

$$\begin{aligned}
\frac{\delta}{\delta\beta} \sum_{i=1}^N (y_i^* - X_i \times \beta)^2 &= 0 \\
\sum_{i=1}^N \frac{\delta}{\delta\beta} (y_i^* - X_i \times \beta)^2 &= 0 \\
\sum_{i=1}^N -2X_i (y_i^* - X_i \times \beta) &= 0 \\
2 \sum_{i=1}^N X_i (X_i \times \beta - y_i^*) &= 0 \\
\sum_{i=1}^N X_i X_i \beta - \sum_{i=1}^N X_i y_i^* &= 0 \\
X^T X \beta - X^T y^* &= 0 \\
X^T X \beta &= X^T y^* \\
(X^T X)^{-1} X^T X \beta &= (X^T X)^{-1} X^T y^* \\
\beta &= (X^T X)^{-1} X^T y^*
\end{aligned}$$

$$\begin{aligned}
\Sigma &= \frac{1}{N} \left(\sum_{i:y_i^*=-1} (X_i - \mu_{-1})^T \times (X_i - \mu_{-1}) + \sum_{i:y_i^*=1} (X_i - \mu_1)^T \times (X_i - \mu_1) \right) \\
\Sigma &= \frac{1}{N} (\sum_{i:y_i^*=-1} X_i^T X - \sum_{i:y_i^*=-1} X_i^T \mu_{-1} - \sum_{i:y_i^*=-1} X_i \mu_{-1}^T + \sum_{i:y_i^*=-1} \mu_{-1}^T \mu_{-1} \\
&\quad + \sum_{i:y_i^*=1} X_i^T X - \sum_{i:y_i^*=1} X_i^T \mu_1 - \sum_{i:y_i^*=1} X_i \mu_1^T + \sum_{i:y_i^*=1} \mu_1^T \mu_1) \\
\text{Because the sum } \sum_{i:y_i^*=-1} X_i^T X + \sum_{i:y_i^*=1} X_i^T X &\text{ covers every possible case in N,}
\end{aligned}$$

$$\text{this sum can be simplified as } \sum_{i=1}^N X_i^T X.$$

$$\mu_{-1} = \frac{1}{N_{-1}} \sum_{i:y_i^*=-1} X_i, \mu_1 = \frac{1}{N_1} \sum_{i:y_i^*=1} X_i, \text{ and } N_{-1} = N_1 = \frac{N}{2} \text{ lead to}$$

$$\begin{aligned}
\sum_{i:y_i^*=-1} X_i^T \mu_{-1} + \sum_{i:y_i^*=1} X_i^T \mu_1 &= \frac{2}{N} \sum_{i=1}^N X_i^T X. \\
\frac{1}{N} (\sum_{i:y_i^*=-1} \mu_{-1}^T \mu_{-1} + \sum_{i:y_i^*=1} \mu_1^T \mu_1) &\text{ simplifies to } \frac{1}{2} (\mu_{-1}^T \mu_{-1} + \mu_1^T \mu_1).
\end{aligned}$$

This leaves us with

$$\Sigma = (N-2) \sum_{i=1}^N X_i^T X - N \left(\sum_{i:y_i^*=-1} X_i \mu_{-1}^T + \sum_{i:y_i^*=1} X_i \mu_1^T \right) + \frac{1}{2} (\mu_{-1}^T \mu_{-1} + \mu_1^T \mu_1)$$

$$\begin{aligned}
\Sigma &= \frac{1}{N} \left(\sum_{i:y_i^*=-1} (X_i - \mu_{-1})^T \times (X_i - \mu_{-1}) + \sum_{i:y_i^*=1} (X_i - \mu_1)^T \times (X_i - \mu_1) \right) \\
\Sigma \times \beta + \frac{1}{4} (\mu_1 - \mu_{-1})^T \times (\mu_1 - \mu_{-1}) \times \beta &= \frac{1}{2} (\mu_1 - \mu_{-1})
\end{aligned}$$