AI5002: Assignment 1

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Download all C codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/blob/main/Assignment_1/Gaussian%20numbers.c

and latex-tikz codes from

https://github.com/Debolena/AI5002−Probability−and−Random−Variables/tree/main/Assignment_1

1 Problem

Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty \tag{1.0.1}$$

Find the mean and variane theoritically.

2 Solution

Clearly, this is the pdf of Gaussian distribution with mean 0 and variance 1.

$$E(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} dx \tag{2.0.1}$$

$$= 0 \quad (\because odd function) \tag{2.0.2}$$

$$E(X^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2}} dx \quad (even function) \tag{2.0.3}$$

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} x^2 e^{-\frac{x^2}{2}} dx \tag{2.0.4}$$

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} \sqrt{2u}.e^{-u} du \quad \left(Let \frac{x^2}{2} = u\right) \tag{2.0.5}$$

$$= \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-u}.u^{\frac{3}{2}-1} du \tag{2.0.6}$$

$$= \frac{2}{\sqrt{\pi}}.\Gamma\left(\frac{3}{2}\right) \tag{2.0.7}$$

$$= \frac{2}{\sqrt{\pi}}.\frac{1}{2}.\Gamma\left(\frac{1}{2}\right) \quad \because \Gamma(n) = (n-1)\Gamma(n-1); \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \tag{2.0.8}$$

$$= 1 \tag{2.0.9}$$

Thus Variance is:

$$V(X) = E(X)^2 - E^2(X) \tag{2.0.10}$$

$$= 1 - 0 \tag{2.0.11}$$

$$= 1 \tag{2.0.12}$$

Thus, theoritically the mean and variance of the given distribution is 0 and 1 respectively, as seen in the C program computationally