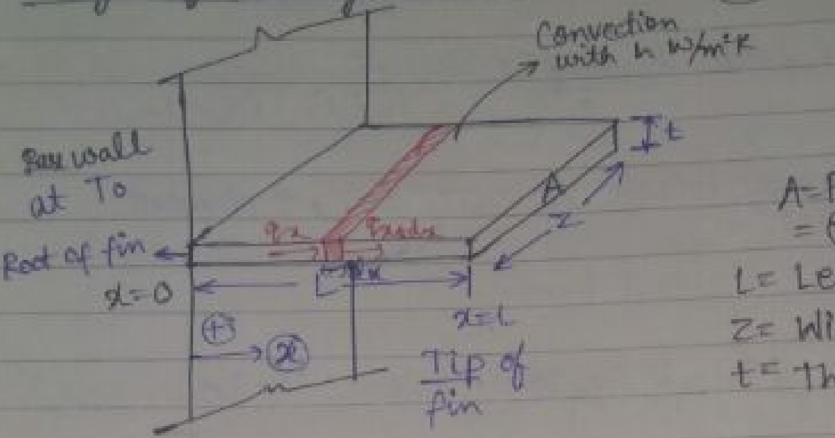
Because liquids are already having high 'h'. No need of fins.

Note: Fins are always used only with the gases (air) i.e. when the convect. HT coeff. are relatively low.

Analysis of Rectangular Fin-



A- Profile Area $= (Z \times t)\,m^2$.
L = Length of fin
Z = Width of fin
t = Thickness of fin

Note: The actual mechanism of HT in any fin is first heat gets conducted into the fin at its root ($x=0$) and then while conducting along the length of the fin i.e. in the x-dirⁿ, heat is also simultaneously convecting from the surface of the fin into the ambient fluid at T_∞ with a convective HT coeff. is h W/m²K.

Objectives:
(i) To get temp. distribution within the fin i.e. $T = f(x)$.
(ii) To get heat transfer rate two fin $q_{fin} = ?$

Assume: Steady state H.T conditions i.e. $T \neq f(\text{time})$

Consider a differential element of fin of length dx at location of x from root of fin where the temp. of the fin is T.

Let q_x = Heat conducted into the element $= -KA\frac{dT}{dx}$.

q_{x+dx} = Heat conducted out of the ~~fluid of~~ element $= q_x + \frac{\partial}{\partial x}(q_x)\,dx$.

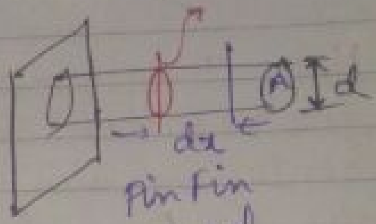
Heat convected from the surface element
$= q_{conv} = h(P\,dx)(T - T_\infty)$.

Writing the energy balance for steady state of element

$$q_x = q_{x+dx} + q_{conv}.$$

$$\cancel{q_x} = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)\,dx + h(P\,dx)(T - T_\infty)$$

$$0 = \frac{\partial}{\partial x}\left(-KA\frac{dT}{dx}\right)\cancel{dx} + hP\cancel{dx}(T - T_\infty).$$



Pin fin
$P = \pi d$
$A = \frac{\pi}{4}d^2$.

$$\frac{d^2T}{dx^2} - \frac{hP}{KA}(T - T_\infty) = 0.$$

Let $T - T_\infty = \theta = f(x)$.

$$\frac{dT}{dx} = \frac{d\theta}{dx}$$

$$\frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$$

And Put $m^2 = \frac{hP}{KA}$.

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0,$$

This is a standard format of second order differential eq^n in θ whose sol^n can be directly given as

$\theta = C_1 e^{-mx} + C_2 e^{mx}$. where $m = \sqrt{\frac{hP}{kA}}$ /metre

C_1 and C_2 are constants of integration that are to be obtained from boundary conditions.

→ One common boundary condition is at
At $x=0$ (ie. at the root of fin) ⇒ $T = T_0$ (base temp.)
& $\theta = \theta_0 = T_0 - T_\infty$.

→ The second boundary condition depends upon 3 diff. cases of fin.

<u>Case 1</u>- Fin is infinitely long or very long fin.

Then the temp. at the tip of the fin will be essentially that of the ambient fluid.

ie. At $x = \infty$ ⇒ $T = T_\infty$ and hence $\theta = 0$.

Then the sol^n for temp. distribution within the fin in Non-dimensional format is -

$$\boxed{\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}}$$

For any fin case,
HT Rate thro fin = q_{fin} = Heat conducted into the fin at its root/base.

$$q_{fin} = -KA\left(\frac{dT}{dx}\right)_{at\ x=0}$$

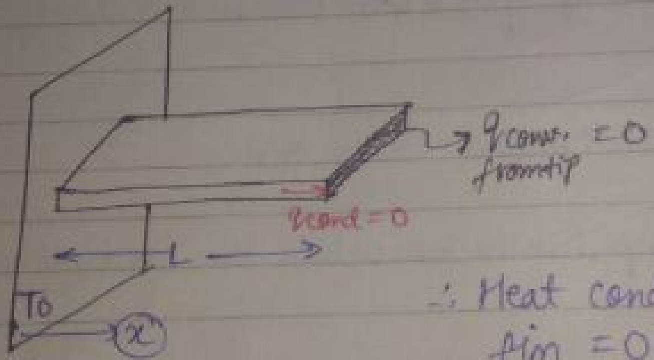
$$\because T = (T_0 - T_\infty)e^{-mx} + T_\infty$$

$$\Rightarrow \frac{dT}{dx} = (T_0 - T_\infty)e^{-mx} \cdot (-m).$$

$$\therefore \boxed{q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\ \text{Watt}}$$

* Practically not useful bcoz fins not even long in practical applications.

**
Case II – Fin is finite in length but its tip is insulated –



$\therefore$ Heat conducted into the tip of fin $= 0$.

$$\Rightarrow -KA\left(\frac{dT}{dx}\right)_{x=L} = 0 \Rightarrow \left(\frac{dT}{dx}\right)_{x=L} = 0$$

$$\Rightarrow \left(\frac{d\theta}{dx}\right)_{x=L} = 0$$

Then the solution for temp. distribution within the fin is –

$$\boxed{\frac{T-T_\infty}{T_0-T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}}$$

Note: In practice we never insulate the tip of the fin rather we neglect the convection heat loss from the tip.

$$q_{\substack{conv.\\ from\ tip}} = \underbrace{h A_{tip} (T_{x=L} - T_\infty)}_{\text{Product is too small}}.$$

→ In any fin problem, if no case is mentioned, by default assume case 2.

Then, q_{fin} = HT Rate thro' fin

$$= -KA\left(\frac{dT}{dx}\right)_{at\ x=0}$$

$$\boxed{q_{fin} = \sqrt{hPKA}\,(T_0-T_\infty)\tanh mL} \text{ Watt.}$$

Case 3- Fin ~~too~~ is finite in Length but tip is ~~insul~~ uninsulated (Convection Heat loss from tip also).

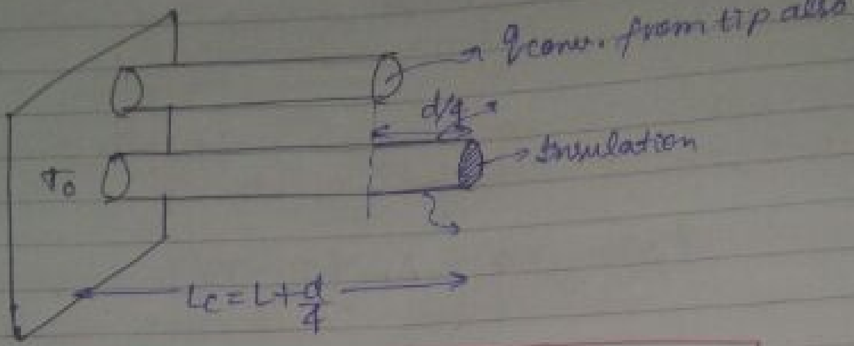
Then the solution for temp. distribution within the fin is -

$$\boxed{\frac{T-T_\infty}{T_0-T_\infty} = \frac{\cosh m(L_c-x)}{\cosh mL_c}}$$

where L_c = ~~Connected~~ Corrected length of fin

$= L + t/2$ (for Rectangular fin)

$= L + \frac{d}{4}$ (for pin fin)



$$\boxed{q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh mL_c \text{ watt}}$$

54 (b) $q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh mL$

$$= \sqrt{40 \times \pi \times 5 \times 10^{-3} \times 400 \times \frac{\pi}{4} \times (5 \times 10^{-3})^2} \times 100 \times \tanh m \times 0.1$$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{40 \times \pi d \times 4}{\underset{10}{\cancel{400}} \times \pi d^2}} = 8.$$

$$= 4.66 \simeq \boxed{5.0}$$

54 $L_c = 30 + 0.1 = 0.301$ ~~cm~~ m.

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{15 \times \cancel{12}\ 0.604}{204 \times \cancel{0.08}\ 6 \times 10^{-4}}} = 8.603.$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L_c - x)}{\cosh(mL_c)} = \cancel{\frac{4.99}{6.69}}\ \cancel{\approx 0.745}$$

$$\frac{T - 30}{270} = \cancel{0.745}\ 0.15$$

$$\boxed{T = 70.3°C}$$

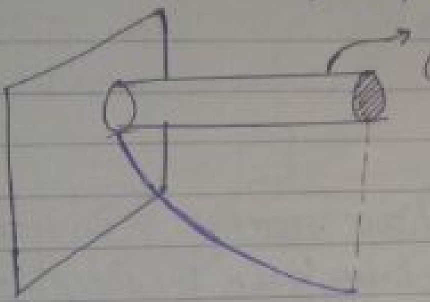
$$q_{fin} = \sqrt{hPKA}\ (T_0 - T_\infty)\tanh mL_c \text{ watt.}$$

$$= \boxed{281.1 \text{ watt.}}$$

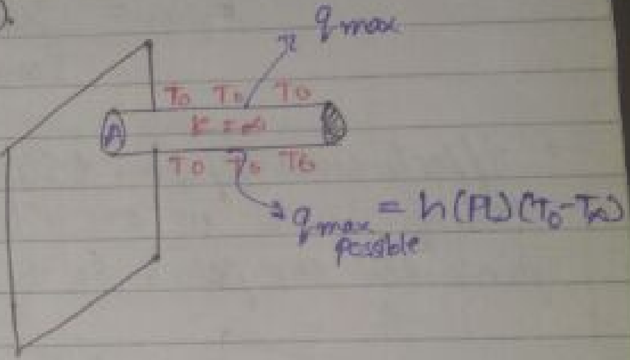
→ **Fin efficiency-**

Fin efficiency is defined as the ratio b/w actual HT rate taking place through the fin and the max. possible HT rate that can occur through the fin i.e. when the entire fin surface is at its root or base temp.

$$\eta_{fin} = \frac{q_{actual}}{q_{max.\ possible}}$$



$q_{act} = \sqrt{hPKA}\ (T_0 - T_\infty)\tanh mL$ watt (Case 2).



→ The entire fin will be at its ~~room~~ root temp only if the material of the fin has infinite thermal conductivity.

$$\therefore\ \eta_{fin} = \frac{\sqrt{hPKA}\ \cancel{(T_0 - T_\infty)}\tanh mL}{h \times (PL)\ \cancel{(T_0 - T_\infty)}}$$

$$= \frac{\tanh mL}{\sqrt{\frac{hP}{KA}}\ L} = \boxed{\left(\frac{\tanh mL}{mL}\right)} \text{ for Case 2.}$$

$$\boxed{\eta_{fin} = \left(\frac{\tanh mL_c}{mL_c}\right)}$$ for case ③.

Since, $\boxed{\eta_{fin} \propto \sqrt{K}.}$

Fin material must have high K.

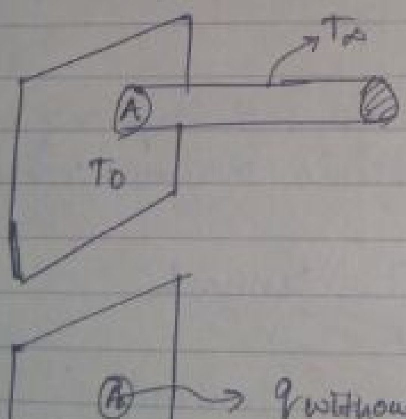
$\underline{5f}$ (iii) $\eta_{fin} = 38.18\%$.

Fin Effectiveness (ϵ_{fin}) -

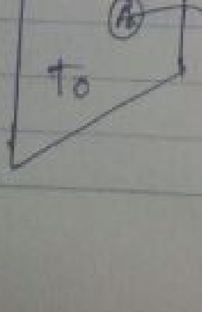
→ Fin effectiveness is defined as the ratio b/w HT rate with fin and the HT rate w/o fin.

$$\epsilon_{fin} = \frac{q_{with}}{q_{without}}.$$

→ Effectiveness of fins tells about how much % ↑ in heat transfer rate we are able to achieve/gain by keeping the fins as compared to the case where there is no fin.



$q_{with\,fin} = \sqrt{hPKA}\,(T_0 - T_\infty) \tanh mL_c$ watt



→ $q_{without\,fin} = hA\,(T_0 - T_\infty)$

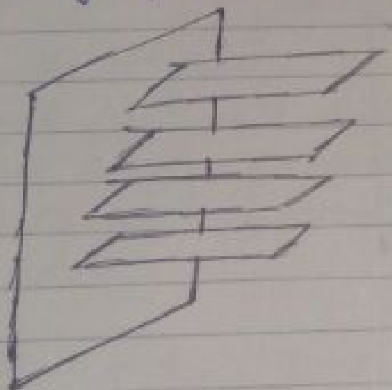
Note: If the effectiveness of fin is low it means that fins are not worth keeping since they do not help us much in ↑ H.T rate. Fins are justified only when the effectiveness is 2 or more than 2. $\epsilon_{fin} \geq 2$.

→ $$\epsilon_{fin} \propto \frac{1}{\sqrt{h}}.$$

This is the reason why fins are never used with water since h with water being higher, effectiveness of fin will be low.

$$\epsilon_{fin} \propto \sqrt{\frac{P}{A}}.$$

∴ To have high effectiveness of fin, fins must be thin & closely spaced.



No. of fins required

$$n = \frac{\text{Total heat i.e. to be transferred}}{\text{H.T rate through each fin.}}$$

$$\epsilon_{fin} \propto \sqrt{K}.$$

∴ Fin material must have high K. (Al).

→ Small thickness and z is kept small. L is also not long.

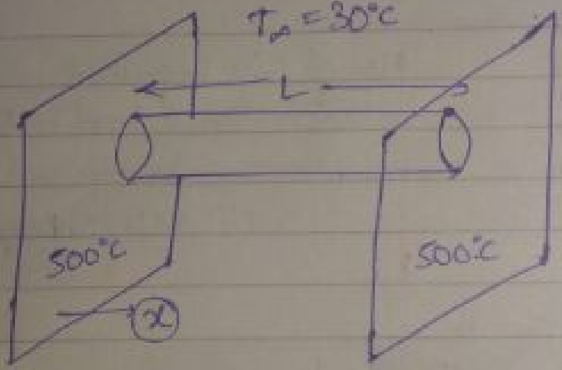
55- $\epsilon = \frac{q_{with}}{q_{w/o}} = \frac{\sqrt{hPKA}\,(T_0 - T_\infty)}{hA\,(T_0 - T_\infty)}$

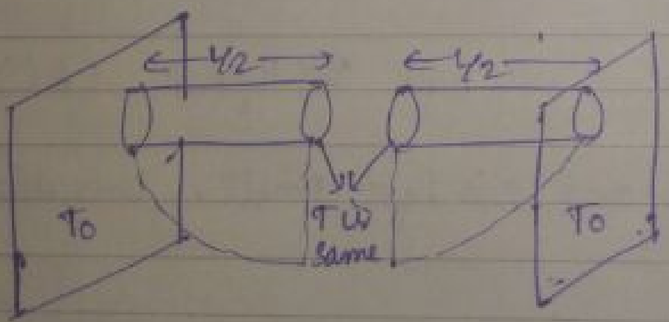
(b) $= \boxed{\left(\frac{PK}{hA_c}\right)^{1/2}}$.

33- 1- Fins are placed according to the gas side or fluid side.
(d) In the case of radiator, if fins are placed once with hot air and then with cold water, they can't be effective equally.

4- Free convection. Hence, flow doesn't matter.
is also possible and most of the case that only happens

21-





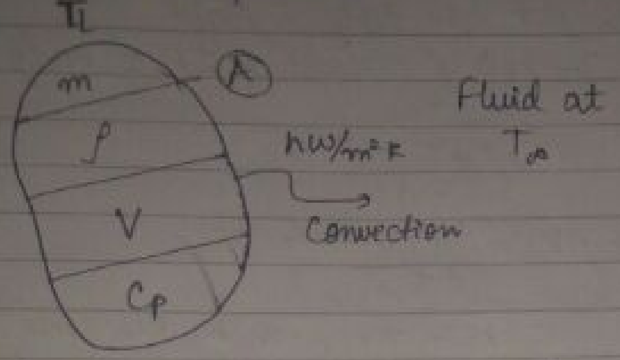
When rods are joined
No heat conduction b/w ends of rods.

$-kA\frac{dT}{dx} = \cancel{}\ 0,$

$\Rightarrow \boxed{\frac{dT}{dx} = 0}$, (d)

→ Unsteady State or Tranient Conduction H.T.-

$T = f(\text{time})$



Consider a solid body of mass 'm', volume V, density ρ, specific heat C_p, which is at an initial temp. of T_i having been heated in a furnace is suddenly taken out of the furnace and exposed to the ambient fluid at T_∞.

Since the body keeps on losing heat by convection to the ambient fluid with a convective HT coeff. of 'h' W/m^2K, the internal energy of the body keeps on decreasing as the time progresses which is manifested by decrease in temp. of the body wrt time.

Let T_i = Initial temp. of the body at the instant of time $\tau = 0$ sec., ie when the body is just exposed to ambient fluid.

Let T = Temp. of the body at any instant of time 'τ' sec. later

$T = f(\tau)$.

Writing the energy balance for the body at any instant of time 'τ' sec.

The rate of convection H.T b/w body & fluid
= The rate of decrease of IE of body wrt time

$$hA(T-T_\infty) = -mC_p\left(\frac{dT}{d\tau}\right) \text{ J/sec.}$$

$$= -\rho V C_p\left(\frac{dT}{d\tau}\right) \text{ J/sec.}$$

(-) bcoz temp. is ↓ with time. If it would be ↑ then (+) would come.

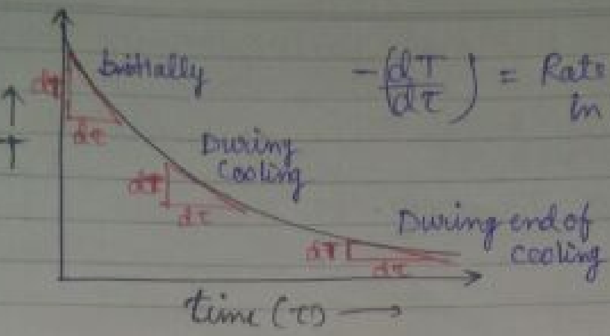
→ Treating all other parameters including 'h' as constant and separating the variable time & temperature, we get

$$\int_0^\tau \frac{hA}{\rho V C_p}\, d\tau = \int_{T_i}^{T} -\frac{dT}{(T-T_\infty)}.$$

$$\frac{hA}{\rho V C_p}\,\tau = \ln(T-T_\infty)\Big|_{T}^{T_i} = \ln\left(\frac{T_i-T_\infty}{T-T_\infty}\right).$$

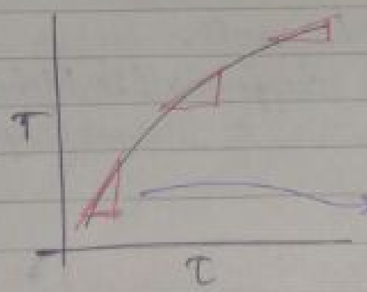
$$\boxed{e^{\left(\frac{hA}{\rho V C_p}\right)\tau} = \frac{T_i-T_\infty}{T-T_\infty}.}$$

→ Hence in any unsteady state HT, the temp. of the body changes exponentially wrt time as shown in fig.



$-\left(\frac{dT}{d\tau}\right)$ = Rate of cooling in K/sec.

Note: During unsteady state heat transfer, the rate of cooling or heating of body itself becomes a function of time. Initially the rate of cooling is very high due to high convection heat transfer rate b/w body and fluid because of large temp. difference existing b/w them. But as the time progresses this rate of cooling keeps on decreasing because of the ↓ in convection heat transfer rate due to the decrease in temp. difference b/w body & fluid.



$\left(\frac{dT}{d\tau}\right)$ = Rate of heating in K/sec.

Initially the rate is high bcoz of temp. difference.

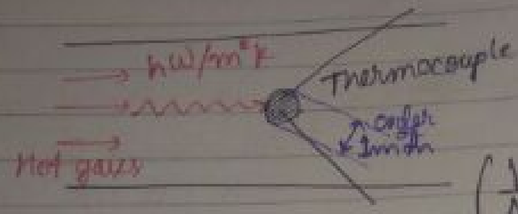
* When temp. diff. is more → Rate is more.

Time constant – Since the group $\left(\frac{\rho V C_p}{hA}\right)$ has got units of 'sec'.

It is called time constant of body.

Note: This time constant of a body ~~too~~ signifies how much time will be taken by a body in approaching the temp. of the thermal ambient (hotter or cooler) when the body is suddenly exposed to the ambient.

Thermocouple or Thermopile-



$$\left(\frac{V}{A}\right)_{\text{Thermocouple Bead (Tiny sphere)}} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3},$$

While measuring the temp. of hot fluids flowing in a duct by using thermocouple, it must have very small time constant for which

(i) The size of the bead must be small.

(ii) The convective heat transfer coeff. must be high.

(iii) The heat capacity of material of thermocouple must be small.

Note: In the above analysis done it is assumed that the temp. of the body is uniform throughout its mass at any instant of time i.e the internal temp gradients within the body are neglected. Such analysis is known as Lumped Heat Capacity Analysis. In this analysis, the temp. of body is a function of time but it is not a f^n of space and we must see the same temp. throughout the body at any specific time.

Criteria for Lumped Heat Analysis-

Biot No. < 0.1 $\longrightarrow$ When biot no. < 0.1, temp. is not a f^n of space.

where, $\boxed{\text{Biot No.} = \frac{hs}{K_{solid}}}$

where $s = \frac{\text{Volume of body}}{\text{Surface of body}} = \left(\frac{V}{A}\right)$

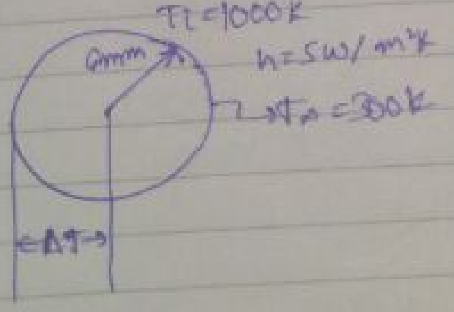
For sphere, $s = \frac{R}{3}$.

$$\text{Biot No.} = \frac{(S/KA)}{(1/hA)} = \frac{\text{Internal conductive resistance offered by a body}}{\text{External convective Resistance}} = \frac{ICR}{ECR}.$$

Note: Low Biot No. values signify that the body offers very little or small conductive resistance for any internal heat transfer within the body as compared to surface convective resistance thereby leveling the temp. diff. that may exist b/w any 2 locations within the body.

eg., For any metallic body of reasonably small size biot no. will be less than 0.1.

36-
(d)



$$\text{Biot No.} = \frac{hS}{k} = \frac{h \times R}{3k}$$

$$= \frac{5 \times 0.006}{3 \times 20} = 0.0005 \ll 0.1$$

$\Rightarrow ICR \ll ECR.$