# Statistical Machine Learning

## Semester 2, 2017

### Workshop #5: Neural Networks

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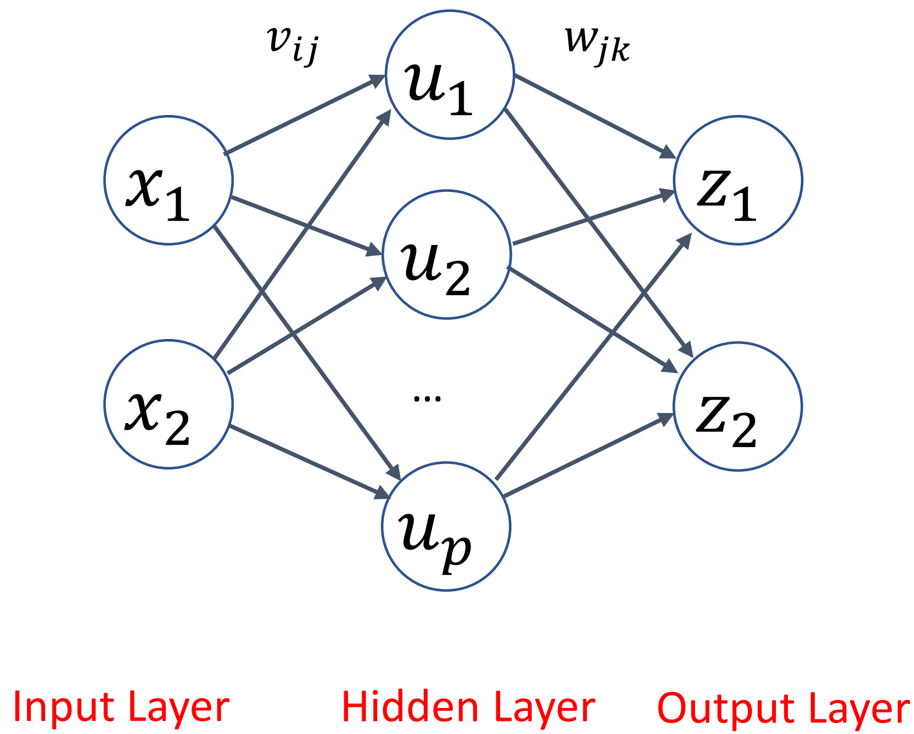
# Neural Network Architecture

Given n inputs and p hidden layers,

- How many weights are connected to each hidden neuron ? m+1
- How many weights should be trained for the whole hidden layer ? p*(m+1)

Given p hidden layers and k output neurons,

- How many weights are connected to each output neuron ? p+1
- How many weights should be trained for the whole output layer ? k*(p+1)

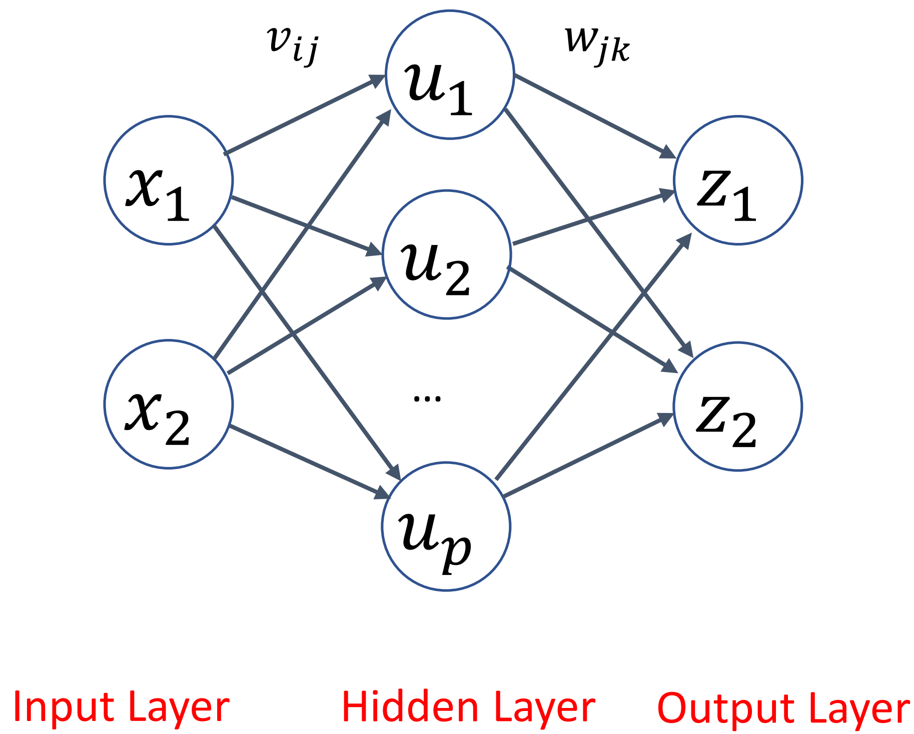


# Hidden Layer forward pass calculations:

$$r_j = v_{0j} + \sum_{i=1}^{2} x_i v_{ij} = \sum_{i=0}^{2} x_i v_{ij}$$

$$u_j = \mathrm{g}(r_j)$$

$$\mathrm{g}(r) = \tanh(r) = \frac{e^r - e^{-r}}{e^r + e^{-r}}$$

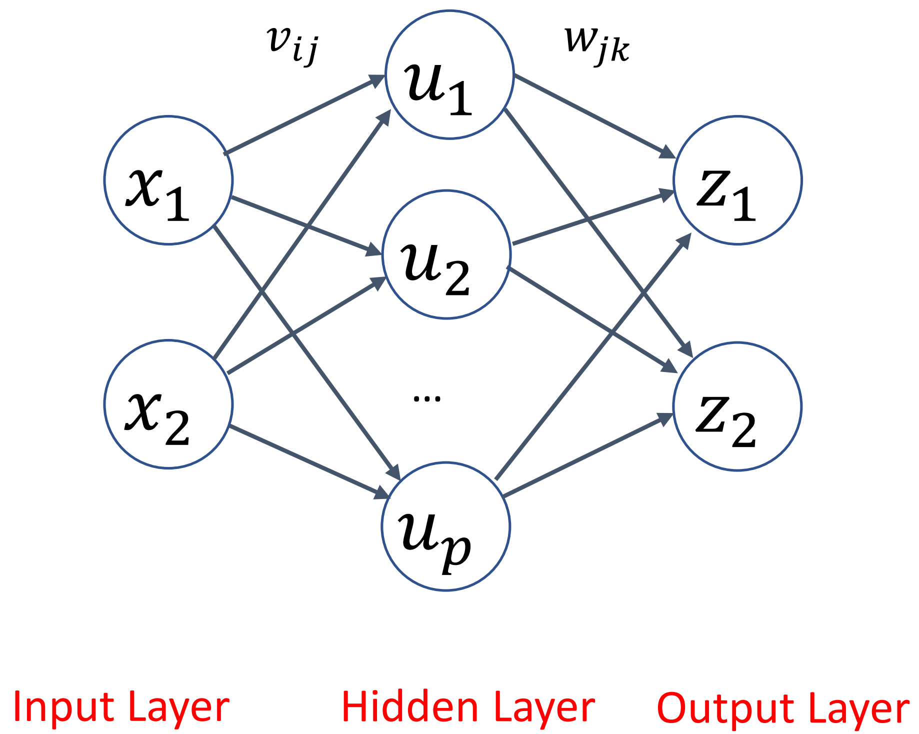


# Output Layer forward pass calculations:

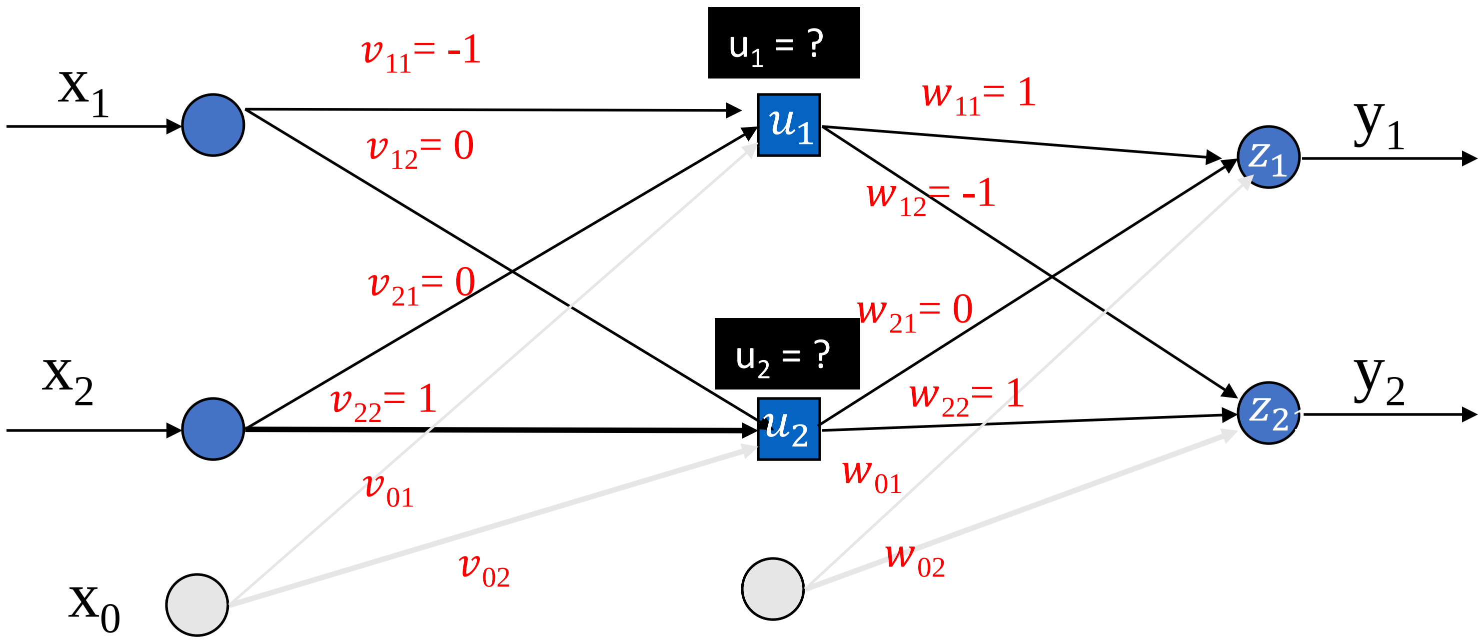
$$s_k = w_{0k} + \sum_{j=1}^{p} u_j w_{jk} = \sum_{j=0}^{p} u_j w_{jk}$$

$$z_k = f(s_k)$$

$$f(s) = \frac{1}{1+e^{-s}}$$



# An example: (Forward pass) – hidden calculations




Use "tanh" activation function (i.e. g(a) = tanh(a))

Have input [0 1] with target [1 0].

All biases set to 1

- $r_1$ = -1x0 + 0x1 +1 = 1 → $u_1$ = tanh($r_1$)= tanh(1) = **0.76**
- $r_2$ = 0x0 + 1x1 +1 = 2 → $u_2$ = tanh($r_2$)= tanh(2) = **0.97**

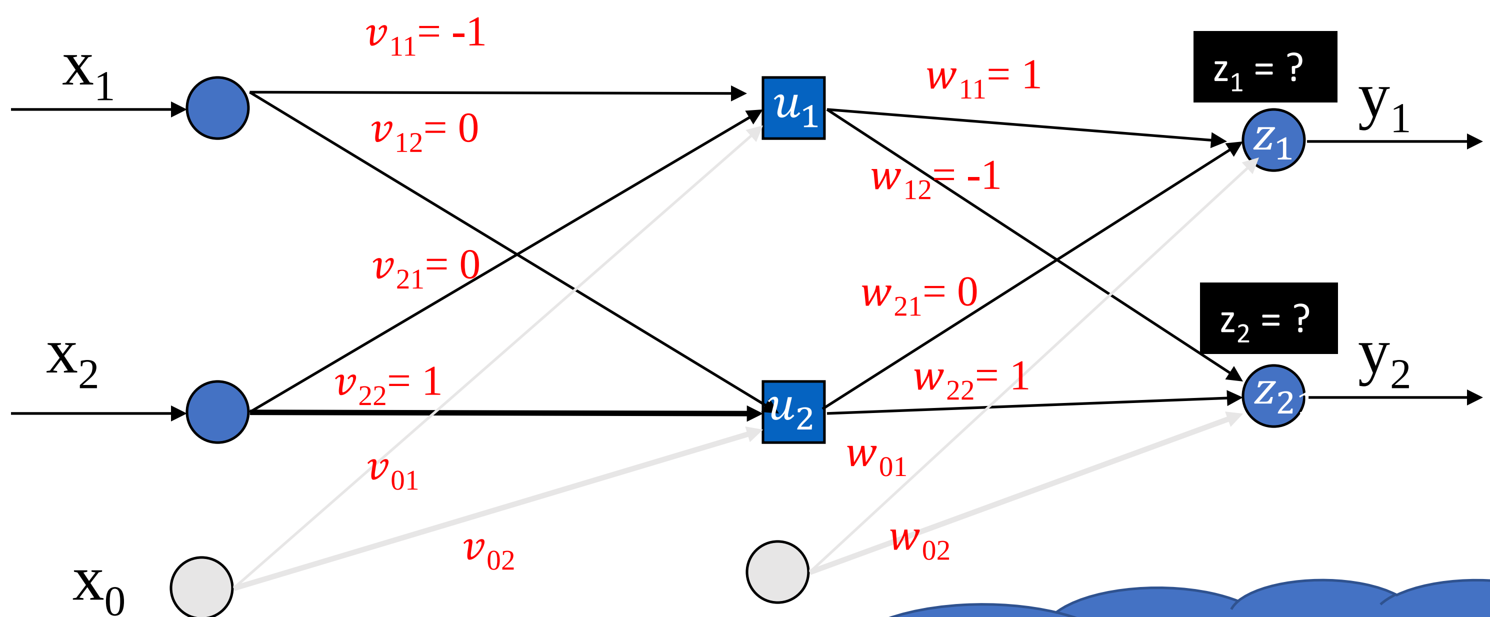
Weight Matrix V

[p x (m+1)]

| $v_{ij}$ | i = 0 | i = 1 | i =2 |
|---|---|---|---|
| j=1 | 1 | -1 | 0 |
| j=2 | 1 | 0 | 1 |

× Input vector x [m+1 x 1] [1 0 1]′ = Vector r [p x 1] [1 2]′

# An example: (Forward pass) – output calculations




Back to tutorial to fill in compute_forward(x,V,W) & ann_predict(X,V,W) functions

Use identity activation function (i.e. g(a) = a)

Have input [0 1] with target [1 0].

All biases set to 1

- $s_1$ = 1x0.67 + 0x0.97 +1 = 2.64 → $z_1 = s_1$ = **2.64**
- $s_2$ = -1x0.67 + 1x0.97 +1 = 1.3 → $z_2 = s_2$ = **1.3**

Weight Matrix W
[k x (p+1)]

| $w_{jk}$ | j = 0 | j = 1 | j =2 |
|---|---|---|---|
| k=1 | 1 | 1 | 0 |
| k=2 | 1 | -1 | 1 |

Input vector u
[p+1 x 1]
[1 0.67 0.97]′

Vector s
[k x 1]
[2.64 1.3]′

# Backpropagation update rule : (1)

- Discrepancy $l = 0.5 \cdot \sum_{k=1}^{c} (y_k - z_k)^2$
- Partial derivatives $\frac{\partial l}{\partial w_{jk}} = \boxed{\frac{\partial l}{\partial s_k}} \frac{\partial s_k}{\partial w_{jk}}$ and $\frac{\partial l}{\partial v_{ij}} = \boxed{\frac{\partial l}{\partial s_k}} \frac{\partial s_k}{\partial v_{ij}}$

let's call $\delta_k$

- $\delta_k = \frac{\partial l}{\partial s_k} = -(y_k - z_k) z_k (1 - z_k)$
- $\frac{\partial l}{\partial w_{jk}} = \delta_k u_j$
- $\frac{\partial l}{\partial v_{ij}} = g'(r_j) x_i \sum_{k=1}^{c} \delta_k w_{jk}$

Stochastic Gradient Descent update rule:

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \eta \nabla D\left(\boldsymbol{\theta}^{(t)}\right)$$

$$w_{jk}{}^{(t+1)} = w_{jk}{}^{(t)} - \eta \frac{\partial D}{\partial w_{jk}}$$

$$v_{ij}{}^{(t+1)} = v_{ij}{}^{(t)} - \eta \frac{\partial D}{\partial v_{ij}}$$

# Backpropagation update rule: (2)

- Discrepancy $l = -\sum_{k=1}^{c} y_k \log(z_k) - (1 - y_k)\log(1 - z_k)$
- Partial derivatives $\frac{\partial l}{\partial w_{jk}} = \boxed{\frac{\partial l}{\partial s_k}} \frac{\partial s_k}{\partial w_{jk}}$ and $\frac{\partial l}{\partial v_{ij}} = \boxed{\frac{\partial l}{\partial s_k}} \frac{\partial s_k}{\partial v_{ij}}$

let's call $\delta_k$

- $\delta_k = \frac{\partial l}{\partial s_k} = (z_k - y_k)$
- $\frac{\partial l}{\partial w_{jk}} = \delta_k u_j$
- $\frac{\partial l}{\partial v_{ij}} = g'(r_j) x_i \sum_{k=1}^{c} \delta_k w_{jk}$
- $= (1 - u_j^2) x_i \sum_{k=1}^{c} \delta_k w_{jk}$
- $= (1 - u_j^2) x_i \delta_1 w_{j1} + (1 - u_j^2) x_i \delta_1 w_{j1}$

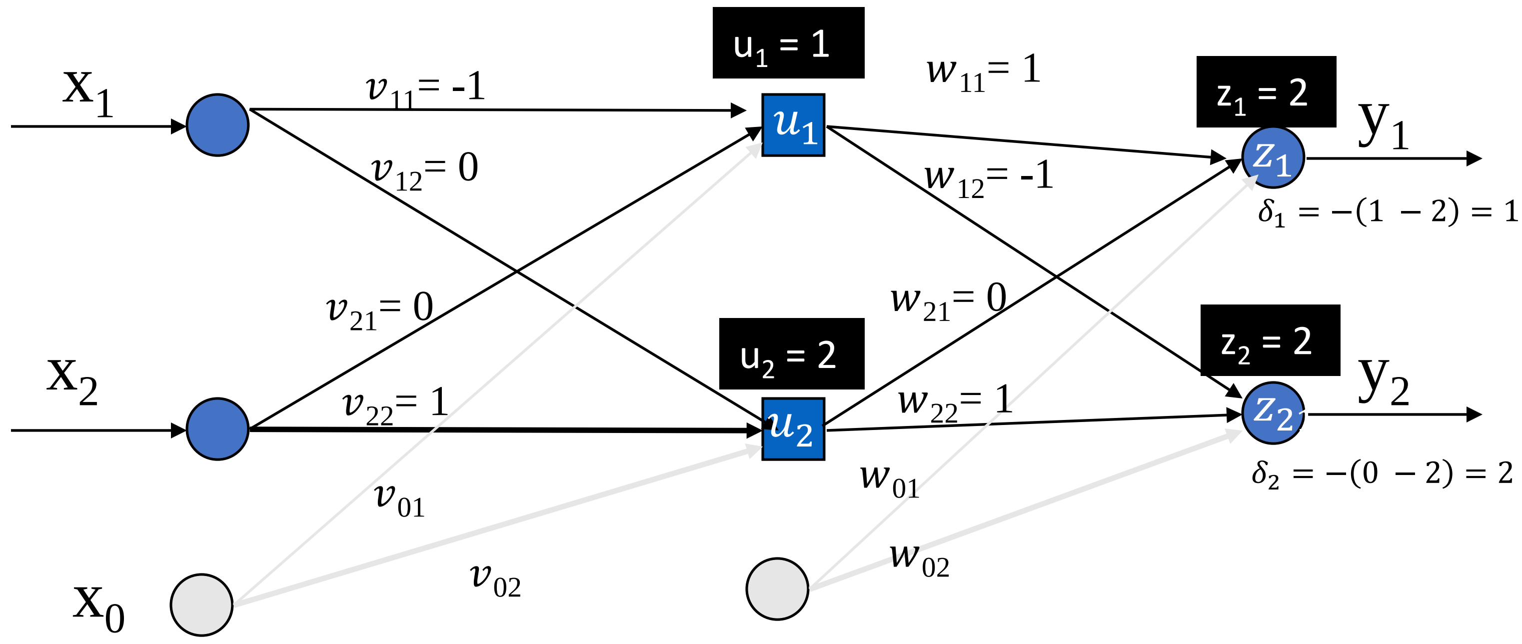
Stochastic Gradient Descent update rule:

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \eta \boldsymbol{\nabla} D\left(\boldsymbol{\theta}^{(t)}\right)$$

$$w_{jk}{}^{(t+1)} = w_{jk}{}^{(t)} - \eta \frac{\partial D}{\partial w_{jk}}$$

$$v_{ij}{}^{(t+1)} = v_{ij}{}^{(t)} - \eta \frac{\partial D}{\partial v_{ij}}$$

# An example: (Backward pass) – output layer




Have input [0 1] with target [1 0]. Learning rate η = 0.1

$$\delta_k = -(y_k - z_k) \; (\frac{\partial f_k}{\partial s_k} = 1 \,)$$

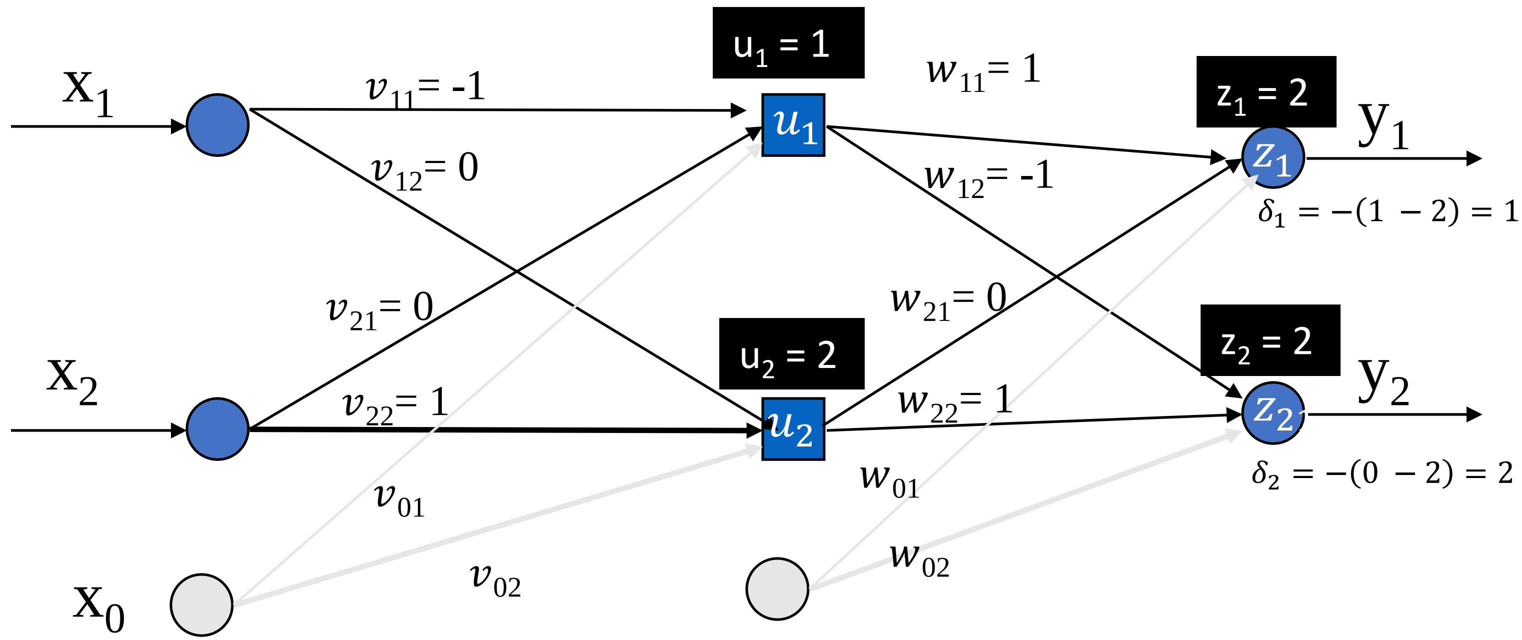
$$w_{jk}{}^{(t+1)} = w_{jk}{}^{(t)} - \eta \frac{\partial D}{\partial w_{jk}}$$

$$= w_{jk}{}^{(t)} - \eta \, \delta_k u_j$$

k=1, j =1 → $w_{11}$= 1 - 0.1 * 1 * 1 = 0.9
k=1, j =2 → $w_{21}$= 0 - 0.1 * 1 * 2 = -0.2
k=1, j =0 → $w_{01}$= 1 - 0.1 * 1 * 1 = 0.9

k=2, j =1 → $w_{12}$= -1 - 0.1 * 2 * 1 = -1.2
k=2, j =2 → $w_{22}$= 1 - 0.1 * 2 * 2 = 0.6
k=2, j =0 → $w_{02}$= 1 - 0.1 * 2 * 1 = 0.8

# An example: (Backward pass) – hidden layer




Have input [0 1] with target [1 0]. Learning rate η = 0.1

$$\delta_k = -(y_k - z_k) \ \left(\frac{\partial f_k}{\partial s_k} = 1\right)$$

$$v_{ij}{}^{(t+1)} = v_{ij}{}^{(t)} - \eta \frac{\partial D}{\partial v_{ij}}$$

$$= v_{ij}{}^{(t)} - \eta x_i \sum_{k=1}^{c} \delta_k w_{jk}$$

j=1, i =1 → $v_{11}$= -1 - 0.1 * -1 * 0 = -1
j=1, i =2 → $v_{21}$= 0 - 0.1 * -1 * 1 = 0.1
j=1, i =0 → $v_{01}$= 1 - 0.1 * -1 * 1 = 1

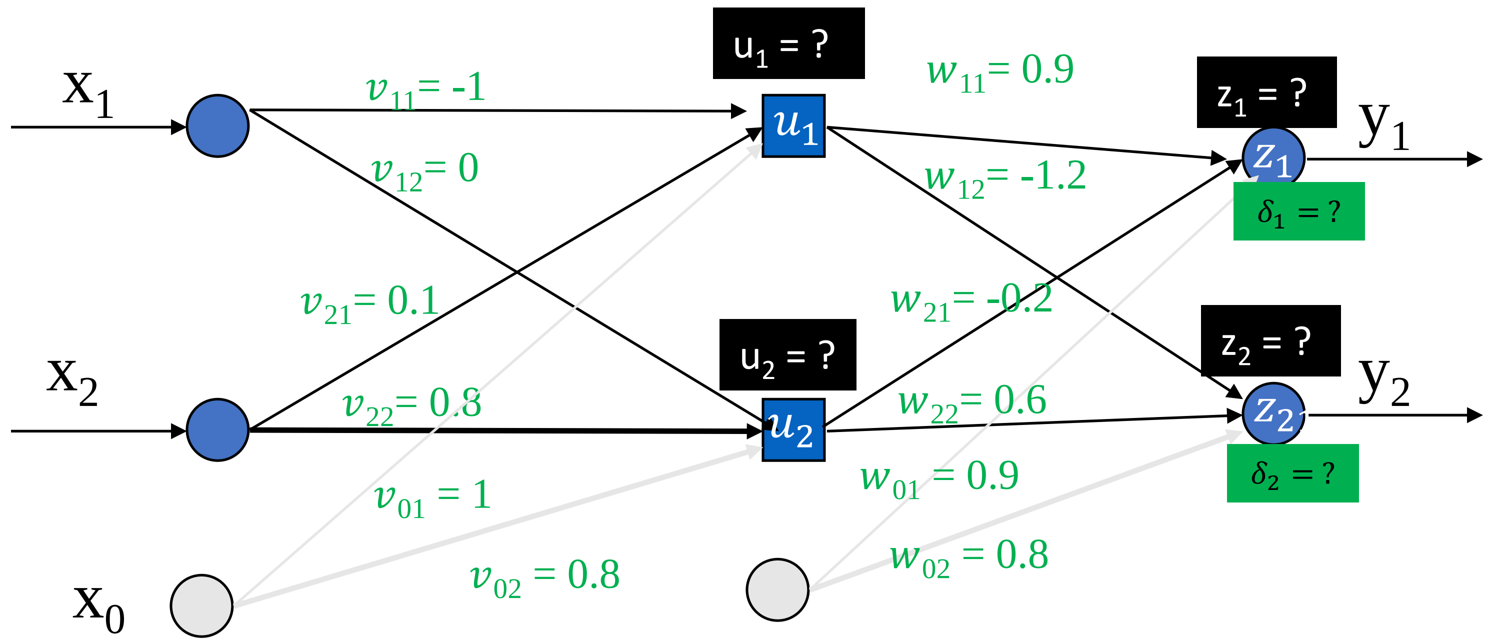
j=2, i =1 → $v_{12}$= 0 - 0.1 * 2 * 0 = 0
j=2, i =2 → $v_{22}$= 1 - 0.1 * 2 * 1 = 0.8
j=2, i =0 → $v_{02}$= 1 - 0.1 * 2 * 1 = 0.8

Note: use old weights $w_{jk}$

$\sum_{k=1}^{c} \delta_k w_{1k}$= 1 x 1 + -1 x 2 = -1

$\sum_{k=1}^{c} \delta_k w_{2k}$= 0 x 1 + 1 x 2 = 2

# An example: updated weights after ONE iteration




Back to tutorial to fill in
update_params(x,y,V,W,eta) &
ann_train(X,y,V0,W0)functions