For Cylindrical pressure Vessel (CPV)-

$$\varepsilon_v = \varepsilon_{long} + 2(\varepsilon_{hoop})$$

$$\Downarrow$$

ε circumferential

$$\varepsilon_{long} = \frac{\delta L}{L} \; ; \; \varepsilon_{hoop} = \frac{\delta D}{D}.$$

Spherical Member-

$$V = \frac{4}{3}\pi R^3 \text{ or } \frac{\pi D^3}{6}. \quad \text{—(1)}$$

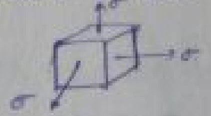
$$\delta V = \frac{\pi}{6}\left[3D^2(\delta D)\right] \quad \text{—(2)}$$

$$\varepsilon_v = \frac{\delta V}{V} = 3\left[\frac{\delta D}{D}\right].$$

SPV- $\varepsilon_v = 3[\varepsilon_{hoop}]$.

* Bulk modulus is used only for hydrostatic state of stress (where pressure is same ~~at all the points~~ in all the dir^ns

$$\sigma_x = \sigma_y = \sigma_z = \sigma.$$



Q- P = 100kN, L = 1m, d = 50mm, E = 200 GPa, $\mu = 0.3$.

$$\varepsilon_{long.} = \frac{PL}{AE} = \frac{10^5 \times 4}{\pi \times 25 \times 10^{-4} \times 200 \times 10^9} = 2.546 \times 10^{-4}.$$

$$\varepsilon_{lateral} = 0.3 \times \varepsilon_{long} = 7.639 \times 10^{-5}.$$

$$\Delta l = l_f - l_0 = \frac{PL}{AE} = 2.546 \times 10^{-4}\,m = 0.2546\,mm.$$

$$\therefore L_f = 0.9997\,m.$$

$$\Delta d = 7.639 \times 10^{-5} \times d = 3.8195 \times 10^{-3}.$$

d_f = 49.996 mm. (Almost negligible)

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_{long} + 2\varepsilon_{lateral} = 0.2546 - 2(0.07639)$$

$$= \cancel{\quad} \; 1.0074 \times 10^{-4}.$$

$$= 0.10074 \times 10^{-3}.$$

$$\delta V = 0.10074 \times 10^{-3} \times \frac{\pi}{4} d^2 \times l$$

$$= \cancel{\quad} \; 0.10074 \times 10^{-3} \times \frac{\pi}{4} \times 25 \times 10^{-4} \times 1$$

$$V_f = V_0 + \delta V.$$

__Note:__ In SOM, engineering stress = True Stress.
In plastic region, both are different.

Engineering Stress, $\sigma_E = \frac{P}{A_0}$.

True Stress, $\sigma_T = \frac{P_i}{A_i}$. i: Instantaneous.

⇒ within elastic region,

$\sigma_E \simeq \sigma_T$ (∵ change in x-s/c dim^n $\simeq 0$)

⇒ ~~ist~~ within plastic region,

$\sigma_E \neq \sigma_T$ (∵ change in x-s/c dim^n $\neq 0$ i.e. plastic deformations are considerable).

दोनों के लिए है

$$\boxed{\sigma_T = \sigma_E (1 + \varepsilon_E)}$$

$$\boxed{\varepsilon_T = \ln(1 + \varepsilon_E)}$$

⇒ $dV = 0$ (i.e. $\mu = \frac{1}{2}$).

$$\boxed{\varepsilon_E = \frac{\delta L}{L_0}}$$

Under tensile loading conditions,

$$\boxed{\sigma_T > \sigma_E} \qquad \boxed{\varepsilon_T < \varepsilon_E}$$

Under compressive loading condⁿ,

$\boxed{\sigma_E > \sigma_T}$ $\boxed{\varepsilon_E < \varepsilon_T}$.

Q- $L_0 = L$, $L_f = 2L$

$\varepsilon_E = \frac{L_f - L_0}{L_0} = 1.$

$\sigma_T = 2\sigma_E.$

$\varepsilon_T = \ln 2 = 0.693.$

$\therefore \sigma_T > \sigma_E$

$\varepsilon_T < \varepsilon_E$

$L_0 = L$, $L_f = L/2$

$\varepsilon_E = \frac{L_f - L_0}{L_0} = -0.5.$

$\sigma_T = \sigma_E/2.$

$\varepsilon_T = \ln(0.5) = -0.693.$

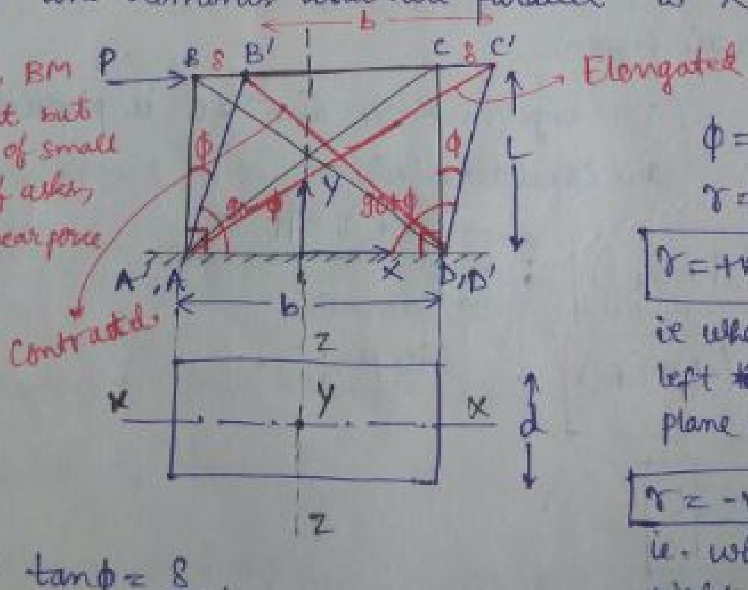
$\therefore \sigma_T < \sigma_E$

$\varepsilon_T > \varepsilon_E.$

Shear Strain [γ] -

Shear strain is defined as the change in initial right angle b/w two line elements which are parallel to X & Y axes.

In this block, BM is also present but neglected coz of small L. But if asked, then both shear force & BM exist.



ϕ = Shear Angle

γ = Shear strain

$\boxed{\gamma = +ve = +(\phi) \Rightarrow CWdirⁿ}$

ie when load applied from left and moves the plane towards right.

$\boxed{\gamma = -ve = -(\phi) \Rightarrow ACWdirⁿ}$

ie. when load applied from right and moves the plane towards left.

$\Delta ABB'$, $\tan\phi = \frac{\delta}{L}$.

$\phi = \frac{\delta}{L}$ [$\because$ for smaller angles, $\tan\phi \approx \phi$]

$\boxed{\gamma = \phi = \delta/L}$.

Theoretically,

1- $\tau_d = \tau_{avg} = \frac{P}{A} =$ ______

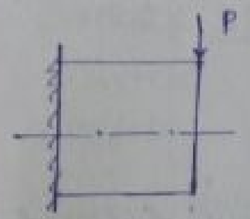
2- $G = \frac{\tau}{\gamma} \Rightarrow \gamma = \frac{\tau_d}{G} =$ ______

3- $\phi = \gamma =$ ______

4- $\delta = \phi L =$ ______

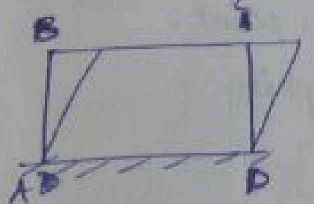
In beams shear stress is neglected and only BM is considered.

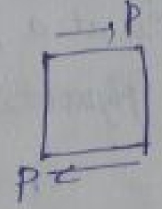
$\sigma_b \propto \frac{M}{Z}$,

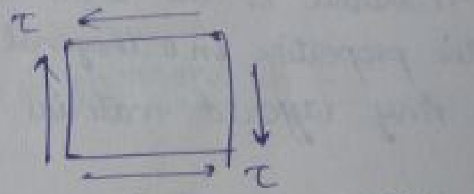


~~Sh~~ BM is neglected because of small length. Only shear stress is considered.

(??)







$$[\sigma]_{2D} = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix} \qquad [\epsilon]_{2D} = \begin{bmatrix} 0 & \gamma/2 \\ \gamma/2 & 0 \end{bmatrix}$$

τ_{xy}, τ_n, σ_y, ϵ_x, $\gamma_{xy}/2$, ϵ_y.

$$(\epsilon_n)_\theta = \frac{1}{2}[\epsilon_x + \epsilon_y] + \frac{1}{2}[\epsilon_x - \epsilon_y]\cos 2\theta + \frac{\gamma_{xy}}{2}(\sin 2\theta).$$

$$= 0 + 0 + \frac{\gamma}{2}(\sin 2\theta).$$

$$\boxed{(\epsilon_n)_{45^\circ} = \frac{\gamma}{2} \text{ (tensile)}} \quad \boxed{(\epsilon_n)_{135^\circ} = -\frac{\gamma}{2} \text{ (compressive)}}$$

$$\boxed{(\gamma_s)_\theta = -[\epsilon_x - \epsilon_y]\sin 2\theta + \gamma_{xy}\cos 2\theta}.$$

$$\boxed{(\gamma_s)_\theta = 0 + \gamma(\cos 2\theta)}$$

$$\boxed{(\gamma_s)_{\theta=45^\circ} = (\gamma_s)_{\theta=135^\circ} = \text{zero}.}$$

Elastic Constants - (EC)

* E.C's are used to obtain stress-strain relationships.
* E.C's are used to determine strain theoretically.
* For a homogeneous & isotropic material the number of E.C's are 4 [ie E, $\underset{\mathcal{E}}{G}$, K & $\underset{\text{or } 1/m}{\mu}$].
* For a homogeneous & isotropic material, the no of independent EC's are 2 [ie E & μ].

Material	Ind. EC
ISOTROPIC	2
ORTHOTROPIC	9
ANISOTROPIC	21

For ESE (GATE)

Orthotropic -

A material is said to be orthotropic when it exhibits different elastic properties in orthogonal dir^n's at a given point.
eg. Any layered material like plywood, graphite.

Relationship b/w EC -

$\Rightarrow E = 2G[1+\mu] \Rightarrow G = \frac{E}{2} \times \frac{1}{1+\mu}$.

$\Rightarrow E = 3K[1-2\mu] \Rightarrow K = \frac{E}{3} \times \frac{1}{1-2\mu}$.

$\Rightarrow E = \frac{9KG}{3K+G}$

* Value of any E.C ≥ 0, but $E, K, G > 0$

$\mu \geq 0$

μ_{cork} = zero → Bottle stoppers.

lateral strain = zero

If $K = +ve \Rightarrow 1-2\mu \geq 0.$

$\Rightarrow \mu \leq \frac{1}{2}.$

$0 \leq \mu \leq \frac{1}{2}.$

$\mu = \frac{1}{2} \Rightarrow \delta V = 0.$

→ Incompressible material or for perfect plastic material ⇒ Dimensions will change but vol. will remain same.

$\mu = \frac{1}{2} \Rightarrow K = \infty.$

* Material — μ

Material	μ
CORK	zero
Concrete	0.1 to 0.2
Metals	$\frac{1}{4}$ to $\frac{1}{3}$
Rubber, Praffin, wax	0.5 ⇒ $\delta V = 0$ ⇓ Incomp. material

* Lower & upper limits for K & G in terms of E -

μ	G	K
zero	E/2	E/3
0.5	E/3	∞ ⇒ Bulk Mod. could be ∞ but E can't be ∞.

* $\mu(\uparrow) \Rightarrow G(\downarrow)$ & $K(\uparrow)$

$$\boxed{E/3 \leq G \leq E/2}.$$

$$\boxed{E/3 \leq K \leq \infty}.$$

→ G is always less than E by E/2 or more.

→ K may or may not be less than E.

For metals - (E > K > G)

μ	G	K
1/4	0.4E	0.67E
1/3	0.37E	E

* if $\mu < \frac{1}{3} \Rightarrow E > K$.

if $\mu = \frac{1}{3} \Rightarrow E = K$

if $\mu > \frac{1}{3} \Rightarrow K > E$

Definitions of Elastic Constants -

(Valid only upto elastic region) ~~(Elastic Region)~~ or proportional limit)

As per Hooke's law, upto P.L (Proportional limit),

$\sigma_x \propto \varepsilon_x$ or $(\varepsilon_{long})_x$

$\sigma_y \propto \varepsilon_y$ or $(\varepsilon_{long})_y$

$\sigma_z \propto \varepsilon_z$ or $(\varepsilon_{long})_z$

$\tau_{xy} \propto \gamma_{xy}$ or G

i.e. corresponding stress $\propto$ corresponding strain

$\sigma \propto \varepsilon_{long} \Rightarrow \sigma = E\,\varepsilon_{long}$

$$\Downarrow$$

$$E = \frac{\sigma}{\varepsilon_{long}} = \frac{\text{Normal Stress}}{\text{Long. Strain}}.$$

E = Slope of σ v/s $\varepsilon_{long.}$ curve upto P.L

⇒ Young's Modulus - Under uniaxial state of stress upto proportional limit the ratio between Normal stress and long. strain is known as Young's Modulus.

⇒ Shear Modulus - Biaxial

Under pure shear state of stress upto P.L, shear modulus is the ratio of ~~stres~~ shear stress and shear strain.

$$\tau \propto \gamma \Rightarrow \tau = G\gamma \Rightarrow G = \frac{\tau}{\gamma \text{ or } \phi} = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$\boxed{G = \text{Slope of } \tau \text{ v/s } \gamma \text{ curve upto P.L}}$$

⇒ Bulk Modulus - Tri-axial

$\sigma \propto \varepsilon_v \Rightarrow \sigma = K\varepsilon_v$.

$$K = \frac{\sigma}{\varepsilon_v} \text{ or } \frac{-P}{\varepsilon_v}$$

ε_v is -ve. Hence, to make K → +ve. (-p) is used.

Under hydrostatic state of stress condition upto P.L the ratio b/w normal stress & volumetric strain.

i.e. $\sigma_x = \sigma_y = \sigma_z = \sigma$.

$\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$.

K = Slope of σ v/s ε_v curve, upto P.L.

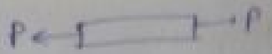
Note - if $\mu = 0.5 \Rightarrow K = \infty \Rightarrow \varepsilon_v = 0 \Rightarrow \delta V = 0$.
↳ Incomp. material.

⇒ Poisson's Ratio -

$$\mu = -\left[\frac{\text{Lateral Strain}}{\text{Long. Strain}}\right] = -\frac{\varepsilon_y}{\varepsilon_x} \text{ or } -\frac{\varepsilon_z}{\varepsilon_x}.$$

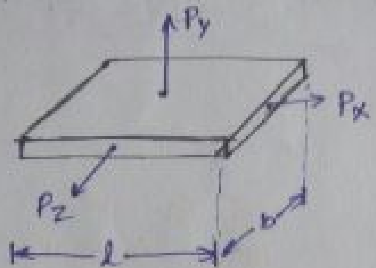
Lateral strain = $-\mu$ [long. strain]
↳ E

Note: To determine Lateral strain both μ & E are required.

For tension test, 

$$\boxed{\mu = -\left[\frac{(\delta d/d_0)}{(\delta l/l_0)}\right] = -\left[\frac{\left(\frac{d_f - d_0}{d_0}\right)}{\left(\frac{l_f - l_0}{l_0}\right)}\right]}$$

Expression for ε_v under tri-axial loading -



$$\sigma_x = \frac{P_x}{bt}$$

$$\sigma_y = \frac{P_y}{lb} \qquad [\sigma] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

$$\sigma_z = \frac{P_z}{lt}$$

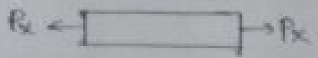
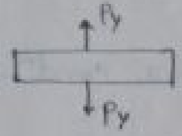
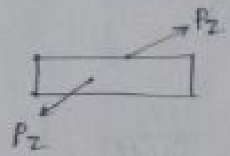
**

$$\boxed{\varepsilon_v = \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z} \quad - (1)$$

$$\left[\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} \\ \varepsilon_y &= \frac{-\mu\sigma_x}{E} \\ \varepsilon_z &= \frac{-\mu\sigma_x}{E} \end{aligned}\right] - (a) \qquad \left[\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} \\ \varepsilon_y &= \frac{\sigma_y}{E} \\ \varepsilon_z &= \frac{\sigma_z}{E} \end{aligned}\right] - (b)$$

Valid under uni-axial state of stress

Valid under tri-axial state of stress if $\mu = 0$ (ie corks)

STRAIN → / LOADING COND^n ↓	x dir^n	y dir^n	z dir^n
	ε_{long} σ_x/E	$-\dfrac{\mu\sigma_x}{E}$	$-\dfrac{\mu\sigma_x}{E}$
	$-\mu\sigma_y/E$	ε_{long} σ_y/E	$-\dfrac{\mu\sigma_y}{E}$
	$-\mu\sigma_z/E$	$-\mu\sigma_z/E$	ε_{long} σ_z/E

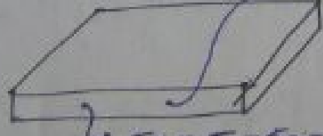
$$\left[\begin{aligned} \varepsilon_x &= (\varepsilon_{Total})_x = \frac{1}{E}\left(\sigma_x - \mu(\sigma_y+\sigma_z)\right) = \delta l/l. \\ \varepsilon_y &= (\varepsilon_{Total})_y = \frac{1}{E}\left(\sigma_y - \mu(\sigma_x+\sigma_z)\right) = \delta t/t. \\ \varepsilon_z &= (\varepsilon_{total})_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x+\sigma_y)\right] = \delta b/b. \end{aligned}\right] \quad \text{— (2)}$$

Above eq^{n}s are used to det. total strain in 3 mutual ⊥r dir^{n}s passing through a point under any state of stress condition (ie. either uni, bi or triaxial).

By substituting above eq^{n}s in eq^n (1),

$$* \boxed{\varepsilon_v = \frac{\delta V}{V} = \left(\frac{1-2\mu}{E}\right)(\sigma_x+\sigma_y+\sigma_z)} \quad \text{— (3)}$$

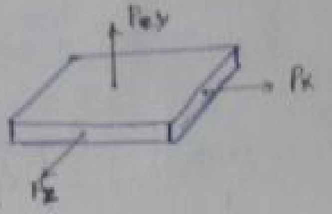
$\Delta T = T°C\,(\uparrow)$



$(\varepsilon_{th})_x \neq 0$
$(\varepsilon_{th})_y \neq 0$
$(\varepsilon_{th})_z \neq 0$ → Strain is present but stress is not present. eq^n (1) will be used.

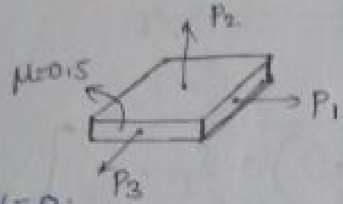
$\sigma_x = \sigma_y = \sigma_z = 0$ $(\because P_x = P_y = P_z = 0)$ $\boxed{\delta V \neq 0}$

Note: If strain is present, it is not necessary that stress will also be there (eg. due to temp., lateral strain) and vice versa (eg. rigid bodies).

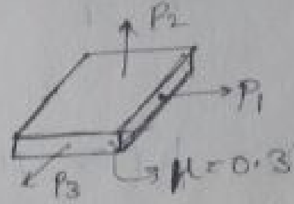


eqⁿ ③ is better to be used, Stress is given.

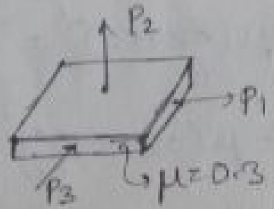
From eqⁿ ③,
if $\mu = 0.5 \Rightarrow \delta V = 0$ under any state of stress condⁿ.
if $\mu \neq 0.5 \Rightarrow \delta V = 0$ when $\sigma_x + \sigma_y + \sigma_z = 0$.



$\delta V = 0$.



$\delta V \neq 0$
(All are tensile)
No chance.



δV may be or may not be zero. Calculations need to be done.

→ If poisson's ratio of the material = 0.5, then change in vol. equal to zero under any state of stress condition.

→ If poisson's ratio of material is less than 0.5, then change in vol. is equal to zero if sum of $\sigma_x, \sigma_y, \sigma_z$ is equal to zero.

For Hydrostatic state of stress condⁿ -

$$\sigma_x = \sigma_y = \sigma_z = \sigma.$$

$$\varepsilon_v = \left(\frac{1-2\mu}{E}\right)(\sigma_x + \sigma_y + \sigma_z).$$

$$\varepsilon_v = \frac{1-2\mu}{E}(3\sigma)$$

$$\frac{E}{3(1-2\mu)} = \frac{\sigma}{\varepsilon_v}$$

$$\therefore \boxed{K = \frac{\sigma}{\varepsilon_v} = \frac{E}{3(1-2\mu)}}$$

Valid under hydrostatic state of stress condⁿ.

For uni-axial state of stress condition -

$\sigma_x = \sigma$, $\sigma_y = \sigma_z = 0$.

$\varepsilon_v = \left(\frac{1-2\mu}{E}\right)(\sigma + 0 + 0)$.

$$\boxed{\frac{\delta V}{V} = \varepsilon_v = \left(\frac{1-2\mu}{E}\right)(\sigma)}$$

if $\mu = 0.5 \Rightarrow \delta V = 0$.

if $\mu < 0.5 \Rightarrow \delta V \neq 0$.

eg. P.A.L $\sigma = P/A$.

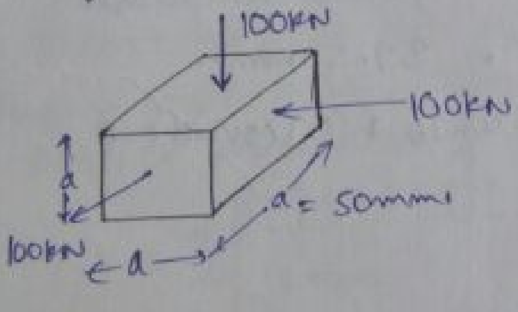
$\varepsilon_v = \frac{\delta V}{AL} = \left(\frac{1-2\mu}{E}\right)(P/A)$.

** $$\boxed{\delta V = \left(\frac{1-2\mu}{E}\right)(PL)}$$

$\delta V = 0$ if $\mu = 0.5$.

$\delta V \neq 0$ if $\mu < 0.5$.

Q- For the cube as shown in the fig. det. the change in vol. of the cube if $E = K = 200\,GPa$.



$\frac{\delta V}{V} = \left(\frac{1-2\mu}{E}\right)(\sigma_x + \sigma_y + \sigma_z)$

$\cancel{E} = 3K[1-2\mu]$

$\frac{1}{3} = 1-2\mu$.

$2\mu = \frac{2}{3}$

$\mu = \frac{1}{3}$.

$\delta V = \frac{100 \times 10^3 (1-2)}{3 \times 200 \times 10^9 \times 25 \times 10^{-4}} \times V$

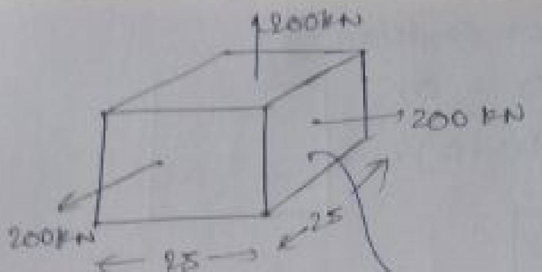
$= \frac{-1}{6 \times 25 \times 10^2} \times 125 \times 10^{-6}$

$= -0.667 \times 10^{-5} \times 125 \times 10^{-6}$

$= -0.8337 \times 10^{-9}\, m^3$

$\delta V = -0.833\, mm^3$.

Q-



$\delta V = ?$

$E = 200\,GPa$

$\mu = 0.25.$

Only vol. of cube changes (i.e. there is no distortion)

[∵ Every plane passing through the point is P.P]

(i.e $(\tau_s)_\theta = 0$). → Hydrostatic state of stress cond^n.

$$E = 3K(1-2\mu),$$

$$K = \frac{200\times10^3}{3(1-0.5)}$$

$$= 133.33\times10^3.$$

$$\cancel{K = \frac{\sigma}{\varepsilon_v}} \qquad \sigma_x = \sigma_y = \sigma_z = \sigma = \frac{200\times10^3}{625\times10^{-6}}.$$

∴ Hydrostatic state of stress.

Note: - In hydrostatic state of stress, every plane is a principal plane becoz shear stress is zero everywhere

$$K = \frac{\sigma}{\varepsilon_v}$$

$$\varepsilon_v = \frac{\delta V}{V} = \frac{\sigma}{K} = \frac{200\times10^3 \times 625\times25\times10^{-9}}{625\times10^{-6}\times133.33\times10^3}$$

$$\delta V = 37.5\ mm^3.$$

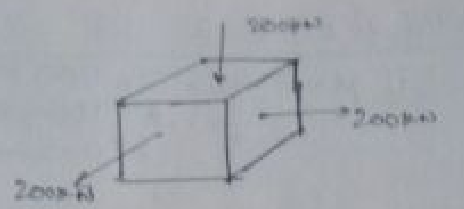
$$* \quad (\tau_s)_\theta = -\frac{1}{2}[\sigma-\sigma]\sin2\theta + (0)\cos2\theta = 0$$

$$\boxed{(\tau_s)_\theta = 0}.$$

$$(\sigma_n)_\theta = \frac{1}{2}[\sigma+\sigma] + \frac{1}{2}[\sigma-\sigma]\cos2\theta + (0)\sin2\theta$$

$$\boxed{(\sigma_n)_\theta = \sigma}$$

→ Indep of 'θ'.



$\sigma_x = \sigma$

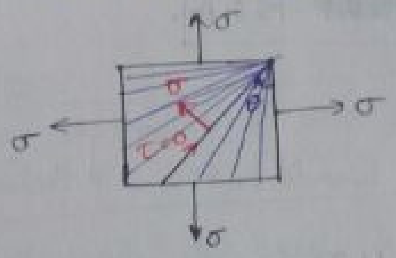
$\sigma_y = -\sigma.$

$\therefore$ * $(\tau_s)_\theta = -\sigma \sin 2\theta.$

$(\sigma_n)_\theta = 0 + \sigma \cos 2\theta + 0.$

$(\sigma_n)_\theta = \sigma \cos 2\theta.$

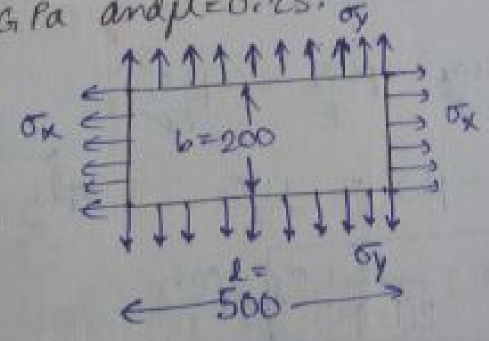
*** Q-



On all the planes $\sigma_n = \sigma$.
& $\tau_s = 0$.

$(\sigma_n)_\theta = \sigma.$
$(\tau_s)_\theta = \text{zero}.$
} Hydrostatic state of stress.

Q- Thin rectangular plate as shown in fig. Det. σ_x & σ_y if change in length = 0.5, change in width = 0.1 mm. Assume $E = 200\,GPa$ and $\mu = 0.25$.



$$\frac{\delta l}{l} = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E}$$

$$E \times \frac{0.5}{500} = \sigma_x - 0.25\sigma_y \quad \text{—①}$$

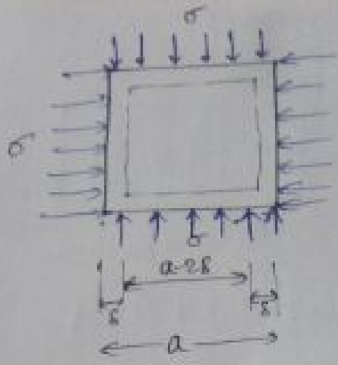
$$\frac{0.1}{200} \times E = \sigma_y - 0.25\sigma_x \quad \text{—②}$$

$\sigma_x = 240\ MPa.$

$\sigma_y = 160\ MPa.$

GATE

Q-



Det. E = ?

if $\mu = 0.25$, $\sigma = 200$ MPa.

$a = 100$ mm

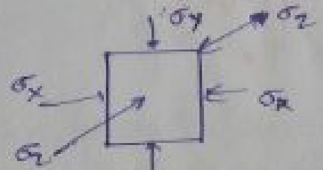
$\delta = 0.5$ mm

$$\frac{\delta a}{a} = -\frac{2\delta}{a} = -\frac{\sigma}{E} + \mu\frac{\sigma}{E}$$

$$-\frac{2\times 0.5\times 10^{-3}}{0.1}\times E = -0.75\times 200\times 10^{6}$$

$$\therefore \boxed{E = 15\,GPa}$$

Ch1 12-



$$\varepsilon_y = \frac{\delta l}{l} = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \mu\frac{\sigma_z}{E} = 0.$$

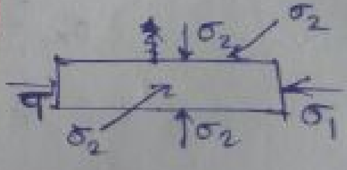
$$(1-\mu)\sigma_y = \mu\sigma_x$$

$$\boxed{\sigma_y = \left(\frac{\mu}{1-\mu}\right)\sigma_x} \text{ . (d)}$$

23-

$$\varepsilon_y = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = -\frac{\mu\sigma_1}{2E}$$

Permitted strain



$$\sigma_2(1-\mu) = \frac{\mu\sigma_1}{2}$$

$$\boxed{\sigma_2 = \frac{\mu}{2(1-\mu)}\sigma_1} \text{ . (c)}$$

H.W.

Ch1- 5, 20, 30, 31, 32, 34,

*** BARS IN SERIES & PARALLEL

$$\sigma_a = \frac{P}{A} = \frac{4P}{\pi d^2} \quad \text{—} ①$$

$$\delta = \delta_L = \frac{PL}{AE} = \frac{4PL}{\pi d^2 E} \quad \text{—} ②$$

Condⁿ to be satisfied for the above eqⁿs -

1- Bar should be prismatic.
2- —— " —— under PAL.
3- Bar shoud made of same material.

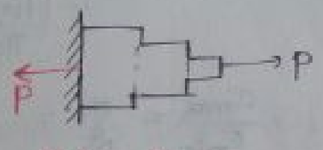
⇒ Series -

Conditions -

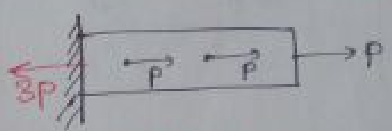
1- Change in lengths or axial deformations of bars are cumulative. i.e. $(\delta L)_{total} = (\delta L)_1 + (\delta L)_2 + (\delta L)_3 + \dots$

2- Axial loads are equal & like in nature
i.e. $P_1 = P_2 = P_3 = \dots = P_n = P$.
(Valid when axial loads are applied at the extreme ends only).

eg.,



Prismatic ×



P.A.L ×
Axial loads are equal ×

⇒ Parallel -
(Composite Bars)

Conditions -

1- Axial loads are cumulative (ie. $P_T = P_1 + P_2$).
2- Axial Defⁿs are equal & like in nature (ie $\delta_1 = \delta_2 = \delta$).

~~Best~~ SERIES -

CASE I -

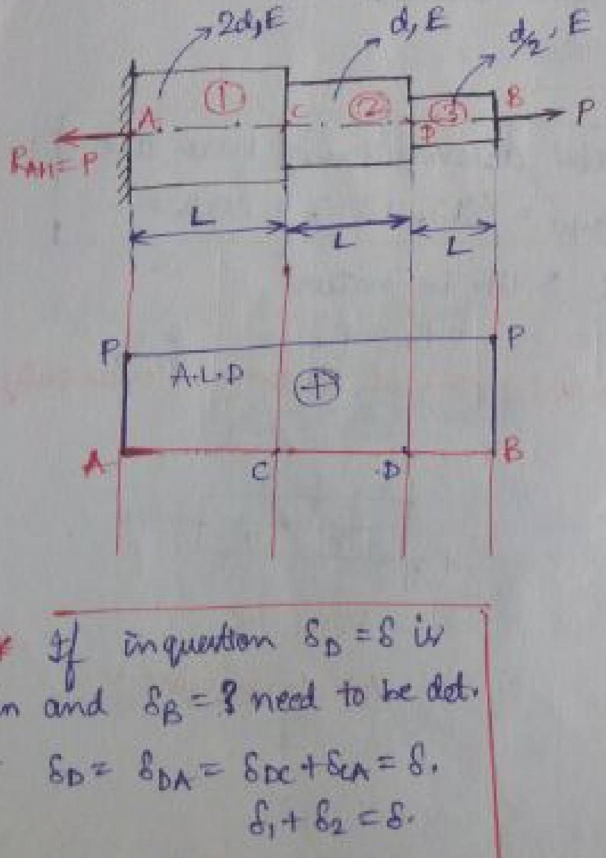
~~Ex~~Q -

For the stepped bar as shown in the fig. Det.

(i) Max. axial stress.

(ii) Ratio of max. & min. axial stresses.

(iii) Change in length of the stepped bar.



$P_1 = P_2 = P_3 = P$

$\sigma_{max} = \sigma_3 = \frac{P_3}{A_3} = \frac{4P_3}{\pi d_3^2}$

$= \frac{4P_3}{\pi d^2} \times 4$

$= \frac{16 P_3}{\pi d^2}$

$\frac{\sigma_{max}}{\sigma_{min}} = \frac{\sigma_3}{\sigma_1} = \frac{16P}{\pi d^2} \Big/ \frac{P}{\pi d^2}$

$= 16.$

$(\delta L)_{Total} = (\delta L)_1 + (\delta L)_2 + (\delta L)_3$

$= \frac{\sigma_1 L}{E} + \frac{\sigma_2 L}{E} + \frac{\sigma_3 L}{E}$

$= \frac{L}{E}\left(\frac{16P}{\pi d^2} + \frac{P}{\pi d^2} + \frac{4P}{\pi d^2}\right)$

$= \frac{21 PL}{\pi d^2}$

* If in question $\delta_D = \delta$ is given and $\delta_B = ?$ need to be det.

Soln - $\delta_D = \delta_{DA} = \delta_{DC} + \delta_{CA} = \delta.$

$\delta_1 + \delta_2 = \delta.$

$\delta_B = \delta_{BA} = \underbrace{\delta_{BD}}_{?} + \underbrace{\delta_{CD} + \delta_{AC}}_{\delta}$

$\frac{\delta_{BD}}{\delta_{DC} + \delta_{AC}} = \frac{\delta_3}{\delta_2 + \delta_1} = \frac{(PL/AE)_3}{(PL/AE)_1 + (PL/AE)_2}$

$\frac{\delta_{BD}}{\delta} = \frac{1/A}{\frac{1}{4A} + \frac{1}{16A}} = \frac{16}{5}$

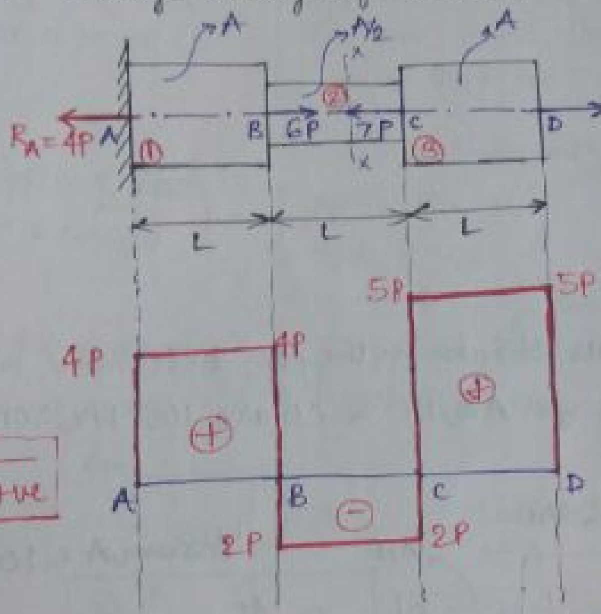
$\delta_{BD} = \frac{16}{5}\delta.$ $\therefore$ $\boxed{\delta_B = \frac{16\delta}{5} + \delta = \frac{21}{5}\delta}$

CASE - 2

Q - For stepped bar as shown in the fig. Det.

(i) Max. stress induced.

(ii) Change in length of the bar.



$P_1 = P_A = 4P$ (Tensile)

$P_2 = -7P + 5P = -2P$.
$= 2P$ (Comp).

$P_3 = 5P$ (Tensile).

$\sigma_1 = \frac{P_1}{A_1} = \frac{4P}{A}$. (Tensile).

$\sigma_2 = \frac{P_2}{A_2} = \frac{-2P \times 2}{A} = \frac{-4P}{A}$ (C).

$\sigma_3 = \frac{P_3}{A_3} = \frac{5P}{A}$ (T).

$\boxed{\sigma_{max} = \frac{5P}{A}}$.

(ii) Change in length of bar -

$(\delta l)_{total} = (\delta L)_1 + (\delta L)_2 + (\delta L)_3$

$= \frac{L}{E}(\sigma_1 + \sigma_2 + \sigma_3)$.

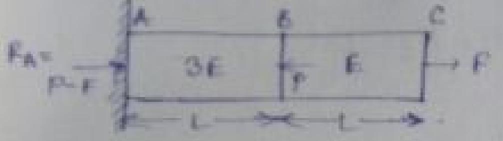
$= \frac{L}{E}\left(\cancel{\frac{4P}{A}} - \cancel{\frac{4P}{A}} + \frac{5P}{A}\right)$

$= \frac{5PL}{AE}$ mm (elongation).

Ch 1 - 6, 35, 38.

5 - c, 6 - d, 20 - d, 30 - d, 31 - 0.3, 32 - 0.2, 34 - 769, 35 - 9, 38 - 4.

38-



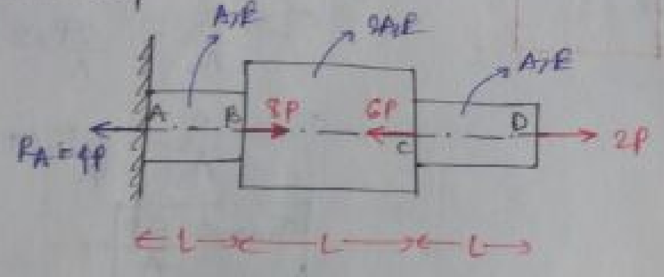
$P_A = -(P-F)$

$P_B = -P+F+P = F.$

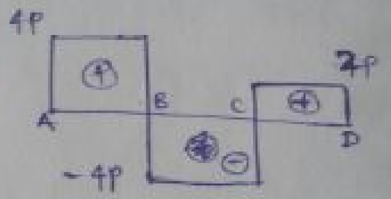
$P_C = F.$

$(\delta L)_{total} = 0.$

Q- For the stepped bar as shown in the fig. Determine value of P if permissible stresses for AB, BC & CD are 100 MPa, 50 MPa & 150 MPa resp.



Assume, $A = 100\,mm^2$.



$P_{AB} = P_A = 4P.$

$P_{BC} = 4P - 8P = -4P.$

$P_{CD} = -4P + 6P = 2P.$

⇒ Safe condⁿ for AB,

$\sigma_{AB} \le \sigma_{per}$

$\left(\frac{P}{A}\right)_{AB} \le 100.$

$\frac{4P}{100} \le 100.$

$P \le 2.5\,kN.$

⇒ Safe condⁿ for BC,

$\sigma_{BC} \le \sigma_{per}$

$\left(\frac{P}{2A}\right)_{BC} \le 50$

$\frac{4P}{200} \le 50$

$P \le 2.5\,kN.$

⇒ Safe condⁿ for CD,

$\sigma_{CD} \le \sigma_{per}$

$\frac{2P}{100} \le 150$

$P \le 7.5\,kN.$

∴ Safe value of P = min of above values

$= 2.5\,kN.$

* $\begin{pmatrix}\text{Height of AFD}\\ \text{at a x-S/c}\end{pmatrix} = \begin{bmatrix}\text{Applied Horizontal force at}\\ \text{the centroid of that}\\ \text{x-S/c}\end{bmatrix}$

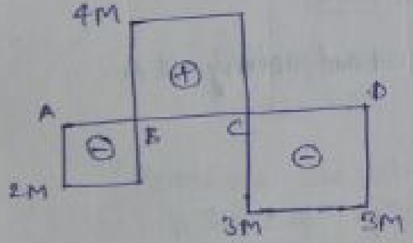
$\begin{pmatrix}\text{Height of SFD}\\ \text{at a x-S/c}\end{pmatrix} = \begin{bmatrix}\text{Applied vertical force}\\ \text{(or) conc. point load}\\ \text{at the x-S/c}\end{bmatrix}$

Bending couple (or) concentrated moment.

$\begin{pmatrix}\text{Height of BMD}\\ \text{at a x-S/c}\end{pmatrix} = \begin{bmatrix}\text{Applied B.C at}\\ \text{that x-S/c}\end{bmatrix}$

Twisting couple

$\begin{pmatrix}\text{Height of TMD}\\ \text{at a x-S/c}\end{pmatrix} = \begin{bmatrix}\text{Applied T.C at}\\ \text{that x-S/c}\end{bmatrix}$



Load acting = A conc. moment of 2M at A

— " — B = A conc. moment of 6M.

— " — C = A conc. moment of 7M.

— " — D = — " — 3M.

But BM at A = 2M (Hogging).

BM at B = Larger Value = 4M (S).

BM at C = — " — = 4M (S)

BM at D = 3M (H)

* No. of vertical lines in A.L.D = No. of A.L applied on a member.

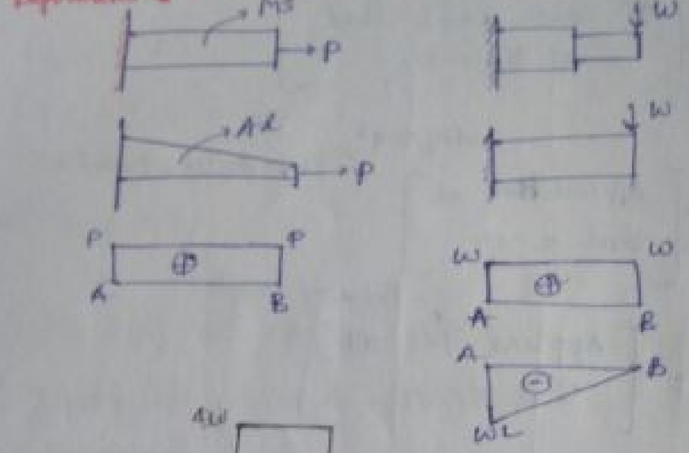
— " — in SFD = No. of conc. point loads — " —

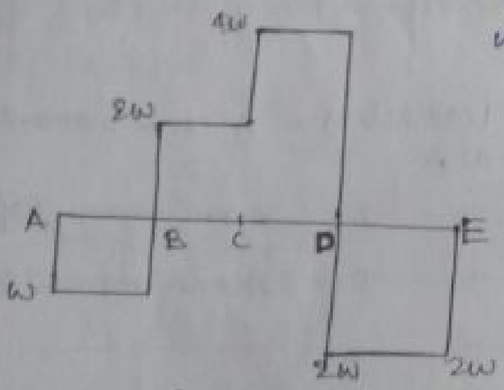
— " — in BMD = No. of conc. moments or B.C applied on a member.

— " — TMD = No. of conc. moments or TC applied on a member.

* Loading Diagrams never depend upon x-s/c dimensions & material.
* Stress depends upon x-s/c dimensions but ind. of material.
* Strain depends upon ~~...~~ x-s/c dim^n as well as material.

Deformations





~~Load acting at A~~

SFD

⇒ Load acting at A = A conc. point Load of w (↓)

$(SF)_A = w$.

⇒ Load acting at B = A conc. point Load of 3W (↑)
(i.e. Ht. of vertical line at B)

$(SF)_B = 2W$ [larger of $-w$ & $2w$].

⇒ Load acting at C = A conc. point Load of 2W. (↑)
(Ht. of vertical line at C)

$(SF)_C = 4W$ [larger of 4W & 2W].

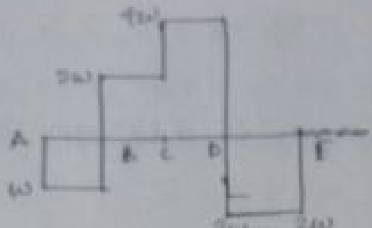
⇒ Load acting at D = Ht. of vertical line at D = 6W. (↓)

$(SF)_D =$ ~~2W~~ $4W$ [larger of 2W & 4W].

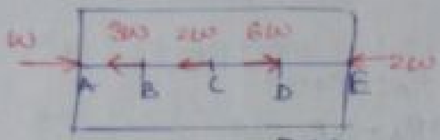
⇒ Load acting at E = 2W. (↑)

$(SF)_E = 2W$.

ALD



When diagram goes downward
= P towards ~~East~~ Right (⟶).
When diagram goes upward.
= P towards Left (⟵).



BAR

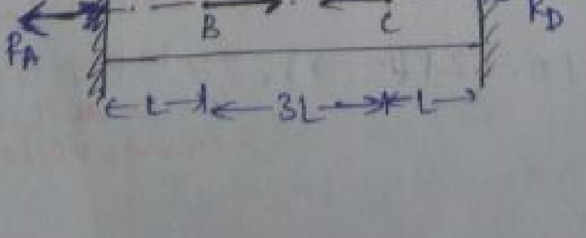
Case III - BARS FIXED AT BOTH ENDS -
(Statically Indeterminate Bars).

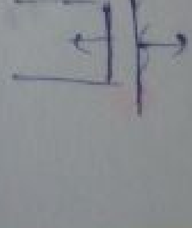
↳ (Total no. of reactions in the bar are greater than no. of useful static eq^m eq^{n}s.)

$\begin{pmatrix}\text{No. of reactions}\\ \text{in the bar}\end{pmatrix} > \begin{pmatrix}\text{No. of useful}\\ \text{Static eq}^m\text{ eq}^n\text{s}\end{pmatrix}$.

→ Reactions are det. with the help of
(i) Static eq^m eq^{n}s.
(ii) Compatibility eq^{n}s
↳ In terms of deformations or deflections.

Q - For the prismatic bar as shown in the fig. Det.
(i) Reactions at the supports.
(ii) Max. ΔL.
(iii) Ratio of max. & min. axial stresses.
(iv) Deformations at B & C.





Ist Method -

(1) Net Axial Load = $3P (\longrightarrow)$

(2) Introduce reactions at the fixed ends in a dir^n opp. to the dir^n of net axial load.

(3) Axial loads:- $P_1 = R_A$

$P_2 = R_A - 6P$.

$P_3 = -R_D$ or $R_A - 6P + 3P$

$\Rightarrow R_A - 3P$.

(4) $\Sigma H = 0$

$R_A - 6P + 3P + R_D = 0$.

$R_A + R_D = 3P$ — ①

(No sign convention. Just check the dir^n like towards left ($\leftarrow$) is taken as +ve then ($\rightarrow$) will be -ve and vice versa.

(5) $(\delta L)_{total} = \delta_1 + \delta_2 + \delta_3 = 0$ — ②

↳ Compatibility eq^n.

⑥ Reactions → from eq^n ②,

$$\frac{1}{AE}\left[P_1 L_1 + P_2 L_2 + P_3 L_3\right] = 0.$$

Axial Rigidity

$\frac{1}{AE} \neq 0$. $\therefore R_A L + (R_A - 6P)3L + (R_A - 3P)L = 0$.

$L\left[R_A + 3R_A - 18P + R_A - 3P\right] = 0$.

$L \neq 0 \therefore 5R_A = 21P$.

$R_A = \frac{21}{5} = \underline{\underline{4.2P}} (\longleftarrow)$.

From ①,

$R_A + R_D = 3P$.

$4.2P + R_D = 3P$.

$R_D = -1.2P$ or $\underline{\underline{1.2P}} (\longrightarrow)$.

* We can't say compression/elongation/deformation/contraction for these pts. of junction bcoz they belong two sections. We words "towards left" or "towards right".

(7) $P_1 = R_A = 4.2P$ (T).

$P_2 = R_A - 6P = -1.8P$ or $1.8P$ (C).

$P_3 = -R_D = -(-1.2P)$

$= 1.2P$ (T).

(8) Max. Tensile load $= 4.2P$.

Max. Comp. load $= 1.8P$.

(9) $\frac{\sigma_{max}}{\sigma_{min}} = \frac{P_{max}}{P_{min}} = \frac{4.2}{1.82}$.

$\delta_B = \delta_{BA}$ or $\delta_{BD} = \delta_{BC} + \delta_{CD}$

$= \delta_1$ or $\delta_2 + \delta_3$

$= \delta_1 = \left(\frac{PL}{AE}\right)_1 = \frac{4.2\,PL}{AE}$ mm ($\longrightarrow$).

$\delta_C = \delta_{CD}$ or $\delta_{CA} = \delta_{CB} + \delta_{BA}$

$= \delta_3$ or $\delta_{CA} = \delta_2' + \delta_1$

$= \delta_3 = \left(\frac{PL}{AE}\right)_3 = \frac{1.2\,PL}{AE}$ mm ($\longleftarrow$)

IInd Method-

$A_1E_1 = A_2E_2 = A_3E_3$.

(1) Net axial load $= 3P$ ($\longrightarrow$).

(2) Introduce rxns in a dir^n opp. to the dir^n of net axial load.

(3) $R_A = \frac{(6P)(4L) + (-3P)(L)}{5L} = \frac{21}{5}P = 4.2P$ ($\longleftarrow$).

$R_D = \frac{(-3P)(4L) + (6P)(L)}{5L} = \frac{-6P}{5} = -1.2P$ or $1.2P$ ($\longrightarrow$).

or $\Sigma H = 0$.

$4.2P - 6P + 3P + R_D = 0$.

$R_D = -1.2P$ or $1.2P$ ($\longrightarrow$).

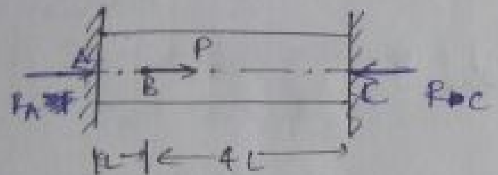
(4) $P_1 = P_A = 4.2P(T)$. ⇒ Max. Tensile load.

$P_2 = 4.2P - 6P = -1.8P$
$= 1.8P(C)$. ⇒ Max. comp load.

$P_3 = P_D = 1.2P(T)$

Note: When Rxn & applied load are opp. in dir^n ⟶ +ve sign be used and vice versa.

Q-



Det. Axial load in AB
— " — BC

$R_A + P = R_C$ —①.

$R_A \times 5L = -P \times 4L$

$R_A = -\frac{4P}{5}$.

$= \frac{4P}{5} (\leftarrow)$.

$-\frac{4P}{5} + P = R_C$

$R_C = \frac{P}{5}. (\leftarrow)$.

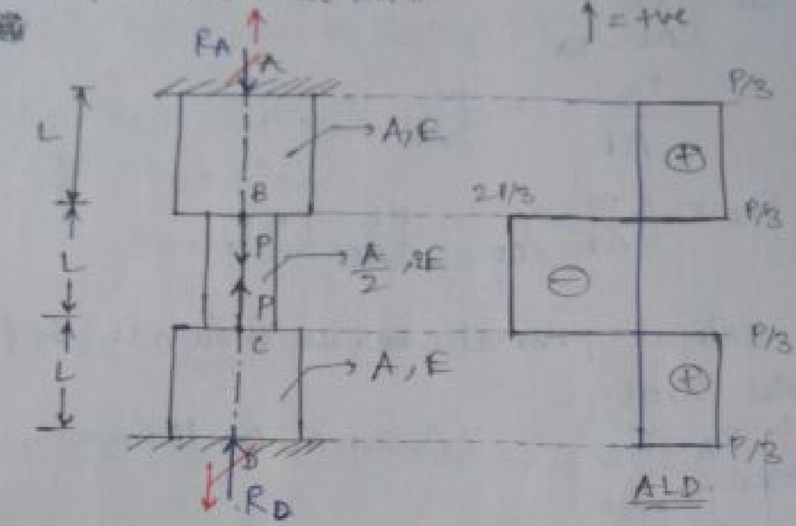
Axial load in AB $= 0.8P$ (T)

— " — BC $= 0.2P$ (C).

Q- For a stepped bar as shown in the fig. det (i) max. tensile and compressive loads.

(ii) Deformation at B&C.

$\uparrow = +ve$



Here, net load = $P - P = 0$.

But this doesn't mean that there will no rxns.

Rxns will be there and will be equal & opp. in dirn.

OR We can calculate.

$$R_A \times 3L + P \times 2L - P \times L = 0.$$

$$R_A = -\frac{P}{3} \text{ or } \frac{P}{3} (\uparrow).$$

$$\Sigma Y = 0 \Rightarrow R_A + P - P + R_D = 0.$$

$$-\frac{P}{3} + R_D = 0.$$

$$R_D = \frac{P}{3} (\downarrow).$$

$P_{AB} = P_A = P/3 \ (T) \Rightarrow$ Max. tensile load.

$P_{BC} = P/3 - P = -2P/3$ or $2P/3 \ (C) \Rightarrow$ Max. comp. load.

$P_{CD} = P/3 \ (T)$.

$$\sigma_{max} = \sigma_{BC} = \left(\frac{P}{A}\right)_{BC} = \frac{-2P/3}{A/2} = \frac{-4P}{3A}$$

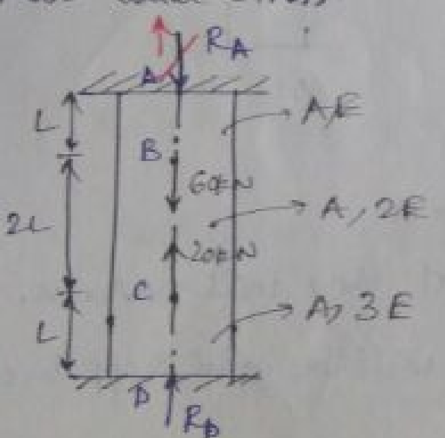
$$= \frac{4P}{3A} \ (C).$$

$$\delta_B = \delta_{BA} = \left(\frac{PL}{AE}\right)_{BA}$$

$$= \frac{P/3\, L}{AE} = \frac{PL}{3AE} \ (\downarrow).$$

$$\delta_C = \delta_{CD} = \left(\frac{PL}{AE}\right)_{CD} = \frac{PL}{3AE} \ (\uparrow).$$

Q- For the vertical prismatic bar as shown in the fig. Det. i) Max. axial stress.



Assume, $A = 100 mm^2$.

$$R_A + 60 - 20 - R_D = 0.$$

$$R_A - R_D = -40. \quad \text{—(1)}$$

$$P_1 = R_A.$$

$$P_2 = R_A + 60.$$

$$P_3 = R_A + 60 - 20 = R_A + 40.$$

$$\delta_1 + \delta_2 + \delta_3 = 0$$

$$\frac{L}{A}\left(\frac{R_A}{E} + \frac{(R_A+60)2}{2E} + \frac{R_A+40}{3E}\right) = 0$$

$$R_A + R_A + 60 + \frac{R_A}{3} + \frac{40}{3} = 0.$$

$$2R_A + \frac{R_A}{3} + \frac{220}{3} = 0.$$

$$\frac{7R_A}{3} = -\frac{220}{3}$$

$$R_A = -\frac{220}{7}. \text{ or } \frac{220}{7} (\uparrow).$$

$$R_A - R_D = -40.$$

$$-\frac{220}{7} - R_D = -40$$

$$R_D = 40 - \frac{220}{7} = \frac{60}{7} (\uparrow).$$

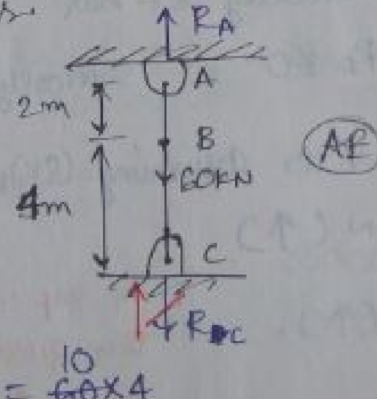
$$\therefore P_1 = R_A = -\frac{220}{7}. (T).$$

$$P_2 = R_A + 60 = -\frac{220}{7} + 60 = \frac{200}{7} (C).$$

$$P_3 = R_D = \frac{60}{7} (C).$$

$$\sigma_{max} = \sigma_1 = \frac{P_1}{A_1} = \frac{220}{7 \times 100} = 314.28 MPa.$$

Q- For the stepped bar as shown in the fig. Det. max. axial stress.



$$R_A \times 6 = \overset{10}{\cancel{60}} \times 4$$

$$R_A = 40.$$

$$R_A - 60 \mp R_C = 0.$$

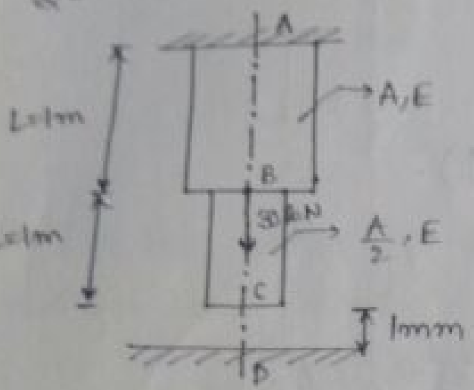
$$R_C = -20 \text{ or } 20 (\uparrow).$$

$P_1 = R_A = 40\,kN$ (T)

$P_2 = R_C = 20\,kN$ (C)

$\sigma_{max} = \frac{40}{A}$ (T).

*** Q-



Assume : $A = 100\,mm^2$
$E = 200\,GPa$.

Can't say R_D is zero or non-zero. It will depend upon the elongation. If elongation is less than 1mm ⇒ R_D is zero.
If elongation is more than 1mm ⇒ $R_D \neq 0$.
⇓
Statically indeterminate bar.

If $(\delta L)_{bar} \leq gap \Rightarrow R_D = 0$
⇓
Statically det. bar

If $(\delta L)_{bar} \geq gap \Rightarrow R_D \neq 0 \Rightarrow$ Statically indet. bar.

CASE I - Assume $R_D = 0$ (ie. assuming $(\delta L)_{bar} \leq gap$).

$\therefore R_A = 30\,kN\,(\uparrow)$

$P_1 = R_A = 30\,kN\,(\uparrow)$.

$P_2 = -R_D = $ zero.

$(\delta L)_{bar} = \delta_1 + \delta_2$

$= \left(\frac{PL}{AE}\right)_1 + \left(\frac{PL}{AE}\right)_2$

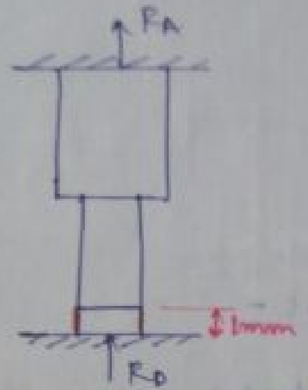
$= \frac{30000 \times 1}{100 \times 200 \times 10^3} + 0 = 1.5\,mm$.

If gap would be less than 1mm then question gets completed here that AB is having Load while BC is not under load.

$R_D \neq 0$ [$\because (\delta L)_{bar} >$ gap or 1mm).

$\therefore$ Bar is indeterminate.

Case II -



$P_1 = R_A$.

$P_2 = -R_D$ or $(R_A - 30)$ kN.

$\Sigma V = 0 \Rightarrow R_A - 30 + R_D = 0$.

$R_A + R_D = 30\text{ kN}$ — (1).

$(\delta L)_{bar} = \delta_1 + \delta_2 = \underline{gap}$. → permitted elongation

$$\frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} = 1\text{mm}$$

$$\frac{R_A \times (1000)}{100 \times 200 \times 10^3} + \frac{(R_A - 30000) \times (1000)}{50 \times 200 \times 10^3} = 1.$$

$$\frac{1}{2 \times 10^4}\left(R_A + 2R_A - 60000\right) = 1$$

$$3R_A = 80000$$

$$R_A = \frac{8}{3} \times 10^4 = 26.66\text{ KN}.(\uparrow)$$

$\therefore R_D = 3.34\text{ KN}.(\uparrow)$

$P_1 = R_A = 26.66\text{ KN (T)}$

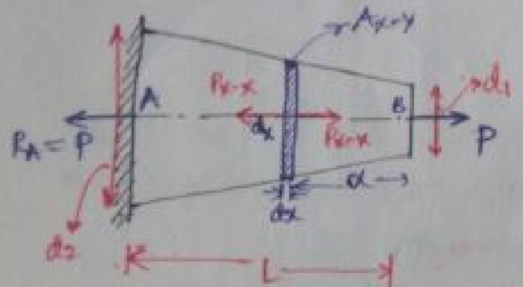
$P_2 = -R_D = -3.34\text{ KN (C)}$

$$\sigma_1 = \frac{P_1}{A_1} = \frac{26.66 \times 1000}{100}$$
$$= 266.6 \text{ MPa (T)}.$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{3.34 \times 1000}{50}$$
$$= 66.8 \text{ MPa(C)}.$$

$$\boxed{(\sigma_{max})_{bar} = 266.6 \text{MPa (T)}}.$$

Elongation of a tapered bar under A.L –



is treated as assembly of n bars of diff. dia which are in series.

$P_1 = \ldots = P_n = P.$

$$\boxed{\sigma_{max} = \sigma_B = \frac{4P_B}{\pi d_B^2} = \frac{4P}{\pi d_1^2} = \frac{4P}{\pi (d_{smaller})^2}}$$

$$\boxed{\frac{\sigma_{max}}{\sigma_{min}} = \frac{\sigma_B}{\sigma_A} = \left(\frac{d_A}{d_B}\right)^2 = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{d_{larger}}{d_{smaller}}\right)^2}.$$

$$(\delta L)_{AB} = \delta L_1 + \delta L_2 + \ldots \delta L_n.$$

$$(\delta L)_{AB} = \int_0^L (\delta L)_{strip} = \int_0^L \frac{(P_{x-x})dx}{(A_{x-x})E_{x-x}}$$

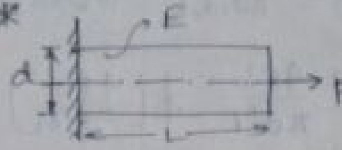
$$= \int_0^L \frac{P\,dx}{\frac{\pi}{4} d_x^2 E}$$

$$d_x = d_1 + (d_2 - d_1)x.$$

$$(\delta L)_{T.B} = \int_0^L \frac{4P\,dx}{\pi\left[d_1 + (d_2-d_1)(x/L)\right]^2 E}$$

$$\boxed{(\delta L)_{T.B} = \frac{4PL}{\pi d_1 d_2 E}}$$

*



$$(\delta L)_{PB} = \frac{4PL}{\pi d^2 E}$$

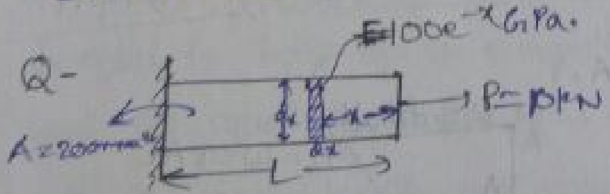
$$(\delta L)_{TB} = \frac{4PL}{\pi d_1 d_2 E}.$$

if $d^2 = d_1 d_2$ or $d = \sqrt{d_1 d_2}$. → Geometric mean of dia T.B.

$$\boxed{(\delta L)_{P.B} = (\delta L)_{T.B}}.$$

Note: Elongation of a tapered bar under an axial load is equal to elongation of an identical prismatic bar, if the diameter of PB is equal to Geometric mean of diameters of tapered bar (ie. $d = \sqrt{d_1 d_2}$)

Q-

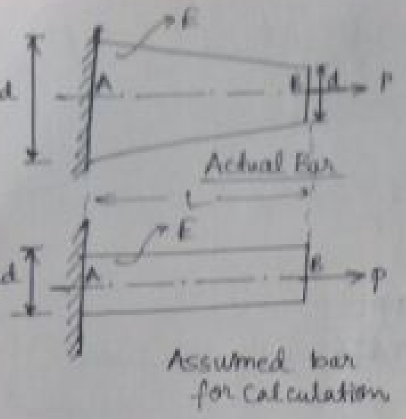


$$(\delta L)_{P.B} = \int_0^L \frac{P\,dx}{AE}$$

$$= \int_0^L \frac{P \times dx \times 4}{\pi d^2 \times 100 e^{-x}}$$

$$= \frac{4P}{\pi d^2 \times 100} \int_0^L e^x\,dx = \cancel{\frac{4P}{\pi d^2 \times 1000}\left(e^L - 1\right)}$$

$$= \frac{P \times 10^3}{200 \times 100 \times 10^3}\left(e^L - 1\right) = 0.859\,mm$$



Det.:

(a) % of error in cal. elongation.

(b) % of error in cal. max. stress.

$$(\delta L)_{actual} = (\delta L)_{TB} = \frac{4PL}{\pi d_1 d_2 E} = \frac{1}{3}\left[\frac{4PL}{\pi d^2 E}\right].$$

$$(\delta L)_{cal} = (\delta L)_{PB} = \frac{4PL}{\pi d^2 E} = \frac{1}{4}\left[\frac{4PL}{\pi d^2 E}\right],$$

% error in cal. elongation $= \frac{\delta_{actual} - \delta_{cal}}{\delta_{actual}} \times 100$

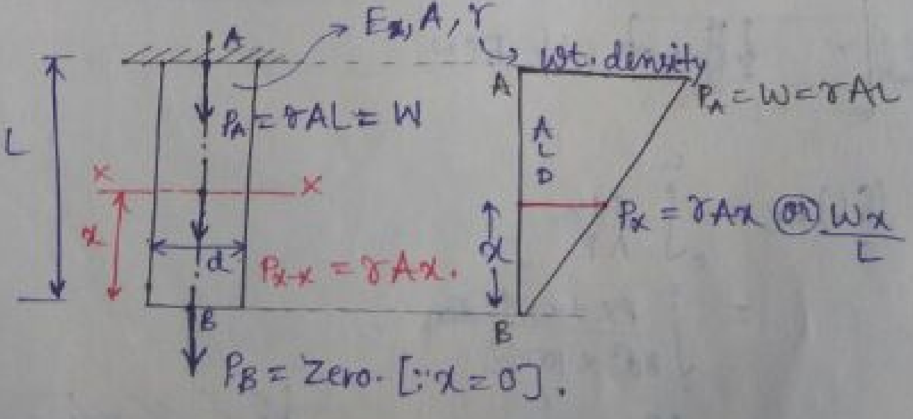
$$= \frac{1/3 - 1/4}{1/3} \times 100 = 25\%.$$

$$(\sigma_{actual})_{max} = \frac{4P}{\pi d^2_{smaller}} = \left[\frac{4P}{\pi d^2}\right].$$

$$(\sigma_{cal})_{max} = \left(\frac{4P}{\pi d^2}\right)_{P.B} = \frac{1}{4}\left[\frac{4P}{\pi d^2}\right].$$

% of error in cal max. stress $= \frac{\sigma_{act} - \sigma_{cal}}{\sigma_{act}} \times 100 = \frac{1 - 1/4}{1} \times 100 = 75\%.$

ELONGATION OF A P.B. UNDER ITS SELF-WEIGHT -



$$P_{x-x} = \gamma A x \quad \text{or} \quad \frac{Wx}{L} \quad \text{--- (1)}$$

where $W = \gamma A L$.

$x = 0 \Rightarrow P_B = \text{zero}$.

$x = L \Rightarrow P_A = \gamma A L = W$.

$$(\sigma_{axial})_{x-x} = \frac{P_{x-x}}{A_{x-x}} = \gamma x \quad \text{--- (2)}.$$

$$\boxed{(\sigma_{max}) = \sigma_A = \gamma L \text{ or } \frac{W}{A}} \quad \text{--- (3)}$$

$$(\delta L)_{bar} = \delta_1 + \delta_2 + \dots + \delta_n.$$

or $$(\delta L)_{bar} = \int_0^L \delta_{strip} = \int_0^L \frac{(P_{x-x})(dx)}{AE_{x-x}}$$

$$= \int_0^L \frac{(\gamma A x)(dx)}{AE} = \int_0^L \left(\frac{\gamma x}{E}\right) dx.$$

Self wt. = γAL.

$$\boxed{(\delta L)_{bar} = \frac{\gamma L^2}{2E} \text{ or } \frac{WL}{2AE}} \quad \text{--- (4)}$$

Conclusion- Elongation of the prismatic bar under its self weight is equal to half of the elongation of identical P.B under an axial load viz equal to self weight of the bar.

* Change in length and max. stress are independent of area from eq^n (3) & (4).

* From eq^n (3), $\sigma_{max} \propto L$.
  From eq^n (4), $(\delta L)_{bar} \propto L^2$.

  σ_{max} & δL are independent of A (ie x-s/c dim^n).
  Bcoz it is due to body force.

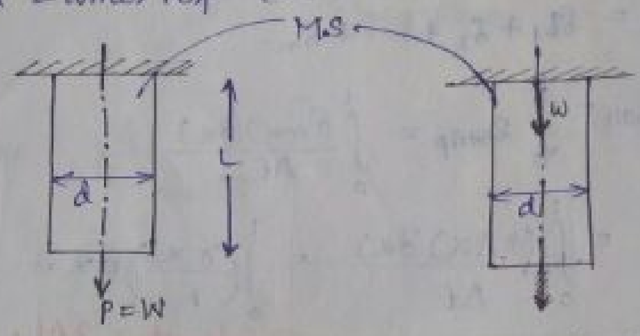
Stress due to Body force ind. of area.

" Surface force $\propto \frac{1}{\text{Area}}$.

* Max. stress induced and change in length of the bar are ind. of x-s/cal ~~dia~~ dimensions bcoz self weight is a <u>body force.</u>

* Max. stress ~~due~~ or max. axial stress due to self weight is directly proportional to length of the bar but elongation of the bar due to self. wt. is $\propto L^2$.

* If all the dimensions of a prismatic bar becomes double then elongation and max. stress due to self weight increases by 4 times & 2 times resp. (bcoz $\delta L \propto L^2$ and $\sigma_{max} \propto L$).

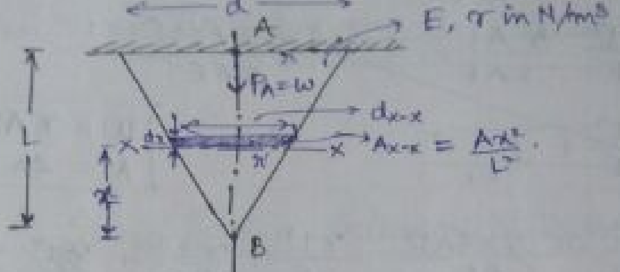


<u>P.B under PAL</u>	<u>PB under Self wt.</u>
→ A.L.D is a rectangle.	→ ALD is a triangle
→ $\sigma_a \propto \frac{1}{A}$	→ (σ_a) is ind. of 'A'.
→ σ_a is ind. of material.	→ $\sigma_a \propto \gamma$.
→ σ_a is ind. of L.	→ $(\sigma_{max})_{axial} \propto L$
→ $\delta L \propto L$	→ $\delta L \propto L^2$
→ $\delta L \propto 1/A$	→ δL ~~is~~ is ind. of 'A'.

* ~~Self~~ When member is vertical ⇒ Self wt. is axial (passes through Centroidal Axis)
⇒ Uniformly varying Load. (UVL)

When member is horizontal, self. wt is T.S.L.
⇒ UDL.

Elongation of a Conical Bar under its self weight -



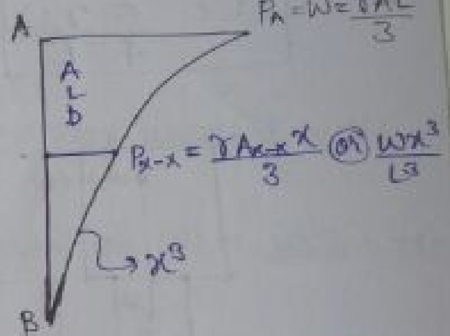
$\frac{y}{r} = \frac{x}{L}$

$\pi y^2 = \pi r^2 \frac{x^2}{L^2}$

$A' = \frac{Ax^2}{L^2}$.

$(\delta L)_{CB} = \frac{\gamma L^2}{6E}$ $\rightarrow$ wt. of C.B $= W = \frac{\gamma A L}{3}$.

$$\boxed{(\delta L)_{CB} = \frac{WL}{2AE}}$$



$P_{x-x} = \gamma \left[\frac{A_{x-x}}{3} . x\right] = \frac{\gamma A x^3}{3L^2} \times \left(\frac{L}{L}\right)$

$P_{x-x} = \left(\frac{\gamma A L}{3}\right) \cdot \left(\frac{x^3}{L^3}\right)$

$$\boxed{P_{x-x} = \frac{W x^3}{L^3}}$$

$(\sigma_{axial})_{x-x} = \frac{P_{x-x}}{A_{x-x}} = \frac{\gamma A_{x-x} x}{3} \times \frac{1}{A_{x-x}} = \frac{\gamma x}{3}$.

$$\boxed{(\sigma_{max})_{axial} = \sigma_A = \frac{\gamma L}{3} \text{ or } \frac{W}{A}.}$$

$$(\delta L)_{CB} = \int_0^L \frac{(P_{x-x})(dx)}{A_{x-x}\, E_{x-x}} = \int_0^L \frac{\gamma A_{x-x}\, x/3}{(A_{x-x}) E}(dx) = \int_0^L \left(\frac{\gamma x}{3E}\right) dx.$$

$$\boxed{(\delta L)_{CB} = \frac{\gamma L^2}{6E} \text{ or } \frac{WL}{2AE}} \quad \text{where } W = \frac{\gamma A L}{3}.$$

Conclusion -

* Change in length of a CB under its self weight is equal to $(1/3)^{rd}$ of elongation of an identical P.B. under its self wt. because of volume of a conical bar $= \frac{1}{3}$ (Volume of a cylinderical bar)
* Max. stress is at A bcoz Load is max.

37- $\delta L = \underbrace{\frac{WL}{2AE}}_{\text{1st bar due to self wt.}} + \underbrace{\frac{WL}{2A'E}}_{\text{2 bar due to self wt.}} + \underbrace{\frac{WL}{AE}}_{\text{2nd bar due axial load equal to self wt. of 2nd bar}}$.

$= \cancel{\frac{\gamma L^2}{2E}} \; \cancel{\frac{\gamma A L^2}{2A'E} + \frac{(\gamma AL + \gamma A' \times AL) \times L}{2A'E}}$

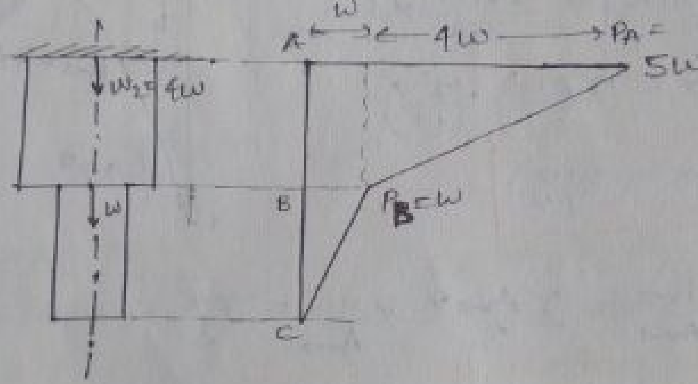
$= \cancel{\frac{\gamma L^2}{2E} + \frac{5\gamma L^2}{2E}}$

$$\boxed{\begin{aligned} W &= \gamma AL \\ A' &= 4A \end{aligned}}$$

$= \cancel{\frac{\gamma A L^2}{2AE} + \frac{\gamma \times 4AL^2}{2E}} \quad \frac{\gamma L^2}{2E} + \frac{\gamma L^2}{2E} + \frac{\gamma A L^2}{4AE}$

$= \frac{\gamma L^2}{E} + \frac{\gamma L^2}{4E}$

$$\boxed{\delta L = \frac{5}{4}\frac{\gamma L^2}{E}}$$



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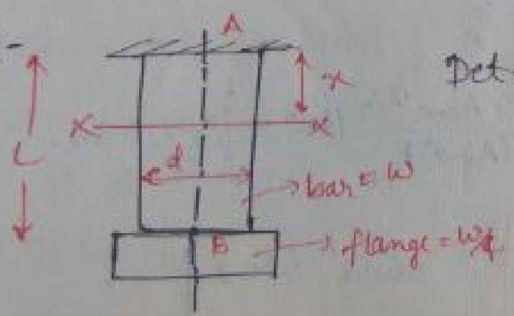

Det. (a) $(A \cdot L)_{x-x}$
(b) $(\delta L)_{bar}$
(c) $[(\sigma_{max})_{axial}]_{bar}$

Note: Weight of flange will act as axial load for bar when centroidal axis of bar coincides with the centroid of flange. Otherwise, it will act as eccentric ~~axial~~ load.

(a) $(AL)_{x-x} = WAx + \frac{W}{4}$.

(b) $(SL)_{max} = \frac{WL^2}{2EA} + \frac{WL}{2E} + \frac{WL}{4E \times x}$

Strain Energy, Resilience, & Toughness

Strain Energy -

Strain energy is the energy absorbed by the member when work done by the load deforms that member.

Resilience - It is defined as the ~~ene~~ energy absorbed by the member within the elastic region.

Resilience = Area of load v/s deformation curve within the elastic limit.

Proof Resilience - It is defined as the max. energy absorbed by a component within the elastic region.

Proof Resilience = Area of load v/s deformation curve upto elastic limit.

$$\boxed{P.R = \frac{1}{2} P_{E.L} \times \delta_{E.L} \text{ or } \frac{1}{2}(\sigma_{E.L}\, \epsilon_{E.L})(AL) \text{ or } \frac{(\sigma_{E.L})^2}{2E} \times (Vol.)}$$

Elastic limit

P.R (↑) ⇒ (a) Selecting a material with higher E.L.

(b) Selecting — " — lower Young's mod.

(c) Providing more vol. for that component.