Exercise Class Solutions 6

1 Systems of Linear Equations

1.2 Gaussian Elimination

1.2.16

Find the circle $x^2+y^2+ax+by+c=0$ passing through the following points.

b) $(1,1)$, $(5,-3)$, and $(-3,-3)$

Solution

Start by finding our system of equations by inserting the points into our equation.

$$\begin{aligned} 1^2+1^2+a+b+c&=0 \\ a+b+c&=-2 \end{aligned}$$

$$\begin{aligned} 5^2+(-3)^2+5a-3b+c&=0 \\ 5a-3b+c&=-34 \end{aligned}$$

$$\begin{aligned} (-3)^2+3^2-3a+3b+c&=0 \\ -3a-3b+c&=-18 \end{aligned}$$

We can now set up our augmented matrix and solve it with Gauss elimination and get

$$\begin{aligned} c&=-6 \\ b&=6 \\ a&=-2 \end{aligned}$$

2 Matrix Algebra

2.1 Matrix Addition, Scalar Multiplication, and Transposition

2.1.2

Compute the following:

b) $3\begin{bmatrix} 3 \\ -1 \end{bmatrix} - 5\begin{bmatrix} 6 \\ 2 \end{bmatrix} + 7\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Solution

$\begin{bmatrix} -14 \\ -20 \end{bmatrix}$

2.1.3

Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$. Compute (if possible)

f) $(A + C)^T$

Solution

$$(A + C) = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix}$$

$$(A + C)^T = \begin{bmatrix} 5 & 2 \\ 0 & -1 \end{bmatrix}$$

2.1.4

Find A if

b) $3A + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 5A - 2\begin{bmatrix} 3 \\ 0 \end{bmatrix}$

Solution

$$A = \begin{bmatrix} 4 \\ 1/2 \end{bmatrix}$$

2.2 Matrix Multiplication

2.2.7

Write each of the following systems of linear equations in matrix form.

b)

$$\begin{aligned} -x_1 + 2x_2 - x_3 + x_4 &= 6 \\ 2x_1 + x_2 - x_3 + 2x_4 &= 1 \\ 3x_1 - 2x_2 + x_4 &= 0 \end{aligned}$$

Solution

$$\begin{bmatrix} -1 & 2 & -1 & 1 \\ 2 & 1 & -1 & 2 \\ 3 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$$

2.3 Matrix Inverses

2.3.1

In each case, show that the matrices are inverses of each other.

a) $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 7 & 2 & -6 \\ -3 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$

Solution

a)

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

c)

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 7 & 2 & -6 \\ -3 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.3.2

Find the inverse of each of the following matrices.

b) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix}$

Solution

a) Set up a matrix with our matrix on the left and the identity on the right. $\left[\begin{array}{cc|cc} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$ Use Gauss to get the identity on the left side and what remains on the right side is our inverse. $\left[\begin{array}{cc|cc} 1 & 0 & \frac{2}{5} & \frac{-1}{5} \\ 0 & 1 & \frac{-3}{5} & \frac{4}{5} \end{array}\right]$

d) Set up a matrix with our matrix on the left and the identity on the right. $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ -5 & 7 & -11 & 0 & 1 & 0 \\ -2 & 3 & -5 & 0 & 0 & 1 \end{array}\right]$ Use Gauss to get the identity on the left side and what remains on the right side is our inverse. $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 3 \\ 0 & 1 & 0 & 3 & 1 & -1 \\ 0 & 0 & 1 & 1 & 1 & -2 \end{array}\right]$

2.3.3

In each case, solve the systems of equations by finding the inverse of the coefficient matrix.

b)

$$\begin{aligned} 2x - 3y &= 0 \\ x - 4y &= 1 \end{aligned}$$

Solution

Set up the matrix to find the inverse

$$\left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 1 & -4 & 0 & 1 \end{array}\right]$$

Solve it to get that the inverse is

$$\frac{1}{5}\begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix}$$

We can then get that

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{-3}{5} \\ \frac{-2}{5} \end{bmatrix}$$

2.3.4

Given $A^{-1} = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \\ -1 & 1 & 0 \end{bmatrix}$:

a) Solve the system of equations $AX = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$.

Solution

Multiply the inverse on both sides from the left to get

$$X = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+1+9 \\ 2+0+15 \\ -1-1+0 \end{bmatrix} = \begin{bmatrix} 11 \\ 17 \\ -2 \end{bmatrix}$$