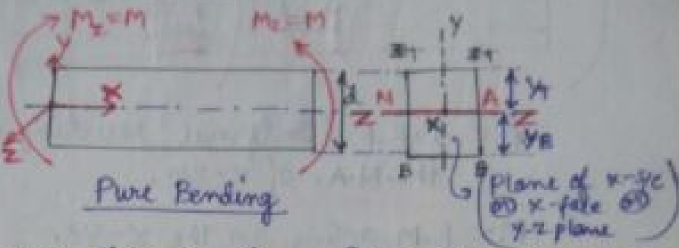
Chapter-5

Bending & Shear Stresses in Beams *

Bending Eq^n -

$$\boxed{\frac{M}{I_{NA}} = \frac{\sigma_b}{y} = \frac{E}{R}}$$



Pure Bending

→ Derived by assuming pure ~~Bex~~. Bending. [ie, SF = TM = AL = zero & BM = Const.]

⇓

ie. Uniaxial state of stress [∵ $\underset{= My/I}{\sigma_x = \sigma_b}$, $\sigma_y = \tau_{xy} = 0$].

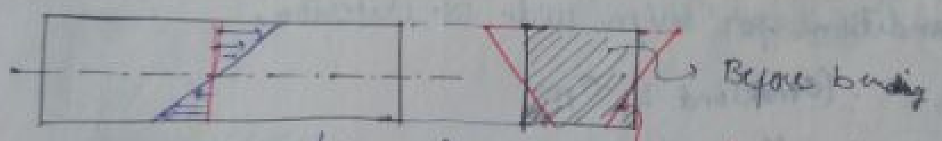
& triaxial state of ~~stress~~ Strain [ie $\varepsilon_x = \varepsilon_{long} = \frac{\sigma_x}{E}$; $\gamma_{xy} = \gamma_{xz}$

$\varepsilon_y = \varepsilon_z = -\mu\frac{\sigma_x}{E}$; $= \gamma_{yz} = 0$]

* For square, circle etc → σ_b max. at top & bottom.

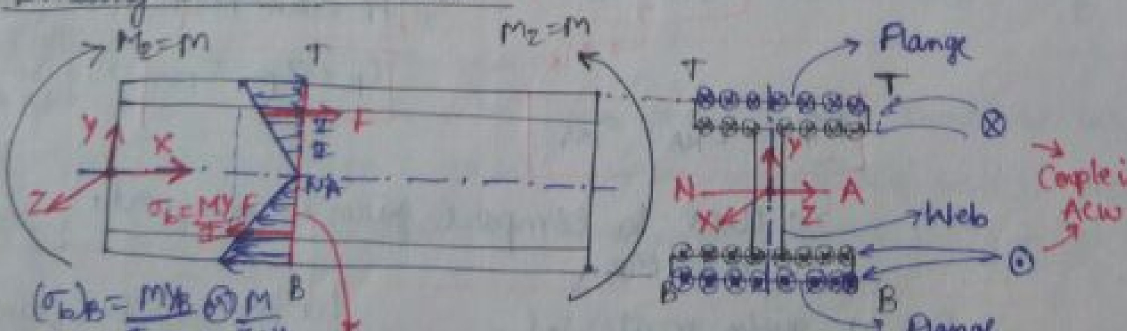
.. For triangle → σ_{max} at top (bcoz centroid is near to base).

For T-section → σ_b max. at bottom (bcoz centroid is near to top).



$$(\sigma_x = \sigma_b) = \frac{M}{Z_{NA}}$$

Bending Stress Distribution -



$(\sigma_b)_B = \frac{M y_B}{I_{NA}}$ (or) $\frac{M}{Z_{NA}}$

Right Hand Cut Face

Right Hand Side Cut Surface

(Plane of x-s/c (or) x-face (or) y-z plane)

that's why left side force लगा रहे हैं tension और comp. के ।

For a given x-s/c Area,

$(Z)_{I} > (Z)_{T} > (Z)_{\blacksquare} > (Z)_{\blacksquare} > (Z)_{\bullet} > (Z)_{\blacklozenge}$

For a given Area & Material,

$(MR)_{I} > (MR)_{T} > (MR)_{\blacksquare} > (MR)_{\blacksquare} > (MR)_{\bullet} > (MR)_{\blacklozenge}$

$[\because MR \propto Z]$

For a given Area & B.M (M),

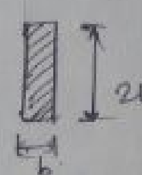
$(\sigma_b)_{\blacklozenge} > (\sigma_b)_{\bullet} > (\sigma_b)_{\blacksquare} > (\sigma_b)_{\blacksquare} > (\sigma_b)_{T} > (\sigma_b)_{I}$

$[\because (\sigma_b)_{max} \propto \frac{1}{Z}]$,

Note - If square is written in the question then assume it to be SS. When written then only SD will be considered.

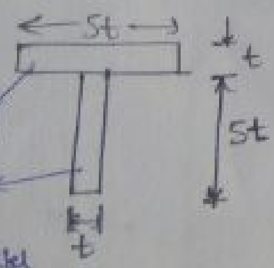
Q- Det. the Z, MR & $(\sigma_b)_{max}$ for following x-s/c, if $A = 1000\,mm^2$, $\sigma_{per} = 100\,MPa$, $M = 500\,N\text{-}m$.

(a) S.D, (b) Circular, (c) S.S, (d)



(e)

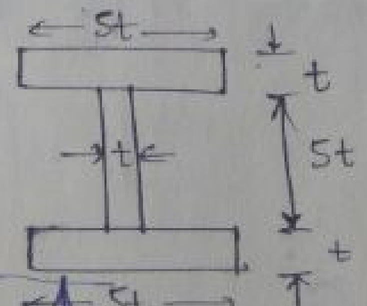
(f)



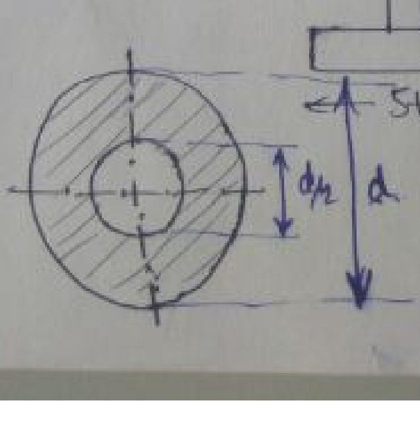
Z of this + Z of this

(Added by using parallel axis theorem i.e. $+MR^2$)

(g)



(h)

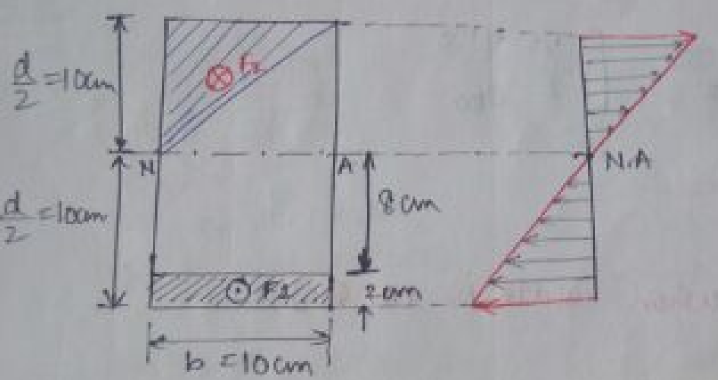


Q- Det. the following for the rectangular x-s/c as shown in fig. when it is subjected to sagging BM of 20kNm.

(i) Bending stress developed at a fibre located at a distance of 2cm from the bottom fibre.

(ii) Tensile force developed on the rectangular hatched area as shown in the fig.

(iii) Comp. force developed on the triangular hatched area.



(i) $(\sigma_b) = \frac{MY}{I_{NA}} = \frac{20\times10^3\times80}{\frac{1}{12}\times100\times200^3}$

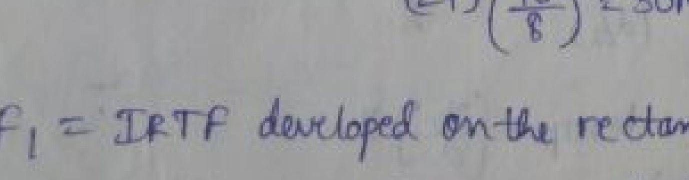
$$\boxed{\sigma_b = 24\,MPa}.$$

(ii) $(\sigma_b)_{max} = \frac{M}{Z_{NA}}$ (or) $(\sigma_b)\left[\frac{Y_{max}}{y}\right]$.

$(24)\left(\frac{10}{8}\right) = 30MPa.$

(iii) F_1 = IRTF developed on the rectangular hatched area.

$\sigma_{avg} = F/A \Rightarrow F = \sigma_{avg}\times A$ (valid when stress is uniformly distributed).



$dF = (\sigma_b)\,dA$.

$dF = \left(\frac{My}{I_{NA}}\right)(dy)(b_{strip})$

$$F = \int_{y_1}^{y_2} dF = \left(\frac{M}{I_{NA}}\right)\int_{y_1}^{y_2} (b_{strip})(y)(dy)$$

$$F_1 = \frac{M(b)}{I_{NA}} \int_{80}^{100} (y)\,dy.$$

$$F_1 = \frac{200\times10^6\times100}{\frac{1}{12}(100)(200)^3}\left[\frac{y^2}{2}\right]_{80}^{100}$$

$$\boxed{F_1 = 54\,kN}$$

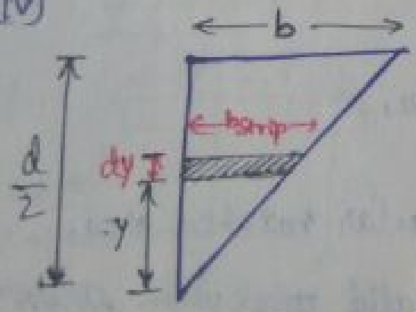
II[nd] Method - (Valid when width is const.)

$$F_1 = (\sigma_{avg})(A_1)$$

$$= \left[\frac{24+30}{2}\right]\times[100\times20]$$

$$\boxed{F_1 = 54\,kN}.$$

(iv)



$$\frac{b}{(d/2)} = \frac{b_{strip}}{y}.$$

$$b_{strip} = \left[\frac{2b}{d}\right]y.$$

$$F_2 = \frac{M}{I_{NA}}\int_0^{100} (y)(y)\,dy$$

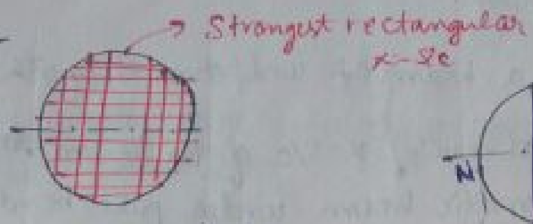
$$F_2 = \frac{20\times10^6}{\frac{1}{12}\times100\times200^3}\left[\frac{y^3}{3}\right]_0^{100} = \underline{100\,kN.}$$

IInd Method -

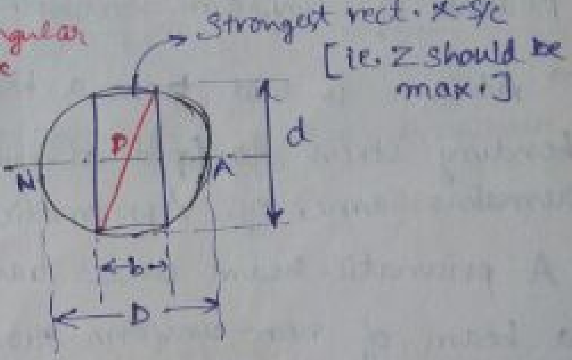
$$F_2 = (\sigma_{avg}) \times (A)$$
$$= \left(\frac{0+30}{2}\right) \times \left(\frac{1}{2} \times 100 \times 100\right)$$
$$= 75 \text{ kN}$$

IInd method is not valid when.

Ch5 - 6-



x-s/c of a circular log of wood of dia (D)



$$b^2 + d^2 = D^2$$
$$d^2 = D^2 - b^2 \quad \text{--- (1)}$$
$$Z_{NA} = \frac{bd^2}{6} = \frac{b}{6}[D^2 - b^2]$$
$$Z_{NA} = \frac{bD^2}{6} - \frac{b^3}{6} \quad \text{--- (2)}$$

for a strongest rectangular x-s/c, Z must be max. [ie $\frac{d}{db}(Z_{NA}) = 0$].

$$\frac{d}{db}\left[\frac{bD^2}{6} - \frac{b^3}{6}\right] = 0.$$
$$\frac{D^2}{6} - \frac{3b^2}{6} = 0 \Rightarrow D^2 = 3b^2.$$
$$\boxed{\frac{b}{D} = \frac{1}{\sqrt{3}}} \quad \text{--- (A)}$$

from ①,

$$d^2 = D^2 - \frac{D^2}{3}$$

$$\boxed{\frac{d}{D} = \sqrt{\frac{2}{3}}} \quad — (B)$$

$$\frac{(A)}{(B)} \Rightarrow \quad \boxed{\frac{b}{d} = \frac{1}{\sqrt{2}}} \quad — (C) \qquad (C) \quad \boxed{d = \sqrt{2}\, b}$$

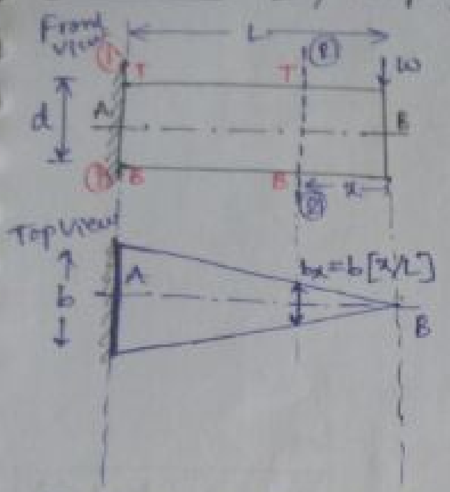
Beams of Uniform Strength -

- A beam is said to be a beam of uniform strength when bending stress developed at every x-s/c of the beam ~~rear~~ remains same, eg. A prismatic beam under pure bending.
- A prismatic beam under transverse shear ~~tod~~ load is said to be a beam of non-uniform strength because bending stress varies from x-s/c to x-s/c due to variable bending moment.
- To obtain a beam of uniform strength under T.S.L, non-prismatic beam should be used.
- ~~To obtain~~ Rectangular x-s/c beams can be made as beam of uniform strength under TSL by following 2 methods -

I[st] Method - By varying width and keeping depth as constant.

II[nd] Method - By varying depth and keeping width as constant.

Method I - [by keeping depth as const. & varying width].



for beam of uniform strength,

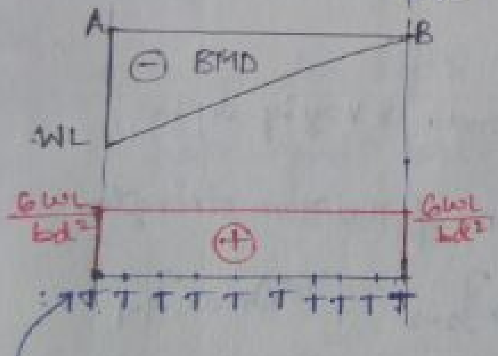
$$[(\sigma_b)_{max}]_{2-2} = [(\sigma_b)_{max}]_{1-1}$$

$$\left(\frac{M}{Z_{NA}}\right)_{2-2} = \left(\frac{M}{Z_{NA}}\right)_{1-1}$$

$$\frac{Wx}{\frac{1}{6} b_x d^2} = \frac{WL}{\frac{1}{6} b d^2}.$$

$$\boxed{b_x = b[x/L]}.$$

* Every single plane is a critical plane bcoz all the planes have $(\sigma_b)_{max} = \frac{6WL}{bd^2}$.



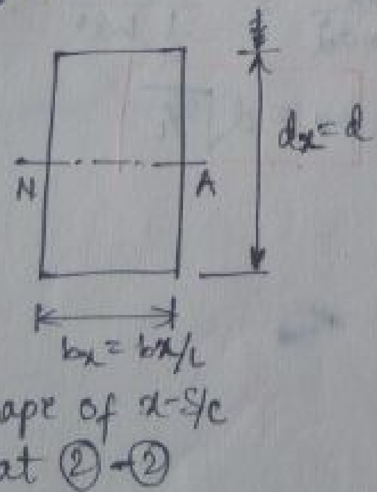
Bending stress variation on top surface.

two similar Δs.

σ_b variation on the x-S/c will be linear to y whereas for whole Top plane σ_b will be same as $(\sigma_b)_{max}$ is constant.

Rectangular.

Q-



Shape of x-S/c at ②-②

$$(I_{2-2})_{NA} = \frac{1}{12} b_x (d_x)^3$$

$$= \frac{1}{12}\left[\frac{bx}{L}\right] d^3.$$

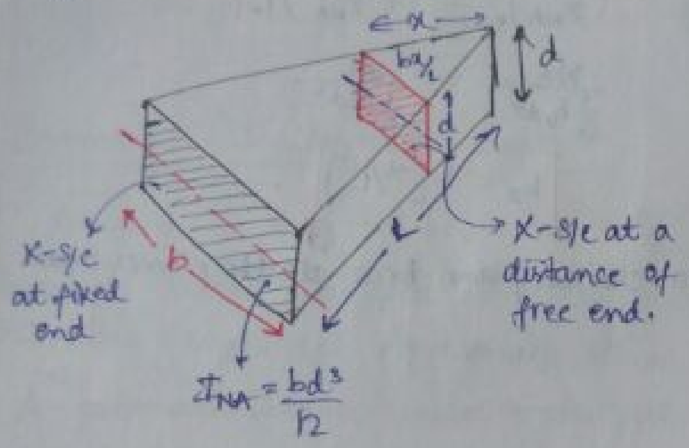
$$U = \int_0^L \frac{(Wx)^2 dx}{2E\left[\frac{bxd^3}{12L}\right]}$$

$$U = \frac{6W^2L}{Ebd^3}\int_0^L \left(\frac{x^2}{x}\right) dx = \frac{3W^2L^3}{Ebd^3}.$$

$$y_B = \frac{\partial U}{\partial W_B} = \frac{\partial}{\partial W}\left[\frac{3W^2L^3}{Ebd^3}\right].$$

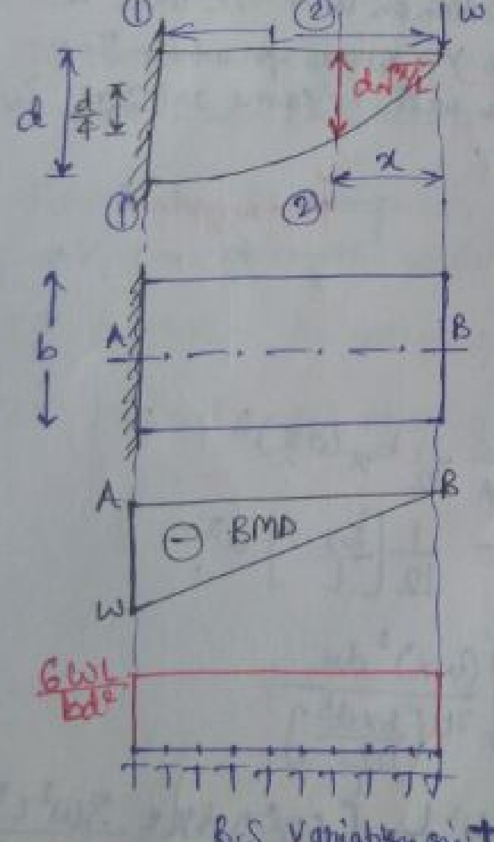
$$\boxed{y_B = \frac{6WL^3}{Ebd^3}}.$$

*



X-s/c at a distance of x from free end. $\Rightarrow \boxed{I_{NA} = \frac{1}{12}\left(\frac{bx}{L}\right)d^3}$

Method-II [By keeping width as const. & varying depth]



B.S variation on the T.S

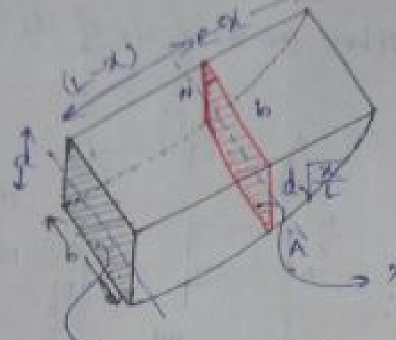
for beam of uniform strength,

$$\left[(\sigma_b)_{max}\right]_{2-2} = \left[(\sigma_b)_{max}\right]_{1-1}$$

$$\left(\frac{M}{Z_{NA}}\right)_{2-2} = \left(\frac{M}{Z_{NA}}\right)_{1-1}$$

$$\frac{Wx}{\frac{1}{6}bd_x^2} = \frac{WL}{\frac{1}{6}bd^2},$$

$$\boxed{d_x = d\sqrt{x/L}}$$



x-s/c at a distance of x from free end $\left[I_{NA} = \frac{1}{12}(b)\left(d\sqrt{x/L}\right)^3\right]$

x-s/c at fixed end $\left[I_{NA} = \frac{bd^3}{12}\right]$

Note: Cross-section remains rectangular no matter width varies or depth.

Expression for depth (d)-

Safe condⁿ wrt. st. criterion,

$$(\sigma_{max})_{ind} \le \sigma_{per}$$

$$\frac{6WL}{bd^2} \le \sigma_{per}$$

$$\boxed{d \ge \sqrt{\frac{6WL}{b(\sigma_{per}\text{ or }f)}}}$$

5- $\sigma_b = \frac{MY}{I_{NA}}$ (or) $\frac{EY}{R}$ (or) $(\sigma_b)_{max}\left[\frac{Y}{Y_{max}}\right]$

6- $(\sigma_b)_{max} = \frac{M}{Z_{NA}}$ (or) $\frac{EY_{max}}{R}$

7- For every x-S/c, σ_b variation consists of two similar Δs.

5- $\tau_s = \frac{PA\bar{Y}}{I_{NA}\, b}$.

6- $\tau_{max} = K\, \tau_{avg}$.

where $\tau_{avg} = P/A$.

$K = \frac{3}{2} \Rightarrow$

$= \frac{4}{3} \Rightarrow$

$= \frac{9}{8} \Rightarrow$

$\frac{\text{big}}{\text{small}} = \frac{3}{2} < \frac{4}{3} < \frac{9}{8}$

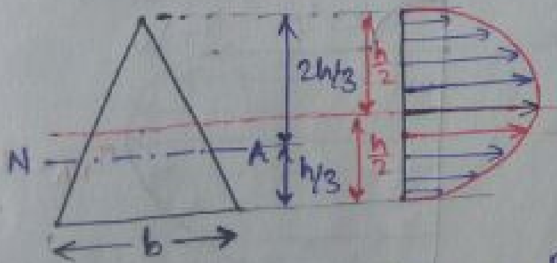
7- Shape of τ_s variations depends on shape of the x-S/c:

→ Uni-axial state of stress is developed (i.e. $\sigma_x = (\sigma_b)_{max}$; $\sigma_y = \tau_{xy} = 0$] at any point on the extreme fibres of the beam.

→ Bi-axial combined state of stress is developed at any point on the ~~mme~~ inner fibre of the beam. [$\sigma_x = \sigma_b$, $\sigma_y = 0$, $\tau_{xy} = \tau$].

→ Pure shear state of stress is developed at any pt. on the N.A of the beam ($\sigma_x = \sigma_y = 0$; $\tau_{xy} = \tau_{max}$ (or) τ]

For triangular x-S/c -

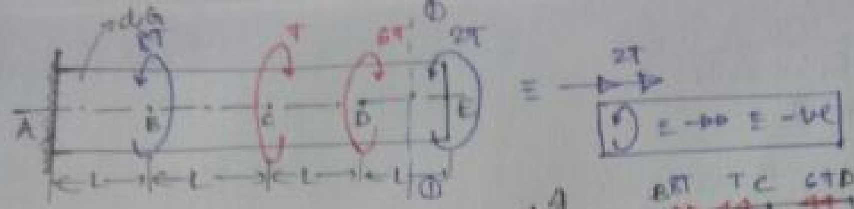


$\tau_{max} = \frac{3}{2}[\tau_{avg}]$

$\tau_{NA} = \frac{4}{3}\tau_{avg}$ (or) $\frac{8}{9}\tau_{max}$

$\tau_{avg} = \frac{P}{A} = \frac{2P}{bh}$.

$$\boxed{\frac{\tau_{max}}{\tau_{NA}} = \frac{9}{8} = 1.125}$$



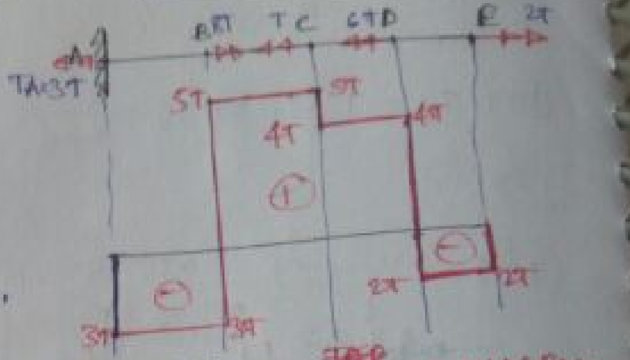
(a) $T_{DE} = -2T$.

$T_{CD} = 4T$.

$T_{BC} = 5T$. $\Rightarrow$ Max. torque.

$T_{AB} = -3T$.

$(T_{max})_{BC} = 5T = 3000\,Nm$.



TORQUE DIAGRAM

(b) d:

Safe condition for design wrt. st. criterion,

$$\tau_{max} \le \tau_{per}$$

$$\frac{T_{max}}{Z_p} \le \tau_{per}$$

$$\left(\frac{16T}{\pi d^3}\right)_{BC} \le 60.$$

$$\frac{16 \times 3000}{\pi \times d^3} \le 60.$$

$$d \ge 29.42\,mm.$$

$$\boxed{d = 30\,mm}.$$

(c) $(\theta\text{-total or } \theta_{EA}) = \theta_{ED} + \theta_{DC} + \theta_{CB} + \theta_{BA}$

$$= \frac{L}{GJ}\left[-2T + 4T + 5T - 3T\right]$$

$$= \frac{4TL}{GJ} = \frac{32 \times 4TL}{\pi G d^4} = \frac{128 \times 60 \times 10^3 \times 500}{\pi (80 \times 10^3)(30)^4}$$

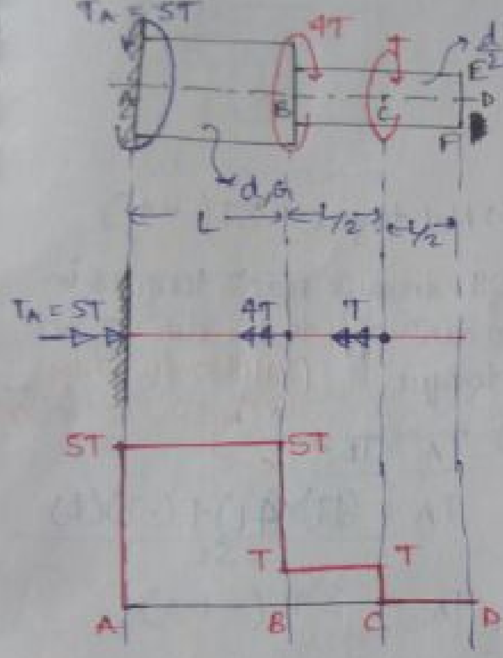
$$= 0.0188 \text{ radians.}$$

Q- For the stepped shaft as shown in the fig. Det.

(i) Angle of twist at D if angle of twist at C is θ.

(ii) Torsional shear stress at D, E, F if max. S.S is τ.

(iii) Exp^n for the max. torsional S.S. of the stepped shaft.



$T_{AB} = 5T = T_{max}$.

$T_{BC} = T$.

$T_{CD} = \text{zero}$.

$(\tau_{max})_{AB} = \left(\frac{16T}{\pi d^3}\right)_{AB} = \frac{80T}{\pi d^3}$.

$(\tau_{max})_{BC} = \left(\frac{16T}{\pi d^3}\right)_{BC} = \frac{16T}{\pi (d/2)^3} = \frac{128T}{\pi d^3}$.

$(\tau_{max})_{CD} = \left(\frac{16T}{\pi d^3}\right)_{CD} = \text{zero} \; (\because T_{CD} = 0)$.

$(\tau_{max})_{\text{stepped shaft}} = \frac{128\,T}{\pi d^3}$.

(i) $\theta_C = \theta$; $\theta_D = ?$.

$\theta_C = \theta_{CA} = \theta_{CB} + \theta_{BA} = \theta$.

$\theta_D = \theta_{DA} = \theta_{CD} + \theta_{CB} + \theta_{BA}$

$= \underbrace{\left(\frac{TL}{GJ}\right)_{DC}}_{0} + \theta = \theta$.

(ii) $\tau_D = \text{zero}$ ($\because$ D is at the centroid).

$\tau_{E,F} = \tau_{max} = \left(\frac{16T}{\pi d^3}\right)_{CD}$

$= \text{Zero}$.

$\Sigma T = 0 \Rightarrow T_A - T + T + T_D = 0.$

$$\boxed{T_A = -T_D}$$

$\theta_1 + \theta_2 + \theta_3 = 0.$

$$\frac{L}{G}\left(\frac{-T_A}{J} + \frac{T - T_A}{J/2} + \frac{-T_A}{J}\right) = 0.$$

$$\frac{L}{GJ}\left(-T_A + 2T - 2T_A - T_A\right) = 0.$$

$$2T = 4T_A$$

$$\boxed{T_A = T/2} \quad (\leftarrow)$$

$$\boxed{T_D = T/2} \quad (\rightarrow)$$

$T_1 = -T_A = -T/2.$

$T_2 = T - T_A = T - T/2 = T/2.$

$T_3 = -T_A = -T/2.$

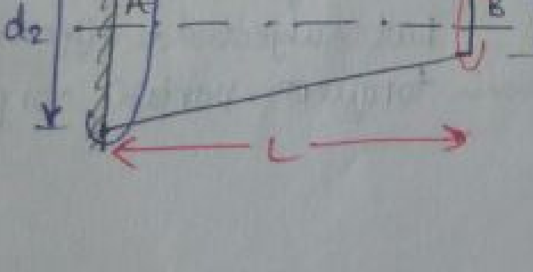
$$\theta_B = \theta_{BA} = \left(\frac{TL}{GJ}\right)_1 = -\frac{TL}{2GJ}.$$

$$\theta_C = \theta_{CD} = \left(\frac{TL}{GJ}\right)_2 = -\frac{TL}{2GJ}.$$

*** Note: If stepped shaft is of same material then apply short-cut method. If the shaft is ~~of~~ of more than one material then apply compatible equation. ($\Sigma\theta = 0$).

Torsion of ~~Stepped~~ Tapered Shaft –

→ Treated as assembly of n shafts which are in series.



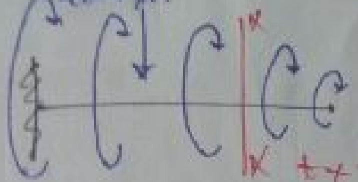
$$U = \int_0^L \frac{(T_{x-x})^2\,dx}{2(GJ)_{x-x}}$$

$$= \frac{1}{2GJ}\int_0^L (tx)^2\,dx.$$

$$= \frac{t^2}{2GJ}\left[\frac{x^3}{3}\right]_0^L$$

$$\therefore \boxed{U = \frac{16t^2L^3}{3\pi G d^4}}$$

WB



$t_{x-x} = \frac{tx}{L}$

$$t_{x-x} = \frac{tx}{L},$$

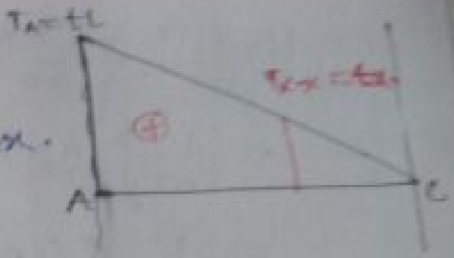
$$T_{x-x} = \int_0^x \left(\frac{tx}{L}\right)dx$$

$$= \frac{tx^2}{2L} \longrightarrow \text{Parabola. BMD.}$$

CA - $[x=0 \text{ to } L]$.

$t_{x-x} = t$ N-m/m.

Torque at x-s/c x-x $= T_{x-x} = \int_0^x t_{x-x}\, dx$.



$T_{x-x} = \int_0^x t\, dx$.

$T_{x-x} = tx$. N-m.

$x = 0 \Rightarrow T_C = 0$.

$x = L \Rightarrow T_A = tL = T_{max}$.

$\tau_{max} = \tau_A = \frac{16 T_A}{\pi d^3} = \frac{16 tL}{\pi d^3}$.

$\theta_C = \theta_{CA} = \int_0^L \theta_{strip}$.

$\theta_C = \theta_{CA} = \int_0^L \frac{(T_{x-x})\, dx}{GJ}$

$= \frac{1}{GJ} \int_0^L (tx)\, dx$

$$\boxed{\theta_C = \theta_{CA} = \frac{tL^2}{2GJ} \text{ or } \frac{16 tL^2}{\pi G d^4}}$$

*** $\theta_B = \theta_{BA} = \int_{L/2}^{L} \theta_{strip}$

$$\boxed{\theta_B = \theta_{BA} = \frac{12 tL^2}{\pi G d^4}}$$

(1) $T_1 = T_2 = T_3 = \dots = T_n = T$.

(2) $(\tau_{max}) = \tau_B = \left(\frac{16T}{\pi d^3}\right)_B = \left(\frac{16T}{\pi d_1^3}\right) = \frac{16T}{\pi (d_{smaller})^3}$.

(3) $\frac{\tau_{max}}{\tau_{min}} = \frac{\tau_B}{\tau_A} = \left(\frac{d_A}{d_B}\right)^3 = \frac{d_2}{d_1} = \left(\frac{d_{larger}}{d_{smaller}}\right)^3$.

(4) $\theta_{T.S} = \theta_1 + \theta_2 + \dots + \theta_n$.

(or) $\theta_{T.S} = \int_0^L (\theta_{strip}) = \int_0^L \frac{(T_{x-x})\,dx}{(GJ)_{x-x}}$.

$J_{x-x} = \frac{\pi}{32} d^4 = \frac{\pi}{32}\left[d_1 + (d_2 - d_1)(x/L)\right]$.

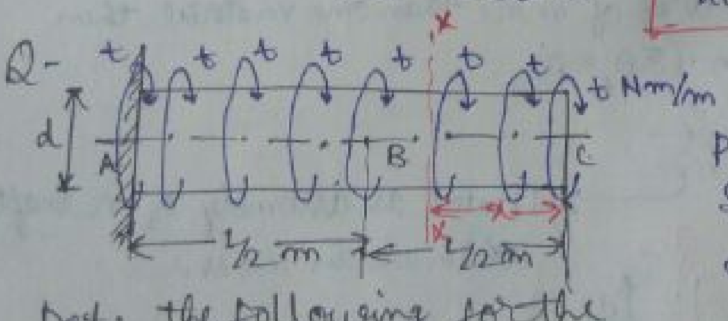
$\theta_{T.S} = \int_0^L \frac{(32)(T)(dx)}{\pi G\left[d_1 + (d_2 - d_1)(x/L)\right]^4}$.

$$\boxed{\theta_{T.S} = \frac{32TL}{\pi G}\left[\frac{d_1^2 + d_1 d_2 + d_2^2}{3 d_1^3 d_2^3}\right]}$$

If $d_1 = d_2 = d \Rightarrow$ T.S. becomes a P.S.

$$\theta_{TS} = \frac{32TL}{\pi G}\left[\frac{3d^2}{3d^6}\right] = \boxed{\frac{32TL}{\pi G}\left[\frac{1}{d^4}\right] = \theta_{P.S}}$$

Q-



Pure torsion condⁿ is not satisfied. 'n' shafts in series of same material & dia but subjected to different torque or variable torque.

Det. the following for the circular & prismatic bar as shown in the fig.

(i) τ_{max}.

(ii) ∠ of twist of the shaft.

(iii) —"— at a x-s/c located at a distance of L/2 from free end.

(iv) Strain energy of the shaft.

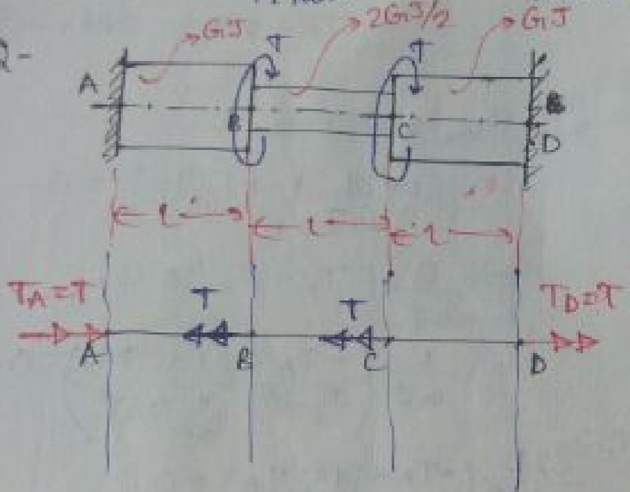
(5) $(\tau_{max})_1 = \left(\frac{16T}{\pi d^3}\right)_1 = \frac{16(-T/17)}{\pi d^3}$

$$= \frac{-16T}{17\pi d^3}$$

(6) $(\tau_{max})_2 = \left(\frac{16T}{\pi d^3}\right)_2 = \frac{16\left(\frac{16}{17}T\right)}{\pi (2d)^3}$

$$= \frac{32T}{17\pi d^3} = \frac{32 \times 2Pd}{17\pi d^3} = \frac{64Pd}{17\pi d^3}$$

Q-



Det θ_B & θ_C = ?

$$T_A = \frac{T(2L) + T(L)}{3L}$$

$$\boxed{T_A = T}$$

(or) By symmetric loading.

$T_{AB} = +T$

$T_{BC} = 0$

$T_{CD} = -T$

$\theta_B = \theta_{BA}$

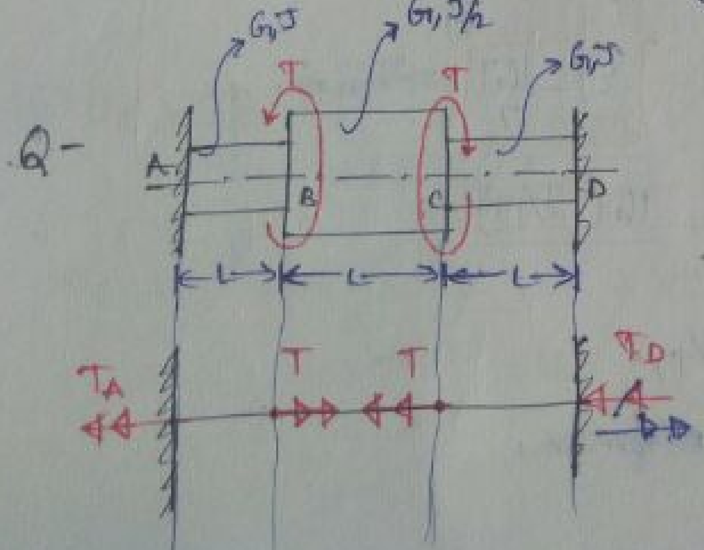
$$= \left(\frac{TL}{GJ}\right)_{AB}$$

$$\theta_B = \frac{TL}{GJ}$$

$\theta_C = \theta_{CD}$

$$= -\frac{TL}{GJ}$$

Q-



$\theta_B = ?$

$\theta_C = ?$

$T_{AB} = -T_A = T_1$

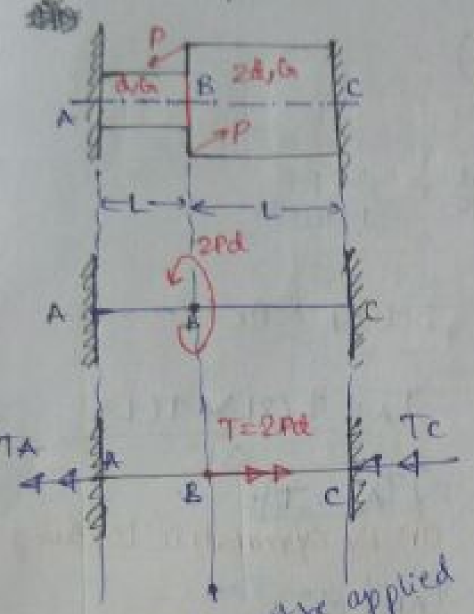
$T_{BC} = -T_A + T = T_2$

$T_{CD} = T_D$ (or) $-T_A + T - T = -T_A$

Q- For the stepped shaft as shown in the fig. Det.

(i) Max. torsional shear stress τ_{max}.

(ii) Ratio of max. & min. torsional shear stress.



This method can't be applied as bar is not prismatic.

① $T_{AB} = -T_A = T_1$

$T_{BC} = T_C$ (or) $2Pd - T_A = T_2$.

② $\Sigma T = 0 \Rightarrow T_A - T + T_C = 0$.

$T_A + T_C = 2Pd$. —(I)

③ $\theta_{Total} = \theta_1 + \theta_2 = 0$.

$= \left(\frac{TL}{GJ}\right)_1 + \left(\frac{TL}{GJ}\right)_2 = 0$.

$= \frac{L}{GJ}(T_1 + T_2) = 0$.

$= \frac{L}{G}\left(\frac{-T_A}{\frac{\pi}{32}d^4} + \frac{2Pd - T_A}{\frac{\pi}{32}(2d)^4}\right) = 0$.

$= \frac{32L}{\pi G d^4}\left(-T_A + \frac{2Pd - T_A}{16}\right) = 0$.

~~$T_A = Pd$.~~

$-16T_A + 2Pd - T_A = 0$.

$\boxed{T_A = \frac{2Pd}{17}}$. (◄◄—)

$\boxed{\text{Let } 2Pd = T.}$

④ from (I), $T_C = T - T_A = T - \frac{T}{17}$

$\boxed{T_C = \frac{16T}{17}}$. (◄◄—)

~~⑤ $(\tau_{max})_1 = \left(\frac{16T}{\pi d^3}\right) = 16(-T/17)$~~

④ $T_1 = -T_A = -T/17$.

$T_2 = T_C = \frac{16T}{17} = T_{max}$,

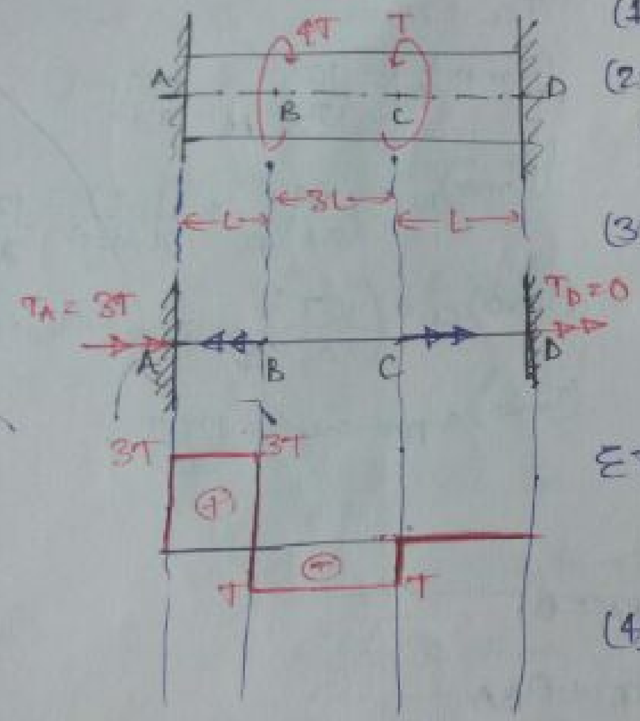
Case III - Statically Indeterminate Shafts

Q2- For the circular & prismatic bar as shown in the fig. Det. the following -

(i) Rxn torques

(ii) $\tau_{max} = ?$

(iii) θ_B & θ_C.



(1) Net torque $= 3T$ (◄◄)

(2) Introduce reac^n torques in a dir^n opp. to the dir^n of net torque. [Valid only when $(GJ)_{AB} = (GJ)_{BC} = (GJ)_{CD}$]

(3) T_A & T_D -

$$T_A = \frac{(4T)\times(4L) + (-T)(L)}{5L}$$

$$T_A = 3T \ (\rightarrow\!\!\blacktriangleright)$$

$$\Sigma T = 0 \Rightarrow 3T - 4T + T + T_D = 0.$$

$$\boxed{T_D = 0}$$

(4) $T_{AB} = 3T = T_{max}$.

$T_{BC} = -T$.

$T_{CD} = \text{zero}$.

(5) $(\tau_{max})_{shaft} = (\tau_{max})_{AB}$

$$= \left(\frac{16T}{\pi d^3}\right)_{AB} = \frac{48T}{\pi d^3}.$$

(6) $$\theta_B = \theta_{BA} = \left(\frac{32TL}{\pi G d^4}\right)_{AB} = \frac{96TL}{\pi G d^4}.$$

$$\theta_C = \theta_{CD} = \left(\frac{32TL}{\pi G d^4}\right)_{CD} = \text{zero}.$$

(i) $\theta = \frac{TL}{G_1J_1 + G_2J_2} = ?$

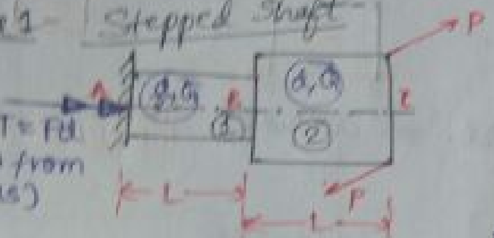
(ii) $\theta_1 = \theta$

$\frac{T_1L_1}{G_1J_1} = \theta \Rightarrow T_1 = ?$

(iii) $T_2 = T - T_1 \Rightarrow T_2 = ?$

Series-

Case I - Stepped Shaft -



Det.

(a) (τ_{max}) stepped shaft

(b) $\frac{\theta_1}{\theta_2} = ?$

** (c) $\theta_C = ?$ if $\theta_B = \theta$.

(a) $T_1 = T_2 = T = Pd.$

$\tau_{max} = \tau_1 = \frac{16T_1}{\pi d_1^3} = \frac{16(Pd)}{\pi (d/2)^3} = \frac{128P}{\pi d^2}$ MPa.

(b) $\frac{\theta_1}{\theta_2} = \frac{(TL/GJ)_1}{(TL/GJ)_2} = \frac{J_2}{J_1} = \left(\frac{d_2}{d_1}\right)^4 = 2^4 = 16.$

(c) $\theta_C = \theta_{CA} = \theta_{CB} + \theta_{BA} = \theta_2 + \theta_1.$

$\theta_B = \theta_{BA} = \theta_1 = \theta.$

$\theta_2 = \frac{\theta_1}{16} = \theta/16$

$\therefore \theta_C = \frac{\theta}{16} + \theta = \frac{17\theta}{16}.$

Case II -

Q- For the circular & prismatic shaft as shown in the fig. Det. the following -

(i) Max. torque

(ii) Diameter of the shaft if per. S.S (τ_{per}) = 60MPa.

(iii) θ of the shaft by using the above diameter.

Assume G = 80GPa.

4 - safe inner dia $= KD = 0.6D$
$= 21 mm.$

Shafts in Series & Parallel -

$$\tau_{max} = \frac{T}{Z_p} = \frac{16T}{\pi d^3} \text{ (or) } \frac{16T}{\pi D^3(1-K^4)} \quad \text{—(1)}$$

$$\theta = \frac{TL}{GJ} = \frac{32TL}{\pi G d^4} \text{ (or) } \frac{32TL}{\pi G D^4(1-K^4)} \quad \text{—(2)}$$

Conditions to be satisfied for the above eq^{ns} -

(i) Shaft should be circular & prismatic.
(ii) —— " —— under pure torsion.
(iii) —— " —— made of same material.

Series -

Cond^{ns} -

(i) Angle of twists are cumulative.
[ie $\theta_{Total} = \theta_1 + \theta_2 + \dots + \theta_n$].

(ii) Torques are equal & like in nature.
[ie. $T_1 = T_2 = \dots = T_n = T$].

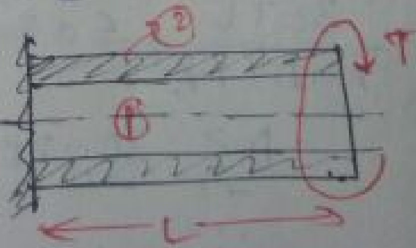
Valid when torques are applied at the extreme ends only.

Parallel - (Composite shafts)

Cond^{ns} -

(i) Torques are cumulative. (ie. $T_1 + T_2 = T$).

(ii) Angle of twists are equal & like in nature.
[ie. $\theta_1 = \theta_2 = \theta$].



Q- Design a hollow circular shaft for the following cond's -

(i) power to be transmitted 100 kW at 960 rpm.

(ii) Dia. ratio, 0.6

(iii) $\tau_{per} = 60 MPa$.

(iv) f.o.s = 2.

(v) Permissible angle of twist is 0.5° over a length of 500 mm.

$G = 80 GPa$.

1- $T = \frac{P \times 60}{2\pi N} \times 10^6$ (or) $\frac{P}{2\pi N'} \times 10^6$ (or) $\frac{P}{\omega} \times 10^6 =$ ______ Nmm

kW, rpm, kW, rRs or Hz, kW, rad/sec.

$T = \frac{10 \times 60}{2\pi \times 960} =$ ______ Nmm

2- Safe condn wrt. st. criterion.

$(\tau_{max})_{ind.} \leq \tau_{per}$ (or) S_{ys}/N

$\frac{T}{Z_p} < \tau_{per}$

$\frac{16T}{\pi D^3(1-k^4)} \leq 60$

0.6

$D \geq 21.327$ mm.

3- Safe condn for design wrt. rigidity criterion,

$\theta_{max} \leq \theta_{per}$

$\frac{TL}{GJ} \leq 0.5 \times \frac{\pi}{180}$

$\frac{32TL}{\pi G D^4(1-k^4)} \leq 0.5 \times \frac{\pi}{180}$

500, 80×10^3

$D \geq 30.217$ mm.

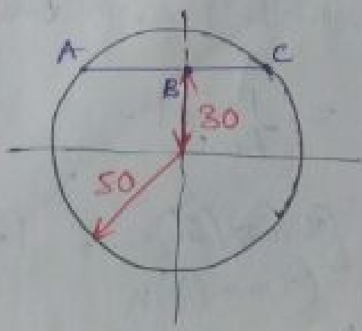
Q- X-S/c of a solid circular shaft as shown in fig. Det. the following when max. torsional S.S is 100MPa due to a twisting moment of T Nm.

(i) Torsional shear stress at various pts. on X-S/c as shown in fig.

(ii) Shear angle at various pts. on the X-S/c as shown in the fig.

(iii) Angle of twist at various pts. on the X-S/c.

Assume G = 100 GPa, length of the shaft under pure torsion is 1m. & the X-S/c shown in the fig. is at the free end.



X-S/c of shaft at free end.

$G = 100\times10^3$.

(i) $\tau_A = \tau_C = \tau_{max} = 100 MPa$,

$$\tau_B = \tau_{max}\left[\frac{r}{R}\right] = \frac{\cancel{100}^{20} \times 3\cancel{0}}{\cancel{5\cancel{0}}} = 60 MPa,$$

(ii) $\phi = \gamma = \tau/G$

$$\phi_A = \phi_C = \cancel{100}\times10^6/\cancel{100}\times10^9 = 0.001 = \phi_{max}$$

$$\phi_B = \tau/G = \frac{\cancel{60}\times10^6}{\cancel{100}\times10^9} = 0.6\times10^{-3}.$$

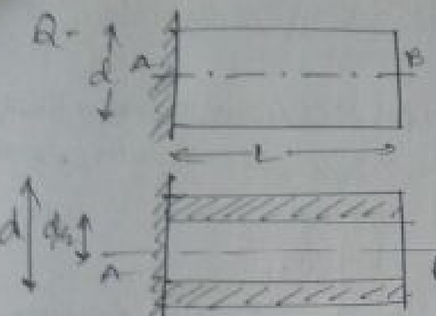
(iii) $\theta = \frac{\tau\times L}{G r}$ θ changes from fixed end to free end. And max. at free end.

~~$\theta_A = \theta_C = 100\times10^6\times$~~

$$\theta_A = \theta_B = \theta_C = \theta_{max} = \frac{\phi_{max} L}{R} = \frac{10^{-3}\times1000}{50}$$

$$= 0.02 \text{ radians}.$$

Q-



$$\frac{(Z_p)_s}{(Z_p)_H} = \frac{J_s}{J_H} = \frac{16}{15}.$$

Det

(a) $\frac{(\tau_{max})_s}{(\tau_{max})_H} = ?$ (b) $\frac{(\phi_{max})_s}{(\phi_{max})_H} = ?$

(c) $\frac{(K_t)_s}{(K_t)_H} = ?$

If both the shafts are twisted by angle θ.

(a) $$\frac{(\tau_{max})_s}{(\tau_{max})_H} = \frac{(GR\theta/L)_s}{(GR_o\theta/L)_H} = \frac{R}{R_o} < 1.$$

(b) $$\frac{(\phi_{max})_s}{(\phi_{max})_H} = \frac{(\gamma_{max})_s}{(\gamma_{max})_H} = \frac{(\tau_{max}/G)_s}{(\tau_{max}/G)_H} = 1.$$

(c) $$\frac{(K_t)_s}{(K_t)_H} = \frac{(T/\theta)_s \text{ or } (GJ/L)_s}{(T/\theta \text{ or } GJ/L)_H} = \frac{(GJ/L)_s}{(GJ/L)_H} = \frac{16}{15}.$$

Q- Same as above, If Torque & Area are also same.

Det. (d) $\frac{\theta_s}{\theta_H} = ?$ $\quad \frac{J_s}{J_H} = \frac{1-k^2}{1+k^2} = \frac{3}{5} = 0.6$

$$\frac{(Z_p)_s}{(Z_p)_H} = \frac{\sqrt{1-k^2}}{1+k^2} = 0.693 \simeq 0.7$$

(a) $$\frac{(\tau_{max})_s}{(\tau_{max})_H} = \frac{(T/Z_p)_s}{(T/Z_p)_H} = \cancel{\frac{15}{16}}\ \frac{1}{0.7} = \frac{10}{7}.$$

(b) $$\frac{(\phi_{max})_s}{(\phi_{max})_H} = \frac{(\gamma_{max})_s}{(\gamma_{max})_H} = \frac{(\tau_{max}/G)_s}{(\tau_{max}/G)_H} = \frac{1}{0.7}.$$

(c) $$\frac{\theta_s}{\theta_H} = \frac{(TL/GJ)_s}{(TL/GJ)_H} = \frac{1}{0.6} = 1.67.$$

(d) $$\frac{(K_t)_s}{(K_t)_H} = \frac{(T/\theta)_s}{(T/\theta)_H} = \frac{3}{5} = 0.6.$$

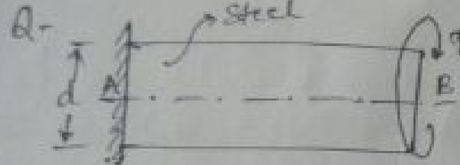
For a given x-s/c area & $k = 1/2$ -

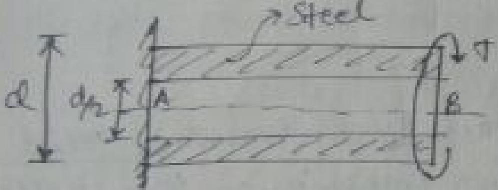
$$\frac{d_s}{d_h} = \sqrt{1-k^2} = 0.87.$$

$$\frac{I_s}{I_h} = \frac{J_s}{J_h} = \frac{1-k^2}{1+k^2} = \frac{3}{5} = 0.6.$$

$$\frac{Z_s}{Z_h} = \frac{\sqrt{1-k^2}}{\sqrt{1+k^2}} = 0.693 \simeq 0.7.$$

Q -





Det. (i) $\frac{(\tau_{max})_s}{(\tau_{max})_h} = ?$

(ii) $\frac{(\phi_{max})_s}{(\phi_{max})_h} = ?$

(iii) $\frac{\theta_s}{\theta_h} = ?$

(iv) $\frac{(K_t)_s}{(K_t)_h} = ?$

$$\frac{J_s}{J_h} = \frac{(Z_p)_s}{(Z_p)_h} = \frac{1}{1-k^4} = \frac{16}{15}.$$

(i) $$\frac{(\tau_{max})_s}{(\tau_{max})_h} = \frac{(T/Z_p)_s}{(T/Z_p)_h} = \frac{(Z_p)_h}{(Z_p)_s} = \frac{15}{16}.$$

(ii) $$\frac{(\phi_{max})_s}{(\phi_{max})_h} = \frac{(\gamma_{max})_s}{(\gamma_{max})_h} = \frac{(\tau_{max}/G)_s}{(\tau_{max}/G)_h} = \frac{(\tau_{max})_s}{(\tau_{max})_h} = \frac{15}{16}.$$

(iii) $$\frac{\theta_s}{\theta_h} = \frac{(TL/GJ)_s}{(TL/GJ)_h} = \frac{J_h}{J_s} = \frac{15}{16}.$$

(iv) $$\frac{(K_t)_s}{(K_t)_h} = \frac{(T/\theta \text{ or } GJ/L)_s}{(T/\theta \text{ or } GJ/L)_h} = \frac{\theta_h}{\theta_s} = \frac{16}{15}.$$

K_t = Torsional Rigidity $= \frac{T}{\theta}$.

$d < D$

$$\frac{(PTC)_S}{(PTC)_H} = \frac{T_S}{T_H} = \frac{\frac{\pi}{16} d^3 \tau_{per}}{\frac{\pi}{16} D^3 (1-k^4) \tau_{per}}$$

$$= \left(\frac{d}{D}\right)^3 \left(\frac{1}{1-k^4}\right).$$

$$\frac{(PTC)_S}{(PTC)_H} = \frac{T_S}{T_H} = \frac{\sqrt{1-k^2}}{1+k^2} < 1$$

$(PTC)_S < (PTC)_H$

if $k = \frac{1}{2}$,

$$\frac{(PTC)_S}{(PTC)_H} = 0.69 \simeq \underline{0.7}.$$

→ For a given dia. (when radial strain is contraint), solid circular shafts are preffered for power transmission than hollow C.S. Due to their higher power transmission capacity [∵ $(Z_P)_{solid} > (Z_P)_{hollow}$].

→ For a given x-s/c area, hollow circular shafts are preffered for power transmission, than solid circular shafts due to their higher PTC. [∵ $(Z_P)_{Hollow} > (Z_P)_{solid}$].

For a given dia & $k = 1/2$ -

$$\boxed{\frac{A_S}{A_H} = \frac{1}{1-k^2} = \frac{4}{3} = 1.333}$$

$$\boxed{\frac{I_S}{I_H} = \frac{J_S}{J_H} = \frac{Z_S}{Z_H} = \frac{(Z_P)_S}{(Z_P)_H} = \frac{1}{1-k^4} = \frac{16}{15}}$$

Valid only when dia are same & $k = \frac{1}{2}$.

* If $k = \frac{1}{2}$,

$$\boxed{\frac{(PTC)_s}{(PTC)_H} = \frac{16}{15} = 1.07}$$

if $(d_i = d_1)(\uparrow) \Rightarrow k(\uparrow)$

$\Rightarrow 1 - k^4 (\downarrow)$

$\Rightarrow \frac{1}{1-k^4} (\uparrow)$

$\Rightarrow \frac{(PTC)_s}{(PTC)_H} (\uparrow)$

* Drawback of hollow shaft is that they acquire more space for the same power transmission.

But, $(PTC)_s > (PTC)_H$ Advantage

$(Wt)_s > (Wt)_H$ Disadvantage

(ii)

wt. length, material & rpm are same.

$(Wt)_s = (Wt)_H$

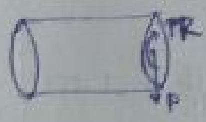
$[(A)(L)(\gamma)]_s = (A L \gamma)_H$

$A_s = A_H.$

$\frac{\pi}{4} d^2 = \frac{\pi}{4} D^2 (1 - k^2)$

$$\boxed{\frac{d}{D} = \sqrt{1-k^2}} \quad \text{—— (1)}$$

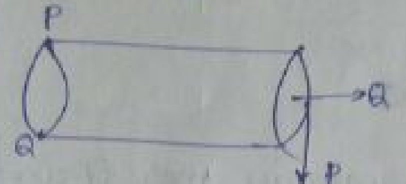
SF = P
B.M = Max = PL.
TM = PR.



$(\tau_d)_{max} = \frac{16P}{3\pi d^2} = \underline{18.86 MPa}.$

$(\sigma_b)_{max} = \frac{PL}{\frac{\pi}{32} d^3} = \frac{32PL}{\pi d^3} = \underline{1886.28 MPa}$

*



Here only P is a critical point.
As it is Hogging, hence P is at tensile side & Q is at comp. side.
On the appⁿ of tensile force Q, P will become the pt. with max. ~~~~ stress.

Q - Det. the ratio of power transmission capacities of solid & hollow circular shafts which are rotating at same rpm under following conditions -

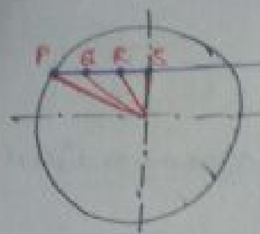
(i) dia, materials, lengths are same for both the shafts.
(ii) Weights, length, materials are same for —— " ——

Material, length, rpm are same.

(i) $\frac{(PTC)_S}{(PTC)_H} = \frac{(T\omega)_S}{(T\omega)_H} = \frac{T_S}{T_H} = \frac{\cancel{\frac{\pi}{16}} d^3 \cdot \cancel{\tau_{per}}}{\cancel{\frac{\pi}{16}} D^3 (1-K^4) \cancel{\tau_{per}}}$ [∵ d = D].

$K = \frac{d_1}{D} < 1$

$\boxed{(PTC)_S > (PTC)_H}$

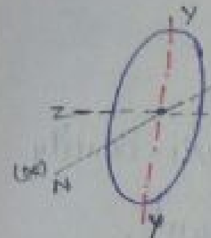


$\rightarrow (\sigma_b)_P = (\sigma_b)_Q = \dots = (\sigma_b)_S = \sigma_b = \frac{My}{I_{NA}}$

$[\because y_P = y_Q = y_R = y_S = y]$.

$\rightarrow [(\tau_t)_P = \tau_{max}] > (\tau_t)_Q > (\tau_t)_R > (\tau_t)_S$.

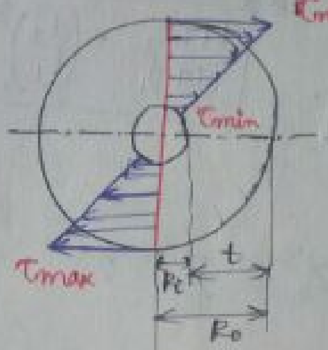
$[\because (r_P = R) > r_Q > r_R > r_S]$.



(x) A $\Rightarrow$ Bending stress is zero on N.A.

z $\Rightarrow$ Torsional shear stress is zero on Z-A.

For Hollow Circular X-S/C-



$\tau_{max} = \frac{T}{Z_P}$ (or) $\frac{G R_0 \theta}{L}$.

$$\boxed{\frac{\tau_{max}}{\tau_{min}} = \frac{R_0}{R_i} = \frac{D}{d} = \frac{1}{K}}.$$

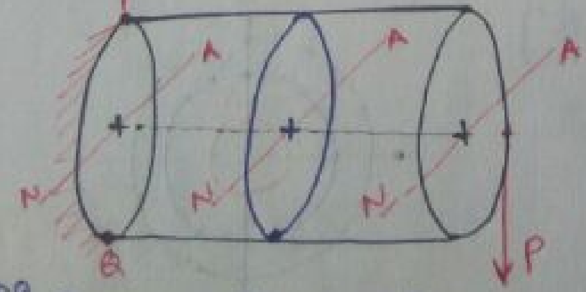
Q- $\tau_{max} = 100\,MPa$,
$(\tau)_{R_i} = ?$
If dia ratio $= 0.6 = k$.

$\frac{\tau_{max}}{\tau_{min}} = \frac{D}{d} = \frac{1}{K}$

* When only hollow is written, assume it to be thick.

$\tau_{min} = \tau_{max} \cdot K$
$= 100 \times 0.6 = \underline{60\,MPa}$.

*



$P = 10\,kN$
$L = 500\,mm$
$d = 30\,mm$

P & Q are critical points,
Neglect the effect of ~~shear~~ direct shear stress.

Moment of Resistance [T_R] -

$$\boxed{T_R = Z_p \, \sigma_{per}} \quad \text{—— ③}$$

Used to compare the shafts. When two shafts of same material is used Z_p need to be compared.

$(T_R)_{\text{solid}} = \frac{\pi}{16} d^3 \tau_{per}$ —— (i)

$(T_R)_{\text{hollow}} = \frac{\pi}{16} D^3 (1-K^4) \tau_{per}$ —— (ii)

PTC → Power transmission capacity

For a given 'ω',

$$\boxed{PTC\,(P) \propto T \propto d^3}$$

$P = \frac{2\pi NT}{60}$.

i.e. $\frac{(PTC)_2}{(PTC)_1} = \frac{T_2}{T_1} = \left(\frac{d_2}{d_1}\right)^3$ (If $N_1 = N_2$).

$$\boxed{\frac{(PTC)_2}{PTC_1} = \left(\frac{d_2}{d_1}\right)^3 \left(\frac{N_2}{N_1}\right)}.$$

$$\boxed{\frac{(PTC)_2}{(PTC)_1} = \left(\frac{d_2}{d_1}\right)^3 \left(\frac{N_2}{N_1}\right)}$$

If $d_2 = 2d_1$; $N_2 = N_1/4$.

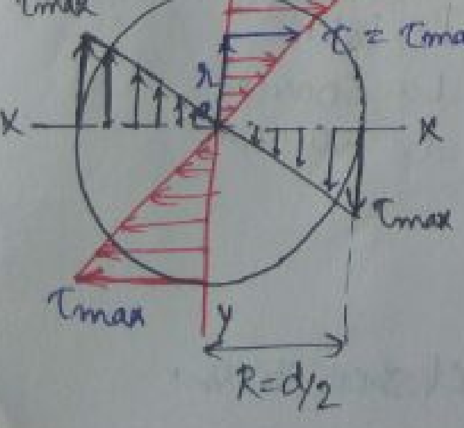
$\frac{(PTC)_2}{(PTC_1)} = 2^3 \times \frac{1}{4} = 2.$

Torsional Shear Stress Variation -

→ For solid circular x-s/c -

$\tau_{max} = \frac{T}{Z_p}$ (or) $\frac{GR\theta}{L}$.

$\tau = \tau_{max}\left[\frac{r}{R}\right]$



Chapter - 6

TORSIONAL SHEAR STRESSES IN SHAFTS

(i.e. Pure Torsion)

T.M = constant , A.L = SF = BM = 0.

Torsion eq^n for circular x-s/c-

$$\boxed{\underset{\text{(A)}}{\frac{T}{J}} = \underset{\text{(B)}}{\frac{\tau_{max}}{R \text{ (or) } R_o}} = \underset{\text{(C)}}{\frac{G\theta}{L}}}$$

(i) $\boxed{\phi' \propto r} \Rightarrow \boxed{\phi' = \gamma'} \Rightarrow \boxed{\tau' \propto \gamma' \propto r}$

(ii) $\cancel{\phi' \propto l}$

$\boxed{\theta' \propto l} \Rightarrow \boxed{\phi = \frac{R\theta}{L}}$

$$\boxed{\tau_{max} = \frac{T}{Z_p} \text{ (or) } \frac{G(R \text{ (or) } R_o)\theta}{L}} \quad \text{— (1)}$$

Should be used when 'T' is known

Should be used when 'θ' is known

$\boxed{\tau_{max} \propto \frac{1}{d^3}}$

$(\tau_{max})_{\text{solid}} = \frac{16T}{\pi d^3}$

$(\tau_{max})_{\text{hollow}} = \frac{16T}{\pi D^3(1-K^4)}$

$(\tau_{max})_{\text{thin}} = \frac{2T}{\pi d^2 t}$

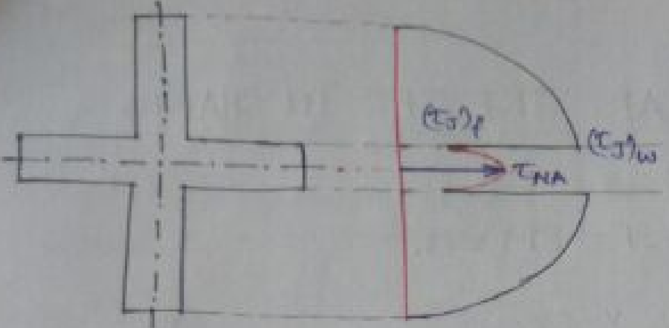
$$\boxed{\theta = \frac{TL}{GJ} = \frac{\phi L}{R \text{ or } R_o}} \quad \text{— (2)}$$

$(\theta)_{\text{solid}} = \frac{32TL}{\pi G d^4} \Rightarrow \boxed{\theta \propto \frac{1}{d^4}}$

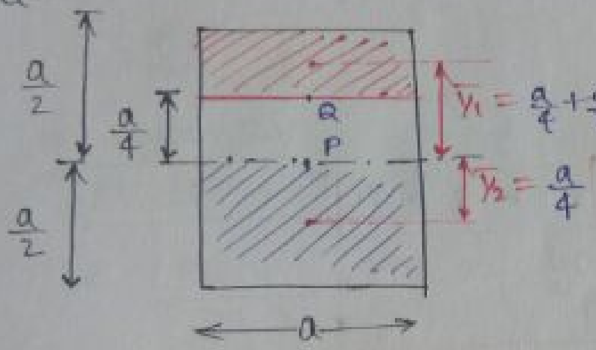
$(\theta)_{\text{hollow}} = \frac{32TL}{\pi G D^4(1-K^4)}$

$(\theta)_{\text{thin}} = \frac{4TL}{\pi d^3 t}$

Note: $\boxed{\tau = \tau_{max}\left[\frac{r}{R}\right]}$



Q-



Det.

(i) $\tau_P / \tau_Q = ?$

(ii) τ_Q in terms of shear force (P).

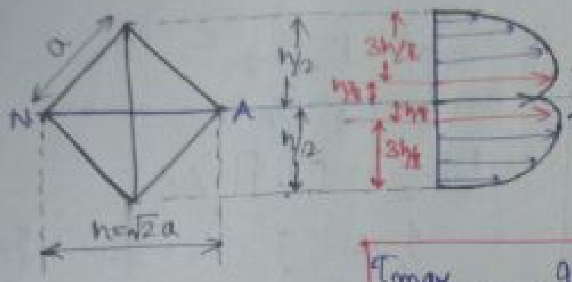
$$\frac{\tau_P}{\tau_Q} = \frac{A_2 \bar{y}_2}{A_1 \bar{y}_1} = \frac{a(a/2)(a/4)}{a(a/4)(3a/8)} = \frac{(1/8)}{(3/32)} = \frac{4}{3},$$

$$\tau_Q = \frac{3}{4}(\tau_P \text{ or } \tau_{max})$$

$$= \left(\frac{3}{4}\right)\left(\frac{3}{2}\tau_{avg}\right)$$

$$\tau_Q = \left(\frac{9}{8}\right)\left(\frac{P}{a^2}\right)$$

For S.D



$\tau_{max} = (9/8)\,\tau_{avg}$

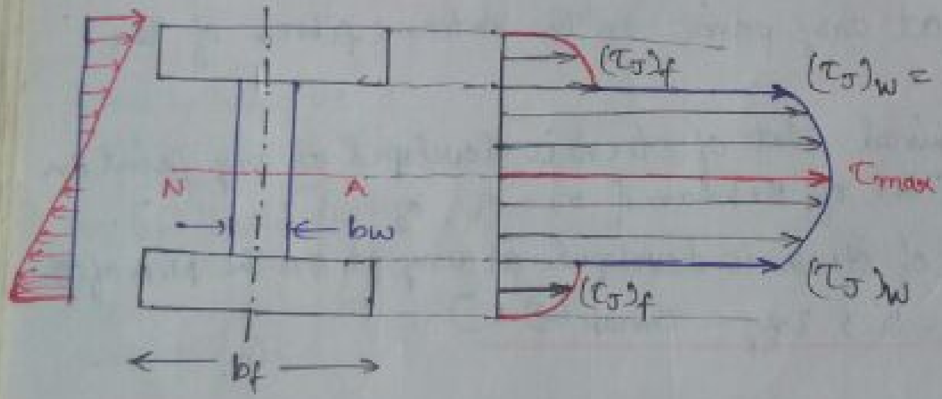
$\tau_{NA} = \tau_{avg}$

τ_{max}

$$\boxed{\frac{\tau_{max}}{\tau_{NA}} = \frac{9}{8} = 1.125}$$

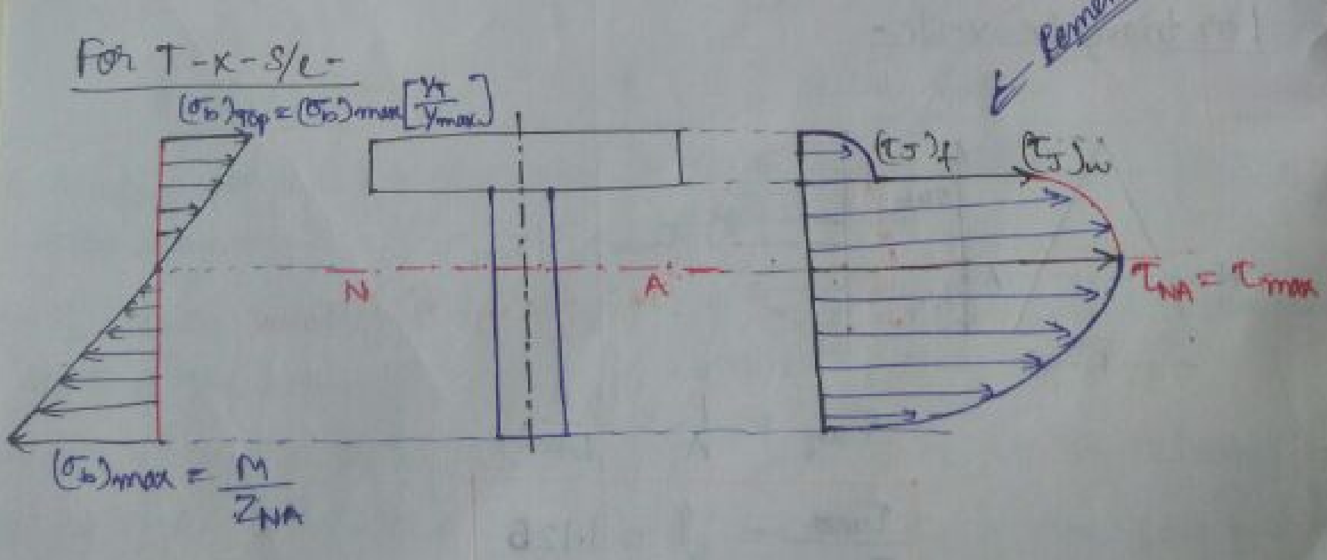
$$\boxed{\tau_{avg} = \frac{P}{A} = \frac{P}{a^2} \text{ or } \frac{2P}{h^2}}$$

For I-s/c-

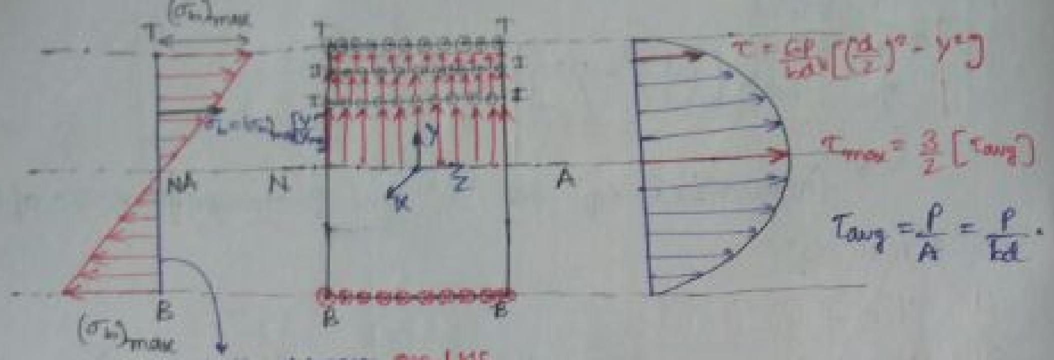


$(\tau_s)_w = (\tau_s)_f \left[\frac{b_f}{b_w}\right]$

For T-x-s/c-



Comparison b/w S.S & B.S-



x-S/c of beam <u>on LHS</u>

Bending stress variation on the rectangular x-S/c

x-S/c of beam
(or) x-face
(or) YZ plane

Shear stress variation on the Rectangular x-S/c.

~~Effect~~
→ Left hand side cutting surface.
(Just by looking the variation we can't tell it's either sagging or hogging. It should be given that which cutting surface we are considering.)
→ Hogging

→ Direction ✓
~~→ Variat~~
→ Magnitude ✓
→ Variation ✗

→ Mag. ✓
→ Direction ✓
→ Variation ✓

Bending Stress	Shear Stress
1- Bending stress is ⊥ to the x-S/c of the beam.	1- Shear stress is ∥ to the x-S/c of the beam.
2- σ_b varies linearly over the depth of the beam.	2- τ_s varies parabolically over the depth of the beam.
3- σ_b max. at extreme fibres.	3- τ_s is zero at the extreme fibres.
4- At N.A. σ_b is zero.	4- At NA, τ_s is non-zero but it becomes max. at neutral axis in case circular, square, rect. I-S/c & T-S/c (ie. except triangular & S.D).

$$\tau = \frac{P}{\left(\frac{bd^3}{12}\right)}\left[\frac{1}{2}\left(\left(\frac{d}{2}\right)^2 - y^2\right)\right]$$

$$\boxed{\tau = \frac{6P}{bd^3}\left[\left(\frac{d}{2}\right)^2 - y^2\right]} \quad \text{--- (2)}$$

Generalised eq^n for 'τ' on Rectangular x-s/c of beam

From eq^n (2),

(i) $\tau \propto f[y^2]$

(ii) As $y \uparrow \Rightarrow \tau \downarrow$. (Opp. to Bending stress).

(iii) At extreme fibres, $\boxed{\tau = 0}$.

To determine location of τ_{max},

$$\frac{d\tau}{dy} = 0 \Rightarrow \boxed{y = 0} \text{ (i.e at N.A).}$$

$$\boxed{\tau_{max} = (\tau)_{y=0} = \frac{3}{2}\left[\frac{P}{bd}\right]}$$

$$\boxed{\tau_{max} = K[\tau_{avg}]} \quad \text{where } \tau_{avg} = \frac{P}{A}$$

Note: $K = \frac{3}{2}$ for ■ ▮ ▲

$= \frac{4}{3}$ for ●

$= \frac{9}{8}$ for S.D.

Note: Shear stress is max. where $\left(\frac{A\bar{y}}{b}\right)$ is max. Except for ▲ & S.D, rest have max. shear stress at N.A.

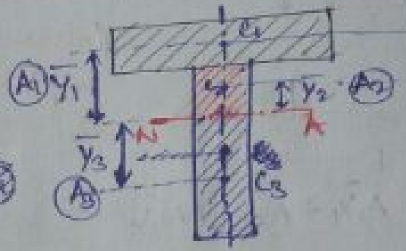
▲ → Max. shear stress at $\boxed{y = \frac{h}{2}}$.

$$\therefore \boxed{\frac{(\tau_J)_w}{(\tau_J)_f} = \frac{b_f}{b_w}} \quad \text{— (3)}$$

* If $b_w = 20$, $b_f = 100$, $(\tau_J)_f = 12\ \text{MPa}$.

$$\therefore \frac{(\tau_J)_w}{12} = \frac{\cancel{100}^{\,5}}{\cancel{20}_{\,1}}$$

$$\boxed{(\tau_J)_w = 60\ \text{MPa}}.$$



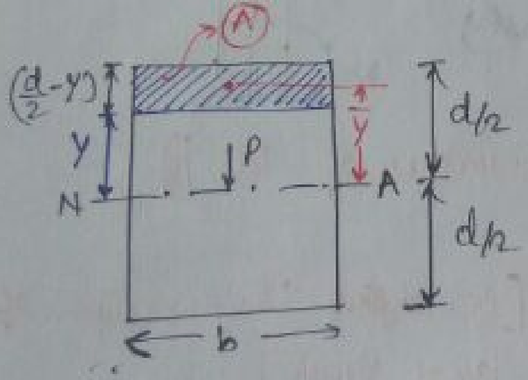
$$\boxed{\tau_{NA} = \frac{P}{I_{NA}}\left[\frac{A_3\bar{y}_3}{b_w}\right]}$$

(or)

$$\boxed{\tau_{NA} = \frac{P}{I_{NA}}\left[\frac{A_1\bar{y}_1 + A_2\bar{y}_2}{b_w}\right]}.$$

Note- At N.A σ_b is zero but shear stress is non-zero.

Generalised eq^n for 'τ' developed on Rectangular x-s/c beam-



S.F on the x-s/c = P.

$$A = b\left[\frac{d}{2} - y\right].$$

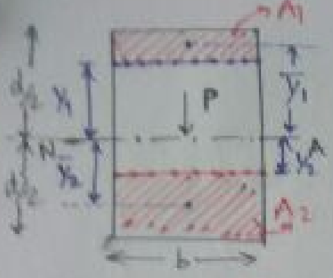
$$\bar{y} = y + \frac{1}{2}\left[\frac{d}{2} - y\right].$$

$$\bar{y} = \frac{1}{2}\left[\frac{d}{2} + y\right].$$

$$A\bar{y} = \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - y^2\right].$$

$$I_{NA} = \frac{bd^3}{12}; \quad b = b.$$

$$\tau = \frac{P}{I_{NA}}\left[\frac{A\bar{y}}{b}\right] \quad \text{— (1)}$$



* $\bar{y}_1 > \bar{y}_2$ & $A_2 > A_1$ → So it is impossible to tell directly where τ is max.

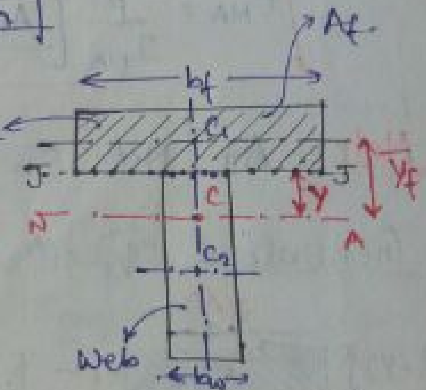
→ $\bar{y}$ should be measured from NA. But y (distance of fibre) can be measured from anywhere either NA or vertex bcoz formula of τ doesn't include y.

$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3$

$\frac{h}{b} = \frac{y}{b_1}$

$\boxed{b_1 = \frac{by}{h}}$

To simplify 'τ' calc^n y should be measured from extreme fibres (i.e. vertex).



$$\tau = \frac{PA\bar{y}}{I_{NA}b}$$

From above eq^n,

$\tau \propto \left(\frac{A\bar{y}}{b}\right)$ [$\because P/I_{NA} = \text{const.}$]

$\tau \propto (A\bar{y})$ [if $b = \text{const.}$] (i.e. in case of)

~~i.e. an ... increase~~

$\tau \propto \frac{1}{b}$ (if $A\bar{y} = \text{const.}$) [i.e. at the junction of flange & web].

* At junction pt. calculate shear stress for 2 times.

$$(\tau_J)_f = \frac{P}{I_{NA}}\left[\frac{A_f\bar{y}_f}{b_f}\right] \quad \text{—(1)}$$

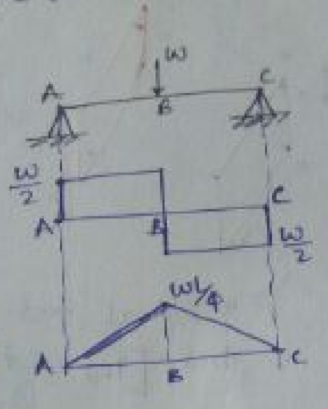
$$(\tau_J)_w = \frac{P}{I_{NA}}\left[\frac{A_f\cdot\bar{y}_f}{b_w}\right] \quad \text{—(2)}$$

→ Area above N.A.

Shear Stresses in Beams

Let τ = Shear stress developed at a fibre on the x-S/c of the beam.

$$\boxed{\tau = \frac{PA\bar{y}}{I_{NA}\, b}}$$



Max SF = $\pm \frac{W}{2}$

Max BM = $\frac{WL}{4}$

where,

P = SF acting on the x-S/c of beam. (obtained from SFD).

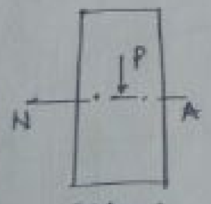
A = Area of hatched portion which is above the fibre where 'τ' is to be determined (if fibre is located above N.A).

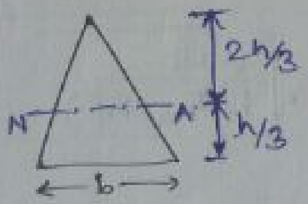
$\bar{y}$ = Distance of centroid of hatched portion from N.A of the x-S/c.

$A\bar{y}$ = First moment of area of hatched portion about N.A of the x-s/c

$I_{NA} = \int y^2 dA$ = Second moment of area of x-S/c about NA of the x-S/c [i.e. MOI of entire x-S/c about N.A].

b = Width of the x-S/c where 'τ' is to be determined.





$\boxed{\tau \propto A\bar{y}}$

$\left(\because \frac{P}{I_{NA}\, b} = \text{Const.}\right)$

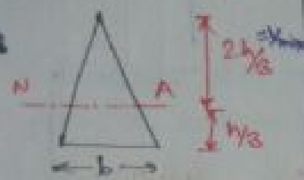
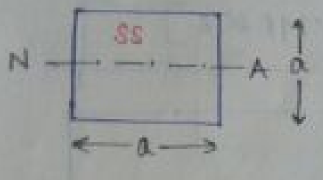
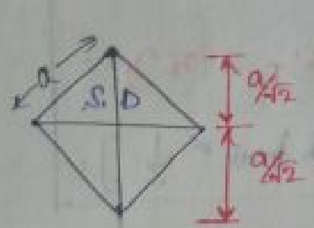
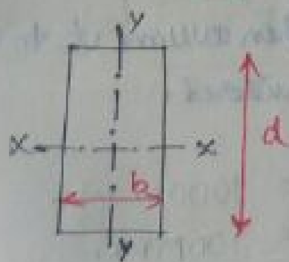
$\boxed{\tau \propto \frac{A\bar{y}}{b}}$

$\left(\because \frac{P}{I_{NA}} = \text{Const.}\right)$

* Can't say which fibre has max. shear stress. The multiple of variables will tell which fibre has max. shear stress.

* Both σ_b & Shear Stresses vary from one fibre to another but const. for one.

* τ is a function of 3 parameters ($A\bar{y}$, b) whereas σ_b is a f^n of only only one variable.

Shape of X-S/c	A	I_{NA}	Z_{NA}
	$\frac{bh}{2}$	$I_{NA} = \frac{bh^3}{36}$, $I_{base} = \frac{bh^3}{12}$,	$\frac{bh^2}{24}$.
	a^2	$I_{xx} = I_{yy} = I_{NA} = \frac{a^4}{12}$	$a^3/6$
	a^2	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}}$.
	bd	$I_{xx} = \frac{bd^3}{12} = I_{NA}$, $I_{yy} = \frac{db^3}{12}$	$Z_{xx} = \frac{bd^2}{6} = Z_{NA}$, $Z_{yy} = \frac{db^2}{6}$,

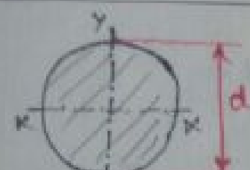
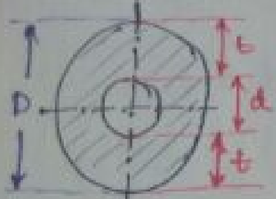
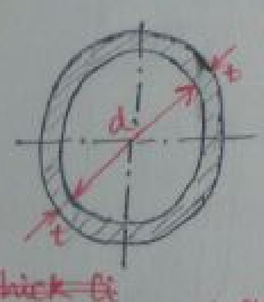
$\frac{bh^3 \times 3}{36 \times 2h}$
$\frac{bh^2}{24}$.

* for a given BM, $\boxed{\sigma_{max} \propto \frac{1}{Z}}$,

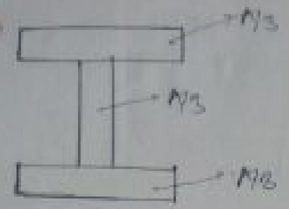
$$\frac{[(\sigma_b)_{max}]_{S.D}}{[(\sigma_b)_{max}]_{S.S}} = \sqrt{2} = 1.414.$$

* for a given material, $\boxed{M_R \propto Z}$

Geometrical Properties -

Shape of x-S/c	Area(A)	MOI (I)	Section Modulus (Z_{NA})	Polar MOI (J)	Polar Section modulus (Z_P)
	$\frac{\pi d^2}{4}$	$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$	$Z_{xx} = Z_{yy} = Z_{NA} = \frac{\pi d^3}{32}$	$\frac{\pi d^4}{32}$	$\frac{\pi d^3}{16}$
 Thick Circular x-S/c ($d/t < 20$)	K = dia ratio $= \frac{d}{D}$ $\frac{\pi D^2}{4}(1-K^2)$	$\frac{\pi D^4}{64}(1-K^4)$	$\frac{\pi D^3}{32}(1-K^4)$	$\frac{\pi D^4}{32}(1-K^4)$	$\frac{\pi D^3}{16}(1-K^4)$
 ~~Thick Ci~~ Thin Circular x-S/c i.e. $d/t \geq 20$	$\pi d t$	$\left(\frac{\pi d^3}{8}\right)t$	$\left(\frac{\pi d^2}{4}\right)t$ $y_{max} = \frac{d}{2} + t \simeq \frac{d}{2}$.	$\left(\frac{\pi d^3}{4}\right)t$	$\left(\frac{\pi d^2}{4}\right)t$ $r_0 = \frac{d}{2} + t \simeq \frac{d}{2}$.

③



This I-section is ~~best~~ best as most of the area is far away from N.A. which increases section modulus (Z) and hence $M_R \uparrow$.

i.e. $\boxed{\begin{array}{l} A_1 = A_2 = A_3 \\ Z_3 > Z_2 > Z_1 \\ M_{R_3} > M_{R_2} > M_{R_1} \end{array}}$

Ex-

Both are square.

Square with sides vertical & horizontal (S.S)

Square with Diagonals vertical & horizontal. (S.D)

$A_{SS} = A_{SD} = a^2$.

$I_{SS} = I_{SD} = \frac{a^4}{12}$.

$(y_{max})_{SS} = a/2$.

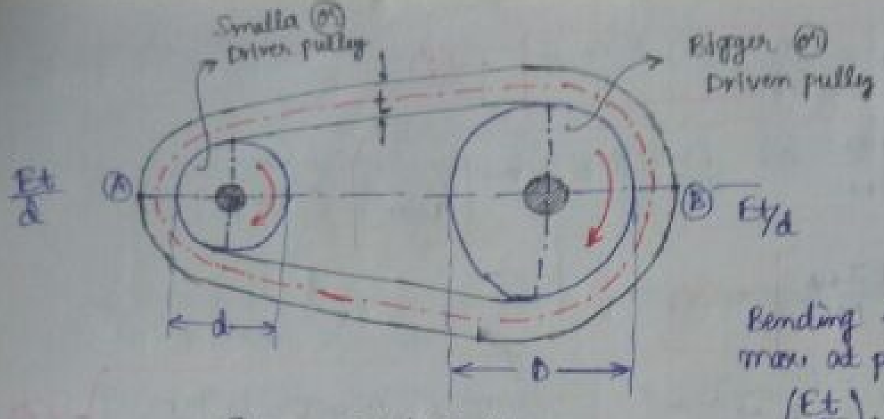
$(y_{max})_{SD} = \frac{a}{\sqrt{2}}$.

$\boxed{(Z_{SS}) = \frac{a^3}{6}}$.

$\boxed{Z_{SD} = \frac{Z_{SS}}{\sqrt{2}} = \frac{a^3}{6\sqrt{2}}}$

$\boxed{\frac{Z_{SS}}{Z_{SD}} = \sqrt{2} = 1.414}$

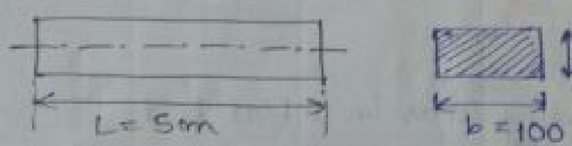
→ A square X-S/c with sides like vertical & Horizontal (i.e. SS) is 41.4% stronger than a Square X-S/c with diagonals vert. & hori (ie SD) under bending (i.e. ~~Reading of~~ ∴ Z_{SS} is $\sqrt{2}$ or 1.414 times Z_{SD}.)



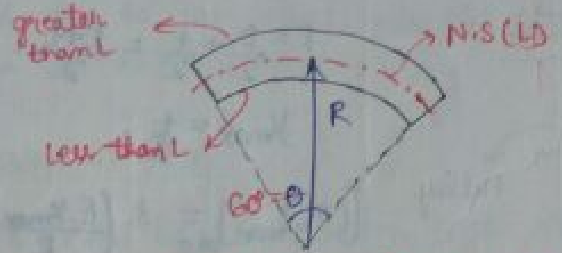
Bending stress is max. at pt. A equal to $\left(\frac{Et}{d}\right)$.

Open Flat Belt Drive

*



Steel Rule



$$\boxed{(\sigma_b)_{max} = \frac{E\,y_{max}}{\left(R \text{ or } \frac{L}{\theta}\right)}} = \frac{E(t/2)\theta}{L}$$

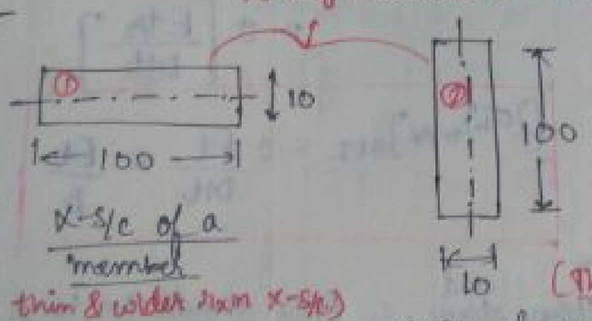
$$= \frac{Et\theta}{2L}$$

$$= \frac{80\times10^3\times10\times60\times\pi}{2\times5000\cdot180}$$

$$= 83.733\ N/mm^2.$$

Q-

Area of materials are same.



X-S/c of a member (i.e. thin & wider form X-S/c)

Depth should be more. (Thick & narrow X-S/c)

X-S/c of a member

$$\frac{(MR)_2}{(MR)_1} = \frac{(Z\sigma_{per})_2}{(Z\sigma_{per})_1} = \frac{Z_2}{Z_1} = \frac{\frac{1}{6}\times10\times100^2}{\frac{1}{6}\times100\times10^2} = 10.$$

② ⇒ Better as beam coz beam shouldn't bend (Z should be high)

① ⇒ Better as belt coz belt should bend (Z should be less)..

Both are better but for different purposes.

* $\boxed{(\sigma_b)_{max} = \frac{M}{Z_{NA}} \text{ (or) } \frac{E\,Y_{max}}{R}}$ — ①

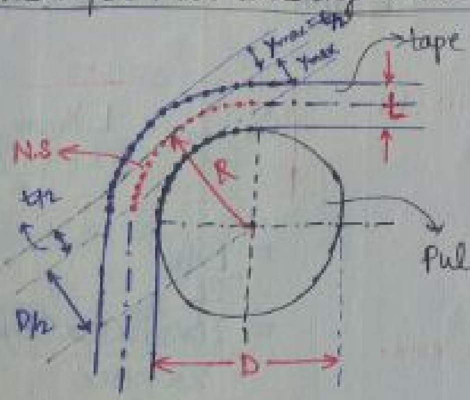
* $\boxed{\sigma_b = \frac{My}{I_{NA}} \text{ (or) } \frac{Ey}{R} \text{ (or) } (\sigma_b)_{max}\left[\frac{y}{Y_{max}}\right]}$ — ②

* $\boxed{R = \frac{E I_{NA}}{M}}$ — ③

* $\boxed{(\epsilon_b)_{max} = (\epsilon_{long})_{max} = \frac{(\sigma_b)_{max}}{E} = \frac{Y_{max}}{R} = \frac{1}{E}\left[\frac{M}{Z_{NA}}\right]}$ — ④

* $\boxed{M_R = Z_{NA} \cdot \sigma_{per}}$ — ⑤

Expression for Max. Bending Stress in a Flat Belt –



$R = \frac{D}{2} + \frac{t}{2} = \frac{D+t}{2}$.

$Y_{max} = \frac{t}{2}$.

$[(\sigma_b)_{max}]_{O\&I} = \pm\left(\frac{E\,Y_{max}}{R}\right)$

$= \pm\left[\frac{E\,t/2}{\frac{D+t}{2}}\right]$

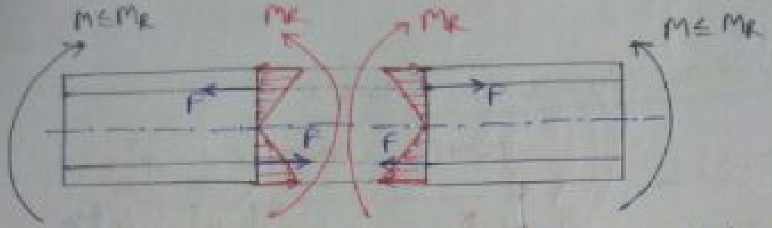
$\boxed{[(\sigma_b)_{max}]_{O\&I} = \pm\frac{Et}{D+t} \approx \frac{Et}{D}}$

Note – For Higher life of belt or better service, $(\sigma_b)_{max}$ Should be less ie. thickness of belt should be less. But as belts are used for the transmission higher power ⇒ higher torque ⇒ Higher ~~stress~~ Load. ∴ Area should be large for less stress. Area = $t \times w$ (where w = width of belt). Hence thin & wide belt is best for transmission of power.

ये भूल जाता है इसलिए।

* $\boxed{[(\sigma_b)_{max}]_{flat\ belt} = \pm\left[\frac{Et}{D_{smaller} \text{ (or) } D_{driver}}\right] \leq \sigma_{per}}$

$\boxed{D_{smaller} \geq \_\_\_\_}$



F = I.R.F (Internal resisting forces developed above & below the N.A. of x-S/c.

M = B.M acting on the x-S/c.

M_R = Moment of resistance @ Resisting bending couple of the x-S/c.

→ Safe Cond^n for bending –

$$\boxed{M \leq M_R}$$

M_R of a x-S/c should be high for resisting the moment applied.

* Moment of resistance is the resisting bending couple which is formed due to two equal, parallel & opp. IRF developed above and below the N.A of that x-S/c.

Expression for M_R –

Safe condition for design wrt. st. criterion,

$$(\sigma_{max})_{ind} \leq \sigma_{per}$$

$$\frac{M}{Z_{NA}} \leq \sigma_{per}$$

$$M \leq \underbrace{Z_{NA} \cdot \sigma_{per}}_{M_R}$$

$$\boxed{M_R = Z_{NA} \cdot \sigma_{per}}$$

Used to compare given beams wrt. their M_R.

Similarly,

$\boxed{F_R = A \cdot \sigma_{per}}$ ⇒ For Axial Load

$\boxed{T_R = Z_P \cdot \tau_{per}}$ ⇒ For Twisting moment

For two shafts having same area, Hollow has more Z_P.
For same dia, solid has more Z_P.

For a given material,

$$\boxed{M_R \propto Z_{NA}}$$