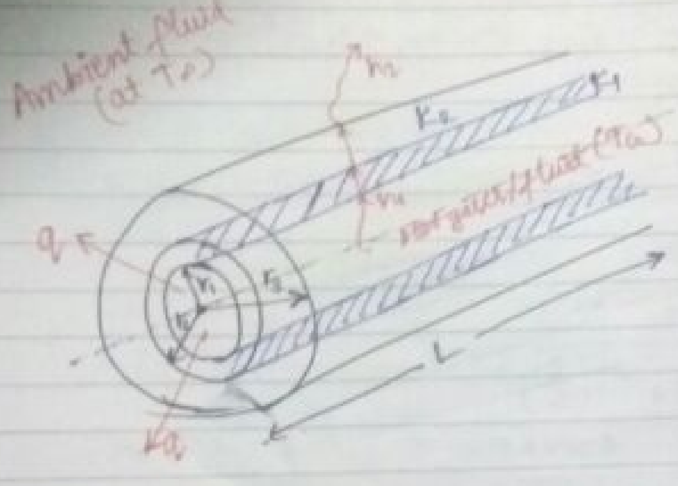
Radial

→ **Conduction- convection H.T through a composite cylinder-**



h_1 = Inside conv. HT coeff.

h_2 = Outside conv. HT coeff.

Assume-

Steady state
One dimensional
Convection conduction heat transfer
b/w hot gases inside and the ambient colder fluid.

Thermal Circuit-

$$\frac{1}{h_1 2\pi r_1 L} \qquad \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_1 L} \qquad \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_2 L} \qquad \frac{1}{h_2 2\pi r_3 L}$$

∴ Rate of H.T b/w hot gases and ambient

$$\Rightarrow q = \frac{T_a - T_\infty}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_2 L} + \frac{1}{h_2 2\pi r_3 L}} \quad \text{— ①}$$

→ Defining overall heat tranfer coeff. (U_i). ie. based on the inside convection heat tranfer area and overall heat tranfer coeff. U_o ie. based on the outside convection heat tranfer area from the equation,

$$q = U_i A_i \Delta T = U_o A_o \Delta T.$$

$$\Rightarrow q = U_i \underbrace{2\pi r_1 L}_{\text{Conv. area inside}} (T_a - T_\infty) = U_o \underbrace{2\pi r_3 L}_{\text{Conv. area outside.}} (T_a - T_\infty). \quad — ②$$

Comparing ① & ②,

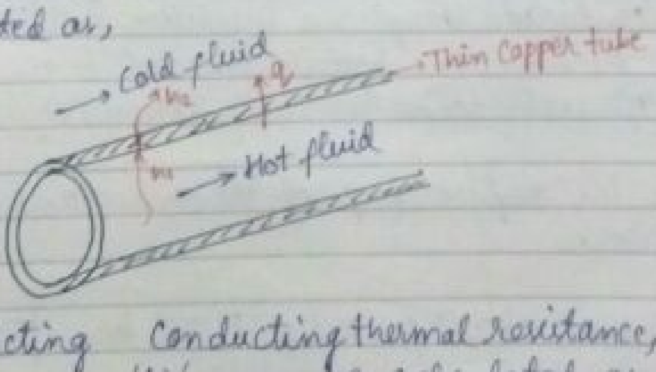
multiplying by $2\pi r_1 L$

$$\boxed{\frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1}{k_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{k_2} \ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{r_3}\left(\frac{1}{h_2}\right)}$$

multiplying by $2\pi r_3 L$

$$\boxed{\frac{1}{U_o} = \left(\frac{r_3}{r_1}\right)\left(\frac{1}{h_1}\right) + \frac{r_3}{k_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_3}{k_2} \ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_2}}$$

→ But in heat exchanger analysis, whether LMTD or effectiveness -NTU (ε-NTU) method, U can be calculated as,

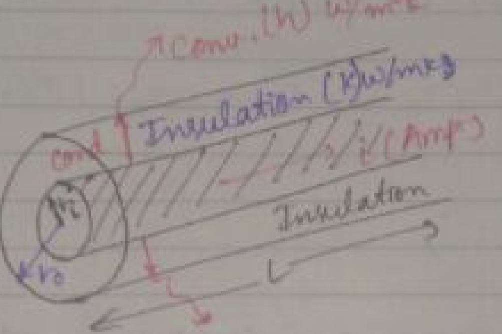


Neglecting conducting thermal resistance, 'U' can be calculated as,

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}}.$$

Critical Radius of Insulation -

Note: for sufficiently thin wires, putting the insulation arround the wire may result in increase of heat transfer rate instead of decrease in heat.



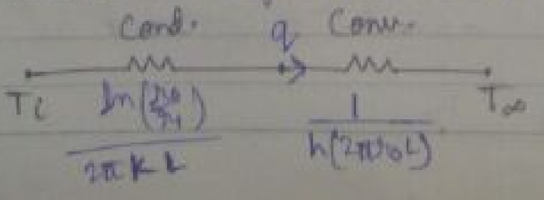
Consider a solid wire of radius r_i inside which heat is being generated by passing electric power.

Let an insulation having a thermal conductivity k is being wrapped around the wire upto the radius r_0.

The heat generated in the wire due to passage of current is radially conducted through the insulation and then from the surface of the insulation, heat is convected to the ambient fluid at T_∞ with convective heat transfer coeff. h w/m²k.

Let T_i be the surface temp of the wire under steady state condition.

Drawing thermal circuit for radial H.T. b/w T_i & T_∞.



∴ Rate of Radial H.T b/w wire & ambient.

$$\boxed{q = \frac{(T_i - T_\infty)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k l} + \frac{1}{h 2\pi r_o l}} \text{ Watt.}}$$

* Assuming all other parameters including 'h' as constant

$$\boxed{q = f(r_o)} \text{ only.}$$

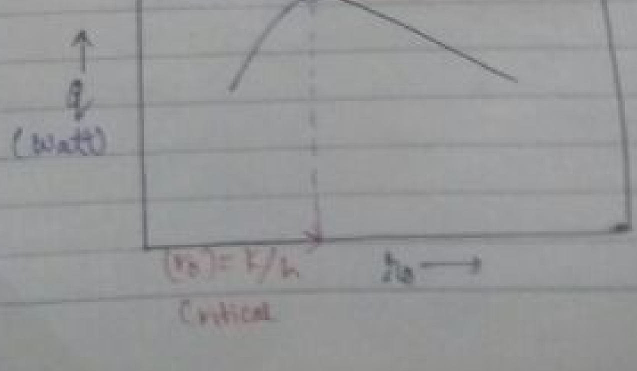
The value of r_o depends upon how much insulation is being wrapped around a wire.

For maximum H.T Rate,

$$\frac{dq}{dr} = 0 \Rightarrow \frac{d}{dr_o}\left[\frac{(T_i - T_\infty)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k l} + \frac{1}{h 2\pi r_o l}}\right] = 0.$$

$$\therefore \boxed{r_o = \frac{k_{ins}}{h}} \rightarrow \text{Critical radius of insulation}$$

Physical Significance of critical radius of insulation -



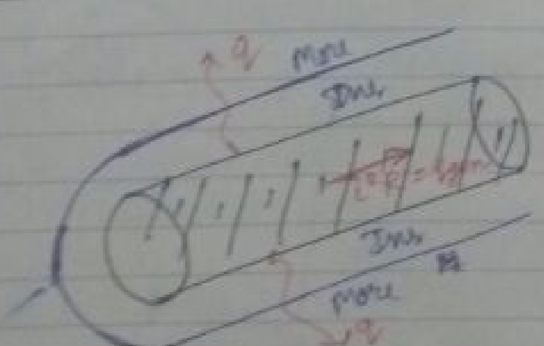
Note: → For sufficiently thin wires whose radius ~~of~~ is lesser than critical radius of insulation, putting the insulation around the wire will result in increase of H.T rate instead of decrease in heat. This happens so because initially when more & more insulation is being wrapped around the wire, there is a rapid decrease in ~~conduction~~ conv. thermal resistance as compared to little increase of ~~therm~~ conduction thermal resistance, the overall effect being decrease in total thermal resistance and hence increase of ~~thermal~~ heat transfer rate.

→ ~~If~~ This continues to happen upto critical radius of insulation beyond which any further insulation added shall ↓ the HT rate.

→ In case if the radius of the wire initially considered ~~o~~ is already more than critical radius of insulation, then any insulation wrapped around it will decrease the H.T rate,

→ Practical Application of Critical Radius -

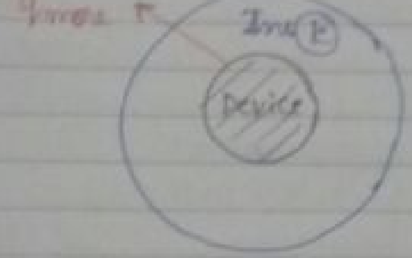
1- Electrical Power transmission Cables -



As $T_{cable}\uparrow \Rightarrow R_{cable,ele}\uparrow$

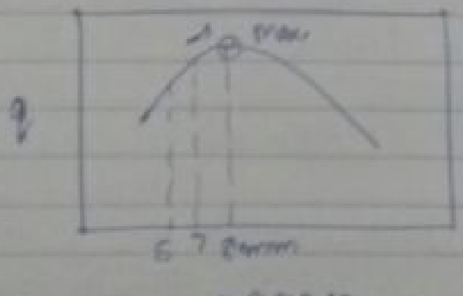
→ Insulation is put up around the electric power transmission cable to increase the heat transfer rate b/w the cable & the ambient so that the temp of the cable can be maintained low, thus its electric resistance can be maintained low thereby transmitting more electric power.

2- Spherical Electronic (Semiconductor) devices -

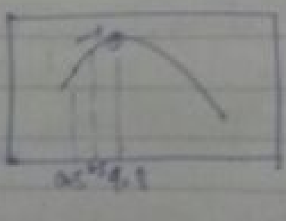


$$\boxed{(r_{critical})_{spherical} = \frac{2k}{h}}$$

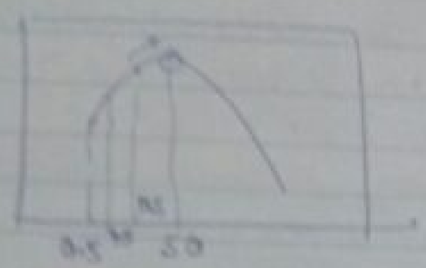
40- $r_{critical} = \frac{0.01}{10} = 1mm.$
(c)



43- $r_c = \frac{\cancel{0.12}\ 0.0049}{25.1} =$ ~~[illegible]~~ 4.8mm.
(a)



32- $r_c = \frac{0.5}{10} = 0.05 = 50 mm$
(a)



* As $q_{\text{from wire}}$ ↑ ⇒ T_{cable} ↓ ⇒ R_{ele} ↓ ⇒ More electric power can be transmitted.

50- $r_c = \frac{0.1}{2} = 0.05$
$= 5 cm$

* [k → of insulation
h → in contact with the insulation]

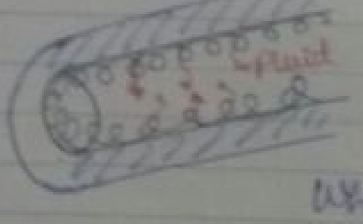
30- ~~$r_c = \frac{0.04}{10} = 4 mm$~~
(b)

$r_c = \frac{2k}{h} = \frac{2 \times 0.04}{10} = 8 mm$

∴ Dia = 16 mm.

38- (c) provided the radius of cable initially taken is lesser than critical radius of insulation.

42-
(A)



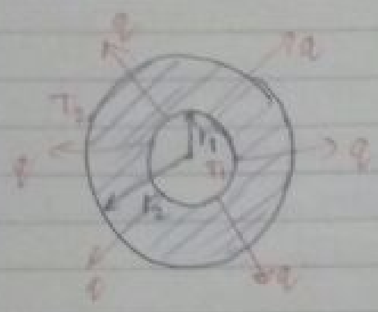
$R_{\text{conv.}} = \frac{1}{hA_{\text{contact}}}$
Surface Resistance

When insulation is kept inside,
since A_{contact} (fluid to solid) decreases ⇒ $R_{\text{conv.}}$ ↑.

Internal Res.

Since both R_{cond} and R_{conv}. ↑ ⇒ HT rate always decreases.

→ Radial Conduction H.T through a hollow sphere -



$T_1 > T_2$

$T = f(r)$

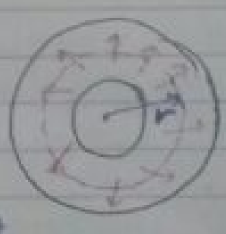
At $r = r_1 \Rightarrow T = T_1$.

At $r = r_2 \Rightarrow T = T_2$.

→ Since temp. gradients are exiting along the radial dirⁿ, heat must conduct radially outwards from the Inner spherical surface at T_1 to outer spherical surface at T_2.

→ Like in case of cylindrical H.T., here also the area of conduction heat transfer changes in the dirⁿ of heat flow. At any radius r, the area of conduction heat transfer is $4\pi r^2$.

ie., $\boxed{A = 4\pi r^2}$.



Fourier's law of conduction,

Rate of Radial H.T = $q = -KA\frac{dT}{dr}$ watt

$$\therefore q = -K4\pi r^2 \frac{dT}{dr}.$$

$$\int_{r_1}^{r_2} q \frac{dr}{r^2} = \int_{T_1}^{T_2} -4\pi K\, dT$$

Assume:

Steady state

One-dimension (radial) H.T. with constᵗ K.

To satisfy steady state condition, $q \neq f(r)$.

$$\text{i.e. } q_{r} = q_{r+dr}.$$

$$\therefore \quad q\left[\frac{-1}{r}\right]_{r_1}^{r_2} = 4\pi k\,(T_1 - T_2).$$

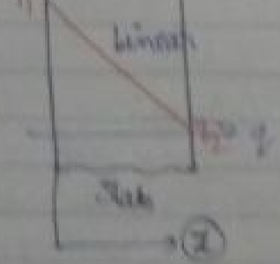
$$\therefore \quad \boxed{q = \left(\frac{4\pi k\,(T_1 - T_2)\, r_1 r_2}{r_2 - r_1}\right) \text{Watt}}$$

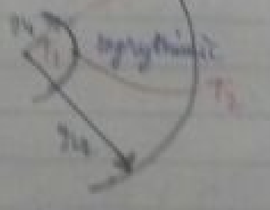
→ The corresponding conduction thermal resistance for hollow sphere is :

$$\left(R_{th}\right)_{\substack{\text{cond.} \\ \text{(hollow sphere)}}} = \frac{T_1 - T_2}{q} \text{ K/watt}$$

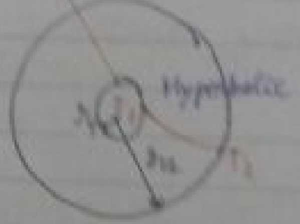
$$= \boxed{\frac{r_2 - r_1}{4\pi k\, r_1 r_2} \text{ K/watt}}.$$

* Temp. profiles-





Hollow Cylinder



Hollow Sphere

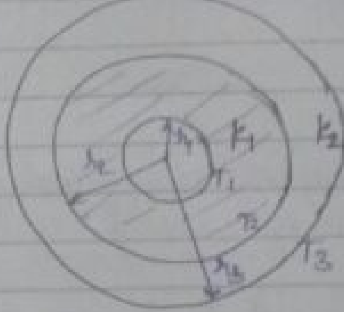
23- $\cancel{q = \frac{T_1 - T_2}{r}}$

(d)

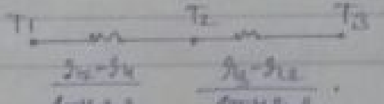
$$\frac{k_1}{k_2} = \frac{1}{2}.$$

$r_2 - r_4 = r_3 - r_2$. (Equal thickness)

$$\frac{r_4}{r_3} = 0.8$$



$$q = \frac{T_1 - T_2}{\frac{r_2 - r_4}{4\pi k_1 r_4 r_2}} = \frac{T_2 - T_3}{\frac{r_3 - r_2}{4\pi k_2 r_2 r_3}}$$

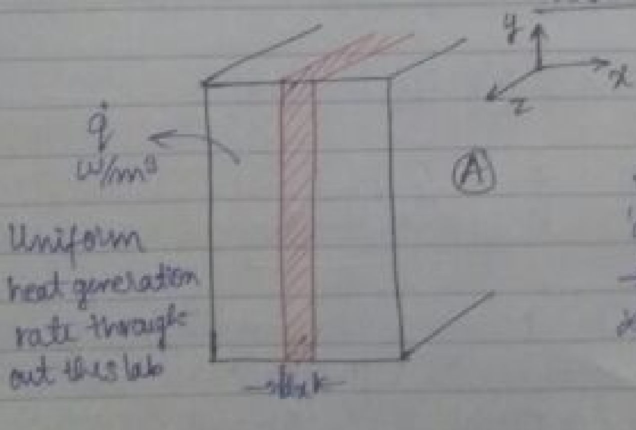


$$(T_1 - T_2) k_1 r_4 = (T_2 - T_3) k_2 r_3.$$

$$(T_1 - T_2)\, 0.8 = (T_2 - T_3)\, 2.$$

$$\frac{T_1 - T_2}{T_2 - T_3} = \boxed{2.5}.$$

Generalised heat conduction (3D, steady or unsteady, with or without heat generation) equation-



Consider a small differential element of the slab of length 'dx' as shown in fig.

~~If q_x is the heat~~

Let q_x = heat conducted into the element along x-dirⁿ

$$= -KA\frac{dT}{dx}.$$

Similarly,

q_{x+dx} = heat conducted out of the element along x-dirⁿ,

$$= q_x + \frac{\partial}{\partial x}(q_x)dx.$$

Heat generated in the element = $\dot{q}$ × Volume of element

$$= \dot{q} \times A\,dx \text{ watt.}$$

→ Writing the energy balance for x-dirⁿ heat conduction through the element, we get

$$q_x + q_{gen} = q_{x+dx} + \text{Rate of change of } \overset{\text{Internal energy}}{\overset{\uparrow}{IE}} \text{ of element} \atop \text{wrt time}$$

$$\cancel{q_x} + \dot{q}A\,dx = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)\,dx + \frac{\partial}{\partial \tau}\underbrace{(mC_pT)}_{\text{IE of element}}$$

where τ = time in sec.

m = mass of element = $\rho A dx$.

$$\dot{q}\cancel{A}\,\cancel{dx} = \frac{\partial}{\partial x}\left(-k\cancel{A}\frac{dT}{dx}\right)\cancel{dx} + \frac{\partial}{\partial \tau}(\rho \cancel{A}\,\cancel{dx}\,C_pT),$$

$$\cancel{\dot{q}}\quad k\frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p\frac{\partial T}{\partial \tau}.$$

→ Writing the energy balance similarly for all the 3 Directional conductions that are occuring along x, y & z dirⁿs simultaneously, we get

$$k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial y^2} + k\frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p\frac{\partial T}{\partial \tau}.$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \left(\frac{\rho C_p}{k}\right)\frac{\partial T}{\partial \tau}.} \rightarrow \text{Rate of heating or cooling.}$$

Thermal Diffusivity (α) – Defining thermal diffusivity (α), a thermophysical prop. of material as the ratio b/w thermal conductivity of the material & its thermal capacity i.e.

$$T.D = \alpha = \left(\frac{\overset{\rightarrow\, W/mK}{k}}{\underset{\rightarrow\, J/kgK}{\rho c_p}}\right),$$

Remember 'J' not 'kJ'

$$\alpha \rightarrow m^2/s.$$

ρC_p → Heat capacity or Heat storage ability.

Diffusion → Passing through or penetrating through.

→ Thermal diffusivity of a material tells about the ability of the material to allow the heat energy to get diffused through the material more rapidly.

→ Higher the conductivity of material & lesser its heat capacity or heat storage ability, more the value of thermal diffusivity.

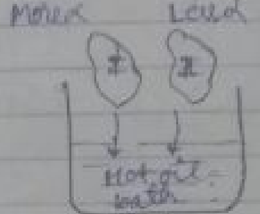
$$\uparrow\alpha = \frac{K\uparrow}{\rho C_p\downarrow}$$

*** $\alpha_{gases} > \alpha_{Liquid}$

ex, $\alpha_{air} > \alpha_{water}$

but $K_{air} < K_{water}$

Capacity is also need to check → $(\rho C_p)_{air} <<< (\rho C_p)_{water}$



Body I will come in eq^n with hot oil faster than II because of its good heat penetrating ability.

$$\therefore \boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}^*}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)}$$

→ If conditions are steady, $\left(\frac{\partial T}{\partial \tau}\right) = 0$.

→ If there is no heat generation, $\dot{q} = 0$.

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0.$$

$\boxed{\nabla^2 T = 0}$. (Laplace eq^n in T).

14-
(d)

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\dot{q}}{k}}^{0} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}.$$

Constant property → ie. α = constant.

$$\therefore \frac{\partial T}{\partial t} \propto \frac{\partial^2 T}{\partial x^2}.$$

22-
(b)

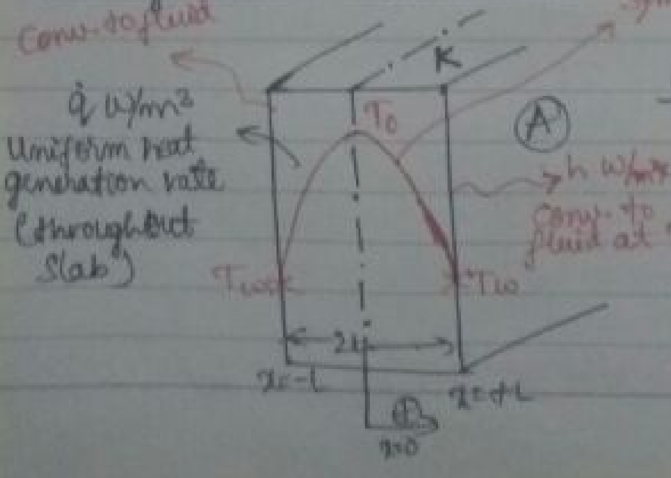
$$\frac{\partial T}{\partial x} = -10 + 40x + 30x^2.$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{x=1} = 40 + 60x = 100.$$

$$100\,\cancel{\alpha} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\boxed{\frac{\partial T}{\partial t} = 0.2}$$

→ **Heat generation in a slab** -



Objective - To get temperature distribution within the slab ie. $T = f(x)$.

Assume:

(i) Steady state H.T. conditions ie $T \neq f(\text{time})$.

Note: To maintain this steady state conditions of the slab while generating heat, all the heat generated in the slab must be convected to a fluid either from one side of the slab or from both the sides.

(ii) One dimensional heat conduction ie. $T = f(x)$ only.

(iii) Uniform heat generation rate and constant 'K' of material.

$$\therefore \frac{\partial^2 T}{\partial x^2} + \cancelto{0}{\frac{\partial^2 T}{\partial y^2}} + \cancelto{0}{\frac{\partial^2 T}{\partial z^2}} + \frac{\dot{q}}{K} = \underset{\text{Steady}}{\cancelto{0}{\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}}$$

$$\frac{d^2 T}{dx^2} = -\frac{\dot{q}}{K}.$$

Integrating wrt. 'x',

$$\frac{dT}{dx} = \frac{-\dot{q}}{K}x + C_1.$$

Integrating,

$$T = \frac{-\dot{q}}{K}\frac{x^2}{2} + C_1 x + C_2,$$

C_1 & C_2 are the constants of integration which are to be obtained from boundary conditions.

One <u>special</u> boundary conditions is,

At $x = +L$ and $x = -L \Rightarrow T = T_w$.

→ This can be possible only when both sides of the slab are exposed to the same fluid at the same temp. with the same convect to heat tranfer coeff. h.

→ To satify this "special" boundary condition c_1 must be zero. Converse is also true. In case if both sides of the slab are at diff. temp. then c_1 would not be zero.

The temp. of slab is max. when $\frac{dT}{dx} = 0$.

$$0 = -\frac{\dot{q}}{k}x + \cancel{c_1}^{\to 0}$$

$$\Rightarrow x = 0 \text{ (ie. At mid-plane).}$$

ie we see the max. temp. of slab at its mid-plane (Only if both sides of the slab are at same temp.)

→ Let the max. temp. of slab be T_0.
ie. At $x = 0 \Rightarrow T = T_0 \Rightarrow c_2 = T_0$.

∴ The temp. distribution within the slab is –

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + T_0.$$

$$\boxed{T_0 - T = \frac{\dot{q}x^2}{2k}} \quad \text{— ①}$$

→ Parabolic temp. distribution

At $x = +L$ (or) $x = -L \Rightarrow T = T_w$.

$$T_0 - T_w = \frac{\dot{q}L^2}{2k}. \quad —②$$

$$\frac{①}{②} \Rightarrow \boxed{\frac{T_0 - T}{T_0 - T_w} = \left(\frac{x}{L}\right)^2} \rightarrow$$ Non-dimensional format of Temp. distribution.

→ The side wall temp. T_w can be obtained from energy balance equation for steady state conditions of the slab i.e.

$\boxed{\text{heat generation in the slab = Heat convected to fluid.}}$

$$\dot{q} \times (\cancel{2}L \times \cancel{A}) = \cancel{2} \times h\cancel{A}(T_w - T_\infty) \text{ Watt.}$$

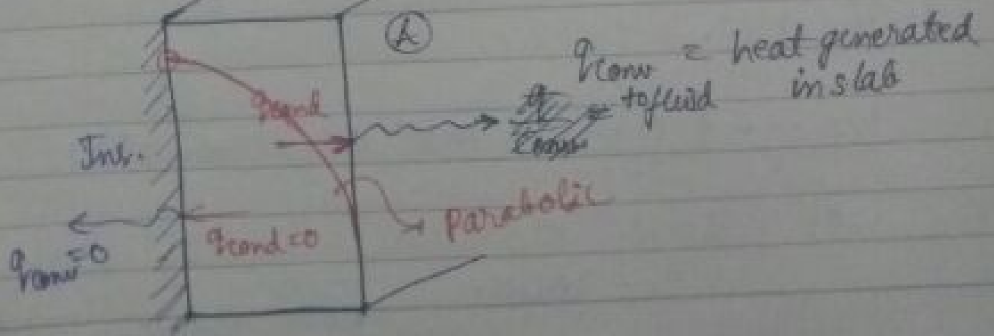
$$\boxed{T_w = \frac{\dot{q}L}{h} + T_\infty}.$$

∴ from eqn ①,

$$\therefore \boxed{T_0 = T_{max} = \frac{\dot{q}L^2}{2k} + \frac{\dot{q}L}{h} + T_\infty}.$$

↓ mid-plane temp.

*) The other extreme case is one side of the slab is perfectly insulated.



To maintain the steady state,
heat conducted at the ~~s~~ insulated surface = 0.

Otherwise, heat would get accumulated at the insulated side.

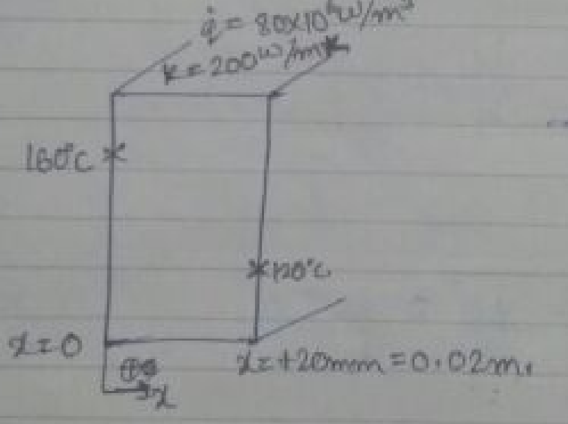
* Always conduction results to surface convection from surface.

$\Rightarrow -KA\frac{dT}{dx} = 0$ at the insulated surface

$\Rightarrow \frac{dT}{dx} = 0$ at the insulated surface.

i.e. T is max. at the insulated surface.

17-



$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{K} = 0.$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{K}.$$

Integrating wrt x,

$$\frac{dT}{dx} = \frac{-\dot{q}}{K}x + C_1.$$

$$T = \frac{-\dot{q}}{2K}x^2 + C_1x + C_2.$$

At x=0 ⇒ T = 160°C.

$\boxed{160 = C_2}$.

At x = 0.020 ⇒ T = 120°C.

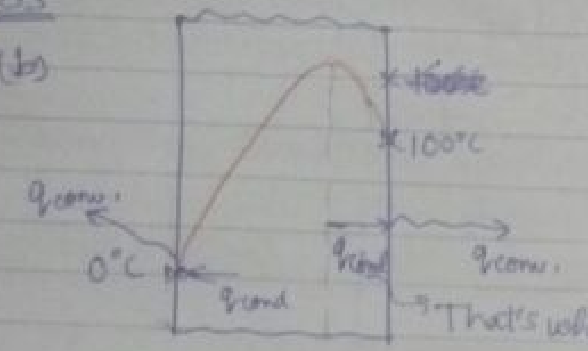
$$120 = -\cancel{\dot{q}}\frac{80\times10^4}{2\times\cancel{200}} \times \cancel{4\times10^{-4}} + C_1 \times 2\times10^{-2} + 160$$

$$\overset{20}{\cancel{40}} = \cancel{2}\times10^{-2}\,C_1$$

$\therefore C_1 = 2000.$

35-

(b)

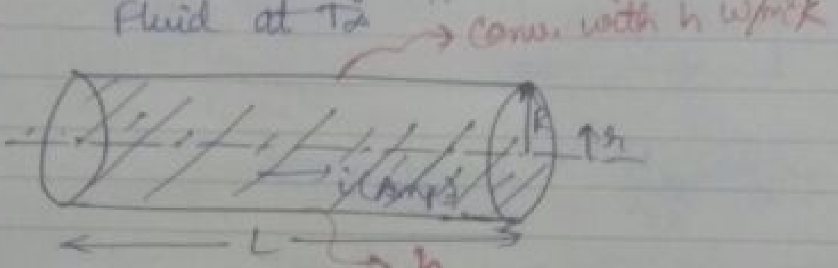


→ That's why not (A)

(A) would be the answer if no heat generation.

28- (a)

→ Heat generation in the cylinder:-



$\dot{q}$ = Uniform heat generation rate.

$= \left(\frac{i^2 R_{ele}}{\pi R^2 L}\right)$ watt/m³.

Objective- To get temp. distribution within the rod ie. $T = f(r) = ?$

Assume- (i) Steady state H.T conditions ie $T \neq f(\text{time})$.

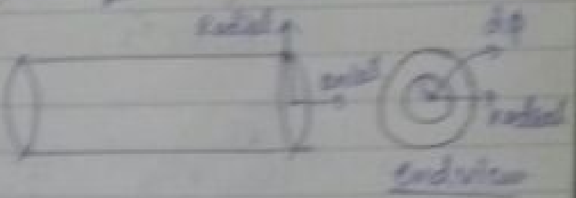
Note: To maintain this steady state conditions of the rod while generating heat, all the heat generated in the rod must be convected to the fluid surrounding the rod at T_∞ with a convect to heat transfer coeff. of 'h'.

(ii) One-dimensional heat conduction, $T = f(r)$ only.

(iii) Uniform heat generation rate i.e. ($\dot{q}$ = const.) and uniform 'k' value.

Heat condⁿ eq. in cylindrical coordinates.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \frac{\dot{q}}{k} = \cancel{\frac{1}{\alpha}\left(\frac{\partial T}{\partial t}\right)}$$



r = Radial dirⁿ
z = Axial Dirⁿ
φ = Azimuthal dirⁿ

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = -\frac{\dot{q}}{k}.$$

$$r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r} = -\frac{\dot{q} r}{k}.$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q} r}{k}.$$

d(uv) = udv + vdu.

Integrating wrt r.

$$r\frac{dT}{dr} = \frac{-\dot{q} r^2}{2k} + C_1.$$

$$\frac{dT}{dr} = -\frac{\dot{q} r}{2k} + \frac{C_1}{r} \quad —①$$

$$\boxed{T = -\frac{\dot{q} r^2}{4k} + C_1 \log r + C_2} \quad —②$$

C_1 & C_2 are constants of integration that are obtained from boundary conditions.

Boundary condⁿ is,

i) At $r=R$ (ie. at the surface of rod) ⇒ $T = T_w$.

ii) For steady state condⁿ of rod,
Heat generated in the rod = Heat conducted Radially at the surface = Heat convected to fluid.

$$\dot{q} \times (\cancel{\pi} R^{\cancel{2}} \cancel{L}) = -K(2\cancel{\pi}\cancel{R}\cancel{L})\left(\frac{dT}{dr}\right)_{r=R}$$

$$\left(\frac{dT}{dr}\right)_{r=R} = \frac{-qR}{2K} \quad \text{③}$$

__Note:__ C_1 can also be said as zero becoz logrythimic fn becomes infinite when $r=0$ (ie at the axis of the rod (but the temp. at the axis can't be infinite. So, $C_1 = 0$.

The temp. of rod be maximum when $\frac{dT}{dr} = 0$.

$$0 = \frac{-\dot{q}r}{2K} + \frac{\cancelto{0}{C_1}}{r} \Rightarrow r = 0 \text{ (ie. At axis).}$$

∴ We see the max. temp. of rod at its axis.

Let the max. temp. of Rod be T_0.

∴ At $r=0 \Rightarrow T=T_0$.
$\Rightarrow C_2 = T_0$.

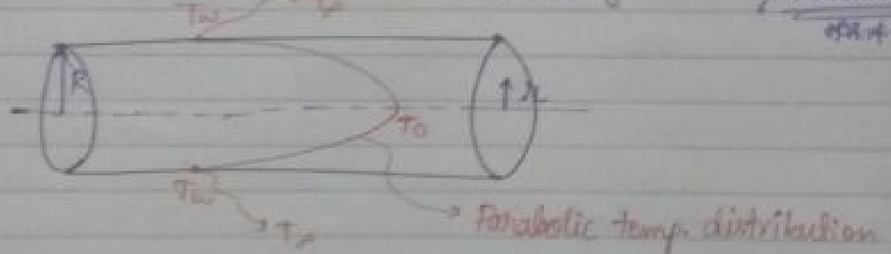
∴ The temp. distribution within the Rod is -

$$T = \frac{-\dot{q} r^2}{4K} + T_0 .$$

$$\boxed{T_0 - T = \frac{\dot{q} r^2}{4K}} \quad \text{—} ③$$

→ Parabolic temp. distribution.

→ Heat generated/Volume.



At $r=R \Rightarrow T = T_w$.

$$∴ T_0 - T_w = \frac{\dot{q} R^2}{4K} . \quad \text{—} ④$$

$$\frac{③}{④} = \boxed{\frac{T_0 - T}{T_0 - T_w} = \left(\frac{r}{R}\right)^2} .$$

→ Non-dimensional format of temp. distribution.

The surface temp of the rod T_w can be obtained from energy balance eqⁿ for steady state conditions of the rod.

i.e. Heat generated in the rod = Heat convected from the rod to fluid.

$$\dot{q} \times \pi R^2 L = h \times 2\pi R L (T_w - T_\infty) \text{ watt.}$$

$$\Rightarrow \boxed{T_w = \frac{\dot{q}R}{2h} + T_\infty}$$

$$\therefore \boxed{T_0 \text{ (or) } T_{max} = \frac{\dot{q}R^2}{4k} + \frac{\dot{q}R}{2h} + T_\infty}$$

i.e at Axis

Note: If the $\dot{q}$ values are very high and if the convect. to HT coeff 'h' are low then the T_0 values may enormously increase which may finally result in melting of the rod and the melting begins at the axis.

56- (C)

→Fins (Extended Surfaces) -

Fins are the projections protruding from a hot surface into ambient fluid and they are meant for increasing HT rate by increasing surface area of heat transfer.

Examples-
(i) Air-cooled IC engines
(ii) Reciprocating Air compressors
(iii) Refrigerator & AC condensor units.
(iv) Electric motors and transformers.
(v) Electronic Devices
(vi) Automobile Radiator