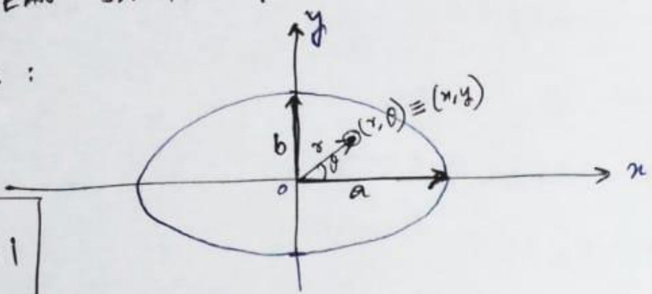
Q1
of Assignment 2
of CS215.

Part (a) is hand-written, an explanation of the technique of random point generation used in this question.
Next page is more description and a formal Algorithm to find the histogram of the data generated.

Q.1 EUCLIDEAN SAMPLING

(a) Mathematics :



$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

for any point inside (or on) the ellipse $(x, y) = (r\cos\theta, r\sin\theta)$

and $S(x, y) \le 0$ (by coordinate Geometry)

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \le 0$$

$$\Rightarrow \frac{r^2\cos^2\theta}{a^2} + \frac{r^2\sin^2\theta}{b^2} - 1 \le 0 \Rightarrow r^2 \le \frac{a^2b^2}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$\Rightarrow \boxed{r \le \frac{ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}}$$ we define, $R_{max} = \frac{ab}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$

given $a=2$, $b=1$ $\therefore$ $\boxed{R_{max} = \frac{2}{\sqrt{4\sin^2\theta + \cos^2\theta}} = \frac{2}{\sqrt{1 + 3\sin^2\theta}}}$

[R_{max} is a function of θ]

Also any point on the edge/rim of the ellipse is of the form $(a\cos\theta, b\sin\theta)$

so, dist from the origin $= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta}$

This is also $= R_{max}$.

But in the algorithm I used the value on the left-hand-side.

Algorithm is simple :

1. Generate 10^7 random values of theta $\in [0, 2\pi)$
2. Corresponding to each value there would be R_max.
3. $r \in [0, R_{max})$. r would be random number in that range.

//so at a given θ, if randomly choose an r, so a random
//point is generated. But when Rmax is small, then the
//points would be crowded. Its reduced somewhat by
// sqrt(rand(.)) trick. so $r = R_{max}$ * sqrt(rand())

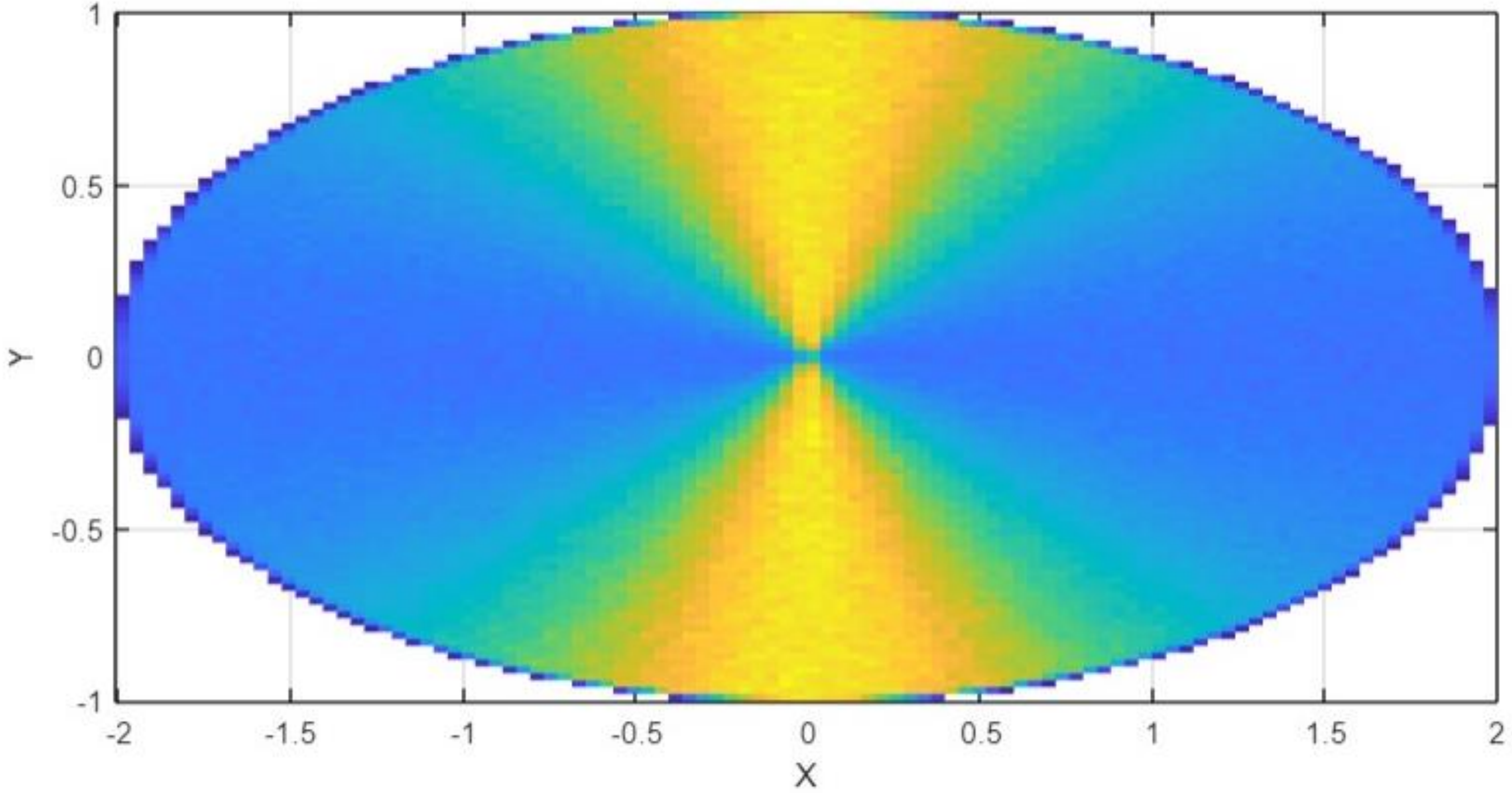
4. $(x, y) = (r\cos\theta, r\sin\theta)$. Obtain (x, y)
5. Plot the histogram(X, Y, 100) . 100 for dividing X & Y axis into 100 parts each, for making histogram tiles.

// More the points that randomly are made to lie in a tile,
// the color of the tile becomes warmer (yellow → orange-red).
// Uniform distribution should have uniform color.

A part of my MATLAB Code to show implementation :

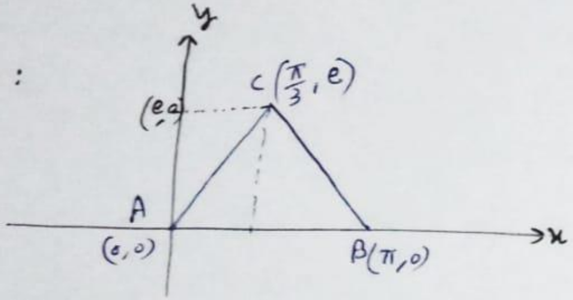
```matlab
clear;
rng(1);
% so that each time same outputs are created
N  = 10000000;

theta = rand(1,N)*2*pi;
% rand() generates numbers from a
%uniform distribution over [0,1)

Max_Radius = 2 ./ sqrt(1 + 3.*(sin(theta)).^2 );

radius = Max_Radius .* sqrt(rand(1,N));

x = radius .* cos(theta);
y = radius .* sin(theta);


histogram2(x,y,100,"DisplayStyle","tile");
axis equal;
xlabel("X");
ylabel("Y");
```

Part (b) : histogram of data points inside the given ellipse :
color – yellow is for dense crowding of data and
blue is for average crowd or accumulation.



Q.1

(c) Mathematics :



line AB : $\boxed{y = 0}$ — (1)

line AC : $\boxed{y = \frac{3e}{\pi} x}$ — (2)

line CB : $(y - e) = \frac{e}{2\pi/3} \cdot \left(x - \pi/3\right)$

$\Rightarrow \boxed{y = \frac{3e}{2}\left(1 - \frac{x}{\pi}\right)}$ — (3)

Algorithm :-

1. Generate random value of $y \in [0, e)$
2. for every 'y' there is a range of x, because of lines AC & CB.

   $x \in \left[\frac{\pi y}{3e} , \pi\left(1 - \frac{2y}{3e}\right) \right)$

   Generate a random value for x in here.

// random number between a and b . is = a + (b-a) * rand(.)

3. Hence a random point is found inside the Δ.
4. Plot the histogram(x, Y, 100) as before.

A part of my MATLAB Code to show implementation :

```
clear;
rng(1);
% so that each time same outputs are created
N  = 10000000;

y = exp(1) * rand(1,N);
% rand() generates numbers from a
% uniform distribution over [0,1)

x_min = pi.*y./(3*exp(1));
x_max = pi.*(1-2.*y./(3*exp(1)));
x = x_min + (x_max - x_min).*rand(1,N);


histogram2(x,y,100,"DisplayStyle","tile");

axis equal;
xlabel("X");
ylabel("Y");
```

Part (d) : histogram of data points inside the given triangle :
color – yellow is for dense crowding of data and
blue is for average crowd or accumulation.
Complete blue color represents good uniform distribution of data points.