-layer Networks (Paul et al.)

   - Orthogonal Linked Matrix Factorization (OLMF)

     The same approach as LMF with the constraint that $P$ is orthogonal, which avoids the need for the regularization term. $\Lambda^{(m)} \in \mathbf{R}^{k \times k}$ s.t.

     $$[\hat{P}, \hat{\Lambda}^{(1 \ldots m)}] = \underset{P^T P = I}{\text{arg min}} \sum_{m=1}^{M} \left\| A^{(m)} - P\Lambda^{(m)} P^T \right\|_F^2$$

   - Baseline comparison to Spectral Clustering and similar methods
   - Tested on Multi-Layer Stochastic Block Model (MLSBM) graphs.

3. Extensions to try:

   - Replacing the adjacency matrix in LMF/OLMF with the *normalized* graph Laplacian. Does this still give the same results as using the adjacency matrix?
   - Using the NBT operator instead. Proven to capture the community structure better (Spectral Redemption) but is this necessarily the case?

4. Other approaches:

   - Tensor methods?
   - Manifold alignment? (discuss)