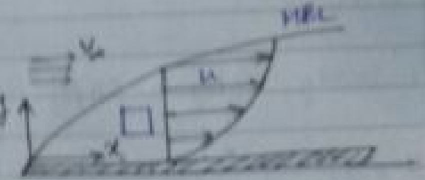
2- Unit depth of element ⊥r to plane of fig.
3- Constant fluid properties.

Newton's II Law of Motion -



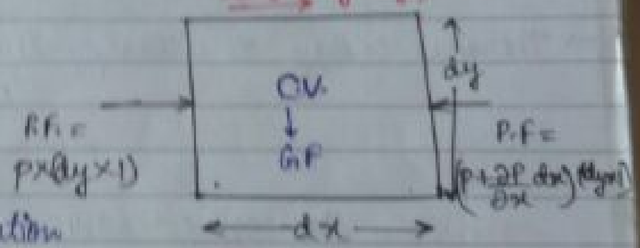
Net algebric sum of all the forces acting on the fluid element along x- direction

= Rate of change of linear momentum of fluid element along x.

$$\overset{①}{\Sigma F_x} = m a_x$$

$V.F = \mu\left[\left(\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\right](dx \times 1).$

⇒ We are considering motion only in x- direction.



$V.F = \tau \times$ Area
$= \left(\mu \frac{\partial u}{\partial y}\right) \times (dx \times 1)$

The resulting momentum equation of HBL along x-direction -

$$\boxed{\underbrace{\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)}_{\text{Convective acc}^n\text{ of fluid along } x.} = \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\rho}\left(\frac{\partial P}{\partial x}\right)}$$

→ Navier-strokes eq^n of motion along x-direction

⇒ For flow over flat plates, $\frac{dp}{dx} = 0$.

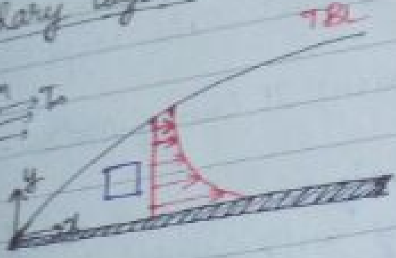
and put $\frac{\mu}{\rho} = \nu$ = K.V. of fluid (m^2/sec).

$$\therefore \boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}}$$ → Momentum eq^n of HBL.

↳ This eq^n of integration & applying boundary cond^n gives.

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3.$$

Energy Equation of Thermal Boundary layer -



Assume -

1- Steady, 2D, incompressible flow.

$\rho = C$

$u = f(x,y)$ $v = f(x,y)$ $T = f(x,y)$

weak

strong

2- Constant fluid properties (ρ, μ, C_p, K).

3- Negligible heat conduction along x-direction.

4- Unit depth of element ($\perp$ to plane of fig.).

⇒ Thermal energy or enthalpy transported or convected by the fluid $= (\dot{m} C_p T)$ J/sec.

Heat convected thro' top face $= \rho (dx \times 1)\left(v + \frac{\partial v}{\partial y} dy\right) C_p \left(T + \frac{\partial T}{\partial y} dy\right)$

Heat conducted thro' Top face $= -K_f (dx \times 1)\left[\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) dy\right]$

Viscous Heat Neglected

C.V

Heat convected thro' left face $= \rho (dy \times 1) u C_p T$

Heat convected thro' right face $= \rho (dy \times 1)\left[u + \frac{\partial u}{\partial x} dx\right] C_p \left[T + \frac{\partial T}{\partial x} dx\right]$

Heat Convected thro' bottom face $= \rho \times (dx \times 1) v C_p T$

Heat conducted thro' bottom face $= -K_f (dx \times 1)\left(\frac{\partial T}{\partial y}\right)$

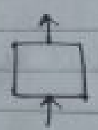
* Change of temp along y-dir^n is much higher than that in x direction

Writing the energy balance for steady state conditions of control volume we get,

Heat conducted thro' bottom face + Heat convected thro' left face + Heat convected thro' bottom face = Heat conducted thro' top face + Heat convected thro' right face + Heat convected thro' top face

The resulting energy equation of TBL is -

$$\underbrace{\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)}_{\text{Net Heat convected into C.V.}} = \underbrace{\left(\frac{k}{\rho C_p}\right)\left(\frac{\partial^2 T}{\partial y^2}\right)}_{\text{Net heat conducted out of CV}}$$



* Large temp. diff. in a very small distance along y-direction leads to conduction along y-dirⁿ.

$$\therefore \boxed{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \underset{\text{T.D.}}{\alpha}\left(\frac{\partial^2 T}{\partial y^2}\right)} \rightarrow \text{Energy eq}^n \text{ of TBL.}$$

Note: Recalling momentum eqⁿ of HBL,

$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \underset{\text{K.V}}{\nu}\left(\frac{\partial^2 u}{\partial y^2}\right)}$$

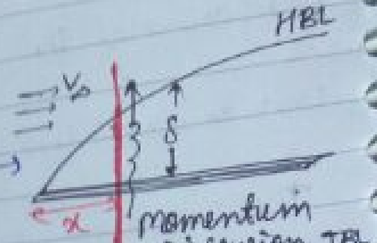
Note: There is a striking similarity b/w the momentum eqⁿ of HBL and the energy eqⁿ of TBL.
The solution to these two second order differential eqⁿs ~~who~~ would exactly be the same if KV of the fluid is equal to its thermal

diffusivity α'.

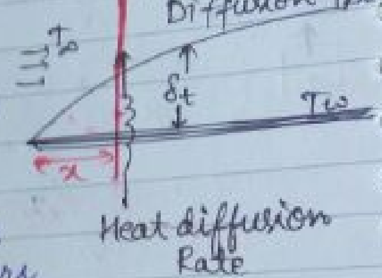
Significance of Prandtl Number (Pr):-

$$Pr = \frac{KV}{TD} = \frac{\nu}{\alpha}.$$

KV(ν) of fluid signifies, Momentum Diffusion → Rate thro' fluid layers



TD (α) of fluid signifies Heat energy diffusion → Rate thro' fluid layers



→ Kinematic Viscosity of fluid tells about the rate at which momentum diffusion occurs through the fluid layers in the normal dir^n to the plate (y-dir^n). Higher the value of KV, faster the momentum diffusion rate in y-dir^n ie. viscous influence of the fluid is felt farther away into the free stream thereby making the thickness of HBL relatively more. → $\delta \propto \nu$

Thermal diffusivity (α) of the fluid tells about the rate at which heat diffusion occurs thro' the fluid layers in the normal direction to the plate. Higher the value of (α), faster the heat diffusion rate in y-dir^n ie. temp. influence of the flat plate is felt farther away into the free stream, thereby making the thickness of TBL relatively more.

Hence Pr No. a ratio b/w ν and α can tell about the relative magnitudes of HBL thickness δ & TBL thickness δ_t at a given location of x measured from leading edge of plate.

If $Pr > 1$	If $Pr < 1$	If $Pr \simeq 1$
$\Rightarrow \nu > \alpha$	$\Rightarrow \nu < \alpha$	$\Rightarrow \nu \simeq \alpha$
$\Rightarrow \delta > \delta_t$	$\Rightarrow \delta < \delta_t$	$\Rightarrow \delta \simeq \delta_t$
$\Rightarrow \frac{\delta_t}{\delta} < 1$	$\Rightarrow \frac{\delta_t}{\delta} > 1$	$\Rightarrow \frac{\delta_t}{\delta} \simeq 1.$

For any given fluid, $\cancel{\frac{\delta_t}{\delta} = \frac{1}{1.026}}$

$$\boxed{\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}}$$

Thus Pr is connecting link b/w HBL & TBL.

2-d, 3-b

6- $Pr = \frac{\mu C_p}{K} = \frac{10^{-3} \times 10^{3}}{1} = 1.$
(c)
$Pr = 1 \Rightarrow \delta_t = \delta = 1mm.$

20- Same temp. $\Rightarrow$ No Heat transfer and hence no TBL formed.
(d) (But even now HBL will exist).

22- a

24- For liquid metals like Hg, Pr no. is very low as K is very high.
(b) $Pr << 1.$
$\nu <<< \alpha$
$\delta <<< \delta_t.$

So - $\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5x}{\sqrt{\frac{Vx}{\nu}}}$

$$= \frac{5 \times 0.5}{\sqrt{\frac{10 \times 0.5}{30 \times 10^{-6}}}}$$

$$= \frac{2.5\sqrt{3}}{10^3\sqrt{0.5}}$$

$\delta = 6.123$ mm.

$\because \frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$ → 0.974

$\therefore \delta_t = \frac{1}{1.026} \times 1 \times 6.123$

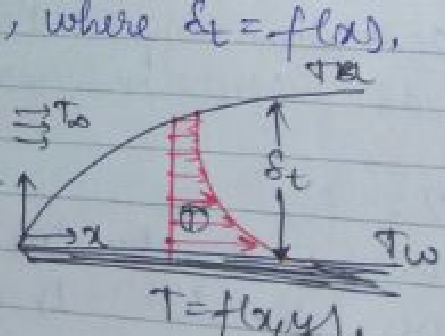
$\boxed{\delta_t = 5.96 \text{ mm}}$.

⇒ By integrating the energy equation of TBL, i.e.

$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right)$, with the help of its boundary conditions, we get the temp. distribution within the TBL as -

At any given x measured from leading edge of plate;

$\boxed{\frac{T - T_\infty}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3}$, where $\delta_t = f(x)$.



$T = f(x, y)$.

$\therefore$ At any given x,

$$\left(\frac{\partial T}{\partial y}\right)_{at\ y=0} = (T_\infty - T_w)\frac{3}{2\delta_t}.$$

$\therefore h_x$ = local convective HT coeff. $= \dfrac{-K_f \left(\frac{\partial T}{\partial y}\right)_{at\ y=0}}{T_w - T_\infty}$ W/m²K.

$\therefore h_x = \dfrac{-K_f\,(T_\infty - T_w)\ 3/2\delta_t}{(T_\infty - T_w)} = \left(\dfrac{3K}{2\delta_t}\right)$ W/m²K.

$$\boxed{h_x = \frac{3K}{2\delta_t}}$$

* We know, $\delta \propto x^{1/2}$.

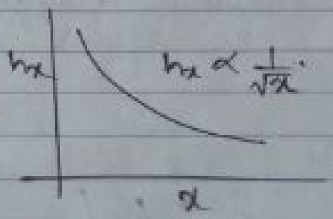
But $\frac{\delta_t}{\delta} \neq f(x)$.

Since $\frac{\delta_t}{\delta} = f(Pr)$.

But $Pr = C$ (as being a property of fluid).

$\therefore \boxed{\delta_t \propto x^{1/2}}$.

$\therefore \boxed{h_x \propto x^{-1/2}}$



Therefore the local convective HT coeff. decreases with the ↑ of x from leading edge to trailing edge.

→ h_x decreases with x because the thicker boundary layers at a greater value of x will offer more thermal resistance against the heat flow b/w the hot plate & the free stream fluid at T_∞.

$h_x \longrightarrow K, \delta_t \longrightarrow K, \delta, Pr \longrightarrow K, x, Re_x, Pr.$

$\uparrow\ \frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$ $\qquad \uparrow\ \delta = \frac{5x}{\sqrt{Re_x}}$.

Let local Nusselt Number = $Nu_x = \frac{h_x x}{K}$.

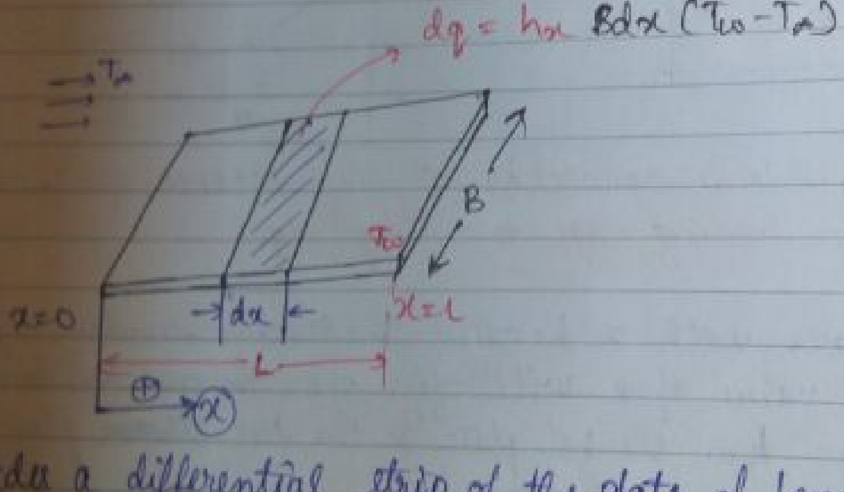
$\therefore$ The local convective H.T coeff. 'h_x' for laminar boundary layer over flat plate can be obtained from-

$$\boxed{Nu_x = \frac{h_x x}{K} = 0.332\, Re_x^{1/2}\, Pr^{1/3}}, \text{ where } Re_x = \frac{V_\infty x \rho}{\mu}.$$

$$\Rightarrow h_x \propto x^{-1/2} \qquad \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}.$$

* But by this h_x we can't get the overall H.T rate as its value changes from point to point. Hence we will take an avg.

Average Convective HT Coefficient ($\bar{h}$):-



Consider a differential strip of the plate of length dx at a distance of x from the leading edge where the local convective H.T coeff is h_x. Then,

Differential H.T Rate from elemental strip to fluid $= dq = h_x B dx (T_w - T_\infty)$ Watt.

Then,

Total H.T Rate from entire plate to fluid $= Q = \int_{x=0}^{L} dq = \int_{x=0}^{x=L} h_x B(T_w - T_\infty) dx$ watt — ①

But in terms of $\bar{h}$,

Total heat transfer rate from entire plate to fluid $= Q = \bar{h}(BL)(T_w - T_\infty)$ watt. — ②

Equating ① & ②, we get

$$\bar{h} \cancel{B} L \cancel{(T_w - T_\infty)} = \int_{x=0}^{L} h_x \cancel{B} \cancel{(T_w - T_\infty)} dx$$

$$\boxed{\bar{h} = \frac{1}{L}\int_0^L h_x \, dx} \quad W/m^2K.$$

Avg. conv. H.T coeff.

We know that, $h_x \propto x^{-1/2}$.

$\therefore h_x = C x^{-1/2}$.

Put $x = L$ on both sides.

$\Rightarrow \boxed{h_{x=L} = C L^{-1/2}}$ Local convective H.T coeff. at the trailing edge.

$\Rightarrow \boxed{C = \left(\frac{h_{x=L}}{L^{-1/2}}\right)}$

$$\therefore \bar{h}_x = \frac{1}{L}\int_0^L h_x \, dx$$

$$\bar{h} = \frac{1}{L}\int_0^L C x^{-1/2}\,dx$$

$$= \frac{C}{L}\left[\frac{x^{-1/2+1}}{-1/2+1}\right]_0^L$$

$$= \frac{h_{x=L}}{L^{-1/2}\cdot L}\left[\frac{L^{1/2}}{1/2}\right].$$

$$\boxed{\bar{h} = 2h_{x=L}}\ W/m^2K.$$

Note: Hence the avg. convective H.T coeff. for the entire plate will be equal to twice the local convective H.T coeff. at the trailing edge (ie $x=L$).

We know that,

$$Nu_x = \frac{h_x x}{K} = 0.332\,Re_x^{1/2}\,Pr^{1/3}.$$

Put $x=L$ on both sides.

$$\frac{h_{x=L}\,L}{K} = 0.332\,Re_L^{1/2}\,Pr^{1/3}$$

where Re_L = Local Re. No. at trailing edge $= \left(\frac{V_\infty L \rho}{\mu}\right)$.

$$\frac{2h_{x=L} \times L}{K} = 0.664\,Re_L^{1/2}\,Pr^{1/3}.$$

$$\frac{\bar{h}L}{K} = 0.664\,Re_L^{1/2}\,Pr^{1/3} \Rightarrow \boxed{\overline{Nu} = \text{Avg. Nu. No.} = 0.664\,Re_L^{1/2}\,Pr^{1/3}.}$$

$$\frac{\bar{h}L}{K}$$

ie $\boxed{\overline{Nu} = 2\,Nu_{x=L}}$

* $\delta \propto x^{1/2}$
$\delta_t \propto x^{1/2}$
$\tau_{wall} \propto x^{-1/2}$
$h_x \propto x^{-1/2}$
$Nu_x \propto x^{1/2}$
$Re_x \propto x$

$Pr \neq f(x)$.

$\frac{\delta_t}{\delta} \neq f(x)$.

26- $h_x = a x^{-0.1}$.

$$\frac{\bar{h}}{h_x} = \frac{\frac{1}{x}\int_{x=0}^{x} h_x dx}{h_x} = \frac{\frac{1}{x}\int_0^x a x^{-0.1} dx}{a x^{-0.1}}$$

$$= \frac{1}{x} \cdot \frac{x^{-0.1+1}}{(-0.1+1)} \times \frac{1}{x^{-0.1}} = 1.11.$$

11- (a) $\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$

$\frac{1}{2} = \frac{1}{1.026} Pr^{-1/3}$

$Pr = 7.4$.

$Nu_p = \cancel{0.332}\ 0.664\ Re_L^{1/2} Pr^{1/3}$

$= \cancel{0.332}\ 0.664 \times 10^2 \times 1.949$

$= \cancel{64.7}\ 129.4$

* when value is given - then relate them together. Don't use the formula

$\left(\frac{\delta_t}{\delta}\right)_P = C\, Pr^{-1/3}$

$2 = C\left(\frac{1}{8}\right)^{-1/3}$

$C = 1$.

$\therefore \left(\frac{\delta_t}{\delta}\right)_Q = C\, Pr^{-1/3}$

$\frac{1}{2} = 1 \times (Pr)^{-1/3}$

$\boxed{Pr = 8.}$

$Nu_p = C' Re^{1/2} Pr^{1/3}$

$35 = C' 10^2 \times \frac{1}{2}$

$C' = 0.7$.

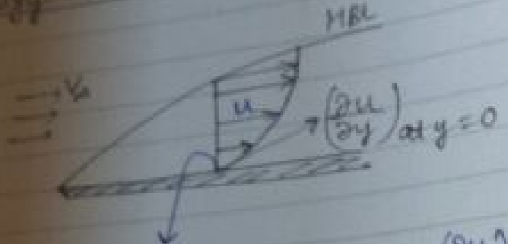
$Nu_Q = C' Re^{1/2} Pr^{1/3}$

$Nu_Q = 0.7 \times 100 \times 2$

$= \boxed{140}$.

(a)

Analogy b/w Fluid Friction and Heat Transfer (Reynold's Colburn Analogy) -



Local wall shear stress = $\tau_{wall} = \mu\left(\frac{\partial u}{\partial y}\right)_{at\ y=0}$

$$= \mu V_0 \frac{3}{2\delta}$$

$$= \mu V_0 \frac{3}{2\times \frac{5x}{\sqrt{Re_x}}}$$

$$= \mu V_0 \times \frac{3}{2\times \frac{5x}{\sqrt{\frac{V_0 x \rho}{\mu}}}} \text{ Pa} \quad \text{—①}$$

But in FM,

$$\tau_{wall} = C_{fx} \times \left(\rho \frac{V_0^2}{2}\right) \text{ Pa} \quad \text{—②}$$

where, C_{fx} = Local skin friction coefficient
(OR)
Local drag coefficient.
(a dimensionless parameter.)
Usually,
$C_{fx} = 0.005$ (OR) 0.004.

Equating ① & ②, we get

$$\boxed{\frac{C_{fx}}{2} = 0.332\, Re_x^{-1/2}}$$

Reynolds (... Analogy) -

$$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}.$$

$$\frac{Nu_x}{Re_x\, Pr} = \frac{0.332\, Re_x^{1/2}\, Pr^{1/3}}{Re_x\, Pr}$$

Stanton No. $= St = \left(\frac{Nu}{Re\, Pr}\right)$

$\therefore$ Local Stanton No. $= St_x = \frac{Nu_x}{Re_x\, Pr}$.

$$\therefore St_x = \frac{\frac{h_x x}{K}}{\frac{V_\infty x \rho}{\mu} \times \frac{\mu C_p}{K}} = \frac{h_x}{\rho V_\infty C_p} = \frac{1}{\pi_2}.$$

⇒ The product of Re & Pr is called Peclet No. (Pe).

$\therefore St = \left(\frac{Nu}{Pe}\right)$ Pe is significant in liquid metal cooling of Nuclear Reactors.

$$\therefore St_x = 0.332\, Re_x^{-1/2}\, Pr^{-2/3}.$$

$$\boxed{St_x\, Pr^{2/3} = \frac{C_{fx}}{2}}$$ → This eq^n is called Reynold's Analogy.

Physical significance of Reynold's Analogy -

Note: From this Reynold's analogy we can predict the value of local convective HT coeff. h_x at some location of x measured from the leading edge just by knowing the local skin friction coeff. C_{fx} at the same location of x even when there is no HT b/w the plate & ~~plate~~ the flowing fluid.

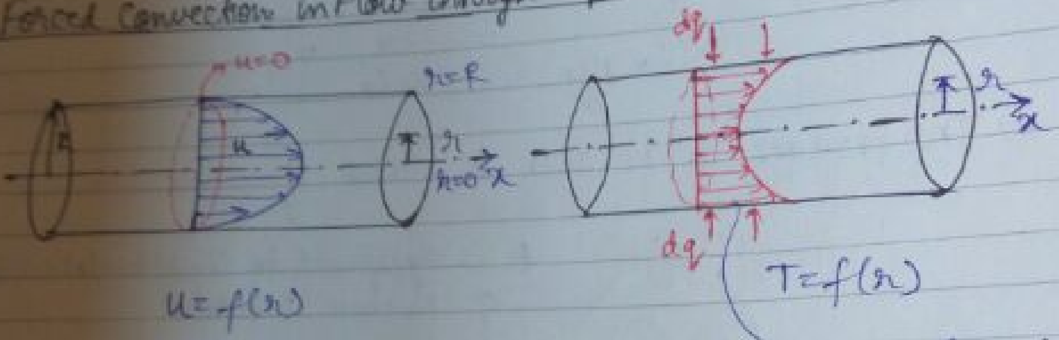
$\underline{31}$ - $St_x \cdot Pr^{2/3} = \frac{C_{fx}}{2}$.

$$\frac{h_x}{\rho V_\infty C_p} \times \left(\frac{\mu C_p}{k}\right)^{2/3} = 0.002.$$

$$h_x = 2\times10^{-3} \times 0.88 \times 50 \times 10^{3} \times 1.001 \left(\frac{2.886\times10^{-5}\times 1.001\times10^{3}}{0.035}\right)^{-2/3}$$

$$\boxed{h_x = 116.89}\ \mathrm{W/m^2\text{-}K}.$$

Forced Convection in Flow through Pipes or Ducts -



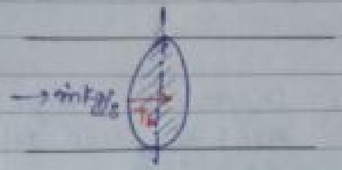
$\dot{m}$ = Mass flow rate = $\rho \times \pi R^2 V_{mean}$ kg/sec.
(Continuity)

Length of the line indicates the temp. at that point.

Just like velocities of fluid layers being a function of 'r' at a given x-s/c of the pipe due to viscous influence of the fluid whenever there is fluid flow taking place in any pipe, in the ~~simmil~~ similar manner, ~~the~~ whenever there is HT ~~and~~ b/w the pipe & the flowing fluid, the temp. of fluid layers also become a function of 'r' at a given x-s/c of the pipe.

Bulk Mean Temp. of fluid (T_b or T_m)-

T_b of fluid at a given x-s/c of pipe is defined as the temp. which takes into account the variation of temp. of fluid layers w.r.t. r at that x-s/c of the pipe by averaging the variation and hence indicates the thermal energy or enthalpy transported by the fluid through that x-s/c.

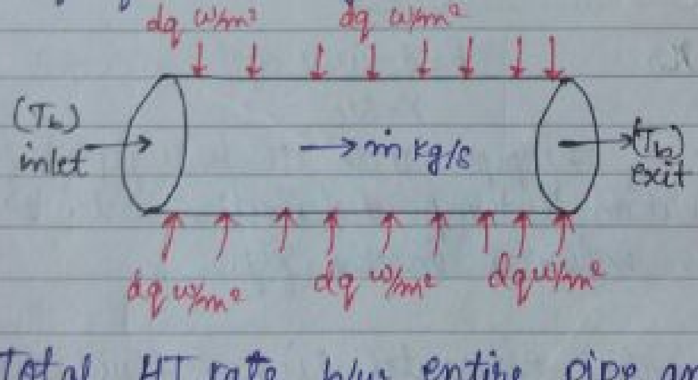


The thermal energy (or) enthalpy transported by fluid through the x-s/c $= (\dot{m} C_p T_b)$ Joule/sec.

$= (\rho \times \pi R^2 V_{mean} \times C_p \times T_b)$ J/sec.

Note: This Bulk mean temp. (T_b) of fluid must change in the dir^n of fluid flow whenever there is HT b/w the pipe and the flowing fluid.

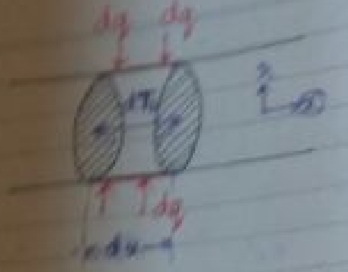
* V_{mean} doesn't change ~~wrt~~ with x if the flow is steady. If flow is steady, $\dot{m}$ is constant that means V_{mean} is constant.
as Ar of x-s/c is same



Total HT rate b/w entire pipe and flowing fluid

Q = The rate of enthalpy change of fluid from inlet to exit

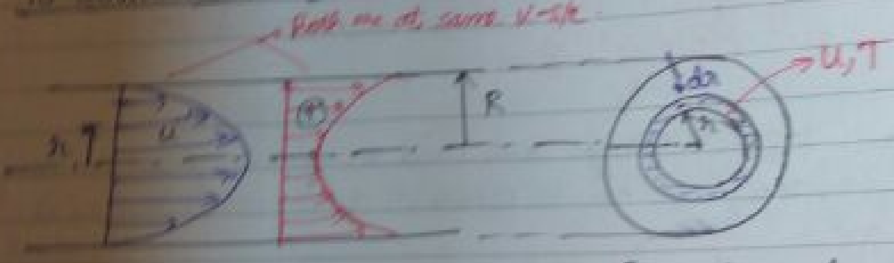
$= (\dot{m} C_p \Delta T_b) = \dot{m} C_p (T_{b_{exit}} - T_{b_{inlet}})$ J/sec



dq = Differential HT Rate between pipe & fluid (for a travel of dx) $= (\dot{m} \, C_p \, dT_b)$ J/sec.

dT_b = Differential change in BMT of fluid due to dq.

To derive a mathematical expression for BMT of fluid -



u = f(r) T = f(r) End-view of X-S/C

Consider a given X-S/C of the pipe with HT b/w pipe & fluid where the velocity distribution & temp. distribution wrt. 'r' at that X-S/C are as indicated in the fig.

Consider a small differential elemental ring of fluid flow in that X-S/C at a radius of 'r' measured from the axis where the velocity of fluid flow is 'u' and its temp is 'T'.

Let dr be the differential radius of elemental ring.

Then,

$d\dot{m}$ = Differential mass flow rate of the fluid through elemental ring.
$= \rho\,(2\pi r\,dr)\,u$ kg/sec.

Then, differential thermal energy (enthalpy) transported by fluid through elemental ring $= d\dot{m}\,C_pT$ J/sec.
$= \rho\,(2\pi r\,dr)\,u\,C_pT$. J/sec.

∴ Total thermal energy transported by fluid through entire x-s/c of pipe $= \int_0^R \rho\,2\pi r\,dr\,u\,C_pT$ — ①

But in terms of T_b,

Total thermal energy transported by fluid through entire x-s/c of pipe $= \dot{m}\,C_pT_b = \left(\rho\pi R^2 V_{mean}\,C_p\,T_b\right)$ J/sec. — ②

Equating ① & ②, we get

$$\cancel{\rho}\times\cancel{\pi}R^2 V_{mean}\,\cancel{C_p}\,T_b = \int_0^R \cancel{\rho}\,2\cancel{\pi}r\,u\,\cancel{C_p}\,T\,dr.$$

$f(r)$

$F(r)$

$$\boxed{\therefore\ T_b = \frac{2\int_0^R r\,u\,\underline{\underline{T}}\,dr}{R^2 V_{mean}}}$$

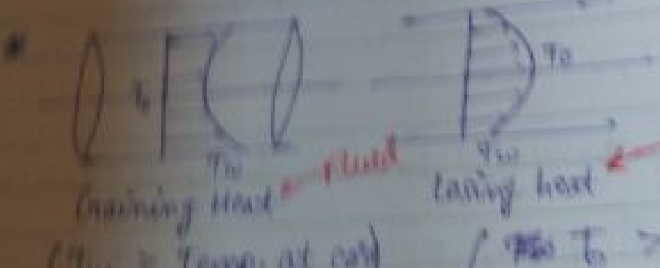
T_b मतलब T और V_{mean} के formula

T_b will change from x-section to x-section.

$$= 0.5 C_2$$

As u is constant $= C_1$

$\therefore U_{mean} = C_1$

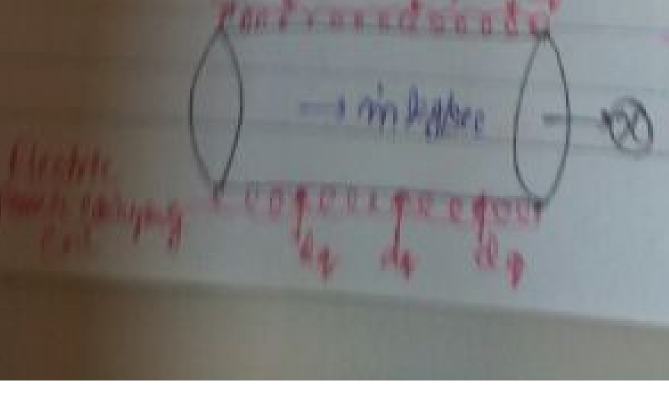


Gaining Heat ($T_w >$ Temp. at centre T_0) — Losing heat ($T_0 > T_w$)

Variation of T_m –

During forced convection HT in flow through pipes or ducts (internal flows) we come across to distinct cases

1. Constant Heat flux Conditions –

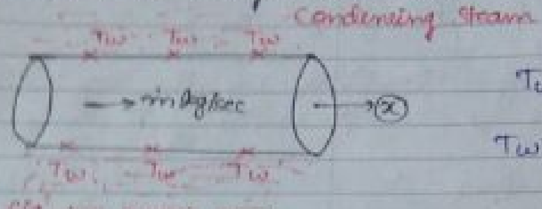


$dq \neq f(x)$

(or) dq = constant

(Local heat flux)

2- Constant wall temp. conditions-



$T_w \neq f(x)$.

T_w = Pipe Wall temp.

(*) See fig. on next page

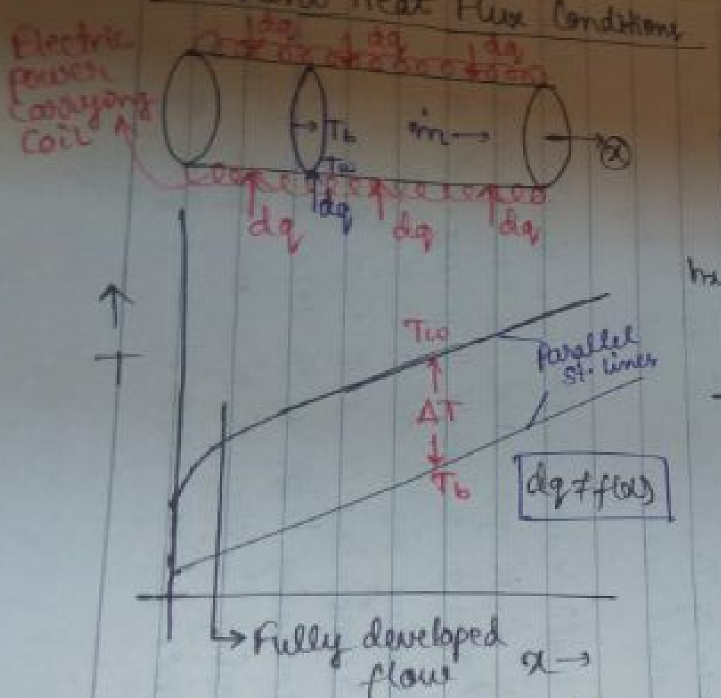
Note: During constant heat flux conditions, since both dq and h_x are remaining constant in the dir^n of ~~heat~~ fluid flow, the value of ΔT also must remain constant in the dir^n of fluid flow.

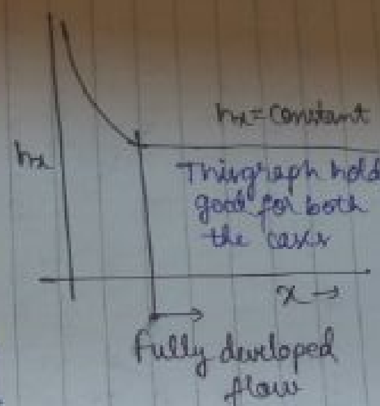
→ Unlike in case of flow over flat plates, for fully developed laminar flow through pipes, the local convective heat transfer coeff. 'h_x' remains constant in the dir^n of fluid flow.

→ During constant heat flux conditions, since ΔT is remaining constant in the dir^n of fluid flow (where $\Delta T = T_w - T_b$) and T_b has to increase in the dir^n of fluid flow (because the fluid is getting heat) the value of T_w also must increase in such a way that $T_w - T_b$ shall remain the same at any x.

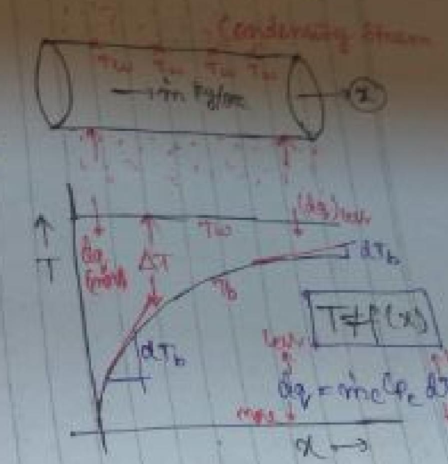
⇒ During constant wall temp. conditions, since T_w is remaining constant in the dir^n of fluid flow and T_b has to increase in the dir^n of fluid flow (because the fluid is getting heat) the value of ΔT must be decreasing in the direction of fluid flow. But since h_x is remaining constant in the dir^n of fluid flow and ΔT is decreasing with x, the local heat flux dq also must be decreasing in the dir^n of fluid flow. This is evident from the decreasing slope of the tangent of dT_b vs x.

Constant Heat Flux Conditions





Constant pipe wall temp.



Newton's law of cooling at any x-

Local Heat flux = $dq = h_x \times A \times (T_w - T_b)$ 1 (calculating flux)

$dq = (h_x \times \Delta T)\ W/m^2$

* For maintaining the constant heat flux then varying dia of coil.

T_w = Pipewall temp.

T_b = Bulk mean temp of fluid

$\Delta T = T_w - T_b$

dq = Local Heat flux

$x \rightarrow$ Direction of fluid flow

Note: When dq = constant (During constant heat flux conditions)
$\Rightarrow T_w$ is increasing with x.

When T_w = constant (Constant wall temp. conditions).
$\Rightarrow dq$ is decreasing with x.

Hence it is just not possible to maintain both constant heat flux conditions and const wall temp. cond^{n}s simultaneously at the same time.

Note: h_x = constant

Local Nu. No. = $Nu_x = \frac{h_x D}{k}$ = constant.

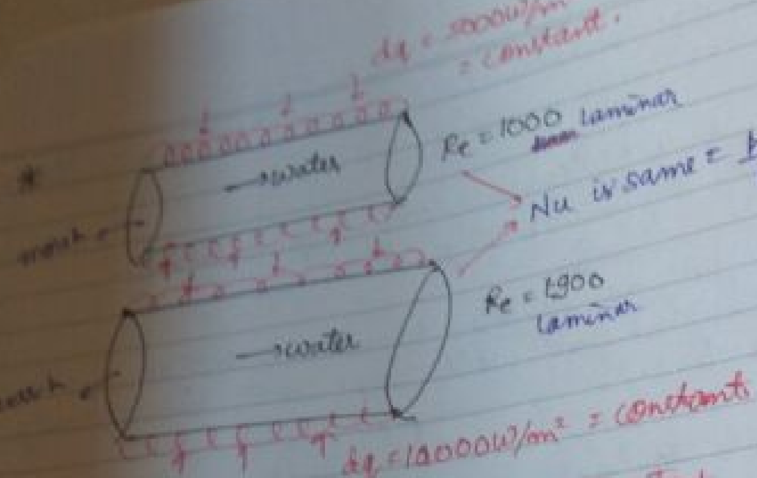
Thus for fully developed laminar flow through pipe, local Nu. No. remains constant in the dir^n of fluid flow during both constant heat flux conditions as well as constant pipe wall temp. conditions.

$\therefore$ For fully developed laminar flow through pipes/ducts ($Re < 2000$).

$\boxed{Nu = \frac{hD}{k} = 4.36}$ (During constant heat flux conditions).

$\boxed{Nu = \frac{hD}{k} = 3.66}$ (During constant wall temp. conditions).

** * These values are applicable for any type of pipe or dimensions. Only condition is flow should be laminar.

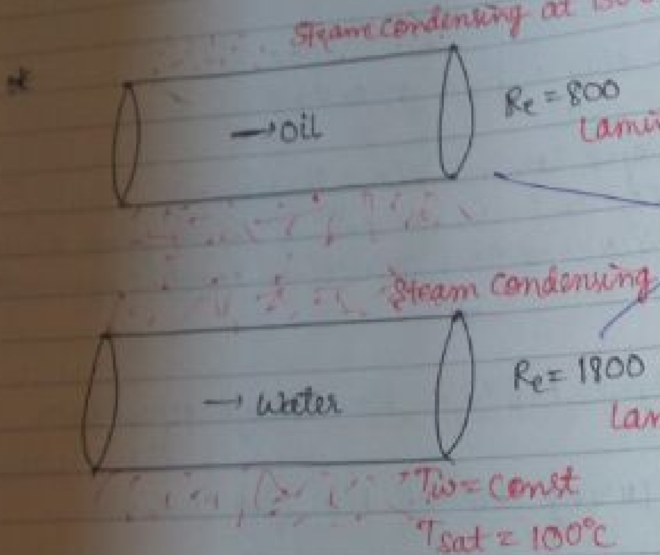


$Re = 1000$ laminar

$Re = 1900$ laminar

Nu is same $= \frac{hD}{K} \Rightarrow$ more D, ↓↓ less h.

Steam condensing at 150°C = T_{sat}



$Re = 800$ laminar

$Re = 1800$ laminar

Nu is same $= 3.66$
$= \frac{hD}{K}$

$h_{with\ water} > h_{with\ oil}$
as
$K_{water} > K_{oil}$

<u>For fully developed Turbulent flow through pipes/ducts</u> ($Re \geq 4000$)

'h' value can be obtained ~~to~~ from

$$\boxed{\frac{hD}{K} = Nu = 0.023\, Re^{0.8}\, Pr^{n}}$$ → McAdam's eq^n or Dittus Boelter's eq^n.

where $n = 0.4$ for heating of fluid
$n = 0.3$ for cooling of fluid

28- $Nu = 0.023 Re^{0.8} Pr^{0.4}$.

$$\frac{hD}{K} = 0.023\left(\frac{\rho V D}{\mu}\right)^{0.8} \times (4.35)^{0.4}$$

$$\boxed{h = 4613\ W/m^2\text{-}K}$$

Z- $Nu = 0.023 Re^{0.8} Pr^{0.33}$

$\cancel{8.4 = 0.023 Re^{0.8} \times Pr}$

$$Re = \frac{\rho V D_h}{\mu} = \frac{\cancel{1.2} \times 10 \times \cancel{2}}{\cancel{3} \times \cancel{1.8} \times 10^{-5}} = 4.44 \times 10^5.$$

$$D_h = \frac{4A_{cs}}{P} = \frac{4 \times \cancel{0.5} \times 1}{\cancel{2} \times \cancel{1.5}} = \frac{2}{3}$$

$\because$ Re is 4.44×10^5.

Since internal flow and $Re > 4000$. Hence flow is turbulent.

$$Nu = \frac{hD}{K} = 0.023 Re^{0.8} Pr^{0.33}$$

$$h = \frac{0.025 \times 3 \times 0.023}{2} (4.44 \times 10^5)^{0.8} \times (0.73)^{0.33}$$

$h = 25.61\ W/m^2 K$.

Rate of H.T. per unit length of Duct $= h A_{conv.} (\Delta T)$ $P \times 1m$

$= 25.61 \times 3 \times 1 \times (30 - 20)$

$= 769$ Watt.

Since heat flux being constant at any x,
Total HT Rate b/w entire pipe and fluid.

$= q_w \times \text{Area of HT} = 5000\pi DL = \dot{m}C_p(T_{be} - T_{bi})$ W

$(T_b)_{exit} = 76°C.$

$q_w = h \times 1(T_{w\,exit} - T_{b\,exit})$ W/m².

$5000 = 1000 \times (T_{w\,\text{at exit}} - 76)$ W/m².

$(T_w)_{\text{at exit}} = 81°C.$ ✓

~~8~~-12- Re < 1500.

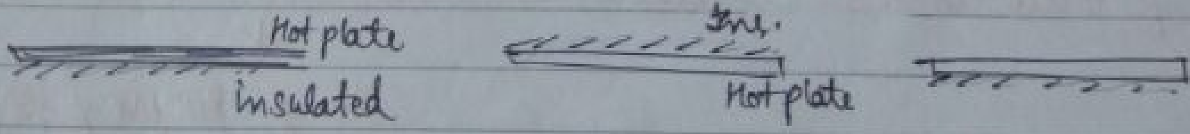
$Nu = \frac{hD}{k} = 4.36.$

$h = 43.6.$

Similarly, $h = 36.6$ W/m²K

<u>Free or Natural Convection</u> Isobaric processes.

No velocity evident but the flow occurs ~~mathematically~~ naturally due to Buoyancy forces arriving out of density changes of fluid bcoz of its ~~the~~ temp. changes.



In any free convection HT, was not considered anywhere in forced convections

$$h = f(g, \beta, \underline{\Delta T}, L, \underbrace{\mu, \rho, C_p, K}_{\text{Thermophysical property of fluid.}}).$$

g = Acc^n due to gravity.
β = Isobaric volume expansion coeff. of fluid.
$= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \; K^{-1}$.

Note: β of a fluid signifies how much volume changes a given fluid may undergo for a given temp. change of the fluid.

$$\beta_{air} > \beta_{water}$$

If β value is more $\Rightarrow (\Delta V)$ is more $\Rightarrow (\Delta \rho)$ is more

$\Downarrow$

Stronger Buoyancy forces.

For ideal gas, like air,

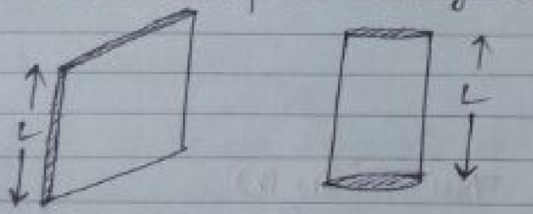
$\boxed{\beta = \frac{1}{T} /K}$ $\boxed{\beta_{air} = \frac{1}{T_{mean} \text{ in } K}}$

$\left(\frac{T_w + T_\infty}{2}\right)$

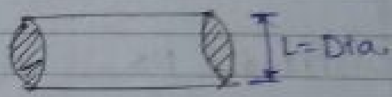
* $\Delta T = (T_w - T_\infty)$.

L = Characteristic dimension of body
(Dimension of body i.e. used in the calculation of dimensionless nos.)

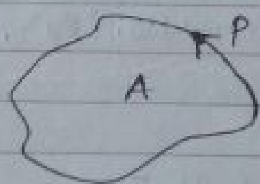
For vertical plates and cylinder,



Horizontal Cylinder

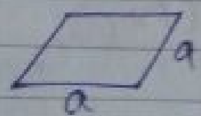


For Horizontal plate,



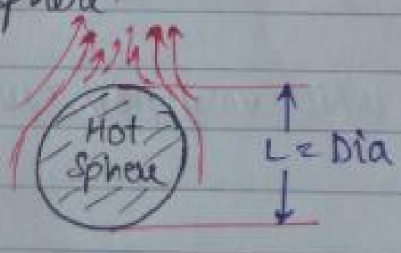
$\boxed{L = \frac{A}{P}}$

eg.,



$L = \frac{a^2}{4a}$

For sphere -



Note! All the variables in free convection HT are grouped into 3 dimensionless numbers from dimensional analysis which are given as

(i) Grashoff No. = $Gr = \frac{g \beta \Delta T L^3}{\nu^2} = \frac{\text{Inertia force} \times \text{Buoyancy force}}{(\text{Viscous force})^2}$

ν → KV of fluid

→ Gr replaces Re in free convection HT.

→ Gr signifies the magnitude of Buoyancy forces (Since it contains β).

→ Take all decimals of Gr. → नहीं तो answers में काफी difference आता है।

(ii) Nusselt No., $Nu = \frac{hL}{k}$.

(iii) Prandtl No., $Pr = \frac{\mu C_p}{k}$.

∴ In any free convection HT, $Nu = f(Gr \times Pr)$

⇓

Product of Gr & Pr. This product is called Rayleigh No. (Ra).

∴ In free convection HT,

$Nu = f(Ra_L)$

↳ means using characteristic dimension.

* Usually the functional relationship appears as -

$Nu = C(Gr\,Pr)^m$

where C & m are constants which vary from case to case.

$m = \frac{1}{4}$ for laminar flow

$m = \frac{1}{3}$ for turbulent flow