$\Sigma T = 0 \Rightarrow T_A - T + T + T_D = 0.$

$$\boxed{T_A = -T_D}$$

$\theta_1 + \theta_2 + \theta_3 = 0.$

$$\frac{L}{G}\left(\frac{-T_A}{J} + \frac{T - T_A}{J/2} + \frac{-T_A}{J}\right) = 0.$$

$$\frac{L}{GJ}\left(-T_A + 2T - 2T_A - T_A\right) = 0.$$

$$2T = 4T_A$$

$$\boxed{T_A = T/2} \quad (\leftarrow\leftarrow)$$

$$\boxed{T_D = T/2} \quad (\rightarrow\rightarrow)$$

$T_1 = -T_A = -T/2.$

$T_2 = T - T_A = T - T/2 = T/2.$

$T_3 = -T_A = -T/2.$

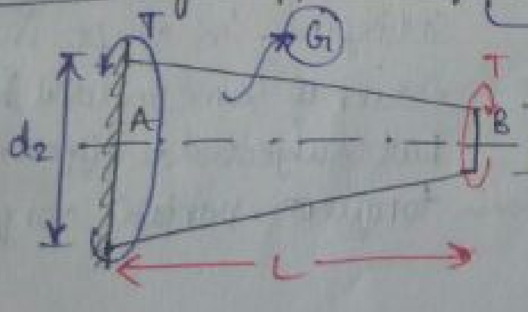
$$\theta_B = \theta_{BA} = \left(\frac{TL}{GJ}\right)_1 = -\frac{TL}{2GJ}.$$

$$\theta_C = \theta_{CD} = \left(\frac{TL}{GJ}\right)_2 = -\frac{TL}{2GJ}.$$

*** **Note:** If stepped shaft is of same material then apply short-cut method. If the shaft is ~~of~~ of more than one material then apply compatible equation. ($\Sigma\theta = 0$).

Torsion of ~~Stepped~~ Tapered Shaft -

→ Treated as assembly of n shafts which are in series.



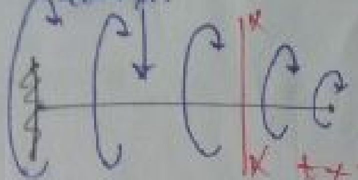
$$U = \int_0^L \frac{(T_{x-x})^2\,dx}{2(GJ)_{x-x}}$$

$$= \frac{1}{2GJ}\int_0^L (tx)^2\,dx$$

$$= \frac{t^2}{2GJ}\left[\frac{x^3}{3}\right]_0^L$$

$$\therefore \boxed{U = \frac{16t^2L^3}{3\pi G d^4}}$$

WB

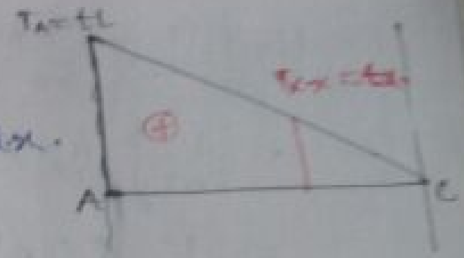


$t_{x-x} = \frac{tx}{L}$

$$t_{x-x} = \frac{tx}{L}.$$

$$T_{x-x} = \int_0^x \left(\frac{tx}{L}\right)dx$$

$$= \frac{tx^2}{2L} \longrightarrow \text{Parabola. BMD.}$$



CA - [$x=0$ to L].

$t_{x-x} = t$ Nm/m.

Torque at x-s/c x-x $= T_{x-x} = \int_0^x t_{x-x}\,dx$.

$$T_{x-x} = \int_0^x t\,dx.$$

$$T_{x-x} = tx. \text{ N-m.}$$

$x=0 \Rightarrow T_C = 0.$

$x=L \Rightarrow T_A = tL = T_{max}.$

$$\tau_{max} = \tau_A = \frac{16T_A}{\pi d^3} = \frac{16tL}{\pi d^3}.$$

$$\theta_C = \theta_{CA} = \int_0^L \theta_{strip}$$

$$\theta_C = \theta_{CA} = \int_0^L \frac{(T_{x-x})dx}{GJ}$$

$$= \frac{1}{GJ}\int_0^L (tx)\,dx$$

$$\boxed{\theta_C = \theta_{CA} = \frac{tL^2}{2GJ} \text{ or } \frac{16tL^2}{\pi G d^4}}.$$

***

$$\theta_B = \theta_{BA} = \int_{L/2}^{L} \theta_{strip}$$

$$\boxed{\theta_B = \theta_{BA} = \frac{12tL^2}{\pi G d^4}}.$$

(1) $T_1 = T_2 = T_3 = \dots = T_n = T$.

(2) $(\tau_{max}) = \tau_B = \left(\frac{16T}{\pi d^3}\right)_B = \left(\frac{16T}{\pi d_1^3}\right) = \frac{16T}{\pi (d_{smaller})^3}$.

(3) $\frac{\tau_{max}}{\tau_{min}} = \frac{\tau_B}{\tau_A} = \left(\frac{d_A}{d_B}\right)^3 = \frac{d_2}{d_1} = \left(\frac{d_{larger}}{d_{smaller}}\right)^3$.

(4) $\theta_{T.S} = \theta_1 + \theta_2 + \dots + \theta_n$.

(or) $\theta_{T.S} = \int_0^L (\theta_{strip}) = \int_0^L \frac{(T_{x-x})\,dx}{(GJ)_{x-x}}$.

$J_{x-x} = \frac{\pi}{32} d^4 = \frac{\pi}{32}\left[d_1 + (d_2 - d_1)(x/L)\right]$.

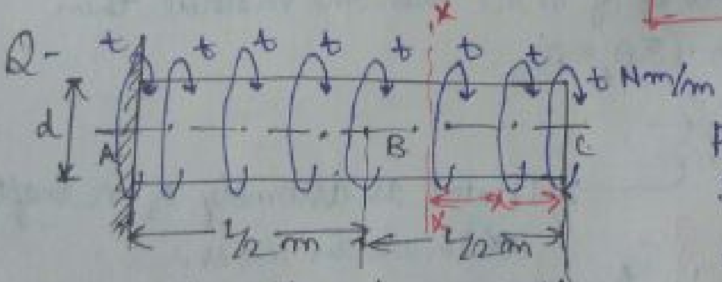
$\theta_{T.S} = \int_0^L \frac{(32)(T)(dx)}{\pi G\left[d_1 + (d_2 - d_1)(x/L)\right]^4}$.

$$\boxed{\theta_{T.S} = \frac{32TL}{\pi G}\left[\frac{d_1^2 + d_1 d_2 + d_2^2}{3d_1^3 d_2^3}\right]}$$

If $d_1 = d_2 = d \Rightarrow$ T.S. becomes a P.S.

$\theta_{TS} = \frac{32TL}{\pi G}\left[\frac{3d^2}{3d^6}\right] = \boxed{\frac{32TL}{\pi G}\left[\frac{1}{d^4}\right] = \theta_{P.S}}$

Q-



Pure torsion cond^n is not satisfied. 'n' shafts in series of same material & dia but subjected to different torque or variable torque

Det. the following for the circular & prismatic bar as shown in the fig.

(i) τ_{max}.

(ii) ∠ of twist of the shaft.

(iii) —"— at a x-s/c located at a distance of L/2 from free end.

(iv) Strain energy of the shaft.

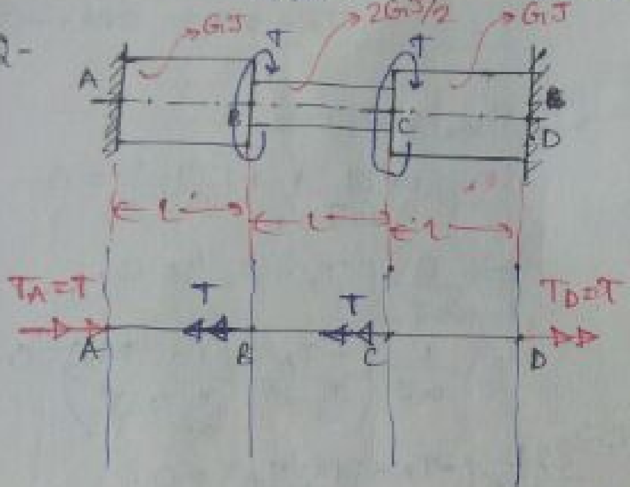
⑤ $(\tau_{max})_1 = \left(\frac{16T}{\pi d^3}\right)_1 = \frac{16(-T/17)}{\pi d^3}$.

$= \frac{-16T}{17\pi d^3}$.

⑥ $(\tau_{max})_2 = \left(\frac{16T}{\pi d^3}\right)_2 = \frac{16\left(\frac{16}{17}T\right)}{\pi (2d)^3}$

$= \frac{32T}{17\pi d^3}, = \frac{32 \times 2Pd}{17\pi d^3} = \frac{64Pd}{17\pi d^3}$.

Q-



Det θ_B & θ_C = ?

$T_A = \frac{T(2L) + T(L)}{3L}$

$\boxed{T_A = T}$.

(or) By symmetric loading.

$T_{AB} = +T$.

$T_{BC} = 0$.

$T_{CD} = -T$.

$\theta_B = \theta_{BA}$

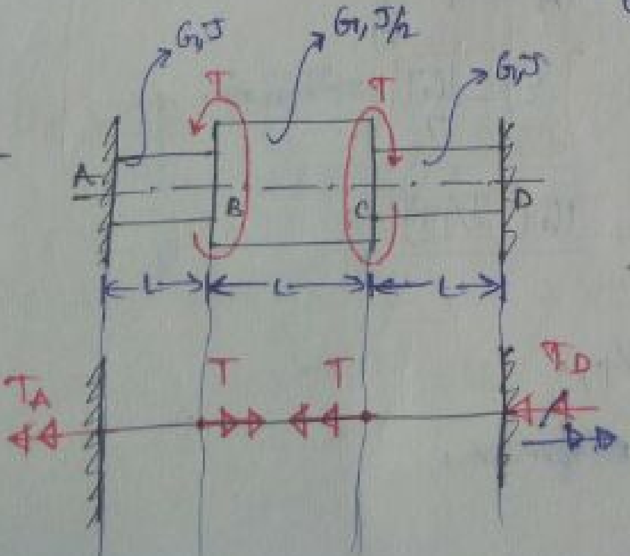
$= \left(\frac{TL}{GJ}\right)_{AB}$

$\theta_B = \frac{TL}{GJ}$.

$\theta_C = \theta_{CD}$

$= \frac{-TL}{GJ}$.

Q-



θ_B = ?

θ_C = ?

$T_{AB} = -T_A = T_1$.

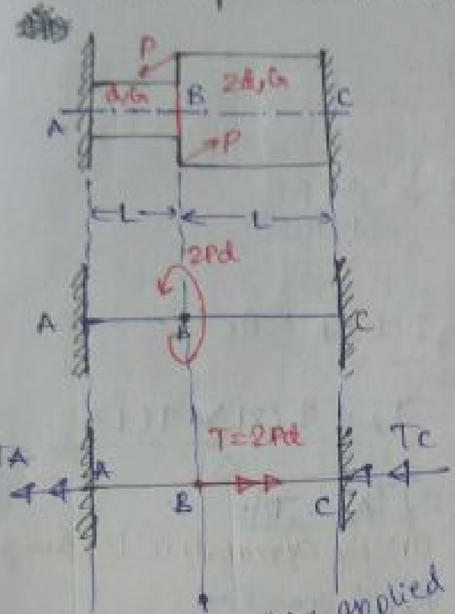
$T_{BC} = -T_A + T = T_2$.

$T_{CD} = T_D$ (or) $-T_A + T - T_2 - T_A$.

Q- For the stepped shaft as shown in the fig. Det.

(i) Max. torsional shear stress τ_{max}.

(ii) Ratio of max. & min. torsional shear stress.



This method can't be applied as bar is not prismatic.

① $T_{AB} = -T_A = T_1$

$T_{BC} = T_C$ (or) $2Pd - T_A = T_2$.

② $\Sigma T = 0 \Rightarrow T_A - T + T_C = 0$.

$T_A + T_C = 2Pd$. — (I)

③ $\theta_{Total} = \theta_1 + \theta_2 = 0$.

$= \left(\frac{TL}{GJ}\right)_1 + \left(\frac{TL}{GJ}\right)_2 = 0$.

$= \frac{L}{GJ}(T_1 + T_2) = 0$.

$= \frac{L}{G}\left(\frac{-T_A}{\frac{\pi}{32}d^4} + \frac{2Pd - T_A}{\frac{\pi}{32}(2d)^4}\right) = 0$.

$= \frac{32L}{\pi G d^4}\left(-T_A + \frac{2Pd - T_A}{16}\right) = 0$.

~~$T_A = Pd$.~~

$-16T_A + 2Pd - T_A = 0$.

$\boxed{T_A = \frac{2Pd}{17}}$. ($\leftarrow\!\!\leftarrow$)

$\boxed{\text{Let } 2Pd = T.}$

~~④~~ from (I), $T_C = T - T_A = T - \frac{T}{17}$

$\boxed{T_C = \frac{16T}{17}}$. ($\leftarrow\!\!\leftarrow$)

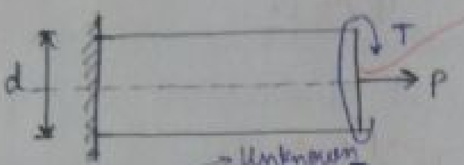
~~⑤ $(\tau_{max})_1 = \left(\frac{16T}{\pi d^3}\right) = 16(-T/17)$~~

④ $T_1 = -T_A = -T/17$.

$T_2 = T_C = \frac{16T}{17} = T_{max}$.

Ch - 07

Principal stresses and ~~Planes~~ Strains -



→ X-S/C is neither a P.P nor a max. τ_s plane. (∵ Combined stresses)

Det.

(i) Max. σ_t (→ On P.P) and τ_s (→ max τ_s plane) developed in the bar.

These are on a X-S/C but $(\sigma_t)_{max}$ is at principal plane. This is max. axial stress.

$$\cancel{(\sigma_t)_{max} = \frac{P}{A} = \frac{4P}{\pi d^2}}$$

$$\cancel{\tau_{max} = \frac{T}{Z_P} = \frac{16T}{\pi d^3}}$$

(ii) (Max σ_t or Max. axial Stress) & (max τ_s or max. torsional shear stress) developed on the X-S/C of the bar.

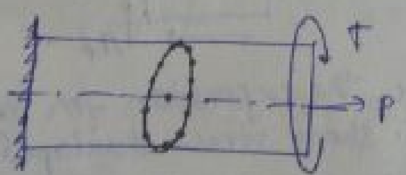
Then the answer will be as above.

$$(\sigma_t)_{max} = \frac{4P}{\pi d^2} \; ; \; (\tau_s)_{max} = \frac{16T}{\pi d^3}.$$

* Under P.A.L, Max σ_t in a member = max. σ_t on the X-S/C. $= \frac{4P}{\pi d^2}$.
(∵ X-S/C is a P.P).

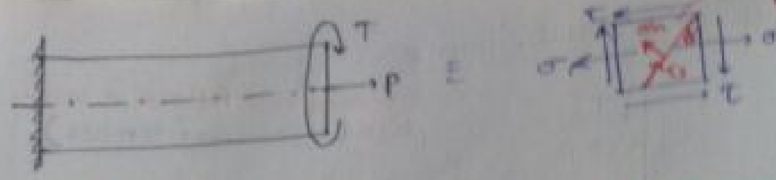
* Under Pure-torsion, Max. τ_s in a member = Max τ_s on the X-S/C. $= \frac{16T}{\pi d^3}$.
(∵ X-S/C is a max. τ_s plane).

→ ~~Fist~~ First of all find the critical point.

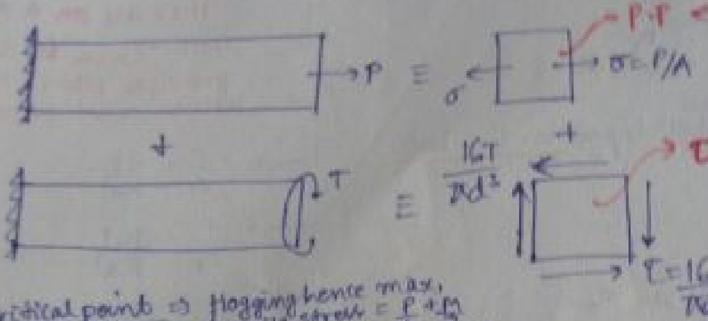


At all the X-S/C axial ~~in~~ stress is max. or same. Whereas τ_s is max at the periphery. Hence all the points on the periphery of the X-S/C ~~is~~ are critical points.

Note:
- → Draw all the stress distribution. Axial, tensile, bending, shear etc.
- → Determine at which point all these stresses are max. ie the critical point.
- → Draw a biaxial state of stress at critical points.



Combined stress hence P.P and τmax planes are unknown.

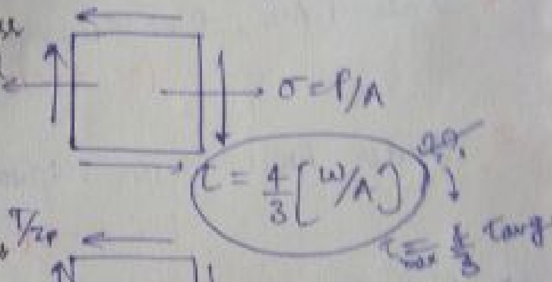


If loads are applied individually.

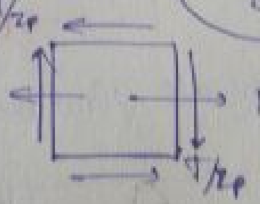
Critical point ⇒ Hogging hence max. tensile stress $= \frac{P}{A} + \frac{M}{Z_{NA}}$

Shear force is zero at critical point.

Note/ 1- Shear force is considered. ⇐ State of stress at the centroid (i.e. at A)



Bending stress is zero. ⇐ State of stress at pt. F



1- Aim of this chapter is to derive the expression for major & min principal stress and max & min. shear stress developed at a critical point on a member under bi-axial or combined state of stress.

2- These eqns. are used in the design of a component under bi-axial state of stress i.e. in theories of failures eq's.

3- Basic strength of material eqns like $\sigma_a = P/A$, $(\sigma_b)_{max} = \frac{M}{Z_{NA}}$, $\tau_{max} = \frac{T}{Z_P}$, $(\tau_d)_{max} = K\,\tau_{avg}$, will represent max. stresses in a member under corresponding loading only.

4- Under bi-axial and combined stresses, basic SOM eqns. should be used to determine the max. stresses at the x-s/c only.

Steps used to determine σ_1, σ_2 & τ_{max} -

1- Loading diag. should be known.
(To determine stress variation from x-s/c to x-s/c).

2- In a prismatic member critical x-s/c is the x-s/c where A.L, SF, BM & TM are max. This can be determined by using above loading diagrams.

3- Stress distribution should be drawn at critical x-s/c.

4- Critical point on the critical x-s/c is the point where resultant normal and resultant shear stress is max.

5- Graphical representation of state of stress at a critical point should be drawn.

6- Determination of σ_x, σ_y, τ_{xy} by using basic SOM eq^ns.

7- σ_n and τ_s eq^ns (Normal stress and Shear stress developed on an oblique plane inclined at an angle of θ) should be written.

8- Location of P.P [i.e. $(\tau_s)_\theta = 0$ or $\frac{d}{d\theta}(\sigma_n)_\theta = 0$].

9- Det. of P. stresses by substituting corresponding values of θ in (σ_n) eq^n.

10- Location of τ_{max} planes [i.e. $\frac{d}{d\theta}(\tau_s)_\theta = 0$ or if you know P.P then τ_{max} plane is $45° + P.P$].

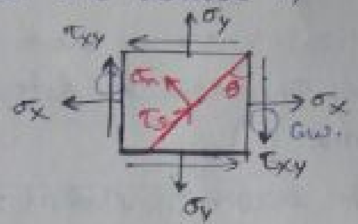
11- Det. of τ_{max} by substituting corresponding value of θ in τ_s eqn.

⇒ σ_n & τ_s eqⁿs on an oblique plane passing through a point under biaxial state of stress -

* $(\sigma_n)_\theta = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta.$ — (1)

* $(\tau_s)_\theta = \frac{1}{2}[\sigma_x - \sigma_y]\sin 2\theta + \tau_{xy}\cos 2\theta.$ — (2)

Above eqns. are derived by assuming ~~flow~~ following state of stress (i.e



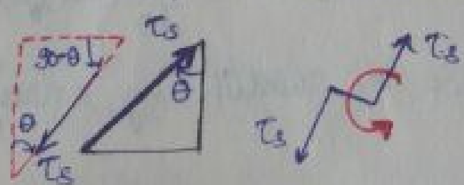
Hence following sign convention is to be used in the above eqⁿˢ -

1- Tensile normal stresses should be treated as +ve and vice-versa.

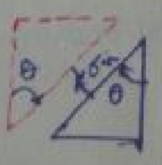
2- Shear stress on X-face (ie. τ_{xy}) should be treated as +ve (i.e vertical shear stress) when it is causeds a couple in C.W. dirⁿ & vice-versa.

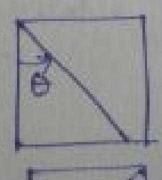
3- Shear stress on an O.P (ie. τ_s) should be treated as +ve when it causes a couple in A.C.W. dirⁿ & vice-versa.

On two parallel planes, the stress will always be equal & oppᵗ.

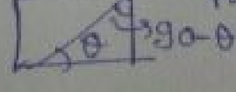


4- Inclination of O.P (ie. θ) should be treated as +ve when it is measured in C.W. dirⁿ from X-face & vice-versa.



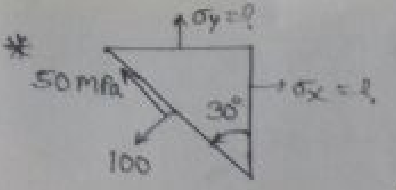


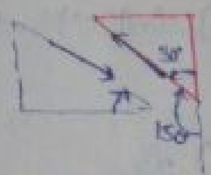
Put $\theta = -\theta$ in eqⁿ.



Put $\theta = 90-\theta$ in eqⁿ.

*





$\theta = -30°$ or $150°$.

Here, $\tau_{xy} = 0$.

$$\therefore (\sigma_n)_{\theta=-30°} = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos(-60°) + (0)\sin(-60°)$$

$$100 = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{4}[\sigma_x - \sigma_y].$$

$$400 = 2\sigma_x + 2\sigma_y + \sigma_x - \sigma_y.$$

$$3\sigma_x + \sigma_y = 400 \quad \text{—(1)}.$$

$$(\tau_t)_{\theta=-30°} = -\frac{1}{2}[\sigma_x - \sigma_y]\sin(-60°) + 0.$$

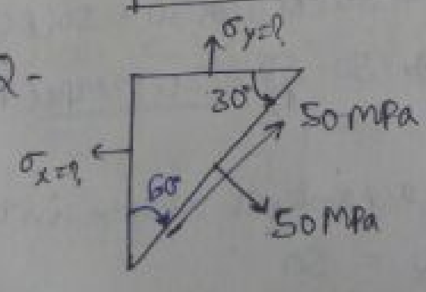
$$50 = +\frac{1}{2}[\sigma_x - \sigma_y]\frac{\sqrt{3}}{2}$$

$$(\sigma_x - \sigma_y) = \cancel{4}\,\frac{200}{\sqrt{3}} \quad \text{—(2)}.$$

$$\cancel{4}\sigma_x = \overset{100}{\cancel{400}} + \frac{\overset{50}{\cancel{200}}}{\sqrt{3}}$$

$$\boxed{\begin{aligned} \sigma_x &= 128.86 \text{ MPa (T)}. \\ \therefore \sigma_y &= 13.39 \text{ MPa (T)}. \end{aligned}}$$

Q -



$$(\sigma_n)_{\theta=60°} = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos(120°) + 0.$$

$$50 = \frac{1}{2}[\sigma_x + \sigma_y] - \frac{1}{4}[\sigma_x - \sigma_y]$$

$$200 = \sigma_x + 3\sigma_y. \quad \text{—(1)}.$$

$-50 = \frac{1}{2}[\sigma_x - \sigma_y] \sin(120°) + 0.$

~~$+00 = \sigma_y$~~

$-50 = (\sigma_x - \sigma_y)\frac{\sqrt{3}}{4}$

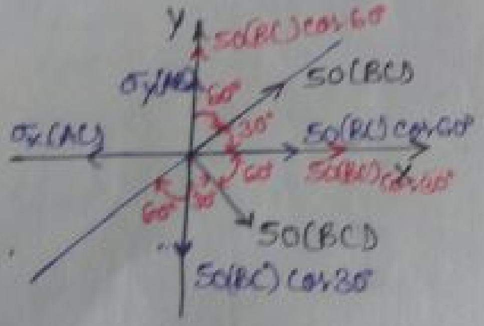
$-\sigma_x + \sigma_y = \frac{+200}{\sqrt{3}}$ — ②.

$\cancel{4}\sigma_y = \overset{50}{\cancel{200}} + \frac{\overset{50}{\cancel{200}}}{\sqrt{3}}$

$\sigma_y = 78.86$ MPa (T).

$\sigma_x = -36.60$ MPa (or) 36.60 MPa (C).

<u>2nd Method-</u>



$(\sigma_x)AC = 50(BC) \times \frac{1}{2} + 50(BC) \times \frac{\sqrt{3}}{2}$,

$\sigma_x = (25 + 25\sqrt{3})\left(\frac{BC}{AC}\right).$

$\frac{1}{\cos 60°}$

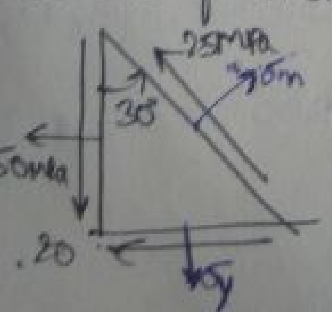
$\sigma_x = (25 + 25\sqrt{3})\frac{2}{1}$

$= 50 + 50\sqrt{3} = \underline{136.6 \text{ MPa (T)}}$

$\sigma_y(AB) = 50(BC)\cos 30 - 50(BC)\cos 60$

$\sigma_y = 50 - \frac{50}{\sqrt{3}} = \underline{21.13 \text{ MPa (T)}}.$

Q- For the biaxial state of stress at a point as shown in the fig. Det. σ_y & σ_n.



$\sigma_x = 50$

$\tau_{xy} = -20.$

$\tau_s = -25.$

$\theta = -30$ or $150.$

$(\sigma_n)_\theta = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta.$

$\sigma_n = \frac{1}{2}[50+\sigma_y] + \frac{1}{2}[50-\sigma_y]\cos(-60) + -20\sin(-60).$

$= \frac{1}{2}[50+\sigma_y] + \frac{1}{4}[50-\sigma_y] + \frac{20\sqrt{3}}{2}$

$\sigma_n - \frac{\sigma_y}{4} = 25 + \frac{25}{2} + 10\sqrt{3}. \quad —(1).$

$(\tau_s)_\theta = \frac{1}{2}[\sigma_x - \sigma_y]\sin 2\theta + \tau_{xy}\cos 2\theta.$

$-25 = \frac{1}{2}[50-\sigma_y]\sin(-60) + -20\cos(-60)$

$-25 = \frac{-\sqrt{3}}{4}[50-\sigma_y] - 10.$

$100 = 50\sqrt{3} - \sqrt{3}\sigma_y + 40.$

$\sigma_y = \frac{50\sqrt{3}-60}{\sqrt{3}}.$?

Ans $\sigma_y = 84.64$ MPa(T).

$(\sigma_n)_\theta = 75.98$ MPa(T).

Q - For a point under bi-axial state of stress, $\sigma_x = 100$ MPa, $\sigma_y = 50$ MPa, $\tau_{xy} \neq 0$.

Det. $(\sigma_n)_{\theta=120°} = ?$ if $(\sigma_n)_{\theta=30°} = 225$ MPa.

$(\sigma_n)_{\theta=30°} = \frac{1}{2}[100+50] + \frac{1}{2}[50]\cos 60° + \tau_{xy}\sin 60°.$

$225 = 75 + 12.5 + \frac{\sqrt{3}}{2}\tau_{xy}$

$\tau_{xy} = 158.77$ MPa

$\therefore (\sigma_n)_{\theta=120°} = 75 + -12.5 + -137.49$

$= -74.99$ MPa. (Comp.).

Shortcut Method -

$$\boxed{(\sigma_n)_\theta + (\sigma_n)_{90+\theta} = \sigma_x + \sigma_y}$$

$(\sigma_n)_{30} + (\sigma_n)_{120} = 100 + 50.$

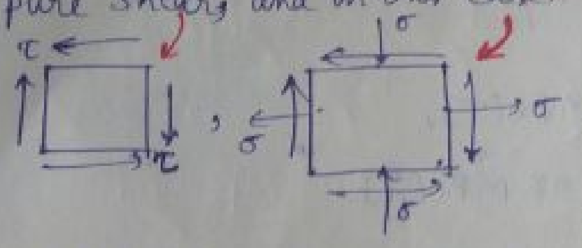
$(\sigma_n)_{120} = -75\text{ MPa}$ (or) 75 MPa (comp.).

(*) $$\boxed{(\tau_s)_\theta + (\tau_s)_{90+\theta} = \text{zero}}$$

Note :

1- Sum of normal stresses on two mutual ⊥r planes passing through a point is always remain same and is equal to sum of σ_x & σ_y (ie. $\sigma_x + \sigma_y$).

2- Normal stresses on two mutual ⊥r planes ~~plan~~ passing through a point are equal & unlike in nature when $\sigma_x + \sigma_y = 0$, eg. pure shear, and in this case.



$$\boxed{(\sigma_n)_\theta = -(\sigma_n)_{90+\theta}}$$

$$\boxed{\sigma_1 = -\sigma_2}$$

3- Shear stresses on 2 mutually ⊥r planes passing through a point (complimentary shear forces) are always equal and opp. in nature because of sum of complimentary shear stresses is zero.

Q- Similar to above question if, $(\tau_s)_{\theta=30^\circ} = 225\text{ MPa}$, Det. $(\tau_s)_{\theta=120^\circ}$.

$(\tau_s)_{\theta=120^\circ} = -225\text{ MPa}.$

Principal planes & principal stresses -

(OR) Planes of zero τ_s.

(OR) — " — Pure comp. normal stresses

(OR) — " — max. & min. — " — (when $\sigma_{1,2}$ are like in nature).

(OR) — " — Max. σ_t & max σ_c (when $\sigma_{1,2}$ are unlike in nature)

⇒ I/P Data → Bi-axial state of stress at a point.

Location of P.P -

$(\tau_s)_\theta = 0$ (OR) $\frac{d}{d\theta}(\sigma_n)_\theta = 0$.

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}} \longrightarrow \text{I}^{st} \text{ Step.}$$

$2\theta =$ ___ , ___ .

$\theta, \theta' =$ ___ , ___ .

$\boxed{\theta' = \theta + 90}$.

II step -

$(\sigma_n)_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$.

$(\sigma_n)_\theta =$ ______ MPa.

III step -

$(\sigma_n)_{\theta'} = \sigma_x + \sigma_y - (\sigma_n)_\theta =$ ______ MPa.

$\sigma_1 =$ Major principal stress at a given point
$=$ Larger of $[(\sigma_n)_\theta$ & $(\sigma_n)_{\theta'}]$.

$\sigma_1 =$ ______ MPa.

$\theta_1 =$ Location of major P.P at the given point.

σ_2 = Minor principal stresses at the given point.
= Smaller of $[(\sigma_n)_\theta$ & $(\sigma_n)_{\theta'}]$.
= ______ MPa.

θ_2 = Location of minor P.P at the given point.

eg. $\Rightarrow (\sigma_n)_\theta = 100$ MPa.
$(\sigma_n)_{\theta'} = -200$ MPa.
$\sigma_1 = -200$ MPa. (Max. comp. stress).
$\theta_1 = \theta'$
$\sigma_2 = 100$ MPa. (Max. tensile stress).
$\theta_2 = \theta$.

$\Rightarrow (\sigma_n)_\theta = 100$ MPa.
$(\sigma_n)_{\theta'} = 50$ MPa.
$\sigma_1 = 100$ MPa (Max. tensile stress)
$\theta_1 = \theta$.
$\sigma_2 = 50$ MPa (min. tensile stresses).
$\theta_2 = \theta'$.

II^nd Method for $\sigma_{1,2}$ -

$$\boxed{\sigma_{1,2} = \frac{1}{2}\left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]}$$

eg. $\sigma_{1,2} = 200$ MPa, 300 MPa (assuming)

$\sigma_1 = 350$ MPa $= (\sigma_t)_{max}$

$\sigma_2 = 200$ MPa $= (\sigma_t)_{min}$

$$\boxed{\begin{array}{l}\theta_1 = 120^\circ \\ \theta_2 = 30^\circ\end{array}}$$

May or may not be correct.

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$\theta = 120^\circ, 30^\circ$

Both results are not co-related, Hence we can't say directly which is θ_1 & θ_2.

* IInd method should be used to det. principal stresses only.

* 2 to det. principal stresses & corresponding plane locations, always it is better to use Ist method only.

$$\boxed{\sigma_{1,2} = \frac{1}{2}\left[(\sigma_x+\sigma_y) \pm \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right]}$$

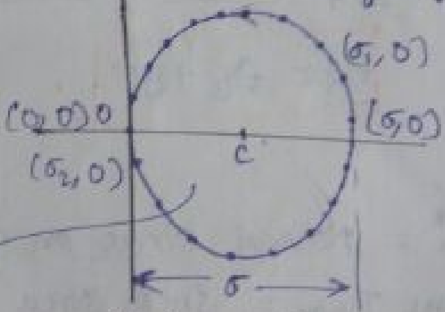
$$\sigma_1+\sigma_2 = \sigma_x+\sigma_y$$

$$\therefore \boxed{\sigma_1-\sigma_2 = \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$ = Dia. for Mohr's circle for stress.

Radius of Mohr's circle for stress $= \dfrac{\sigma_1-\sigma_2}{2} = \dfrac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$.

⇒ For Uni-axial state of stress, (i.e. $\sigma_x = \sigma$, $\sigma_y = \tau_{xy} = 0$).
eg P.A.L, P. Bending, E.A.L, extreme fibre of any beam.

$$\boxed{\text{Radius} = \frac{\sigma_x}{2} = \sigma_1}$$



Is a circle viz tangential to y-axis. Radius $= \frac{\sigma}{2}$.

Mohr's circle for uni-axial state of stress.

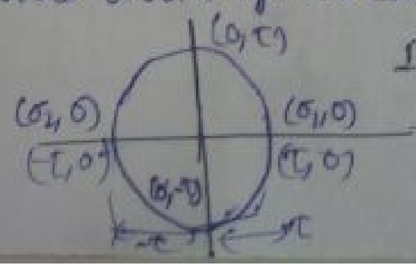
⇒ For pure shear state of stress, (i.e. $\sigma_x = \sigma_y = 0$; $\tau_{xy} = -\tau$)

⇒ Shaft subjected to pure torsion,

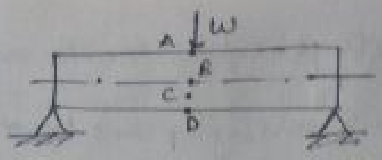
⇒ Any point on the N.A of beam = $\boxed{\text{Radius} = \pm\tau}$

Here Mohr's circle is circle with origin as centre and radius $= \tau$.

$$\boxed{\begin{aligned}\sigma_1 &= \tau\\ \sigma_2 &= -\tau\\ \sigma_1/\sigma_2 &= -1.\end{aligned}}$$



Mohr's circle for pure shear stress.

⇒ 

At A → Uni-axial state of stress (Comp.)

At B → Pure shear (Case III). (At N.A ⇒ Bending stress zero).

At C → Combined state

At D → Uni-axial state of stress (tensile) Case II.

Max τ_s planes, In-plane τ_{max} & Absolute τ_{max} –

Location of τ_{max} plane –

$$\frac{d}{d\theta}(\tau_s) = 0.$$

$$\boxed{\tan 2\theta = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} = \frac{-1}{\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}}.$$

$2\theta =$ ____, ____

$\theta_{3,4} =$ ____, ____.

$$\boxed{\theta_4 = \theta_3 + 90^\circ}.$$

Let,

$\sigma_n^* =$ Normal stress on max. τ_s planes.

In-plane τ_{max} = Shear stress on max. τ_s planes.

Ist method for σ_n^* & In-plane τ_{max} –

$$\sigma_n^* = (\sigma_n)_{\theta = \theta_3 \text{ or } \theta_4} = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta_3 + \tau_{xy}\sin 2\theta_3$$

$\sigma_n^* =$ ______ MPa.

In-plane $\tau_{max} = (\tau_s)_{\theta = \theta_3 \text{ or } \theta_4}$

$= \frac{1}{2}[\sigma_x - \sigma_y] \sin 2\theta_3 + \tau_{xy} \cos 2\theta_3$.

In-plane $\tau_{max} = \pm$ ______ MPa.

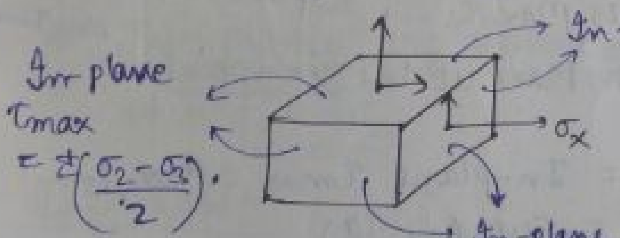
<u>IInd Method for σ_n^* & In-plane τ_{max}*</u>

$$\boxed{\sigma_n^* = \frac{\sigma_1 + \sigma_2}{2} \text{ or } \frac{\sigma_x + \sigma_y}{2}}$$

How??

$$\boxed{\text{In-plane } \tau_{max} = \pm\left(\frac{\sigma_1 - \sigma_2}{2}\right) = \pm \text{ Radius of Mohr's circle for stress.}}$$

<u>Note:</u> In plane τ_{max} is the max. value of τ for a given plane eg., $xy \to (\sigma_x - \sigma_y)$; $yz \to (\sigma_y - \sigma_z)$; $zx \to (\sigma_z - \sigma_x)$, whereas absolute τ_{max} is the larger value of τ_{max} from the given values.



Absolute τ_{max} = larger of $\left|\left(\frac{\sigma_1 - \sigma_2}{2}\right), \pm\left(\frac{\sigma_2 - \sigma_3}{2}\right), \left(\frac{\sigma_3 - \sigma_1}{2}\right)\right|$.

In-plane τ_{max} = radius of Mohr's circle.

For bi-axial state of stress, $\sigma_3 = 0$.

$\therefore$ Abs. τ_{max} = larger of $\left[\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2}{2}, \frac{\sigma_1}{2}\right]$.

Abs. $\tau_{max} = \left[\frac{\sigma_1}{2}\right] \to$ when σ_1, σ_2 are <u>like</u> in nature.

$= \left[\frac{\sigma_1 - \sigma_2}{2}\right] \to$ — " — <u>unlike</u> — " —.

* Absolute τ_{max} represents max. shear stress developed at a given pt. Hence for a do all designed calculations absolute τ_{max} should be considered.

* In-plane τ_{max} represents max. shear stress in the given plane only.

* In-plane τ_{max} & absolute τ_{max} are equal in magnitude under following state of stress condition -

(i) Uni-axial s.o.s condition (In plane $\tau_{max} = \tau_{abs} = \sigma/2$).

(ii) Bi-axial s.o.s cond^n, when σ_1 & σ_2 are unlike in nature {In plane when τ_{max} =

$$\text{In-plane } \tau_{max} = \text{Absolute } \tau_{max} = \frac{\sigma_1 - \sigma_2}{2}.$$

Q- Principal stresses under bi-axial state of stresses are 200 MPa & 100 MPa. Det. (i) Max. τ_s → In-plane τ_{max}

$\underbrace{200 \text{ MPa } \& \text{ } 100 \text{ MPa}}_{\text{Like in nature}}$

(ii) Max. at the given point → Abs. τ_{max}

(a) 50 MPa

(b) 100 MPa

(c) 150 MPa

(d) 200 MPa

(i) $\frac{\sigma_1 - \sigma_2}{2}$ = In-plane τ_{max} = Max. τ_s = 50 MPa. (a)

(ii) $\frac{\sigma_1}{2}$ = Abs. τ_{max} = max. at the given point = 100 MPa. (b)

Q- For a point under bi-axial state of stress, principal stresses are 200 MPa & -100 MPa. Det. (i) Max τ_s. (b)

$\underbrace{200 \text{ MPa } \& -100 \text{ MPa}}_{\text{Unlike in nature}}$

(ii) Max. τ_s at the given point. (b)

(a) 200 MPa

(b) 150 MPa

(c) 100 MPa

(d) 50 MPa.

(i) Max τ_s = In-plane τ_{max} = Abs. τ_{max} $= \frac{\sigma_1 - \sigma_2}{2} = 150$ MPa.

Note -

means $\sigma_n = 0$.

* Plane of pure shear is achieved when σ_1 & σ_2 are unlike in nature.

* Plane of pure shear and τ_{max} plane coincides when σ_1 & σ_2 are equal & unlike in nature.

Planes of pure τ_s (or) planes of zero σ_n:

Not σ_x & σ_y

* These planes will exist when $\sigma_{1,2}$ are unlike in nature.
* —"— are located by equating σ_n eq^n to zero [ie. $(\sigma_n)_\theta =$ ___ $= 0 \Rightarrow \theta_{5,6} = ?$].
* These planes becomes τ_{max} planes when $\sigma_1 = -\sigma_2$.
* $\tau_s^* =$ Shear stress on pure shear plane [ie $\tau_s^* = (\tau_s)_{\theta=\theta_{5,6}}$

$$= -\frac{1}{2}(\sigma_x - \sigma_y)(\sin 2\theta_5) + \tau_{xy}\cos 2\theta_5].$$

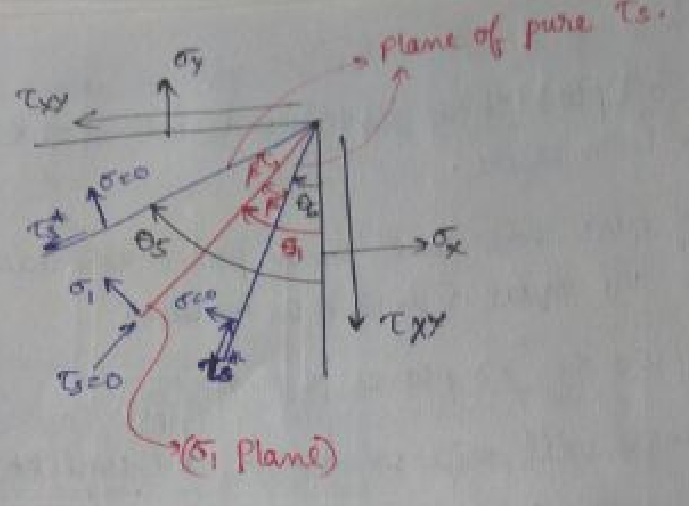
* $[\tau_s^* \le \tau_{max}]$
* These planes are non-complimentary planes except when $\sigma_1 = -\sigma_2$.

IInd Method -

$$\boxed{\tau_s^* = \sqrt{-\sigma_1 \sigma_2}}$$

$$\boxed{\tan\beta = \sqrt{\frac{-\sigma_1}{\sigma_2}}}$$, where $\beta =$ Angle b/w major P.P & planes of pure shear.

$$\boxed{\theta_{5,6} = \theta_1 \pm \beta}.$$



eq., $\sigma_x = ✓$, $\sigma_y = ✓$, $\tau_{xy} = ✓$

1- $\sigma_{1,2} = \frac{1}{2}\left[(\sigma_x+\sigma_y) \pm \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right]$.

2- $\sigma_n^* = \frac{\sigma_1+\sigma_2}{2}$ (or) $\frac{\sigma_x+\sigma_y}{2}$,

In-plane $\tau_{max} = \pm\left(\frac{\sigma_1-\sigma_2}{2}\right)$.

Abs. $\tau_{max} = \left(\frac{\sigma_1}{2}\right)$ (or) $\left(\frac{\sigma_1-\sigma_2}{2}\right)$

} Stresses on τ_{max} plane.

→ $\sigma_{1,2}$ are like in nature.

→ $\sigma_{1,2}$ are unlike in nature.

3- $\tau_s^* = \sqrt{-\sigma_1\sigma_2}$.

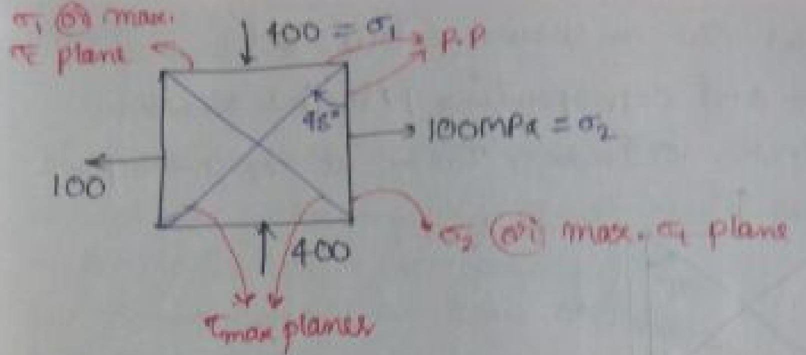
Q- For the biaxial state of stress given below, det.

(i) Principal stresses and corresponding plane locations.

(ii) Resultant stress on max. shear stress planes & correspon[ding] plane location.

(iii) Loc^n of planes of pure shear & shear stress acting on the pla[ne]

(iv) Max. tensile stress, max comp. & max. shear stress at a given point.



(i) $\sigma_1 = -400 \text{ MPa}$; $\theta_1 = 90°$.
$\sigma_2 = 100 \text{ MPa}$; $\theta_2 = 0°$.

Angle from a vertical plane i.e. x-plane.

(ii) $\theta_{3,4} = 45°$ & $135°$.

$$\sigma_m^* = \frac{\sigma_1 + \sigma_2}{2} = -150 \text{ MPa}$$

In plane $\tau_{max} = \pm\left(\frac{\sigma_1 - \sigma_2}{2}\right) = \pm 250 \text{ MPa}$.

Res. stress on τ_{max} plane $= \sqrt{(-150)^2 + (250)^2}$

$= \underline{\underline{291.55 \text{ MPa}}}$.

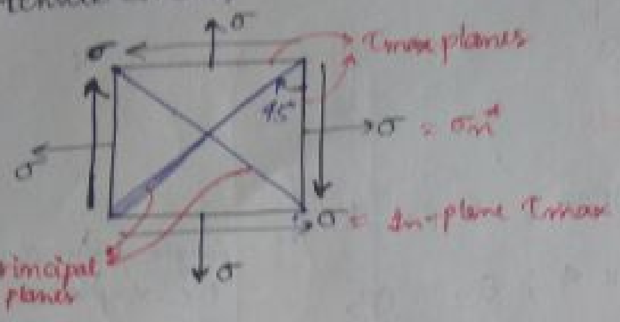
(iii) $\tan\beta = \sqrt{\frac{-\sigma_1}{\sigma_2}} = 2$,

$\beta = \tan^{-1}2$
$\beta = 63.43°$.

$\theta_5 = \theta_1 + \beta = 153.43°$.
$\theta_6 = \theta_1 - \beta = 26.57°$.
$\tau_s^* = \sqrt{-\sigma_1\sigma_2} = \pm 200 \text{ MPa}$.

(iv) Max $\sigma_t = 100 \text{ MPa}$.
max $\sigma_c = 400 \text{ MPa}$.
Abs. τ_{max} = In-plane τ_{max} = $\underline{250 \text{ MPa}}$.

Q- For the bi-axial SOS as shown below, det.
(i) Principal planes and corresponding principal stresses.
(ii) max. tensile stress, $(\sigma_t)_{max}$, τ_{max} at the given point.



$\sigma_x = \sigma_y = \tau_{xy} = \sigma$.

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \infty.$$

$$2\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$\boxed{\theta_P = 45° \text{ or } \& \ 135°}$$

$$(\sigma_n)_{\theta=45°} = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 90° + \tau_{xy}\sin 90°$$

$$= \sigma + 0 + \sigma = \boxed{2\sigma}.$$

$$(\sigma_n)_{\theta=135°} = \frac{1}{\cancel{2}} \times \cancel{2}\sigma + 0 + \sigma \sin (270°)$$

$$= \sigma - \sigma = \boxed{0}.$$

$\therefore \sigma_1 = 2\sigma;\ \sigma_2 = 0.$

$\theta_1 = 45°;\ \theta_2 = 135°.$

(ii) $\theta_{3,4} = 0° \ \& \ 90°.$

$\sigma_n^* = \sigma.$

In-plane $\tau_{max} = \sigma$. ??

(iii) $(\sigma_t)_{max} = 2\sigma$.

$(\sigma_c)_{max} = \text{zero}$.

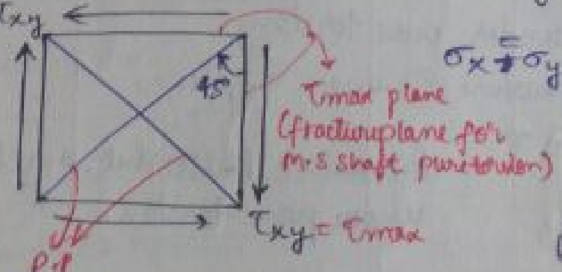
Abs. τ_{max} = In-plane $\tau_{max} = \sigma$.

2 - Det. the following when the pt. is under pure shear S.O.S.

(i) principal planes & corresponding P.S.

(ii) τ_{max} planes & — " — stresses.

(iii) Max. σ_t, Max σ_c, Max τ_s at the given point.



$\sigma_x = \sigma_y = 0$.

eg. (i) Shaft subjected to pure torsion $\left(\tau = \frac{16T}{\pi d^3}\right)$,

(ii) Any point on the N.A of beam.

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \infty.$$

$2\theta = 90°, 270°$.

$\theta = 45°, 135°$.

$$(\sigma_n)_{\theta=45°} = \frac{1}{2}(0) + \frac{1}{2}(0)\cos 90° + \tau \sin 90$$

$= \tau$.

$$(\sigma_n)_{\theta=135°} = \tau \sin(270°)$$

$= -\tau$.

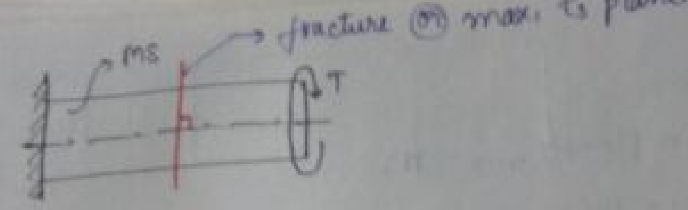
$\therefore \sigma_1 = \tau$; $\theta_1 = 45°$.

$\sigma_2 = -\tau$; $\theta_2 = 135°$.

$$\frac{\sigma_1}{\sigma_2} = -1.$$

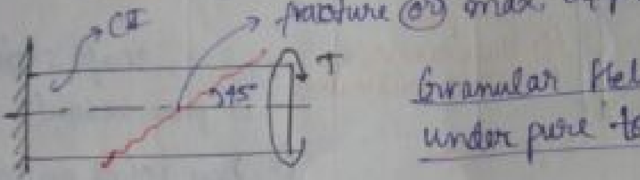
$\sigma_n^* = 0$; in-plane $\tau_{max} = \pm\left(\frac{\sigma_1 - \sigma_2}{2}\right) = \pm\tau = \tau_s^*$.

*



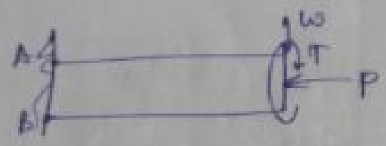
Smooth transverse fracture of M.S. shaft under pure torsion.

*



Granular Helicoidal fracture of CI shaft under pure torsion.

13-



$\frac{P}{A} = -\sigma.$ $\qquad \frac{M}{Z} - \sigma = \sigma. \leftarrow$ At pt. A

$\frac{M}{Z} = 2\sigma.$

[$\frac{M}{Z} = \pm\sigma$ है इस question में σ कि total normal stress है।]

max. Comp. $= -\frac{M}{Z} - \sigma$

$= -2\sigma - \sigma$

$= -3\sigma$

$(\sigma_b)_{max} = \pm\sigma = \sigma_x.$

$\sigma_a = -\sigma = \sigma_x.$

$(\tau_t)_{max} = \sqrt{3}\,\sigma = \tau_{xy}.$

$\sigma_x = \sigma_a \pm \sigma_b = (-\sigma) - (\sigma) = -2\sigma.$

$\sigma_y = 0.$

$\tau_{xy} = \sqrt{3}\,\sigma = (\tau_t)_{max}.$

$$\sigma_{1,2} = \frac{1}{2}\left[(-2\sigma+0) \pm \sqrt{(-2\sigma-0)^2 + 4(-\sqrt{3}\sigma)^2}\right]$$

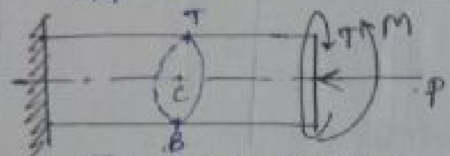
$$= \frac{1}{2}\left[-2\sigma \pm 4\sigma\right]$$

$$= \sigma, -3\sigma.$$

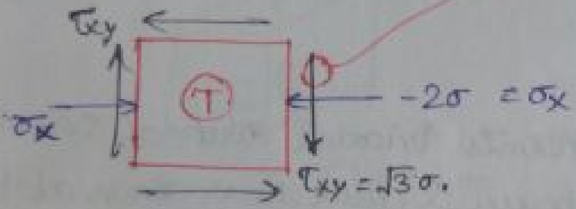
$$\boxed{\begin{aligned}\sigma_1 &= -3\sigma = (\sigma_c)_{max}\\ \sigma_2 &= \sigma = (\sigma_t)_{max}\end{aligned}}$$

OR

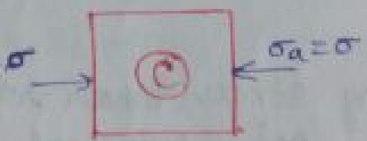
$\sigma_{Top} = -\sigma - \sigma = -2\sigma.$



$\sigma_{Bottom} = -\sigma + \sigma = \text{zero}.$

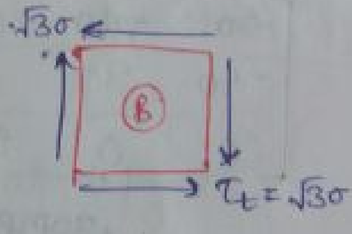


* If this dir^n is taken opp. then the value of $\sigma_{1,2}$ will not change bcoz we have (square) in the formula but it will change the location.



$(\sigma_{1,2})_{Top} = \sigma, -3\sigma.$

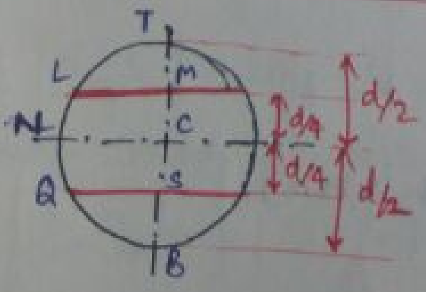
$(\sigma_{1,2})_c = -\sigma, 0.$

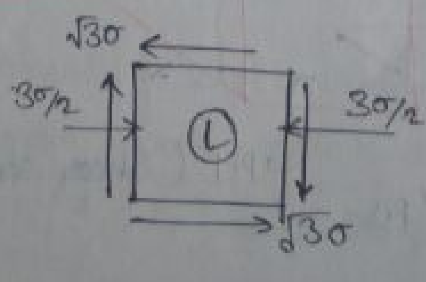


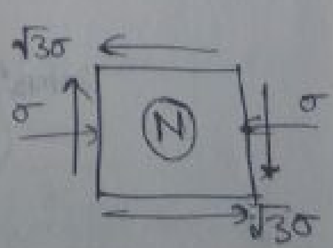
As critical point is at top surface.
Hence max. compressive normal stress $= 3\sigma$.

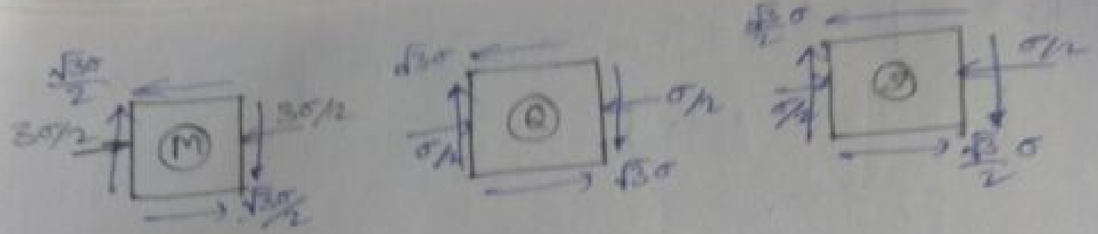
$(\sigma_1)_B = \sqrt{3}\sigma, (\sigma_2)_B = -\sqrt{3}\sigma.$

Question का POST MORTEM-









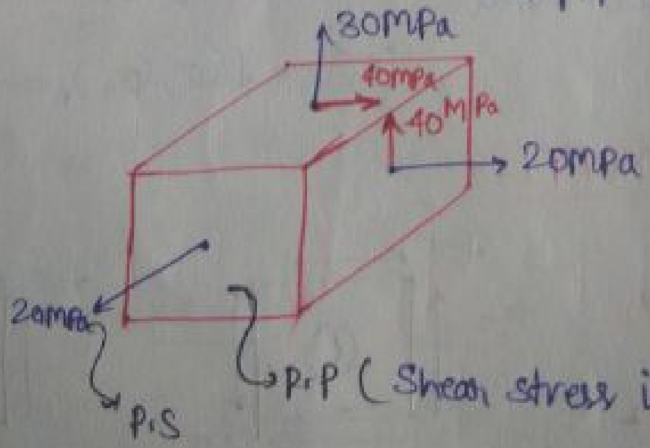
Points on the x-y/c (x-faces)	σ_a	σ_b	$\sigma_r = \sigma_a + \sigma_b = \sigma_x$	$\tau_r = \tau_t = \tau_{xy}$ ($\tau_d = 0$)
T				

[GATE]

Q- Following stress tensor represents triaxial state of stress at a point. Det. principal stresses & max shear stress at the point.

$$[\sigma]_{3D} = \begin{bmatrix} 20 & -40 & 0 \\ -40 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix} \text{MPa.}$$

→ P.P



$$\sigma_{1,2} = \frac{1}{2}\left[(20+30) \pm \sqrt{(20-30)^2 + 4(-40)^2}\right]$$

$$\cancel{= \frac{1}{2}[\,50} = 65.31,\ -15.3\,\text{MPa}.$$

$\cancel{\sigma_1 = 65.3\,\text{MPa}.}$
$\cancel{\sigma_2 = -15.3\,\text{MPa}.}$
$\cancel{\sigma_3 = 20\,\text{MPa}}$

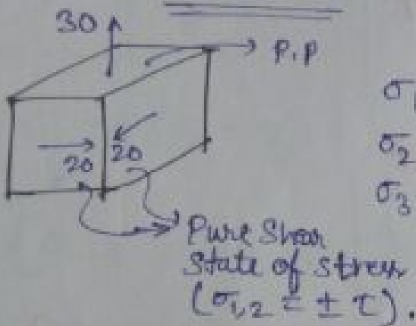
$$\boxed{\begin{aligned}\sigma_1 &= 65.31\ \text{MPa}\\ \sigma_2 &= 20\ \text{MPa}\\ \sigma_3 &= -15.3\ \text{MPa}\end{aligned}}$$

$$\text{Abs. } \tau_{max} = \text{larger of}\left[\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_2-\sigma_3}{2}\right|, \left|\frac{\sigma_1-\sigma_3}{2}\right|\right].$$

$$= \frac{\sigma_1-\sigma_3}{2}$$

$$= 40.31\,\text{MPa}.$$

*



$\sigma_1 = 30$ MPa
$\sigma_2 = 20$ MPa
$\sigma_3 = -20$ MPa.

⇒ Principal strains, In-plane max. shear strain, Absolute max. Shear strain -

⇒ I/P Data → State of strain at a point.

$$[\varepsilon]_{2D} = \begin{bmatrix}\varepsilon_x & \gamma_{xy}/2\\ \gamma_{xy}/2 & \varepsilon_y\end{bmatrix}$$

$\sigma_x \to \varepsilon_x$
$\sigma_y \to \varepsilon_y$ &
$\tau_{xy} \to \frac{\gamma_{xy}}{2}$

Let ε_n & γ_s are the normal & shear strains on an O.P passing through a point under bi-axial state of strain.

$$\boxed{[\varepsilon_n]_\theta = \frac{1}{2}[\varepsilon_x+\varepsilon_y] + \frac{1}{2}[\varepsilon_x-\varepsilon_y]\cos 2\theta + \left(\frac{\gamma_{xy}}{2}\right)\sin 2\theta} \quad \text{(I)}$$

$$\left(\frac{\gamma_s}{2}\right)_\theta = -\frac{1}{2}[\varepsilon_x - \varepsilon_y]\sin 2\theta + \left(\frac{\gamma_{xy}}{2}\right)\cos 2\theta$$

$$\boxed{(\gamma_s)_\theta = -[\varepsilon_x - \varepsilon_y]\sin 2\theta + (\gamma_{xy})\cos 2\theta} \quad \text{— (II)}$$

$$\boxed{(\varepsilon_n)_\theta + (\varepsilon_n)_{90+\theta} = \varepsilon_x + \varepsilon_y} \quad \text{— (III)}$$

$$\boxed{(\gamma_s)_\theta + (\gamma_s)_{90+\theta} = \text{zero.}} \quad \text{— (IV)}$$

P.P. Locations:-

$(\gamma_s)_\theta = 0$ (or) $\frac{d}{d\theta}(\varepsilon_n)_\theta = 0.$

$$\boxed{\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}} \quad \text{— (V)}$$

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}} \quad \text{— (VI)}$$

IInd method for $\varepsilon_{1,2}$:-

$$\boxed{\varepsilon_{1,2} = \frac{1}{2}\left[(\varepsilon_x + \varepsilon_y) \pm \sqrt{(\varepsilon_x - \varepsilon_y) + 4\left(\frac{\gamma_{xy}}{2}\right)^2}\right]}$$

$$\boxed{\varepsilon_1 + \varepsilon_2 = \varepsilon_x + \varepsilon_y} \qquad \boxed{\varepsilon_1 - \varepsilon_2 = \sqrt{(\varepsilon_x - \varepsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2}}$$

= Dia. of Mohr's Circle for strain

IInd Method for Abs. γ_{max}:-

$$\text{Abs. } \frac{\gamma_{max}}{2} = \text{larger of}\left[\left|\frac{\varepsilon_1 - \varepsilon_2}{2}\right|, \left|\frac{\varepsilon_2 - \varepsilon_3}{2}\right|, \left|\frac{\varepsilon_3 - \varepsilon_1}{2}\right|\right],$$

$$\text{Abs. } \gamma_{max} = \text{Larger of}\left[|\varepsilon_1 - \varepsilon_2|, |\varepsilon_2 - \varepsilon_3|, |\varepsilon_3 - \varepsilon_1|\right].$$

In-plane γ_{max} = Dia of Mohr's circle of strain.

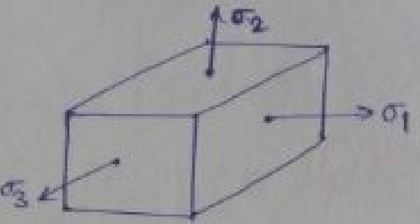
⇒ For 2D,

Abs $\gamma_{max} = |\varepsilon_1| \Rightarrow \varepsilon_{1,2}$ are <u>like</u> in nature.

$= |\varepsilon_1 - \varepsilon_2| \Rightarrow$ —"— <u>unlike</u> —"—

⇒ <u>Relationship b/w Principal stresses & principal strains</u> -

<u>Case I</u> - Expression for principal strains ($\varepsilon_{1,2,3}$) in terms of principal stresses ($\sigma_{1,2,3}$) -



$$\left.\begin{aligned} \varepsilon_1 &= \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2+\sigma_3)\right]. \\ \varepsilon_2 &= \frac{1}{E}\left[\sigma_2 - \mu(\sigma_1+\sigma_3)\right]. \\ \varepsilon_3 &= \frac{1}{E}\left[\sigma_3 - \mu(\sigma_1+\sigma_2)\right]. \end{aligned}\right\} \text{(I)}$$

$$\boxed{\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \left(\frac{1-2\mu}{E}\right)\left[\sigma_1+\sigma_2+\sigma_3\right].} \longleftarrow \text{(II)}$$

For bi-axial state of stress ($\sigma_3 = 0$) -

$$\left.\begin{aligned} \varepsilon_1 &= \frac{1}{E}\left[\sigma_1 - \mu\sigma_2\right] \quad \text{— (a)} \\ \varepsilon_2 &= \frac{1}{E}\left[\sigma_2 - \mu\sigma_1\right] \quad \text{— (b)} \\ \cancel{\varepsilon_v = \varepsilon_1 +}\ \varepsilon_3 &= \cancel{\left(\frac{1-2\mu}{E}\right)} \frac{-\mu}{E}\left[\sigma_1+\sigma_2\right] \quad \text{— (c)} \end{aligned}\right\} \text{(III)}$$

Above set of eq^{ns} are used to det. principal strains at a point when state of stress or principal stresses at that point are known.

Case II – Exp^{ns} for $\sigma_{1,2}$ in terms of $\varepsilon_{1,2}$.

from (a), $\sigma_1 = E\varepsilon_1 + \mu\sigma_2$ — (1)

" (b), $\sigma_2 = E\varepsilon_2 + \mu\sigma_1$ — (2)

by sub. eq^n (2) in eq^n (1),

$$\sigma_1 = E\varepsilon_1 + \mu[E\varepsilon_2 + \mu\sigma_1]$$

$$\boxed{\sigma_1 = \left(\frac{E}{1-\mu^2}\right)[\varepsilon_1 + \mu\varepsilon_2]} \quad — (3)$$

Similarly by sub eq^n (1) in eq^n (2),

$$\boxed{\sigma_2 = \left(\frac{E}{1-\mu^2}\right)[\varepsilon_2 + \mu\varepsilon_1]} \quad — (4)$$

Eq^{n}s (3) & (4) are used to det. $\sigma_{1,2}$ when state of strain or $\varepsilon_{1,2}$ at that point are known.

Q- If $\sigma_x = 200\,MPa$, $\sigma_y = 100\,MPa$, $\tau_{xy} = 50\,MPa$. Assume: $E = 200\,GPa$, $\mu = 0.3$.

Det: (a) $\varepsilon_{1,2}$
(b) Abs. τ_{max}
(c) Abs. γ_{max}.

(i) $\sigma_{1,2} = \frac{1}{2}\left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$

= ____ , ____

(ii) $\varepsilon_1 = \frac{1}{E}[\sigma_1 - \mu\sigma_2]$.

$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \mu\sigma_1]$.

(iii). Abs. $\tau_{max} = \left|\frac{\sigma_1}{2}\right|$ or $\left|\frac{\sigma_1 - \sigma_2}{2}\right|$.

(iv) Abs. $\gamma_{max} = |\varepsilon_1|$ or $|\varepsilon_1 - \varepsilon_2|$.

Q- If $\varepsilon_x = 500 \times 10^{-6}$ or 500μ
$\varepsilon_y = 200 \times 10^{-6}$
$\frac{\gamma_{xy}}{2} = 150 \times 10^{-6}$.
$E = 200$ GPa
$\mu = 0.3$

Det. (a) $\sigma_{1,2} = ?$
(b) Abs. $\tau_{max} = ?$

$$\varepsilon_{1,2} = \frac{1}{2}\left[(\varepsilon_x + \varepsilon_y) \pm \sqrt{(\varepsilon_x - \varepsilon_y) + 4\left(\frac{\gamma_{xy}}{2}\right)^2}\right]$$

$$\sigma_1 = \left(\frac{E}{1-\mu^2}\right)(\varepsilon_1 + \mu\varepsilon_2)$$

$$\sigma_2 = \left(\frac{E}{1-\mu^2}\right)(\varepsilon_2 + \mu\varepsilon_1)$$

Strain Rosettes-

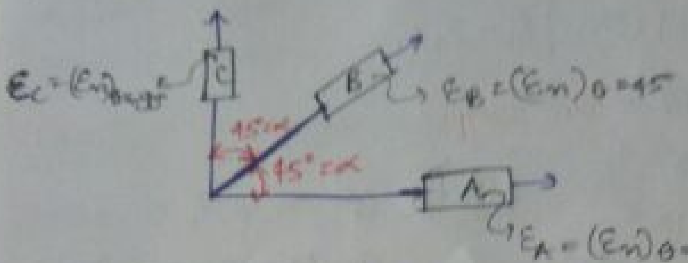
Strain Rosettes is ~~defin~~ defined as an arrangement of 3 strain gauges in 3 arbitrary dirns. These strain gauges are used to measure normal strain in the dirn.

Based on the arrangement of strain gauges, strain rosettes are classified into -

1- Rectangular Strain Rosettes ($\alpha = 45°; \theta = 0°, 45°, 90°$)
2- δ strain Rosette ($\alpha = 60°; \theta = 0°, 60°$ & $120°$).
3- Star Strain Rosette ($\alpha = 120°; \theta = 0°, 120°$ & $240°$).

where α = Angle b/w strain gauges.
θ = Inclination of the O.P from reference plane.

Exp^{NS} for ε_x, ε_y, γ_{xy} in terms of rectangular strain rosette -



$$(\varepsilon_n)_\theta = \frac{1}{2}[\varepsilon_x + \varepsilon_y] + \frac{1}{2}[\varepsilon_x - \varepsilon_y]\cos 2\theta + \left(\frac{\gamma_{xy}}{2}\right)\sin 2\theta \quad \text{— ①}$$

$$(\varepsilon_n)_{\theta=0°} = \frac{1}{2}[\varepsilon_x + \varepsilon_y] + \frac{1}{2}[\varepsilon_x - \varepsilon_y](1) + 0 = \varepsilon_A$$

$$\boxed{\varepsilon_x = \varepsilon_A}$$

$$(\varepsilon_n)_{\theta=90} = \frac{1}{2}[\varepsilon_x + \varepsilon_y] + \frac{1}{2}[\varepsilon_x - \varepsilon_y]\cos(180) + 0 = \varepsilon_C$$

$$\boxed{\varepsilon_y = \varepsilon_C}$$

$$(\varepsilon_n)_{\theta=45} = \frac{1}{2}[\varepsilon_x + \varepsilon_y] + 0 + \frac{\gamma_{xy}}{2} = \varepsilon_B$$

$$\gamma_{xy} = 2\varepsilon_B - \varepsilon_x - \varepsilon_y$$

$$\boxed{\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C}$$

$$[\varepsilon]_{2D} = \begin{bmatrix} \varepsilon_A & \frac{2\varepsilon_B - \varepsilon_A - \varepsilon_C}{2} \\ \frac{2\varepsilon_B - \varepsilon_A - \varepsilon_C}{2} & \varepsilon_C \end{bmatrix}$$

- Det. (a) $\varepsilon_{1,2}$; (b) $\sigma_{1,2}$, If rect. strain gauge readings as follows -

$(\varepsilon)_{\theta=0°} = 1000 \times 10^{-6} = \varepsilon_x$ $\qquad$ $E = 200\ GPa$

$(\varepsilon)_{\theta=45°} = 300 \times 10^{-6}$ $\qquad$ $\mu = 0.25$,

$(\varepsilon)_{\theta=90°} = 800 \times 10^{-6} = \varepsilon_y$