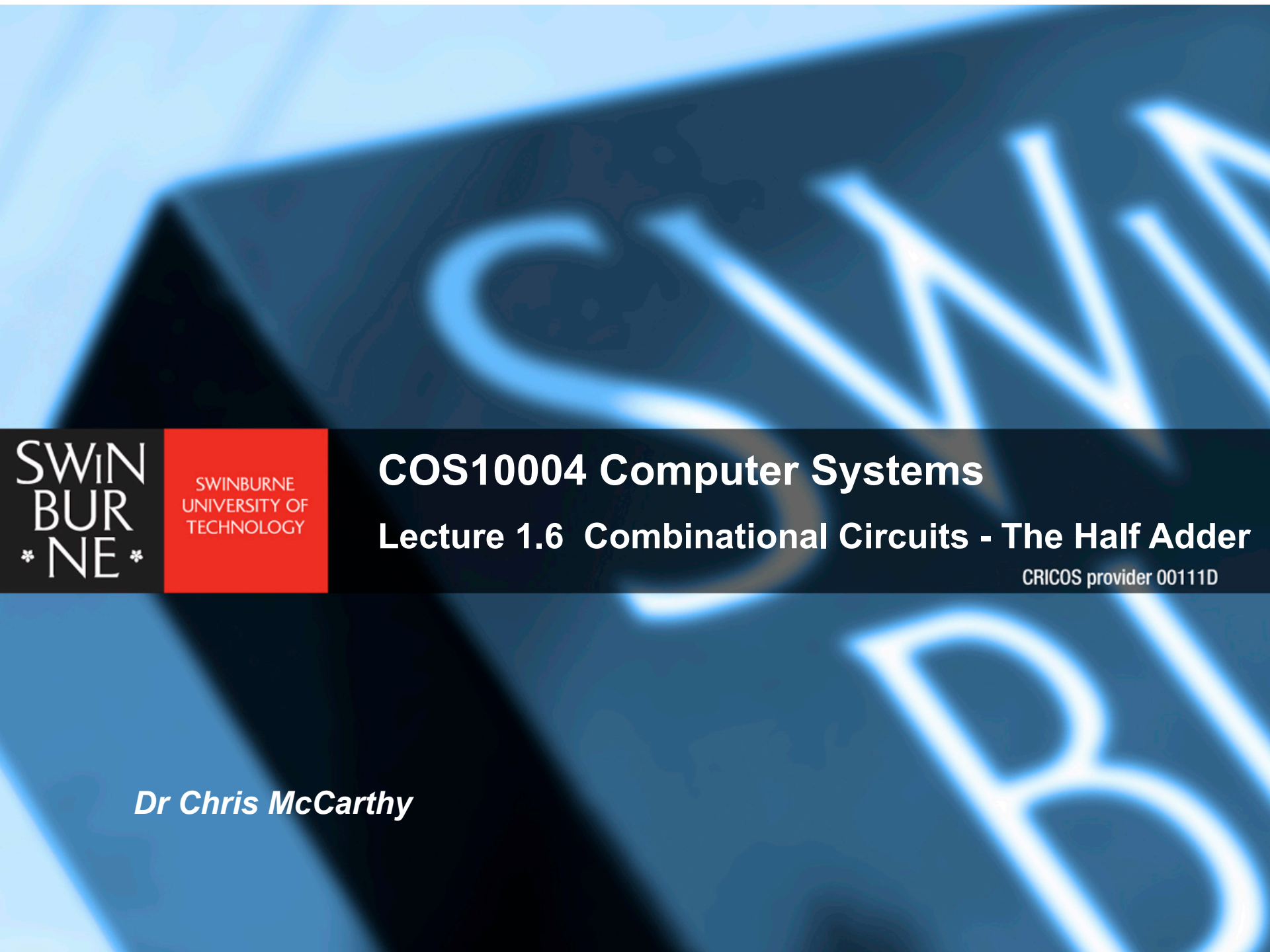


SWINBURNE UNIVERSITY OF TECHNOLOGY

COS10004 Computer Systems

Lecture 1.6 Combinational Circuits - The Half Adder

CRICOS provider 00111D

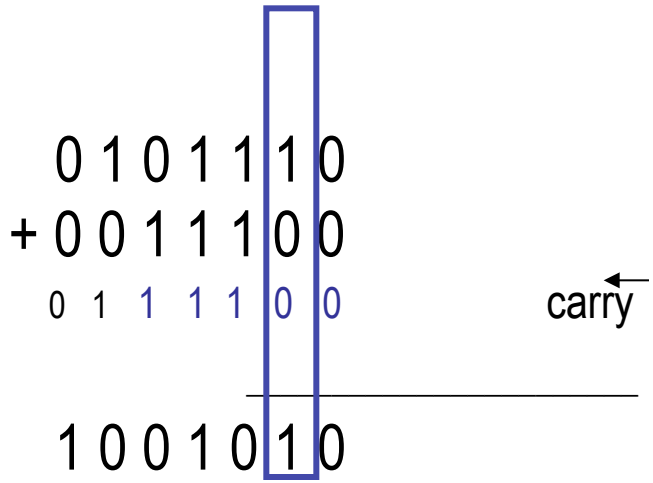
Dr Chris McCarthy

COMBINATIONAL CIRCUIT

- A combinational circuit is a connected arrangement of logic gates with the set of inputs and outputs.
- A combinational circuit can be described by a truth table showing the binary relationship between *n* input variables and *m* output variables.
- Two examples: half adder, full adder
- These circuits serve as basic building blocks for the construction of more complicated arithmetic circuits.

BINARY ADDITION

- Consider:

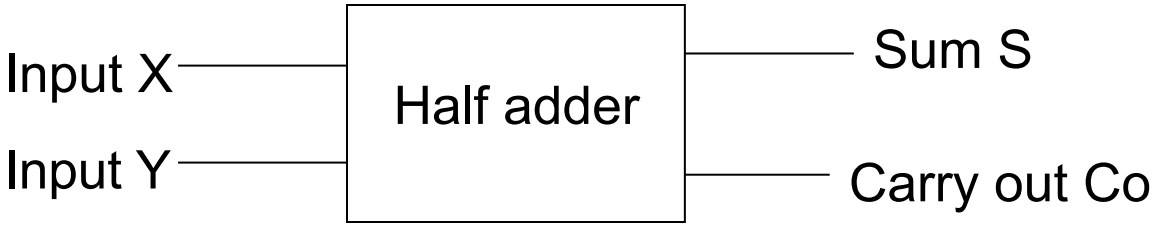


- We need 1-bit adders…

HALF-ADDER

- The most basic arithmetic circuit is the addition of two binary digits.
- A combinational circuit that performs the arithmetic addition of two bits is called a half-adder.
- It consists of two inputs and two outputs:
  - Two inputs get summed
  - Two outputs: sum and carry bits
- It is necessary to have two output bits because the sum 1 + 1 is binary 10, which has two digits.

A half-adder



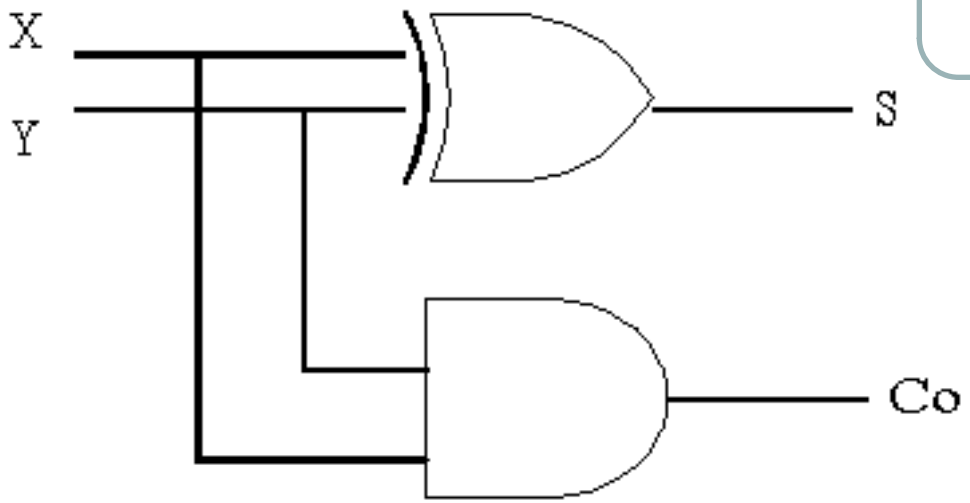
X	Y	S	Co
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$S = X \oplus Y$

$Co = X \bullet Y$

The plus-circle (modulo addition of 1-bit values) **means XOR, the dot means AND**

Half-adder



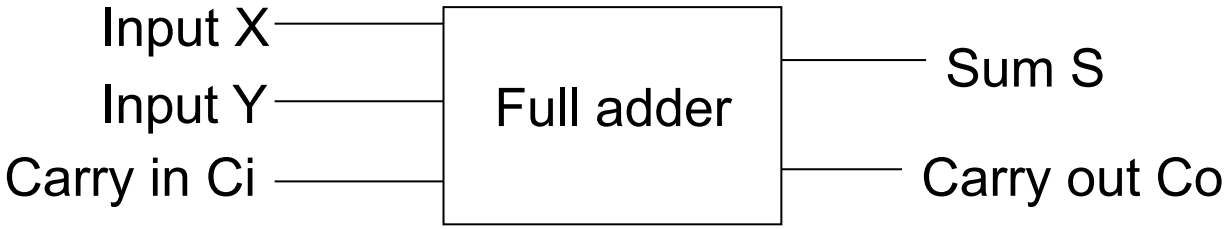
The XOR gate calculates the sum

The AND gate calculates the carry

FULL-ADDER

- A full-adder is a combination circuit that forms the arithmetic sum of three input bits.
- It consists of three inputs and two outputs.
- Two of the inputs represent the two bits to be added.
- The third input represents the carry from the previous lower significant position.
- One of the outputs will be the carry into the next higher significant position, i.e. into the next full adder.
  - Of course two output bits are necessary because the arithmetic sum of three binary digits ranges from 0 to 3, and 2 and 3 need two bits.

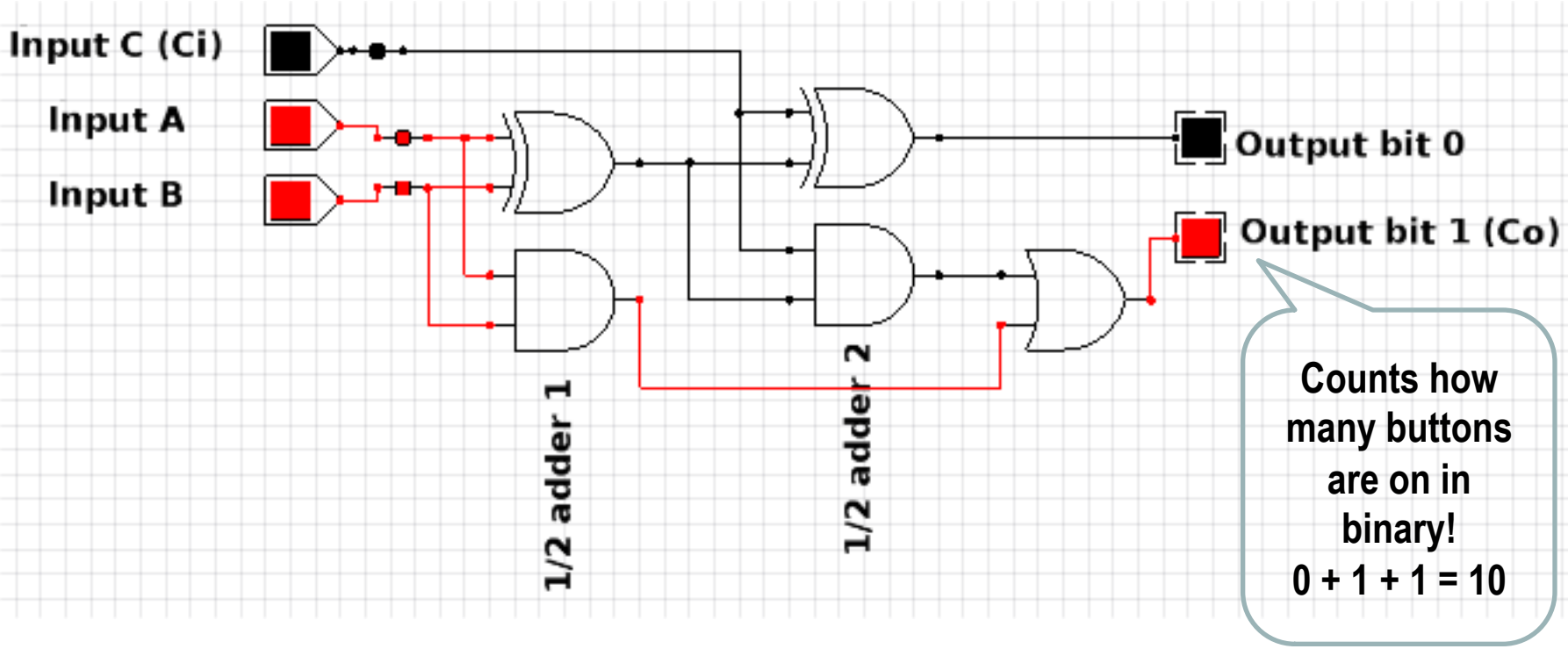
A full adder



	Ci	X	Y	S		Co	
a	0	0	0	0	$S = X \oplus Y$	0	$Co = X \bullet Y$
b	0	0	1	1		0	
c	0	1	0	1		0	
d	0	1	1	0		1	
e	1	0	0	1	$S = \overline{X \oplus Y}$	0	$Co = X + Y$
f	1	0	1	0		1	
g	1	1	0	0		1	
h	1	1	1	1		1	

A full adder

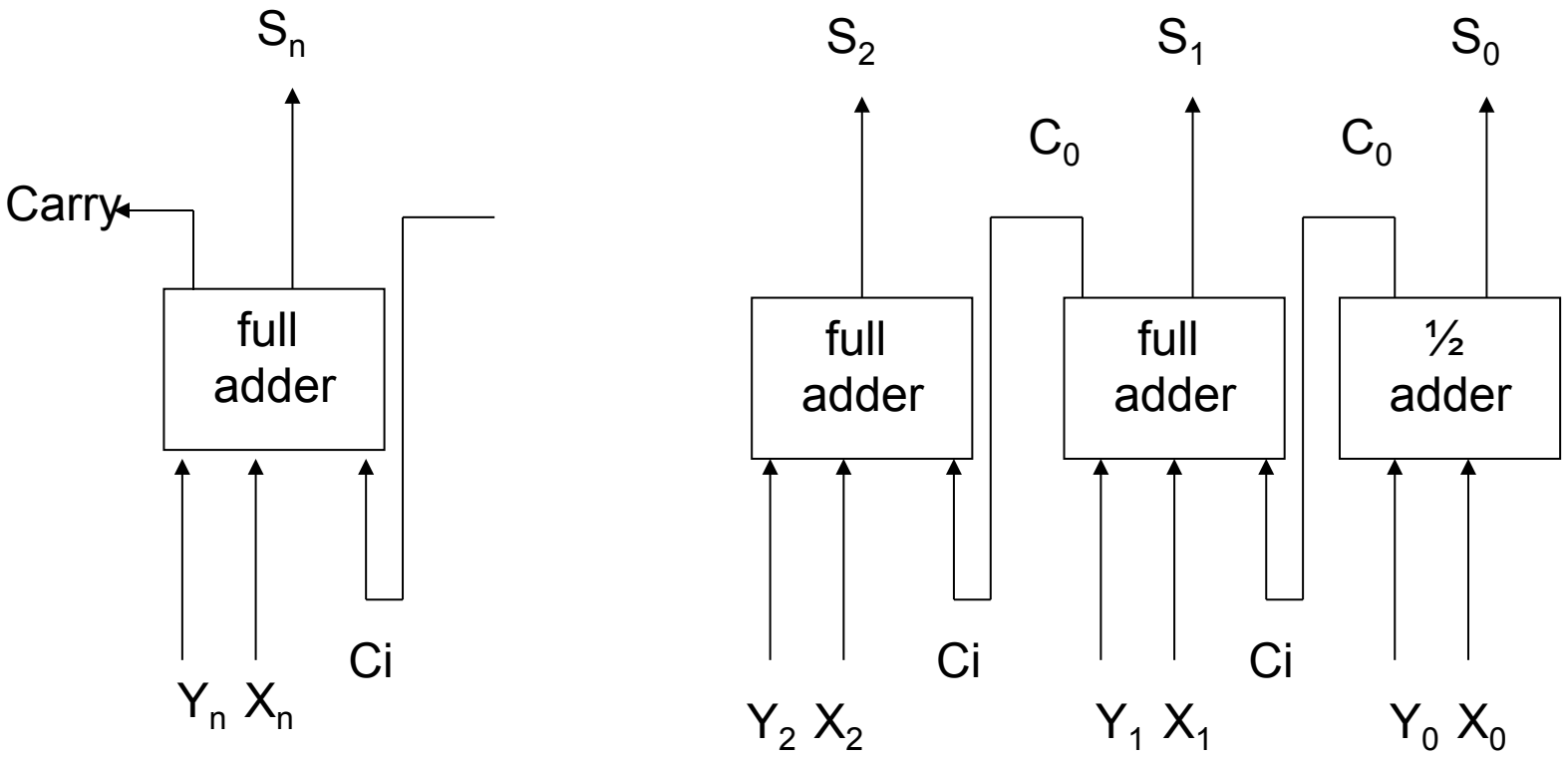
- We could make this from two half-adders:



Time to Imagine

- A full-adder is a combination circuit that forms the arithmetic sum of three input bits.
- Suppose we added a second full adder (A2) to the output of the first adder (A1), and connected the Ci pin of A2 to Co of A1?
- Wouldn't we be able to add two 2-bit numbers?
- A 3rd adder?
- A 4th....

Multi bit addition - combine 8 adders with carry to get 8-bit addition (of two 8-bit numbers).



In the Lab...

- Play with Gates (in a “sim”) and construct truth tables.
- Design a half adder
- I made some video tutorials that might also help:
  - Logisim introduction: https://youtu.be/fwPw28O5ORI
  - Half-adder tutorial: https://youtu.be/BpvlMgb40U4
  - Full-adder tutorial: https://youtu.be/1qLAk0AsH40

Summary of Week 1

- Computer Systems:
  - We're building a computer up conceptually!
  - Get your Pi and hardware
  - Check Canvas regularly
- Information and bits
- Digital Circuit Design:
  - Gates, Half-Adders and Full-Adders
- Next week: more complex circuits and Flip Flops (memory)!