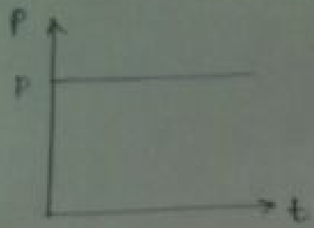


Fig: Dead Load

↳ Wt. of Structural & m/c components

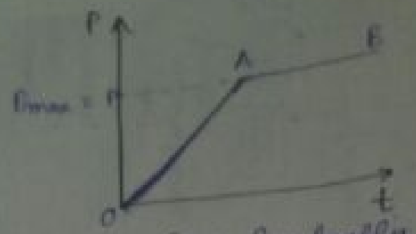
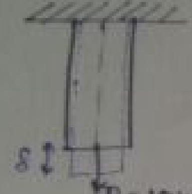


Fig: Gradually Applied Load

↳ All external forces are assumed to be gradually applied load in SOM.

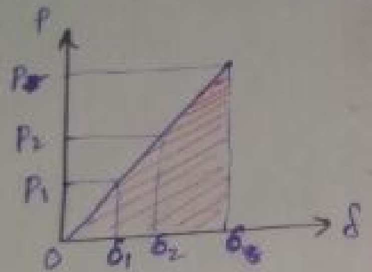


P = 10 kN = Pmax

→ gradually applied load varying from (0 to 10) gradually.

$$\boxed{W.D \text{ by } P = \frac{1}{2} P \cdot \delta}$$

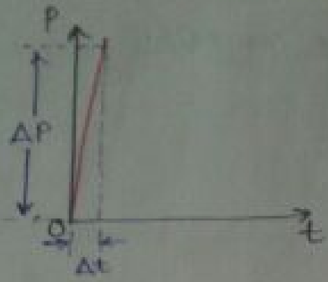
↳ Avg. of P is taken bcoz P is gradually increased.



$$\boxed{W.D. \text{ by } P = \text{Area of } P \text{ v/s } \delta \text{ curve}}$$

Impact Loads – Loads which are acting for a short interval of time.

eg., Springs used in shock absorber;
IC engine components during power stroke like connecting rod & piston;
Spur gear tooth at a higher velocities;
Components used in punching & forging operations.



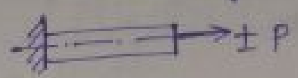
* Impact loads are directly dependent on the velocity of appⁿ.

Fatigue Load - Fatigue Load is a load whose magnitude or dirⁿ or both magnitude & dirⁿ changes wrt. time & same load is repeatedly applied.

→ Only dirⁿ changes - (Completely reversed fatigue load)

⇓

Pulling & pushing with Same mag.

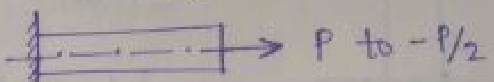


→ Only mag. changes - Fluctuating fatigue Load

⇓

→ Both changes - Alternating fatigue Load

⇓



Impact Load -

$$\delta_{IMPACT} = \delta_{static} \times I.F.$$

$$\sigma_{IMPACT} = \sigma_{STATIC} \times I.f.$$

where, I.f = Impact factor

$$= 1 + \sqrt{1 + \frac{2h}{\delta_{static}}} \qquad h = \frac{v^2}{2g}$$

* δ_{static} & σ_{static} are obtained by using SOM eqⁿs.

$$\boxed{I.F. \geq 2}$$

$$\boxed{\sigma_{impact} \geq 2\,\sigma_{static}}$$

$$\boxed{\delta_{impact} \geq 2\,\delta_{static}}$$

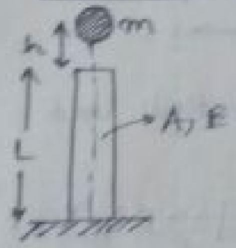
If $h \to 0 \Rightarrow I.F. \simeq 2$.

↳ Impact load is known as ~~Static~~ Suddenly applied load or Instantaneous load.

$$\boxed{\sigma_{SAL} = 2\,\sigma_{static}}$$

$$\boxed{\delta_{SAL} = 2\,\delta_{static}}$$

Q-

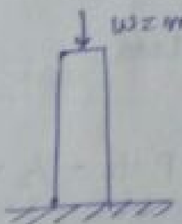


$mgh = \frac{1}{2} mg \cdot \delta$ ✗

$\delta_{static} = 2h$.

$mg(h+\delta_2) = WD$ by W.

Can't we Avg. WD bcoz this is not the case of static loading



Considering static load,

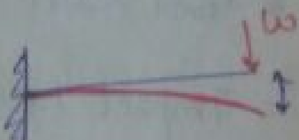
$$\delta_{static} = \frac{WL}{AE}$$

$$I.f. = 1 + \sqrt{1 + \frac{2h}{\delta_{static}}}$$

$$\therefore \delta_{impact} = I.f \times \delta_{static}$$

Ex-

$\equiv$



$\delta_{static} = \frac{WL^3}{3EI}$.

WD by W = $W[h+\delta_I]$.

$= \underbrace{\frac{(\sigma_I)^2}{2E} \times Vol.}_{\text{Resilience of bar}}$

~~$W[h+\delta_I] = \frac{(\sigma_I)^2}{2E} \times$~~

WD by W = Resilience of bar

$W[h+\delta_I] = \frac{\sigma_I^2}{2E} \times A \times L.$

$\Downarrow$

$\frac{\sigma_I L}{E}$.

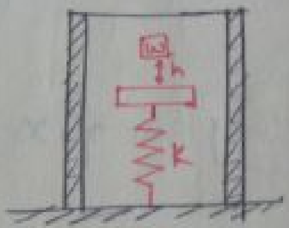
$\sigma_I = \underbrace{\frac{W}{A}}_{\sigma_{static}}\left[1+\sqrt{1+\frac{2h}{\underbrace{\left(\frac{WL}{AE}\right)}_{\delta_{static}}}}\right]$

Impact load ($\downarrow$) ⇒ (a) selecting a material with lower E.

(b) Providing more length

(c) — " — area.

Ex-



$\downarrow W = 2kN$

$K = 1000 N/mm$

Det. δ_{impact}?

If $W = 2kN$

$K = 1000 kN/mm$

$h = 10mm$

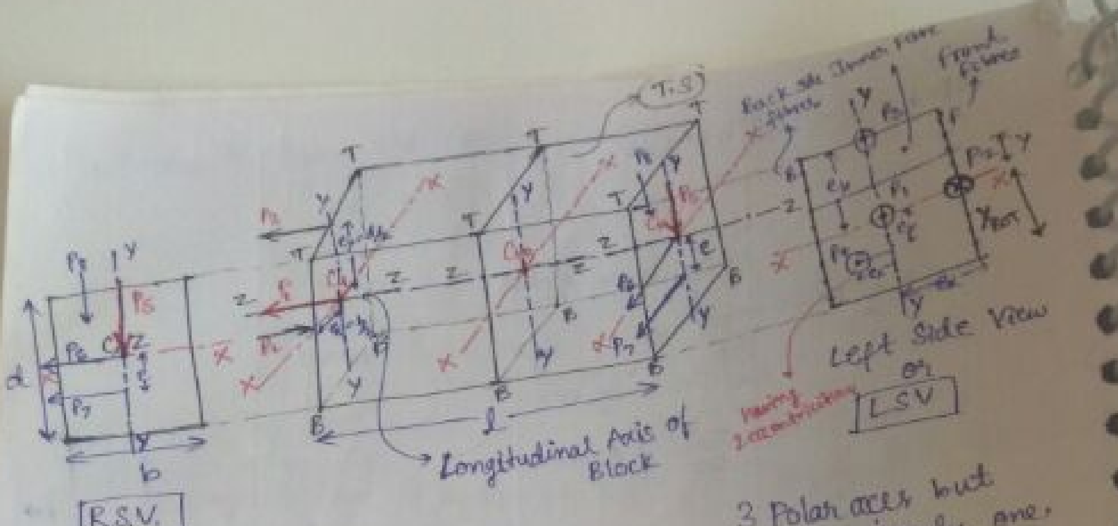
$\delta_{static} = \frac{W}{K} = 2mm.$

$I.F. = 1+\sqrt{1+\frac{2\times10}{2}}$

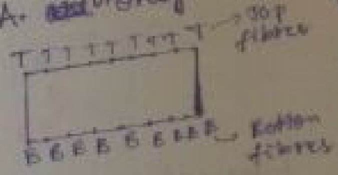
$= 4.316.$

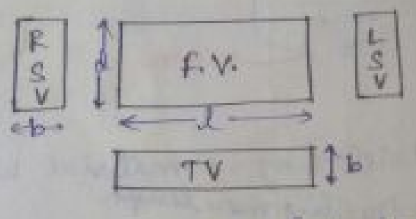
$\delta_{impact} = \delta_{st} \times I.F$

$= 8.632 mm.$



3 Polar axes but L.A. is only one.

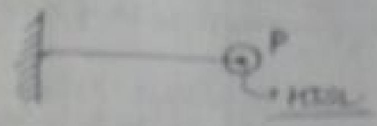




I-Angle Projection

* X-X, Y-Y & Z-Z are the centroidal Axis of a X-S/c which are mutually ⊥ to each other.
* X-X & Y-Y are the CA which are in the plane of X-S/c
* X-X is a horizontal CA (H.CA)
  Y-Y is a vertical CA (V.CA.)
* Z-Z is a polar axis (P.A)
  ↳ CA is ⊥ to plane of X-S/c
* L.A of a member coincides with polar axis of the X-S/c
* X, Y, Z are arbitrary and can be changed acc. to the question.

Eccentricity- It distance b/w centroid on longitudinal axis from the line of action of ~~force~~ load.



$P_1 \Rightarrow$ ATL

$P_2 \Rightarrow$ EACL ($e_x = e = b/2$)

P_3 & $P_4 \Rightarrow$ EATL ($e_x = e_y$)

$e_y = d/2$

$P_5 \Rightarrow$ VTSL

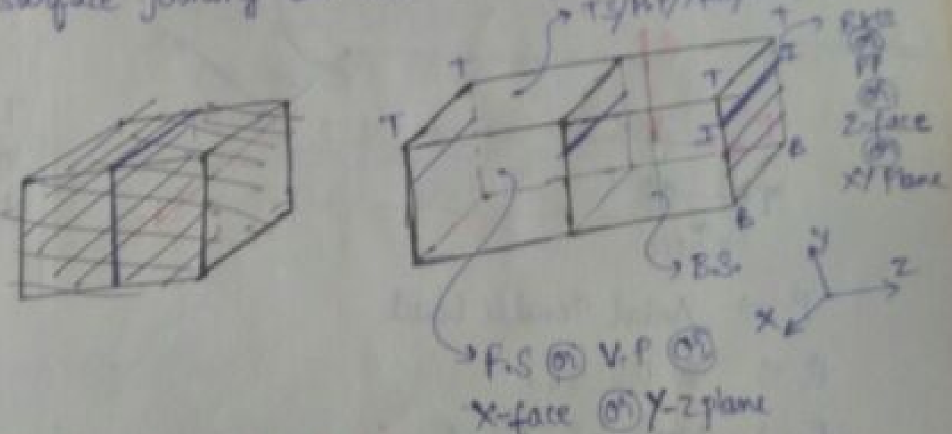
$P_6 \Rightarrow$ HTSL

$P_7 \Rightarrow$ HETSL

$P_8 \Rightarrow$ VETSL

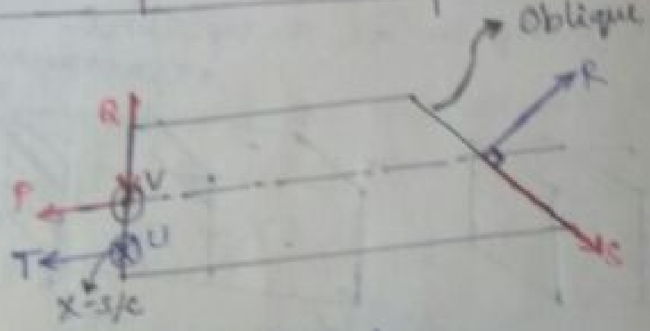
$\odot^P \Rightarrow$ A force which is ⊥ & away from the given fig.

Top Surface- A surface joining ~~the~~ all the top fibres.



Type of load	Plane of X-S/c	L.A. of Member	C.A. of X-S/c
Axial load	⊥r to P.O. X-S/c & passes through the centroid of X-S/c.	Along the L.A of member.	Along the P.A (Polar axis) of X-S/c.
Eccentric Axial load	⊥r to plane of X-S/c but away from the centroid of X-S/c.	‖ to L.A of member.	‖ to PA of X-S/c.
Transverse Shear load	‖ to PO X-S/c & passes through the centroid of X-S/c.	⊥r to LA of member & intersects LA.	⊥r & intersects PA.
Eccentric Transverse Shear load	‖ to PO X-S/c but away from the centroid of the X-S/c.	⊥r to LA but doesn't intersects LA.	⊥r to PA but doesn't intersects PA of X-S/c.
Bending Couple	⊥ to PO X-S/c	Along the L.A.	Acts about either HCA or VCA.
Twisting Couple	‖ to P.O. X-S/c	⊥ to L.A.	Acts about PA.

Eg:



T → EATL
V → HTSL
U → HETSL

P ⇒ Axial Tensile Load

Q ⇒ VTSL.

R ⇒ Normal Load (But not a tensile load bcoz of not acting along the L.A & not ⊥ to X-S/c).

S ⇒ Shear Load (But not a transverse load bcoz of not acting ⊥r to the LA & not ‖ to X-S/c).

∴ Normal Loads → P & R
(P ↓ ATL)

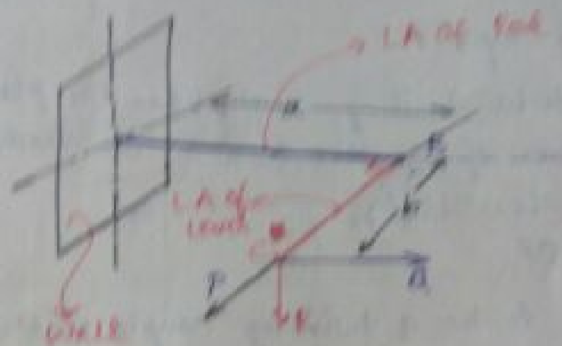
Shear Loads → Q, & S
(Q ↓ VTSL)

Note:-

\# Every Normal load is not an Axial load but every axial load is a normal load.

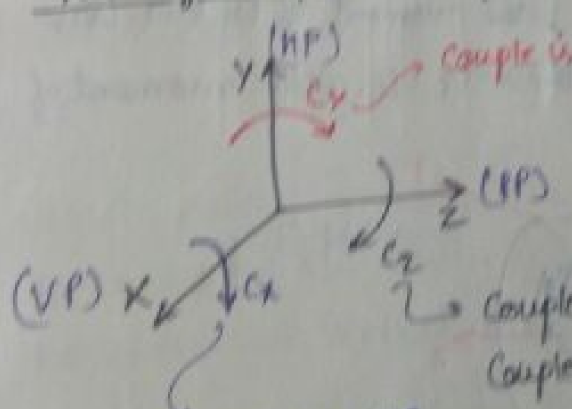
\# Every TSL is a shear load but converse is not true.

Q- For the rod & lever assembly as shown in the fig. Det. the type of load w.r.t. lever & rod.



For Lever	For Rod
P ⇒ ATL	P ⇒ HTSL
Q ⇒ HTSL	Q ⇒ EATL (e=b).
R ⇒ VTSL	R ⇒ VETSL (e=b).

Twisting Couple or Torque or Twisting moment-



Couple is in a plane which is || to H.P.

Couple is acting about 'z' axis.
Couple is in a plane viz || to P.P.

Couple is in a V.P.
i.e. couple is in a plane which is || to VP.

* To find twisting and bending couple, check the plane of cross-section first. Twisting couple will lie in this plane of x-sec and couples in rest two planes will contribute to bending.

* Since twisting couple acts about the axis ⊥ to the cross section i.e. polar axis that's why we always take polar moment of inertia in calculations.

Definitions of Twisting couple-

(a) A couple is said to be twisting couple when the plane of the couple is ∥ to the plane of the x-section of the member.
(Twisting couple is a shear load).

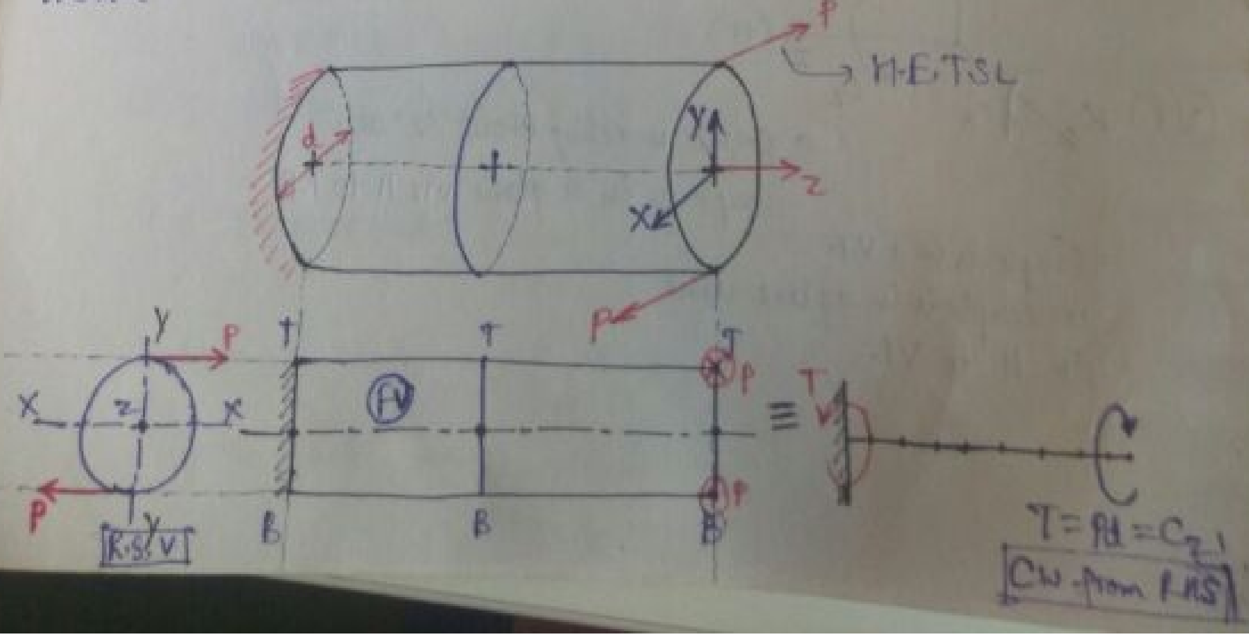
OR

(b) A couple is said to be a twisting couple when the plane of the couple is ⊥ to longitudinal axis of the member.

OR

(c) A couple is said to be a twisting couple when the couple is acting about a centroidal axis which is ⊥ to plane of x-section.
(i.e. about Polar Axis of the x-s/c)

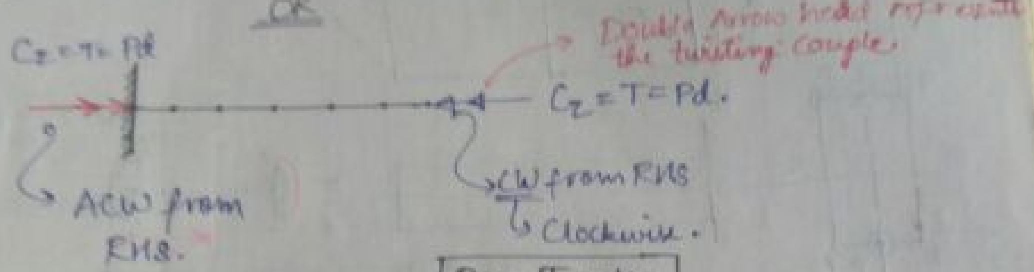
Hence, in presence of twisting couple, the moment of inertia has to be considered about polar axis of the x-s/c i.e. polar moment of inertia.



$\curvearrowleft$ Semi-closed Elipse → Twisting Couple

$\curvearrowleft$ Arcuf Circle → Bending Couple

OR



$\boxed{\text{Pure Torsion}}$

* Twisting couple will always act bcoz of two Ecc. transverse shear load.

Bending Couple -

(a) A couple is said to be a bending couple when plane of the couple is ⊥ to the plane of x - s/c of the member. (ie. Bending couple is a Normal Load).

OR

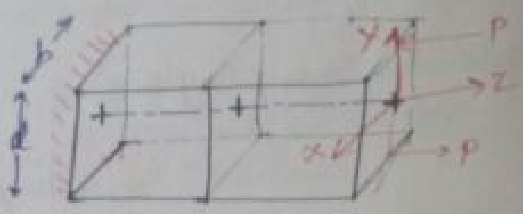
(b) A couple is said to be a bending couple when it is acting in a plane viz along the LA of the member.

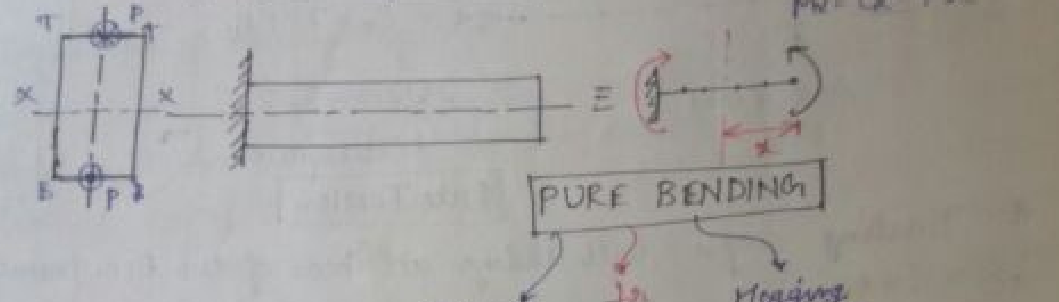
OR

(c) A couple is said to be a bending couple when it is acting about a centroidal axis viz in the plane of x-s/c (ie. About either horizontal or Vertical centroidal Axis).

→ In presence of Bending couple the moment of inertia has to be considered about either horizontal C.A or Vertical CA. ~~Hence moment~~

When a horizontal member undergoes bending in horizontal plane then MOI has to considered about vertical C.A. & vice versa.





Case II - P'-P' couple is in horizontal plane & is about Y-axis
∴ We have to take I_{yy} & Reaction couple is $M_R = P \times b$.

⇓

Front plane will be Contracted in the given case.

⇓

-ve Bending.

* If the line of action of eccentric axial load lies on horizontal C.A. then bending couple will act about vertical C.A. Hence MOI has to be considered about vertical CA & vice versa.

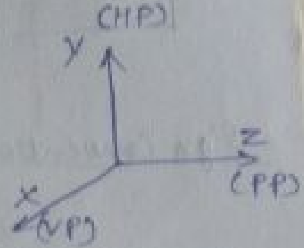
* Left face contracted → +ve   Front face cont. → -ve.
Right —"— → -ve   Back surface —"— → +ve.

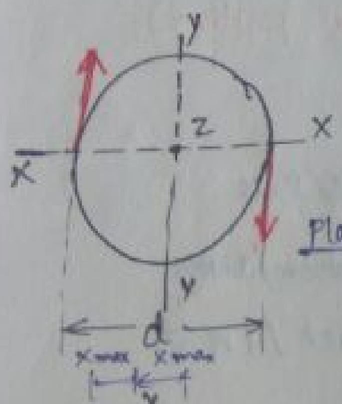
* T.S (Top Surface) gets contracted (sagging) = +ve Bend

Back surface gets contracted = +ve Bend

Left surface gets contracted = +ve Bend.
?

Plane of couple / Member position	V.P (I_{xx})	H.P (I_{yy})	P.P (I_{zz})
Horizontal (X-S/C is in P.P)	BC (I_{xx})	BC (I_{yy})	TC (I_{zz})
Vertical (X-S/C is in H.P)	BC (I_{xx})	TC (I_{yy})	BC (I_{zz})
⊥r to plane of Board (X-S/C is in V.P)	TC (I_{xx})	BC (I_{yy})	BC (I_{zz})



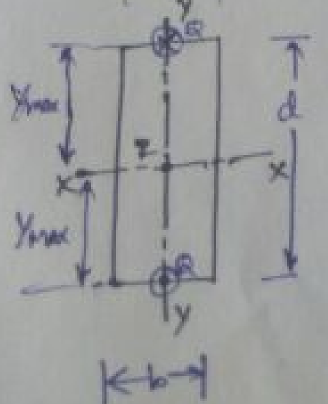


Plane of X-S/C (V.P)

$$I_{xx} = I_{yy} = \frac{\pi}{64} d^4.$$

$$I_{zz} = I_{xx} + I_{yy}$$
$$= \frac{\pi}{32} d^4.$$

$C_z = Pd$ (ie. in VP).
⇓
T.C ⟹ I_{zz} or J.



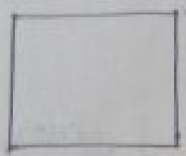
Plane of X-S/C (V.P)

$C_z = Qd$ (ie in PP)
↳ B.C ⟹ I_{xx}

$$I_{xx} = \frac{1}{12} b d^3.$$
$$I_{yy} = \frac{1}{12} d b^3.$$

$$Z_{xx} = \frac{I_{xx}}{Y_{max}} = \frac{1}{6} bd^2.$$
$$Z_{yy} = \frac{I_{yy}}{X_{max}} = \frac{1}{6} db^2.$$

} Section Modulus



Sign Convention for the Loads acting on the X-S/C -
(i.e. A.L, S.F, BM & TM).

→ As per method of section [M.O.S] -

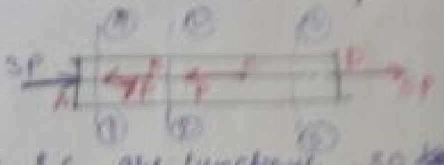
Load acting at any arbitrary X-S/C of the member is equal to algebraic sum of corresponding loads either on LHS of that X-S/C or on the RHS of that X-S/C.

(It is by assuming member in horizontal position).

⇓ Generalise

Member Posn	X-S/C Plane	
Horizontal	P.P	LHS/RHS
Vertical	HP	Bottom/above
⊥ to the board	VP	Back / Front

Q- For the prismatic bar as shown in the fig., determine the max. tensile load & max. compressive load acting on the bar.



* ~~load~~ B & C are junctions, so ~~load~~ load cannot be decided as compressive or tensile.

$(A.L)_{AB} = (AL)_{1-1} = -3P$ or $3P$ (comp) (LHS)

$= -7P - P + 5P = -3P$ (RHS).

$(AL)_{BC} = (AL)_{2-2} = -3P + 7P = 4P$ (Tensile) (LHS).

$= -P + 5P = 4P$ (RHS).

$(AL)_{CD} = (AL)_{3-3} = -3P + 7P + P = 5P$ (RHS)

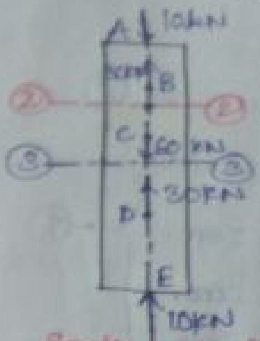
$= 5P$ (LHS).

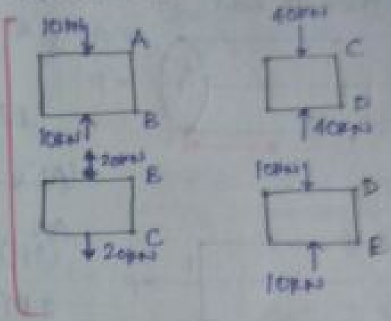
$\therefore$ Max. Tensile Load $= 5P$.

Max. compressive Load $= 3P$.

* Load at any X-S/c of the member is equal to Load acting at the non-junction X-S/c of the member. Hence it is better to use section method for intermediate members only.

Q- Det. (a) Max. tensile Load & Max. comp.
(b) (AL) at A, B, C, D & E.





Free Body Diagram Method

Section Method-

$(AL)_{AB} = (AL)_A = -10kN$ or $10kN$ (C).

$(AL)_{BC} = 30 - 10 = 20kN$ (T).

$(AL)_{CD} = -30 - 10 = -40kN$ or $40kN$ (C).

$(AL)_{DE} = (AL)_E = -10kN$ or $10kN$ (C).

Max. Tensile Load = 20kN.
Max. Comp. Load = 40kN.

$(AL)_A = 10kN$ (C).

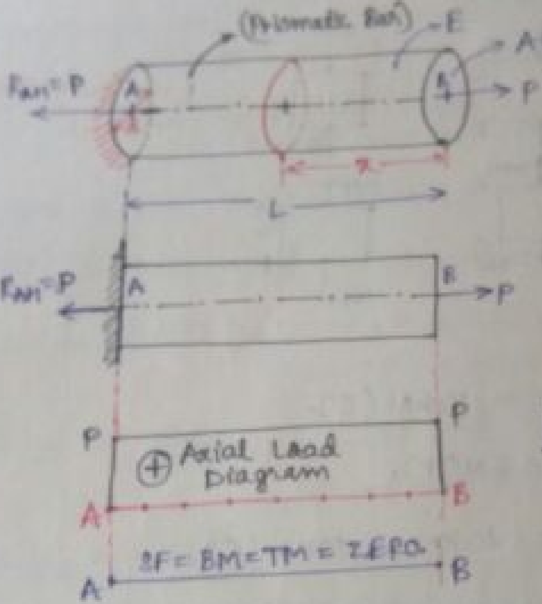
$(AL)_B$ = Larger of $[(AL)_{AB}$ & $(AL)_{BC}] = 20kN$ (T).

$(AL)_C$ = Larger of $[(AL)_{BC}$ & $(AL)_{CD}] = 40kN$ (C).

$(AL)_D$ = — " — $[(AL)_{CD}$ & $(AL)_{DE}] = 40kN$ (C).

$(AL)_E = 10kN$ (C).

Pure Axial Load (P.A.L.) - i.e. AL = const. SF = BM = TM = zero.



BA - [x=0 to L]

$$\left.\begin{aligned} (AL)_{x\text{-}x} &= P = \text{const.} \\ (SF)_{x\text{-}x} &= \text{Zero.} \\ (BM)_{x\text{-}x} &= \text{Zero.} \\ (TM)_{x\text{-}x} &= \text{Zero} \end{aligned}\right\} - (1)$$

Hence, P.B. is under PAL.

$$\boxed{\sigma_a = \frac{P}{A}} \quad - (1) \Rightarrow \text{First two conditions should be satisfied.}$$

$$\boxed{\delta_L = \delta = \frac{PL}{AE}} \quad - (2) \Rightarrow \text{All three conditions should be satisfied.}$$

Conditions to be satisfied for the above eq^ns -

(i) Bar should be prismatic.
(ii) Bar should be under P.A.L.
(iii) Bar should made of same material.

If any of these conditions is not satisfied, split the bar into parts so that these conditions get satisfied.

Note:

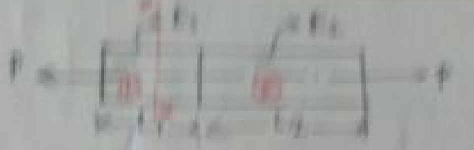
$\sigma_t = \frac{[illegible]}{[illegible]}$

$\sigma_c = \frac{[illegible]}{[illegible]}$

Under static loading cond^n,

$$\boxed{\text{Stress} \propto \left(\frac{\text{Force, Load}}{\text{X-S/c dimensions}}\right)}.$$

Q:



Det. the stress at x-x.

$\sigma_x = \frac{P}{A}$ (const. for whole bar).

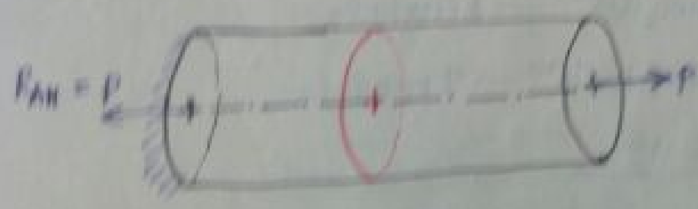
$$\therefore \boxed{\sigma_{x-x} = \frac{P}{A}}.$$

Here, material cond^n is not satisfied. Hence we assumed the bar to be splitted into two

$\delta L_1 = \frac{PL_1}{AE_1}$, $\delta L_2 = \frac{PL_2}{AE_2}$.

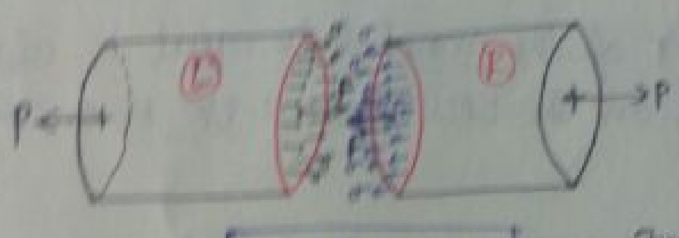
$\delta L = \delta L_1 + \delta L_2$

$$\therefore \boxed{\delta L = \frac{P}{A}\left(\frac{L_1}{E_1} + \frac{L_2}{E_2}\right)}.$$



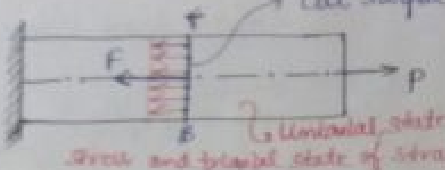
* Internal resistance ~~load~~ force are similar to the applied load but in opposite direction.

Tensile तो Tensile
Comp. तो Comp.
Shear तो Shear



$$\boxed{\sigma = \frac{P}{A} \text{ or } \frac{F}{A}}.$$

Stress is uniformly distributed on the X-S/c.



Cut surface on RHS (As the load is in right dirⁿ).

If the load is in left dirⁿ then the cut surface will be on LHS.

Uniaxial state of stress and triaxial state of strain

⇒ Stress is defined as the intensity or magnitude of Internal resistance force developed or induced at a point under corresponding loading condition.

$$\sigma_a = \frac{P}{A}.$$

$$\epsilon_{long} = \frac{\sigma_a}{E}. \quad \left(\because E = \frac{\sigma_x}{\epsilon_x \text{ or } \epsilon_{long}}\right)$$

$$\frac{\delta L}{L} = \frac{P/A}{E}.$$

$$\boxed{\delta L = \frac{PL}{AE} \text{ or } \frac{\sigma_a L}{E}}.$$

* $\sigma_y = \sigma_z = \text{zero} \; (\because P_y = P_z = 0)$

$$\boxed{\epsilon_y = \epsilon_z = \text{lateral} = -\mu\, \epsilon_{long}}.$$

* If load is acting in one ⊥ dirⁿ → Unidirectional or Uniaxial.

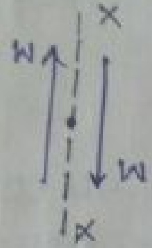
—— " —————— two ⊥ dirⁿˢ → Biaxial.

—— " —————— three ⊥ dirⁿˢ → Triaxial.

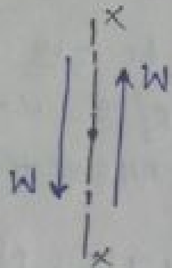
S.F. & its sign Convention-

→ Shear force at any X-S/c of the member is equal to algebraic sum of shear forces either on the LHS of the X-S/c or on the RHS of the X-S/c.

→ SF at any x-s/c of the member is said to be +ve when ~~SF~~ shear forces on either side of the x-s/c causes a couple in the clockwise dir^n & vice versa.



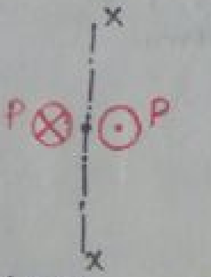
$(SF)_{x-x} = +W$ (i.e causes a couple in CW dir^n).



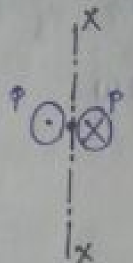
$(SF)_{xx} = -W$ (i.e causes a couple in ACW dir^n).

⊗ → आँख मारी
⊙ → आँख खुली

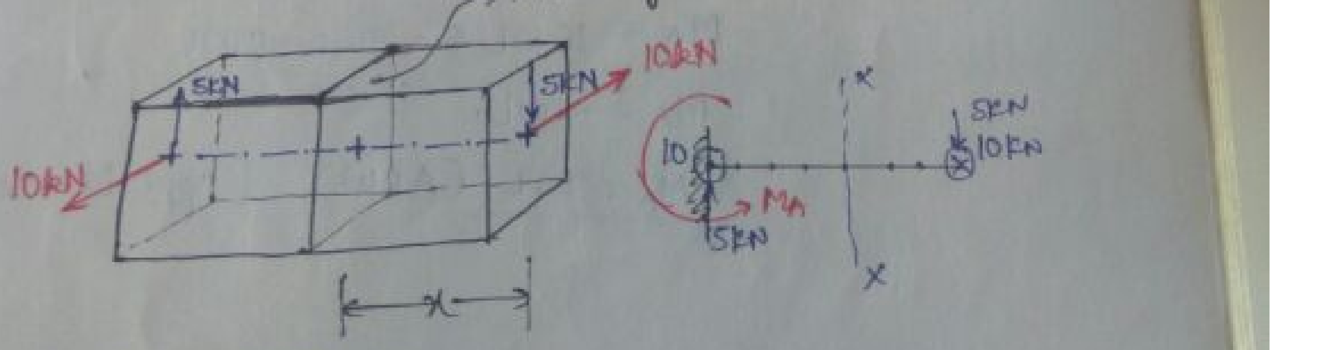
Left आँख मारी → Good ⇒ +ve
Right आँख मारी → Bad ⇒ -ve



$(SF)_{xx} = +P$



$(SF)_{xx} = -P$



$(HSF)_{xx} = -10kN$

$(VSF)_{xx} = 5kN.$

$(RSF)_{xx} = \sqrt{(-10)^2+5^2} = \sqrt{125}\ kN.$

→ S.F. & its sign convention wrt. Beams -

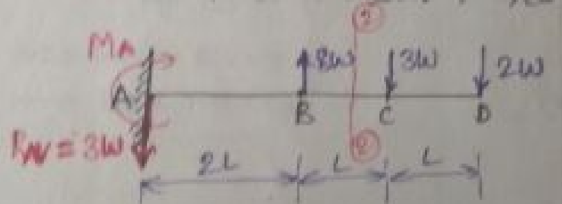
→ SF at any x-s/c of the beam is equal to algebraic sum of vertical forces either on the LHS of the x-s/c or on RHS of the x-s/c.

Sign Convention -

→ SF at any ~~at~~ x-s/c of the beam is said to be +ve when it is acting in the upward dir^n on the LHS of the cross-section or when it is acting in the downward dir^n on the RHS of x-s/c.

→ SF at any x-s/c of the beam is said to be -ve when it is acting in the downward dir^n on the LHS of the x-s/c or when it is acting in the upward dir^n on the RHS of the x-s/c.

Q- For a cantilever beam as shown in the fig. Det. the max. SF and SF at various x-s/cs of the beam.



$$\Sigma M_A = 0 \Rightarrow M_A - 8W(2L) + 3W(3L) + 2W(4L) = 0.$$

$$M_A = 16WL - 9WL - 8WL$$

$$M_A = -WL.$$

$\therefore$ $M_A = WL$. (ACW) [opp. to one assumed].

$(SF)_{AB} = (SF)_A = -3W.$

$(SF)_{BC} = 3W + 2W = 5W$ (RHS)
or ~~B~~ $= -3W + 8W = 5W$ (LHS)

Section method is applied for junction.

$(SF)_{CD} = (SF)_D = 2W.$

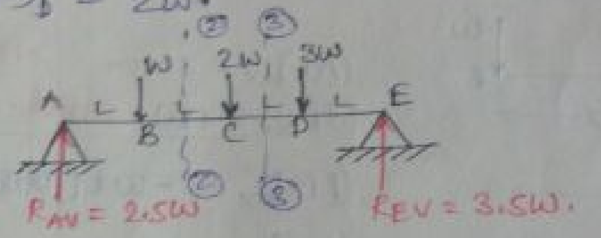
$\therefore$ **Max. SF = 5W.**

$(SF)_A = -3W$; $(SF)_B$ = Larger of $[(SF)_{AB}$ & $(SF)_{BC}] = 5W.$

$(SF)_C$ = Larger of $[(SF)_{BC}$ & $(SF)_{CD}] = \cancel{2}5W.$

$(SF)_D = 2W.$

Q -



$(SF)_{AB} = (SF)_A = 2.5W.$

$(SF)_{BC} = 2.5W - W = 1.5W.$

$(SF)_{CD} = 2.5W - W - 2W = -0.5W.$

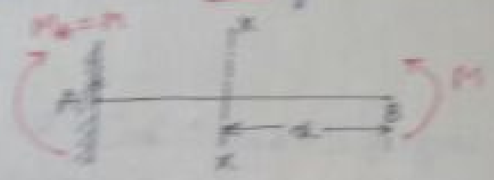
$(SF)_{DE} = (SF)_E = -3.5W.$

$\therefore$ **Max. SF = −3.5W.**

$(SF)_A = 2.5W$, $(SF)_B = 2.5W$, $(SF)_C = 1.5W$, $(SF)_D = -3.5W$
$(SF)_E = -3.5W.$

B.M & its sign convention -

BM at any X-S/C of the beam is equal to algebraic sum of couples & moments [i.e. which are along the LA of beam] either on the LHS of that x-s/c or the RHS of that x-s/c.

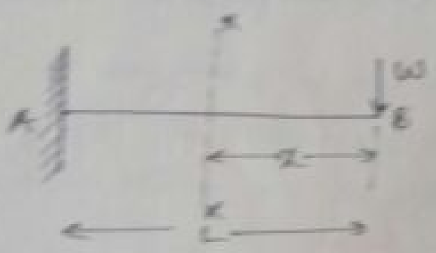


$(AL)_{xx} = (SF)_{xx} = (TM)_{xx} = \text{zero}$.

$(BM)_{xx} = M$ (Sagging)

$= \text{const}$.

$\boxed{\text{PURE BENDING}}$



$(AL)_{xx} = \text{zero}$.

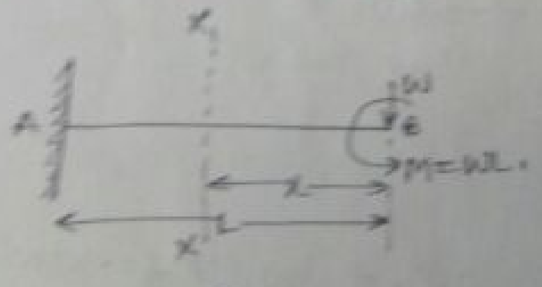
$(SF)_{xx} = +W =$ const. → Rectangle

$(BM)_{xx} = -Wx$ (Variable) → Triangle

$(TM)_{xx} = \text{zero}$.

⇒ B.M = const ⇒ SF = 0 ⇒ Pure Bending.

⇒ BM = variable ⇒ SF ≠ 0.



$(SF)_{xx} = +W$.

$(BM)_{xx} = WL - Wx$.

$x = 0 \Rightarrow (SF)_B = W$

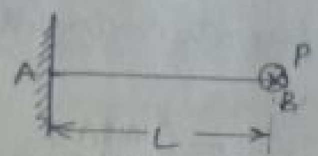
$(BM)_B = WL$

$x = L \Rightarrow (SF)_A = W$

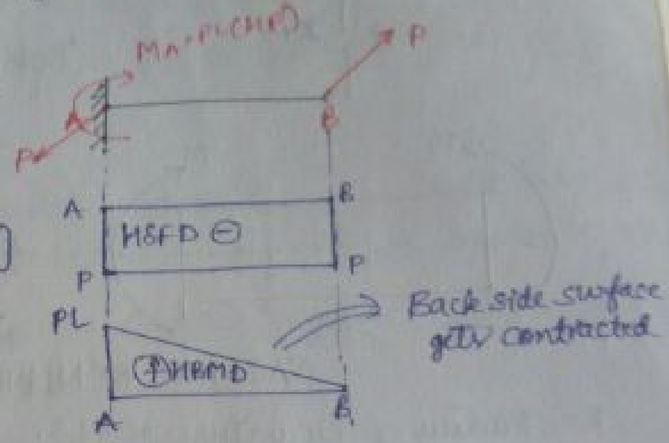
$BM_A = \text{zero}$

→ VTSL ⇒ SF & variable BM (V.P)

HTSL ⇒ SF & —"— (HP)

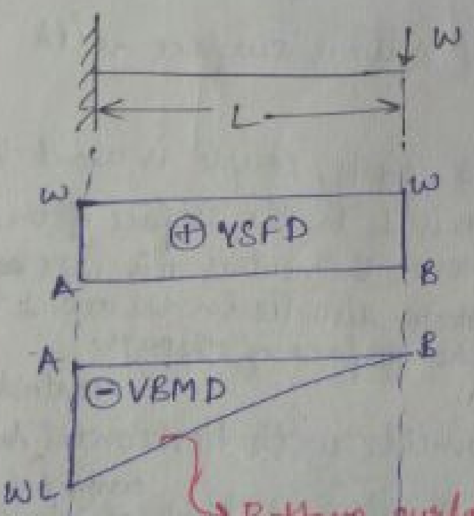


$\equiv$



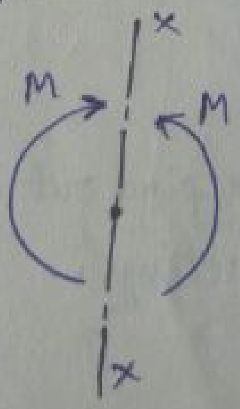
$(SF)_{xx} = -P$ or P [ACW dir^n] (HP)

$(BM)_{xx} = Px$ (HP)



Bottom surface gets contracted (hogging)

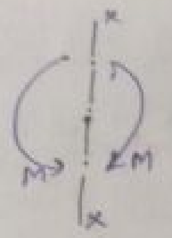
→ <u>Sign Convention</u>-



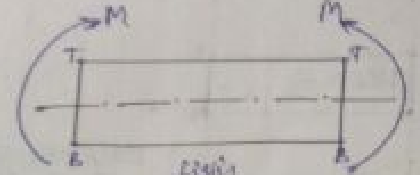
$(BM)_{xx} = +M$ (i.e, Sagging bending)

⇓

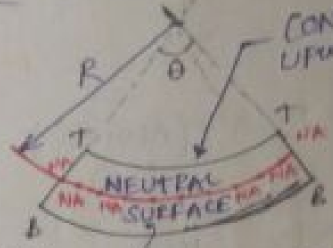
(T.S gets contracted)



$(BM)_{x-x} = -M.$
(Hogging Bending)
⇓
(Bottom surface gets contracted)



fig(i)

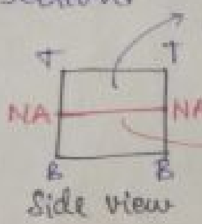


SAGGING BENDING

whose length doesn't changes ~~once~~ after bending

R = Radius of curvature of N.S.

<u>Neutral Axis</u> - Line of intersection of neutral surface with cross-section.



* In fig(i), couple is about the axis ⊥ to the surface of the page. Hence the neutral axis will ~~be~~ also lie in the axis ⊥ to the surface of the page.

* In vertical bending, NA will coincide with Horizontal centroidal Axis. (NS cuts Horizontal CA.)

In Horizontal ——"—— Vertical centroidal Axis. (NS cuts Vertical CA)

<u>Bending</u> -

Rotation of a X-S/C about Horizontal or vertical axis.

<u>Twisting</u> -

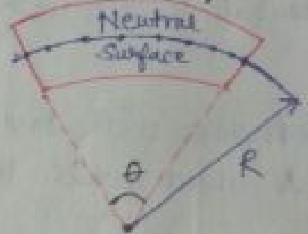
Rotation of a X-S/C about polar axis.

* Horizontal, vertical & polar axis will be find out by checking the plane in which couples are acting.

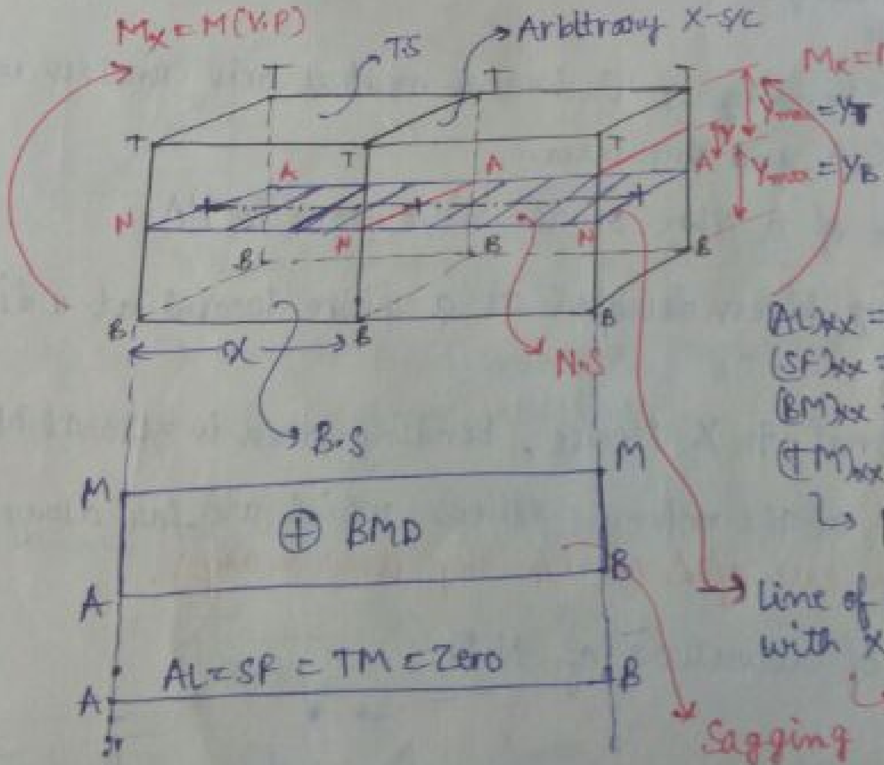
Note :

* Bending moment in any x-s/c of the beam is said to be +ve when B.M is in clockwise dirⁿ on the LHS of the x-s/c or when BM is in a ACW dirⁿ on the RHS of the x-s/c. (ie. Sagging Bending).

* Bending moment in any x-s/c of the beam is said to be -ve. when it is acting in the ACW dirⁿ on the LHS of the x-s/c or when it is acting in the CW dirⁿ on the RHS of the x-s/c. (ie. Hogging Bending).



HOGGING BENDING



$(AL)_{xx} = 0$
$(SF)_{xx} = 0$
$(BM)_{xx} = M =$ const.
$(TM)_{xx} =$ zero.
↳ PURE BENDING

Line of intersection of N.S with X-s/c on RHS.
↳ NA of x-s/c on RHS

* BMD (+ve) ⇒ Sagging Bending ⇒ Compressive stresses at top surface ⇒ -ve stress

* BMD (-ve) ⇒ Hogging Bending ⇒ Tensile stresses at bottom surface ⇒ +ve stress

* At NS → zero stress. Non-uniformly distributed stresses

Bending Eqⁿ –

$$\boxed{\frac{M}{I_{NA}} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

Pure Bending

$$\frac{\sigma_m}{y} = \frac{M}{I} = \frac{E}{R}$$

$$\frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$$

[illegible]

M is equal to Bending Moment acting on the x-s/c of the beam.

NS is a surface whose length before and after bending remains same.

NA is the line of intersection of neutral surface with the x-s/c of the beam.

* In the absence of axial load NA coincides with one of the centroidal axis in the plane of x-s/c & about which bending moment, or couple is acting.

* In vertical bending of the beams neutral axis coincides with Horizontal C.A and vice versa.

* y = Distance of a fibre on the x-s/c from its NA.

* σ_b = Bending stress developed at a fibre located at a distance of y.

* σ_b proportional to y. Hence, bending stress is zero at NA but becomes max. at the extreme fibres which are far away from the NA. (In case of Δ ck-s/c top is far away).

* R = Radius of curvature of N.S.

~~[illegible]~~

* $I_{NA} = \int y^2 \, dA$

  = Second Moment of [illegible] area of X-S/c about N.A.

  or Area M.O.I of X-S/c about N.A.

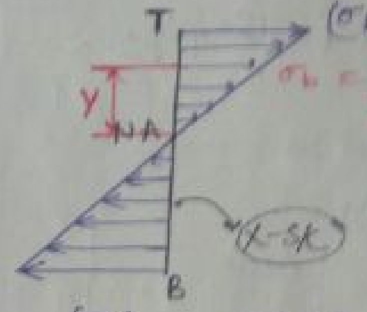
Ⓐ = Ⓑ & Ⓑ = Ⓒ

$$\boxed{\sigma_b = \frac{My}{I_{NA}}} \quad —① \rightarrow \text{when } \underline{BM} \text{ is known.}$$

$$\boxed{\sigma_b = \frac{Ey}{R}} \quad —② \rightarrow \text{when } \underline{R} \text{ is known.}$$

from ① & ②,

$$\boxed{\sigma_b \propto y} \quad \left(\because \frac{M}{I_{NA}} = \frac{E}{R} = \text{const.}\right)$$



$(\sigma_b)_{Top} = (\sigma_b)_{max} = \frac{M}{Z_{NA}}$

$\sigma_b = \frac{My}{I_{NA}}$ or $\frac{Ey}{R}$.

Bending stress variation

(for sagging or [illegible] the cut surface)

$$(\sigma_b)_{Bottom} = (\sigma_b)_{max} = \frac{My_{max}}{I_{NA}} \text{ or } \frac{M}{Z_{NA}}.$$

* If we know the B.S (σ_b) at a single fibre, we can calculate for all due to its linear variation.

$$(\sigma_b)_{max} = \frac{M y_{max}}{I_{NA}} \text{ or } \frac{E y_{max}}{R}.$$

$$= \frac{M}{(I_{NA}/y_{max})} \text{ or } \frac{E y_{max}}{R}$$

$$\boxed{(\sigma_b)_{max} = \frac{M}{Z_{NA}} \text{ or } \frac{E y_{max}}{R}} \quad —③$$

$Z_{NA} = \frac{I_{NA}}{y_{max}}$ = Section modulus of x-s/c about its NA.

y_{max} = larger of [y_t & y_b].

$\Rightarrow \boxed{(\epsilon_{b})_{max} = \frac{(\sigma_b)_{max}}{E} = \frac{y_{max}}{R} = \frac{M}{Z_{NA} \cdot E}}$ —— (4)

(A) = (C)

$\boxed{R = \frac{E I_{NA}}{M}}$ —— (5)

where $E I_{NA}$ = Flexural rigidity of a x-s/c about its NA.

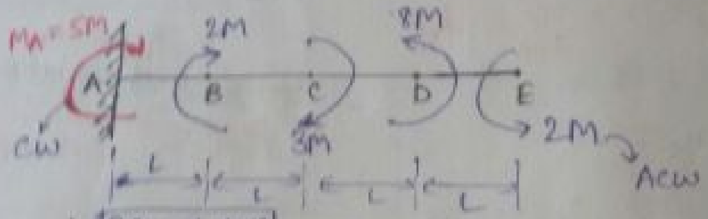
AE = Axial rigidity of a x-s/c.

GJ = Torsional —— " ——

Q- For the cantilever beam as shown in the fig., det.

(i) Max. sagging BM, Max. Hogging BM

(ii) BM at various x-s/cs as shown in the fig.



∴ BM = +ve

$(BM)_{AB} = (BM)_A = +5M$ (S.B)

$(BM)_{BC} = +5+2 = +7M$. (SB)

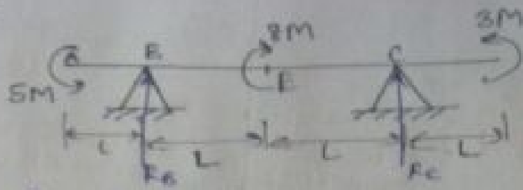
$(BM)_{CD} = +8+2 = +10M$. (SB)

$(BM)_{DE} = (BM)_E = +2M$. (SB)

Max S.B.M = 10M.

Max. HBM = 0.

Q-



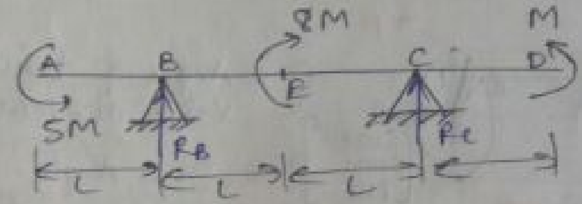
$\Sigma V = 0$.
$\Sigma H = 0$.
$R_B = R_C = 0$.

* Pure Bending case ⇒ Reactions are zero.
Whereas Max. Reactions are 4.

$(BM)_{AE} = (BM)_A = -5M$ (H.B).

$(BM)_{ED} = (BM)_D = +3M$ (S.B).

Q-



$R_B = ?$
$R_C = ?$

$-5M + 8M + R_C \times 2L + M = 0$.

$R_C \times 2L = -\cancel{4}^{2}M$

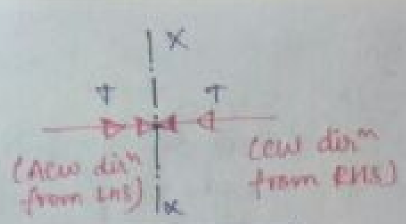
$R_C = \frac{2M}{L}$.

$R_B + R_C = 0$.

$R_B = -\frac{2M}{L}$.

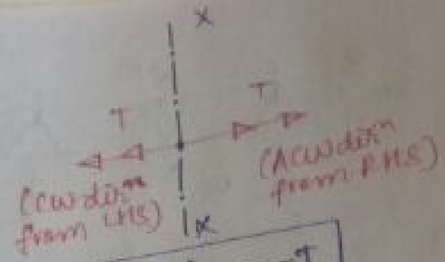
Twisting Moment & its Sign Convention-

TM at any X-S/C of the member is equal to algebraic sum of twisting couples (ie. which are ⊥r to LA of member) either on the LHS of the X-S/C Or on the RHS of that X-S/C



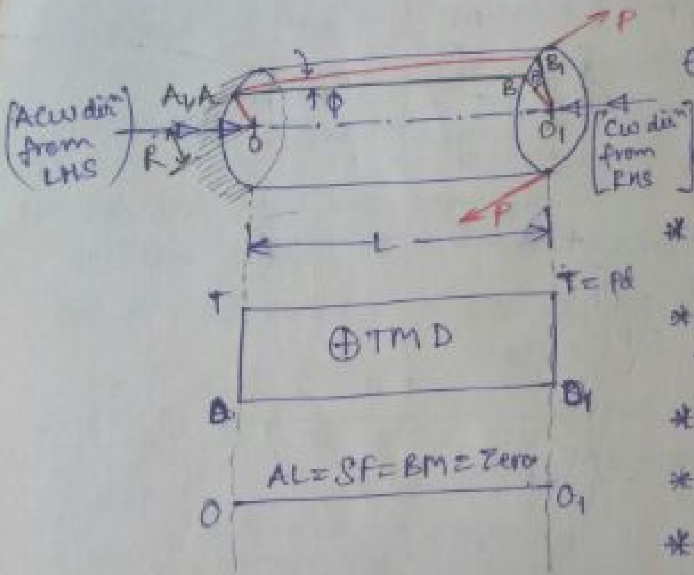
$$\boxed{(TM)_{xx} = +T}$$

X-S/c is twisted in cw dirn by observing from RHS.



$$\boxed{(TM)_{xx} = -T}$$

* Just opposite to Axial Load Sign convention.



θ = Angle of twist of X-S/c at free end

φ = Shear Angle.

* When we move from fixed centre to periphery φ increases.
* When we move from fixed end to free end θ increases.
* φ is max at surface.
* θ is max at free end.
* If one from θ & φ is cal., other can be easily determine.

In ΔABB_1,

$$\tan\phi = \frac{BB_1}{AB} = \frac{R\theta}{L}.$$

$$\boxed{\phi = \frac{R\theta}{L}} \qquad (\because \tan\phi \simeq \phi).$$

Shear strain (γ) = Shear angle (ϕ).

$$\boxed{\gamma_{max} = \frac{R\theta}{L}.}$$

$$\boxed{\tau_{max} = G\gamma_{max} = \frac{GR\theta}{L}} \quad \text{--- (1)}$$

$$\boxed{E = \frac{\sigma}{\epsilon_{long}}} \qquad \boxed{G = \frac{\tau}{\gamma}}$$

$$\boxed{\frac{\tau_{max}}{R} = \frac{G\theta}{L}}$$

$$\therefore \boxed{\frac{T}{J} = \frac{\tau_{max}}{R \text{ or } R_o} = \frac{G\theta}{L}}$$

→ Torsion eqⁿ for circular X-S/C.

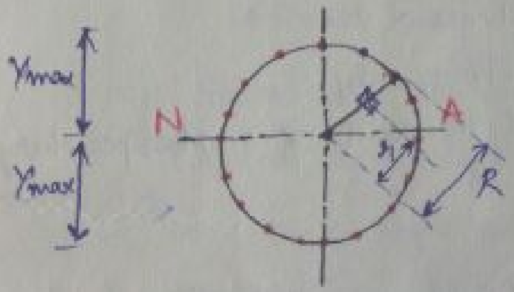
PURE TORSION.

Assumptions-

(i) Prismatic
(ii) Pure torsional
(iii) Circular
(iv) Same material.

$J = \int r^2 dA = I_{xx} + I_{yy}$
= Second moment of Area of X-S/C about polar axis
OR
= Polar MOI of X-S/C.

* Torsion is max. at periphery bcoz it is $\propto$ r.



$\boxed{\sigma_b \propto Y}$

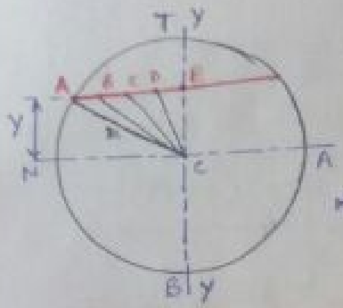
& $\boxed{\tau_t \propto r}$

→ Bending stress is max. at only two points (Top and Bottom).

→ Torsional shear stress is max. at all the points on the periphery.

→ Torsional stress is zero ~~all~~ at only one centroid point.

→ σ_b is zero at whole NA.



* $(\sigma_b)_A = (\sigma_b)_B = \text{———} = (\sigma_b)_E = \frac{EY}{R}$ or $\frac{MY}{I_{NA}}$ or $(\sigma_b)_{max}\left[\frac{Y}{Y_{max}}\right]$

$[\because Y_A = Y_B = \text{———} = Y_E = Y]$.

* $[(\tau_t)_A = \tau_{max}] > (\tau_t)_B > (\tau_t)_C > (\tau_t)_D > (\tau_t)_E$

$[\because (r_A = R) > r_B > r_C > r_D > r_E]$.

* $(\sigma_b)_{max} = (\sigma_b)_T = (\sigma_b)_B = \frac{M}{Z_{NA}}$ or $\frac{EY_{max}}{R}$

$[\because Y_T = Y_B = Y_{max}]$.

* $(\tau_t)_1 = (\tau_t)_2 = \text{———} = (\tau_t)_E = (\tau_t)_{max} = \frac{T}{Z_P}$ [where $Z_P = \frac{J}{R \text{ or } R_0}$]

$(\sigma_b)_a = \text{———} = (\sigma_b)_z = 0$ [They are all on the N.A].

$(\tau_t)_C =$ Zero. $[\because r_C = 0]$.

→ Bending stress $\propto Y$. Torsional stress $\propto r$.

→ Bending stress is max. at top and bottom only.
Torsional shear stress is max. at all the points ~~of~~ on periphery.

→ B.S is zero on all the points of N.A. &
T.S is zero at centroid only.

→ B.S remains same for a given fibre. (Same for all the pts. (A, B, C -- E) of one fibre)
T.S changes from point to point. (Diff. for A, B, C -- E pts. of same fibre).

→ But on all the points of X-S/c, axial stress is same,
Angle of twist is same, (depends on 'L')
Shear angle will diff. (depends on 'r')

Terminology used in torsion eqⁿ -

$T \Rightarrow$ Twisting moment acting on the shaft.

$$\boxed{\frac{T}{J} = \frac{\tau_{max}}{R \text{ or } R_0} = \frac{G\theta}{l}}$$ Torsion eqⁿ for circular X-S/c.

$$T = \frac{P \times 60}{2\pi N} \times 10^6 = \_\_\_\_ \text{ Nmm.}$$

KW ← P; N → rpm

$$= \frac{P}{2\pi N'} \times 10^6 = \_\_\_\_ \text{ Nmm.}$$

KW ← P; N' → rps or Hertz

$$= \frac{P}{\omega} \times 10^6 = \_\_\_\_ \text{ Nmm.}$$

KW ← P; ω → rad/s

$r \Rightarrow$ Distance of an arbitrary pt. on the X-S/c from its centroid or polar axis.

R or $R_0 \Rightarrow$ Distance of a far away point on a X-S/c from its centroid. (ie. any point on the periphery of the X-S/c)

$\phi' \Rightarrow$ Shear angle of an arbitrary point on a X-S/c which is located at a distance of r. ($\phi' \propto r$)

$\phi \Rightarrow$ Max. shear angle on the ~~S/c~~ X-S/c. (ie. any point on the periphery).

$\tau' \Rightarrow$ Torsional shear stress at an ~~at~~ arbitrary pt. on the X-S/c which is located at a distance of 'r'.

τ_{max} = Max. torsional shear stress on the X-S/c (ie. at any pt. on the periphery) $\boxed{\tau' \propto r.}$

$$\underbrace{\tau' \propto \gamma' \text{ or } \phi'}_{\text{Using Hooke's law.}} \propto r$$

$l \Rightarrow$ Distance of an arbitrary X-S/C from fixed end

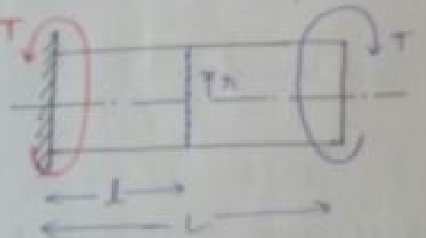
$L \Rightarrow$ Distance of a far away X-S/C from the fixed end (i.e. at the free end).

or

Length of the shaft under pure torsion

$\theta' \Rightarrow$ Angle of twist of a X-S/C located at a distance of l.

$\theta \Rightarrow$ Max. angle of twist of the shaft (i.e. angle of twist of a X-S/C at the free end). $(\theta' \propto l)$.



Shaft is under pure torsion

$J \Rightarrow$ Polar M.O.I of a X-S/C = Second moment of area

$$J = I_{zz} = I_{xx} + I_{yy}$$
$$= 2(I_{xx} \text{ or } I_{yy})$$
$$= \frac{\pi d^4}{32},$$

$$\boxed{\phi = \frac{R\theta}{L}}$$

ϕ depends on θ.

θ depends on the value of torque applied.

* θ is angle of twist.

ϕ is shear angle.

$$\frac{\tau_{max}}{G} = \frac{R\theta}{l}$$

γ or ϕ.

$$\frac{T}{J} = \frac{\tau_{max}}{R \text{ or } R_o} = \frac{G\theta}{L}$$

(A) (B) (C)

(A) = (B) & (B) = (C)

$$\boxed{\tau_{max} = \frac{T(R \text{ or } R_o)}{J} = \frac{T}{Z_p}} \quad — (1)$$

use- when 'T' on the x-s/c is known

where $Z_p = \frac{J}{R \text{ or } R_o}$ = Polar section modulus of the x-s/c.

$$\boxed{\tau_{max} = \frac{G(R \text{ or } R_o)\theta}{l}} \quad — (2)$$

use- when 'θ' of the shaft is known.

(A) & = (C)

$$\boxed{\theta = \frac{TL}{GJ}} \quad — (3)$$

$$\boxed{K_t = \frac{T}{\theta} = \frac{GJ}{L}}$$

Torsional stiffness ⇒ Amt. of torque required to produce unit angle of twist (rad.) at free end.

$$\boxed{K_a = \frac{P}{\delta} = \frac{AE}{L}}$$

Axial stiffness.

For solid circular x-s/c -

* $I_{xx} = I_{yy} = \frac{\pi d^4}{64}$.

* $J = \frac{\pi d^4}{32}$

* $Z_P = \frac{\pi d^3}{16} = \frac{J}{R}$.

* $Z_{N.A} = \frac{\pi d^3}{32} = \frac{I_{N.A}}{y_{max}}$.

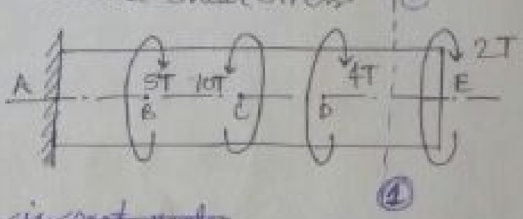
→ Circular & prismatic.
→ Pure torsion.
→ Same material.

Q - For circular & prismatic shaft as shown in the fig. Det.

(i) Max. torque

(ii) Torque at diff. x-s/c as shown in the fig.

(iii) Max. torsional shear stress.



Shaft is not under pure torsion. So we will split the shaft and then apply the torsional eqn.

~~Shaft is not under~~

$T_E = 2T$.

$T_D = 6T$. (Larger)

$T_C = 6T$. (Larger)

$T_B = -4T$. (Larger)

$T_A = T$.

$T_{DE} = 2T$ ($2T$ = +ve).

$T_{CD} = 6T = T_{max}$

$T_{BC} = 6T - 10T = -4T$

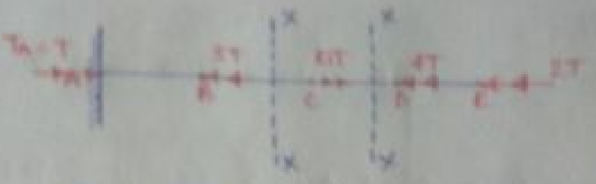
$T_{AB} = -4T + 5T = T$

$$\tau_{max} = (\tau_{max})_{CD} = \left(\frac{T}{Z_P}\right)_{CD}$$

$$= \frac{16 T_{CD}}{\pi d^3_{CD}} = \frac{16(6T)}{\pi d^3} = \frac{96T}{\pi d^3}.$$

($\because$ T is max. in CD portion)

* If d for whole shaft is diff. then we would need to apply torsional eqn separately.



$T_{AB} = T_A = T$

$T_{BC} = T - 5T = -4T$

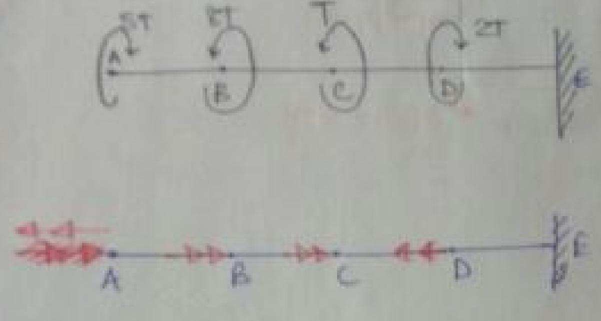
$T_{CD} = 4T + 2T = 6T.$

$T_{DE} = T_E = 2T.$

* Towards shaft → +ve
Away shaft → -ve

Opp. to axial load sign convection.

Q- For circular prismatic shaft as shown in the fig. Det. the max. torque & torques at diff. X-s/cs.



$T_{AB} = -5T.$ (↻ ≡ -ve)

$T_{BC} = -5T + 8T = 3T.$

$T_{CD} = 3T + T = 4T.$

$T_{DE} = 4T - 2T = 2T.$

$\boxed{T_{max} = -5T}$

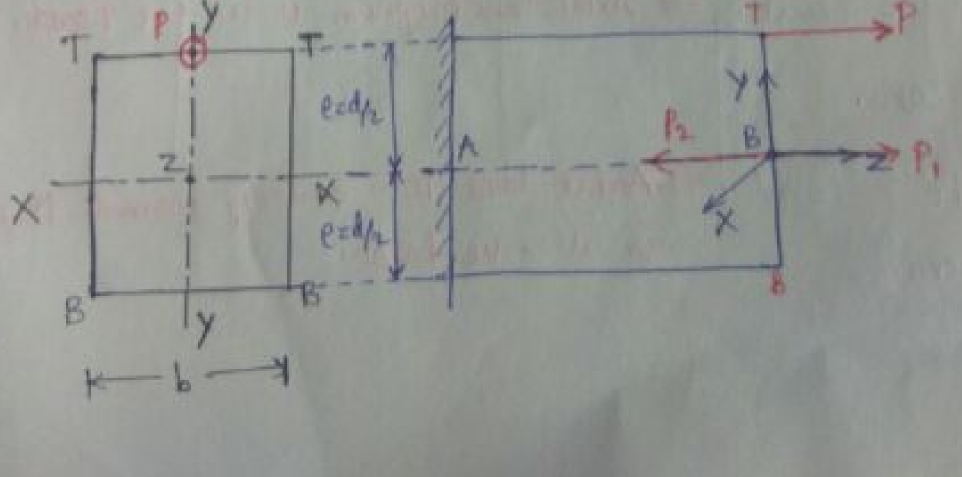
$T_A = -5T.$ $T_c = 4T$

$T_B = -5T$ $T_D = 2T.$

$T_E = 2T.$

Equivalent loads on X-s/c under E.A.L -

(Assuming member is fixed at one end) (Hence B.M & Axial load are same through out the the shaft)



1- Locate centroid of the X-S/C.

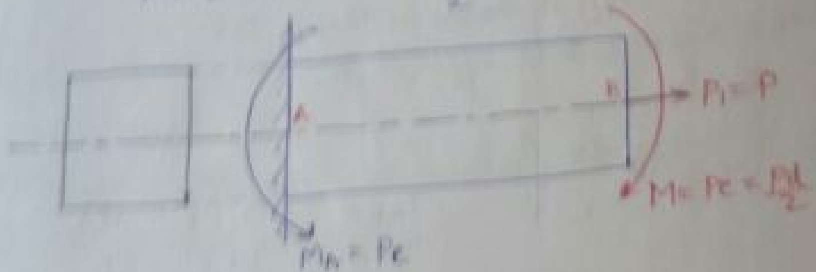
2- Introduce two dummy forces (i.e. P_1 & P_2) at centroid in a dirⁿ
|| to applied load & $P_1 = P_2 = P$.

3- e = ⊥ distance b/w line of action of E.A.L & centroid of the X-S/C.

4- Eqᵐ loads on the X-S/C:

$P_1 = P$ ⇒ ATL.

P & P_2 ⇒ $M_x = Pe = \frac{Pd}{2}$.



$M_A = Pe$
$F_{AY} = 0$
$R_{AH} = P$

Note:-

* If roller support would be there, then axial load would not be balanced.

* If ~~the~~ Hinge support would be there, then there will be no reaction moment and ~~mom~~ external moment will be balanced by the vertical force and shear force will not be zero.

5- $(AL)_{xx} = P$ = const. ⇒ Axial load Diagram is a +ve Rect.

$(SF)_{xx}$ = zero.

$(BM)_{xx} = -Pe$ = const. ⇒ ~~Axial load Dia~~ Bending moment Diagram is a -ve Rect.

$(TM)_{xx}$ = zero.

* Super position principle –

When both loads are of same nature, they can be added or subtracted.

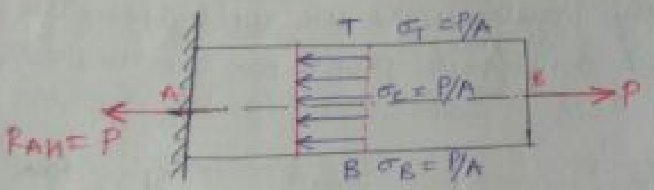
In above eg., (i) Axial Load & B.M (Normal load type) (Normal load type) ✓ can be applied.

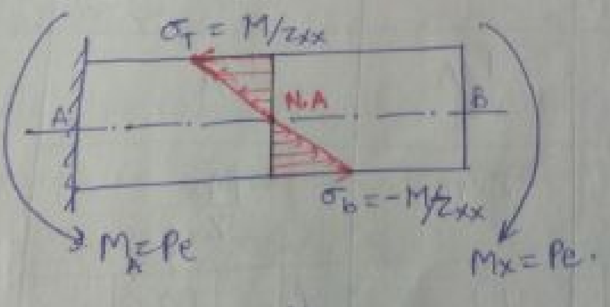
(ii) Axial Load & T.M (Normal load type) (Shear load type) × Can't be applied.

∴ Above C.A $= \frac{P}{A} + \frac{M}{Z}$.

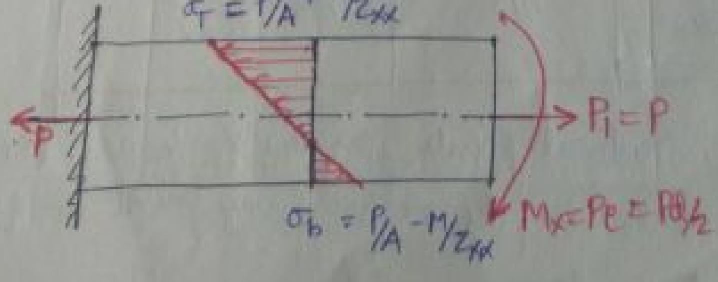
Below C.A $= \frac{P}{A} - \frac{M}{Z}$.

∴ Stress will be zero somewhere below C.A which implies Neutral ~~Axial~~ Axis will be below centroidal axis of X-S/C. ✓ ?





Pure Hogging Bending



$(\sigma_b)_{max} > \sigma_a$.

Whenever load is applied with couple, then N.A ~~loads~~ can't coincide with C.A.

Note:

* In a prismatic member, critical x-s/c is a x-s/c where an axial load, shear force, bending moment & torsional moment are max.

* Under eccentric axial load, every x-s/c is a critical x-s/c (if [illegible] force)

* Critical point on the critical x-s/c is the point where σ_R (Resultant normal stress) & τ_R (Resultant shear stress) is max.
  Axial load & bending — Direct shear & Torsional shear

→ Draw loading diagram (A.L.D, S.F.D, B.M.D, T.M.D) and check the x-s/c where load is max. This will be our critical x-s/c.

→ On this x-s/c find point where σ_R & τ_R are max, which gives us the critical point.

→ Critical points are the only points where we will apply the eqn. as it is impossible to check on infinite points on the member in the case of varying load.

$$Z_{xx} = \frac{I_{xx}}{y_{max}}$$

Stress / Loading load	$(\sigma_t)_{top}$	$(\sigma_t)_{CA}$	$(\sigma_t)_{bottom}$
P.A.L	P/A	P/A	P/A
Pure Bending	$+\frac{M_x}{Z_{xx}}$	Zero	$-\frac{M_x}{Z_{xx}}$
E.A.L	$P/A + \frac{M_x}{Z_{xx}}$	P/A	$P/A - \frac{M_x}{Z_{xx}}$

For this given E.A.L, critical points on the x-s/c are the points on top fibre & critical points in the member any point on the top surface.

$$(\sigma_R)_{max} = \sigma_{TOP} = \sigma_a + (\sigma_b)_{max}$$

$$= \frac{P}{A} + \frac{M_x \text{ or } Pe}{Z_{xx}}$$

$$= \frac{P}{bd} + \frac{P(d/2)}{\frac{1}{6} bd^2}$$

$$= 4P/bd \quad \text{(T)}$$

$$(\sigma_R)_{CA} = \frac{P}{A} = \frac{P}{bd}. \quad \text{(T)}$$

$$(\sigma_R)_{Bottom} = \sigma_a - (\sigma_b)_{max}$$

$$= \frac{P}{A} - \frac{M_x \text{ or } Pe}{Z_{xx}}$$

$$= \frac{P}{bd} - \frac{P(d/2)}{\frac{1}{6} bd^2}$$

$$= \frac{2P}{bd} \quad \text{(Comp.)}$$

* For this given E.A.L condn, N.A (ie. $\sigma_R = 0$) coincides with inner fibres below Horizontal C.A.

* Fast Method to find the dirn of Z ie either Z_{xx} or Z_{yy} -

Check the load is applied on which axis on X-S/C. ~~Then~~

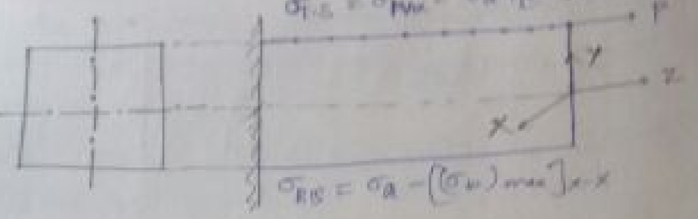
For eg. in above case the load was on Y-axis in the X-S/C.

∴ Z will be taken along X-axis ie Z_{xx}.

Vice, versa.

X-axis में load तो Z_{yy}.

Z-axis में Load तो twisting moment.

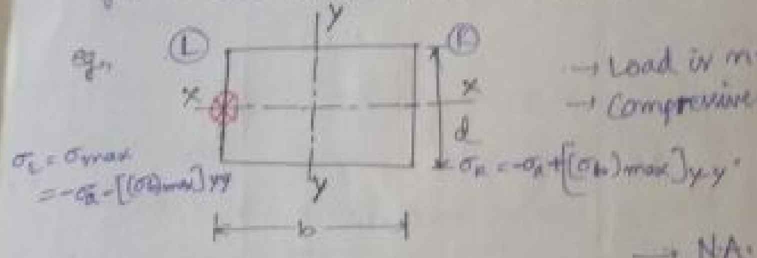


→ give which surface has max stress

1- Check load is near to top fibre or bottom fibre.

2- Check the type of load. As it is ~~the nature~~ tensile in above eq., hence σ_{max} will be tensile.

eg.,



→ Load is near to left side.
→ Compressive ~~s~~ type.

→ $\underline{N.A.}$ → is near to right side.

$$\sigma_{max} = \sigma_L = -\sigma_a - [(\sigma_b)_{max}]_{y-y}$$

$$= (-)\left[\frac{P}{A} + \frac{M_y \text{ or } Pe}{Z_{yy}}\right]$$

$$= (\text{comp})\left[\frac{P}{bd} + \frac{P(b/2)}{1/6\, db^2}\right] = \frac{4P}{bd} \text{ (comp.)}$$

$$Z_{yy} = \frac{I_{yy}}{X_{max}} = \frac{\frac{1}{12} db^3}{b/2} = \frac{db^2}{6}.$$

*Following conclusions can be made when a prismatic member is fixed at one end and subjected to E.A.L at the free end.

(i) ALD & BMD are rectangles.

(ii) SFD & TMD are the lines coinciding with zero.

(iii) Bending moment value is Load × e (P×e).

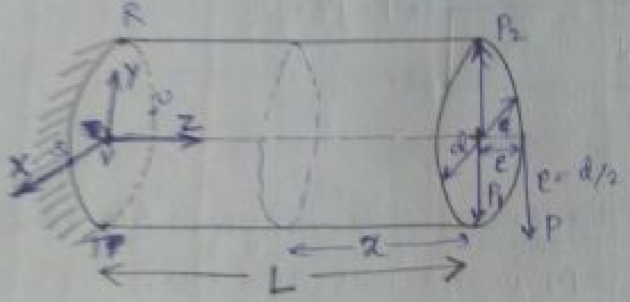
(v) Every x-s/c is the critical x-s/c.

(vi) Critical points or any point on the top fibre or bottom fibre of the x-s/c (it depends on ~~the~~ ~~where~~ the fibre which is nearer to line of action of E.A.L).

(vii) Neutral axis never coincides with the C.A in the plane of x-s/c. (either below or above depends).

(viii) When line of action of EAL lies on vertical CA then bending couple will act along horizonal CA. Hence ~~size~~ section modulus (Z) should be considered about H.C.A. and vice versa.

Equivalent loads on the x-s/c under E.T.S.L -



Couple plane is along the profile plane i.e. the plane of x-s/c. Hence there will be Twisting moment.

At free end,

$P_1 = P \Rightarrow VTSL \Rightarrow S.F.$

$P \& P_2 \Rightarrow T.M \Rightarrow Pe = Pd/2.$

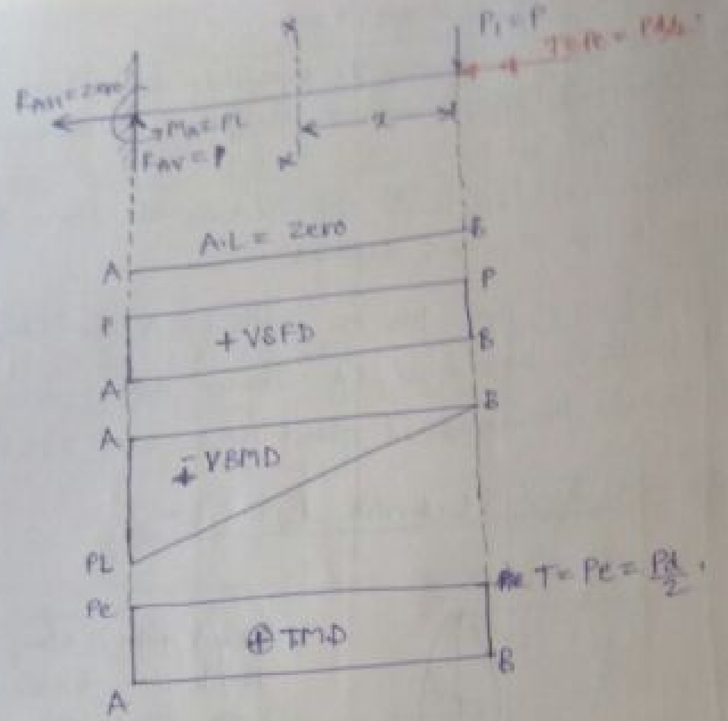
At arbitrary x-s/c,

$(AL)_{x-x} = \text{zero}.$

$(SF)_{x-x} = +P = \text{const}.$

$(BM)_{xx} = -P.x$ (Variable). Case of Hogging. } (Vertical Plane)

$(TM)_{xx} = P.e = \text{Const}.$ (Profile Plane).

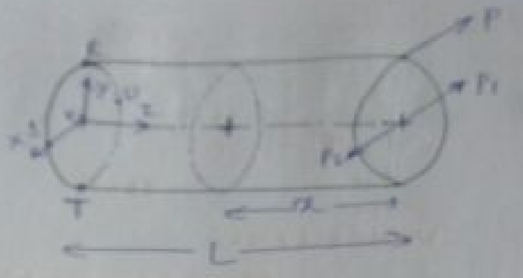


→ Critical ~~plane~~ x-s/c will be ~~…~~ the one at fixed end bcoz ~~BM~~ SF & TM are constant through out and BM is max. at fixed end.

→ Now at fixed ~~point~~ x-s/c if shear force is neglected, TM is max. at all points on periphery and BM is max only at top and bottom. Hence, R & T are the critical points.

→ On A → Both σ_b & torsional shear stress = 0.

Note → If P_o is tensile AL then only R will be the critical point as due to hogging top fibre is in tension & total load (normal) = $(\sigma_b)_{max} + \sigma_a$.



At free end,

$P_1 = P \Rightarrow$ HTSL $\Rightarrow$ S.F.

P & $P_2 \Rightarrow TM = Pe = \frac{Pd}{2}$.

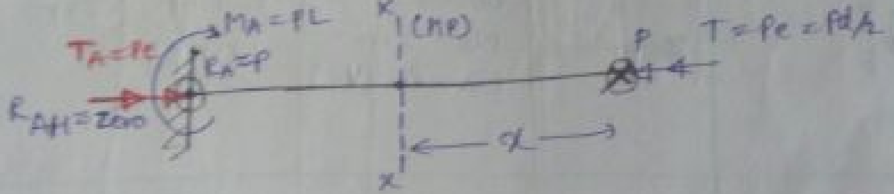
S&U are C.P

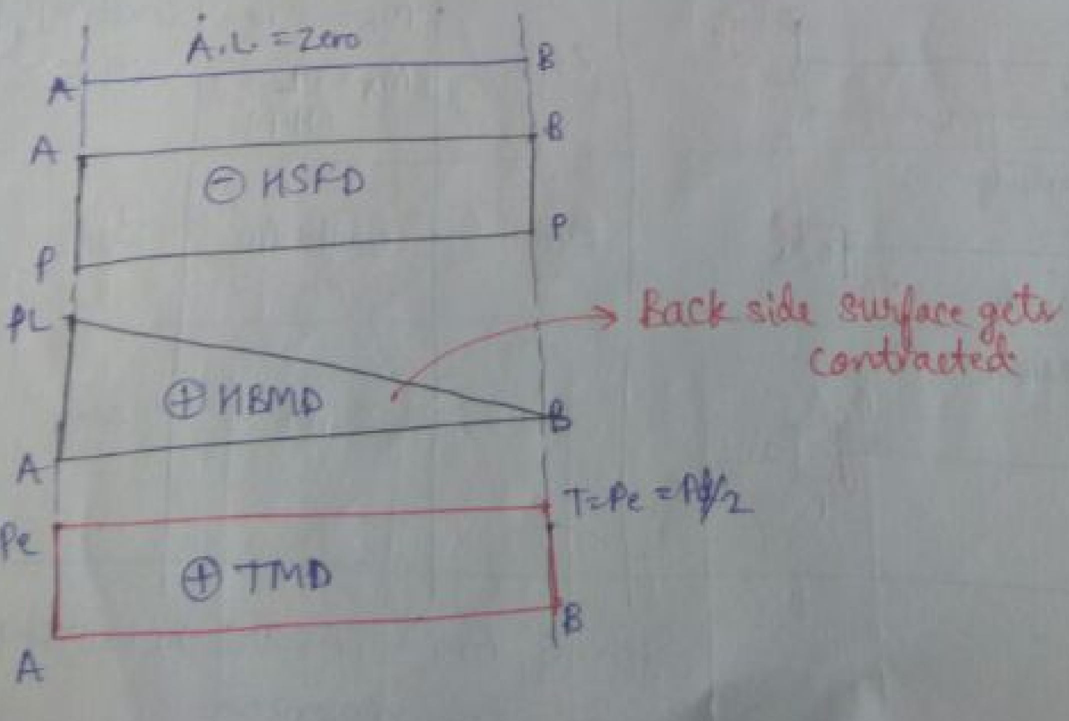
Fixed end → C.X-S/C

At arbitrary x-S/C,

$$\left.\begin{aligned}(AL)_{xx} &= \text{zero.}\\ (SF)_{xx} &= -P = \text{const.}\\ (BM)_{xx} &= +Px \ (\text{variable})\end{aligned}\right\} \text{H.P}$$

$$(TM)_{xx} = Pe = +Pd/2 = \text{const.} \ \} \ (\text{P.P}).$$





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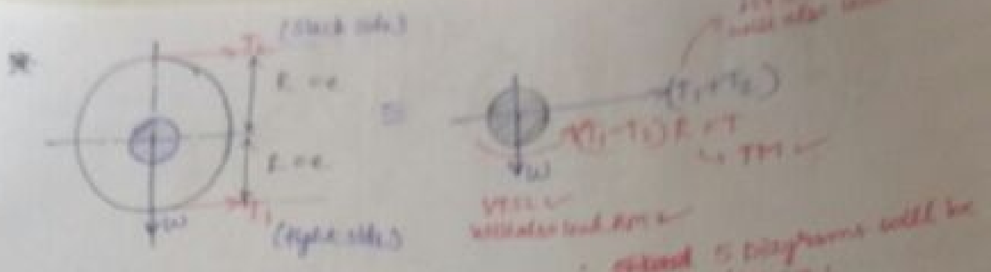
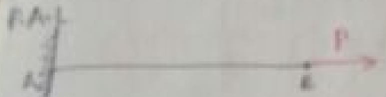
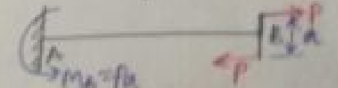
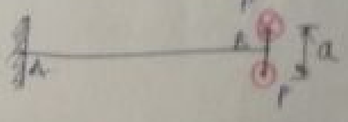


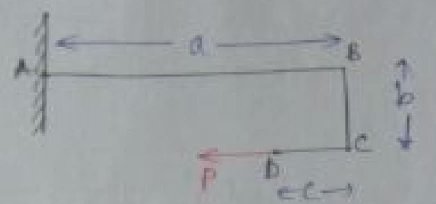
∴ 5 Diagrams will be drawn.

Note -

Eq. Loads on the X-Sc (AB) → Loading Condⁿ ↓	AF ($\sigma_t = \frac{P}{A}$)	BM [$(\sigma_b)_{max} = \frac{M}{Z_{min}}$]	SF ($\tau_s = \frac{P}{A}$)	TM [$(\tau_s)_{max} = \frac{T}{Z_P}$]
F.A.L 	Const. = P	Zero	Zero	Zero
E.A.L 	Const. = P	Const. = Pe (Due to bending couples)	Zero	Zero
TSL 	Zero	Variable M_B = Zero $M_A = P.L$ (M.P)	Const. = P (M.P)	Zero
ETSL 	Zero	M_B = Zero $M_A = PL$ (M.P)	Const. = P (M.P)	Const = Pe
Pure Bending - 	Zero	Const = Pa	Zero	Zero
Pure Torsion - 	Zero	Zero	Zero	Const = Pa.

Sign of BM & TM are not written. Only magnitude.

Q- For the structural member ABCD as shown in the fig. Det.
(i) Axial load, Shearforce, BM & TM on members AB, BC & CD.
(ii) Axial load, shearforce, BM & TM at x-S/Cs A, B, C, D.

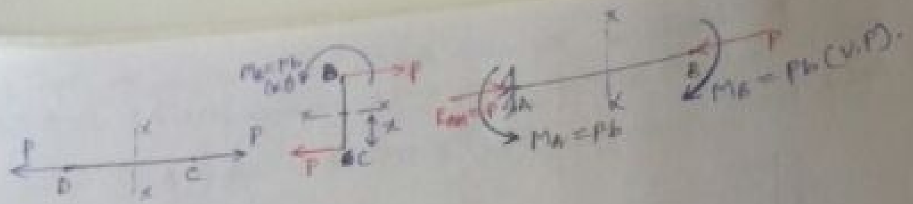


a>b>c

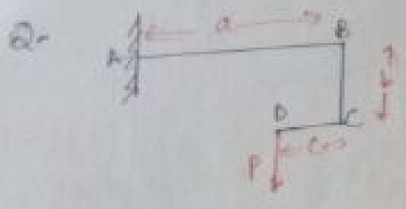
Eq. Loads / Member	A.L	SF	BM	TM
DC (PAL)	P const	0	0	0
BC (TSL)	0	Const.=P	$M_c=0$ $M_B=Pb$ → (+ve) Left side contraction.	0
AB (EAL) e=-b	Const=-P	0	Const=Pe =Pb → (+ve)	0

Eq. Loads (→) / X-S/C (↓)	A.L	S.F	BM	T.M
A	-P	0	Pb	0
B	-P	P	Pb	0
C	P	P	0	0
D	P	0	0	0

$M_c=0$.
$x=0$.

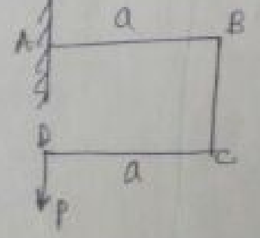


Q-



Eq. Loads / Member	AL	SF	BM	TM
DC (VTSL)	0	P = const.	$M_D = 0$, $M_C = PC$.	0
BC (EATL)	P = const.	0	$M_B = PC$ = const.	0
AB (EVTSL)	0	P = const.	$M_B = PC$, $M_A = P(a-c)$	0

*

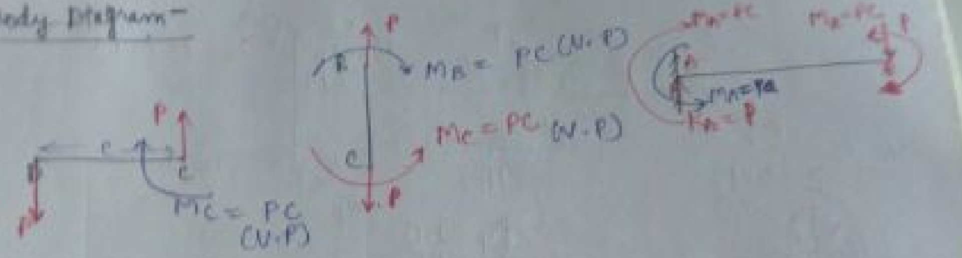


$M_A = ?$

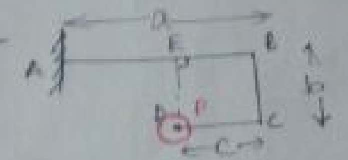
$M_A = 0$. (Lies on the line of action of ~~~~ load P)

Eq. Loads / X-S/C	AL	SF	BM	TM
A	0	P	P(a-c)	0
B	P	P	PC	0
C	P	P	PC	0
D	0	P	0	0

Free Body Diagram -



Q-

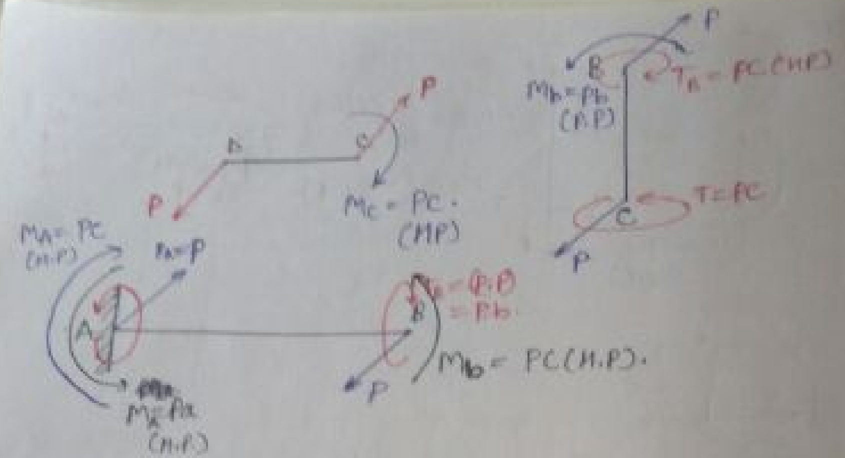


Eq. Loads / Member	AL	SF	BM	TM
CD (HTSL)	0	P	$M_D = 0$ $M_C = PC$ (H.P)	0
BC (EHTSL)	0	P	$M_C = 0$ $M_B = Pb$ (P.P)	Const = PC = PC (H.P)
AB (EVTSL)	0	P	$M_B = PC$ $M_A = P(a-C)$ (H.P)	Const = PC = Pb (P.P)

* BM of CD is TM of BC. & TM of BC is BM of AB
* BM of BC is TM of AB.

Vertically हो तो load shift हुआ है

Eq. Loads / Member	AL	SF	BM	TM
A	0	P	P(a-C)	Pb
B	0	P	Pb	Pb
C	0	P	PC	PC
D	0	P	0	0

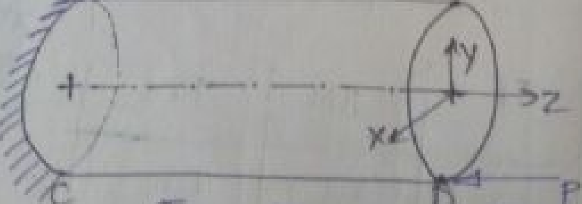


* V.P couple becomes B.C for Horizontal & Vertical Members.
* H.P couple & P.P. couple becomes BC for one member & TC for another.

Ch-3
5, 6, 7, 8, 9, 13, 18, 27 ,, 2, 11

Q- For a prismatic bar, find max. stress developed in x-s/c of bar.

$\sigma_{T.S} = -\sigma_a + [(\sigma_b)_{max}]_{x-x}$



$\sigma_{B.S} = \sigma_{max} = -\sigma_a - [(\sigma_b)_{max}]_{x-x}$.

(a) A & B
(b) B & C
(c) C & D
(d) A & D

Every x-s/c is a critical x-s/c.

*Any point on the bottom surface are critical points.

$$\sigma_{max} = \sigma_{B.S} = -\sigma_a - [(\sigma_b)_{max}]_{xx}$$

$$= -[\sigma_a + \{(\sigma_b)_{max}\}_{xx}]$$

$$= (comp.)\left[\frac{P}{A} + \frac{M_y \text{ or } Pe}{Z_{xx}}\right].$$

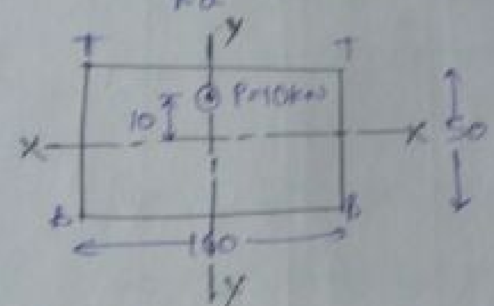
$$= (comp.)\left[\frac{4P}{\pi d^2} + \frac{P(d/2)}{\pi d^3/32}\right].$$

$$= \frac{20P}{\pi d^2} \text{ (comp.)}$$

$$\sigma_{TS} = -\sigma_a + [(\sigma_b)_{max}]_{xx}$$

$$= \frac{12P}{\pi d^2} \text{ (Tensile).}$$

Q-



$$\frac{\sigma_{max}}{\sigma_{axial}} = ?$$

$$\sigma_a = \frac{P}{A} = \frac{10000}{100 \times 50} = 2 \text{ MPa}$$

$$[(\sigma_b)_{max}]_{xx} = \frac{M_y \text{ or } Pe}{Z_{xx}}$$

$$= \frac{10000 \times 10}{\frac{1}{6} \times 100 \times 50^2} = 2.4 \text{ MPa}$$

$$\sigma_{max} = \sigma_a + (\sigma_b)_{max}$$

$$= 4.4 \text{ MPa}$$

$$\frac{\sigma_{max}}{\sigma_{axial}} = \frac{4.4}{2} = 2.2 \text{ MPa}$$

Q- For the member as shown in fig. Det. the stress developed at point A.