Exercise Class Solutions 4

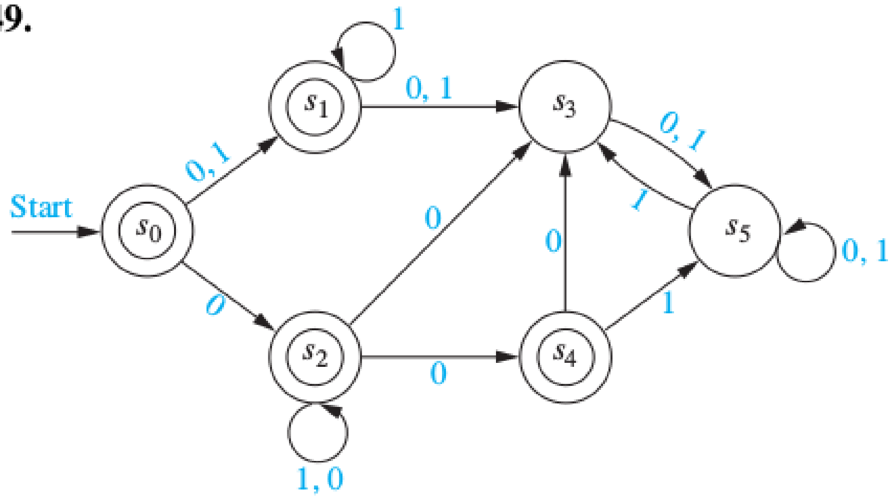
13 Modeling Computation

13.3 Finite-State Machines with no Output

13.3.49

Find the language recognized by the given nondeterministic finite-state automaton.

49.



Solution

All bit strings that start with 0 or contain only 1s. I.e. $\{0\}\{0,1\}^* \cup \{1\}^*$.

13.4 Language Recognition

13.4.1

b) $\mathbf{1^*00^*}$

d) $\mathbf{(1 \cup 00)^*}$

f) $\mathbf{(0 \cup 1)(0 \cup 1)^*00}$

Solution

b) All bit strings that start with some amount of 1s followed one or more 0s.

d) All bit strings where all runs of zeros are of even length.

f) All bit strings of length at least 3 that end with 00.

13.4.5

Express each of these using a regular expression.

a) the set consisting of the strings 0, 11, and 010

b) the set of strings of three 0s followed by two or more 0s

c) the set of strings of odd length

d) the set of strings that contain exactly one 1

e) the set of strings ending in 1 and not containing 000

Solution

a) $0 \cup 11 \cup 010$

b) 000000^*

c) $(0 \cup 1)((0 \cup 1)(0 \cup 1))^*$

d) 0^*10^*

e) $(1 \cup 01 \cup 001)^*(1 \cup 01 \cup 001)$

13.4.7

Express each of these using a regular expression.

a) the set of strings of one or more 0s followed by a 1

b) the set of strings of two or more symbols followed by three or more 0s

c) the set of strings with either no 1 preceding a 0 or no 0 preceding a 1

d) the set of strings containing a string of 1s such that the number of 1s equals 2 modulo 3, followed by an even number of 0s

Solution

a) 00^*1

b) $(0 \cup 1)(0 \cup 1)(0 \cup 1)^*0000^*$

c) $1^*0^* \cup 0^*1^*$

d) $(0 \cup 1)^*11(111)^*(00)^*(0 \cup 1)^*$

13.4.14

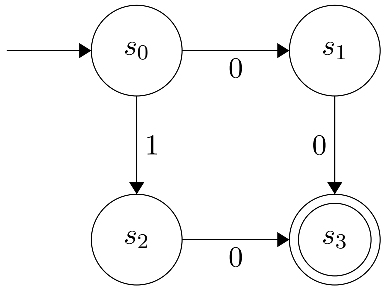
Construct a nondeterministic finite-state automaton that recognizes the language generated by the regular grammar $G = (V, T, S, P)$, where $V = \{0, 1, S, A, B\}$, $T = \{0, 1\}$, S is the start symbol, and the set of productions is

a) $S \to 0A$, $S \to 1B$, $A \to 0$, $B \to 0$.

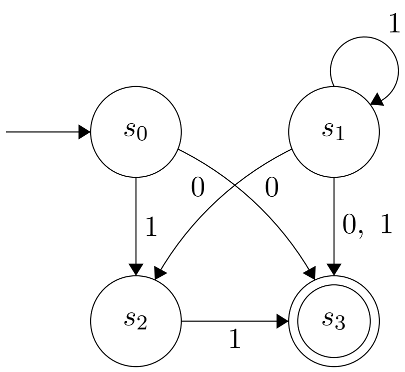
c) $S \to 1B$, $S \to 0$, $A \to 1A$, $A \to 0B$, $A \to 1$, $A \to 0$, $B \to 1$.

Solution

a)

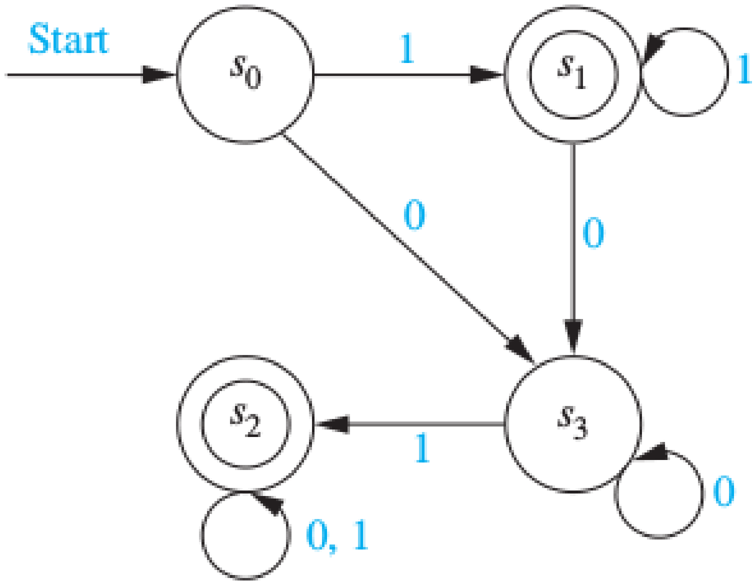


c)



13.4.17

Construct a regular grammar $G = (V, T, S, P)$ that generates the language recognized by the given finite-state machine.



Solution

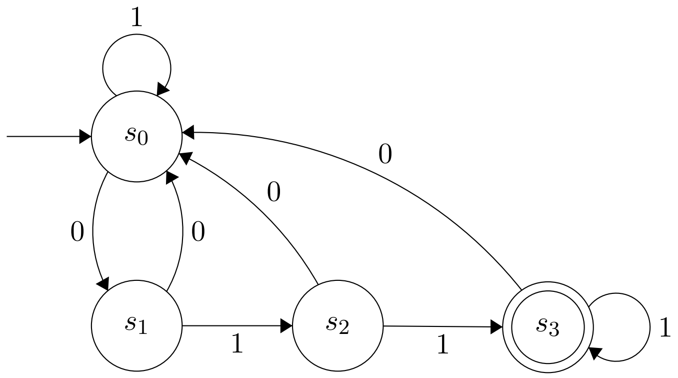
$S \to 1A$
$S \to 0B$
$S \to 1$
$A \to 1A$
$A \to 0B$
$A \to 1$
$B \to 0B$
$B \to 1C$
$B \to 1$
$C \to 0C$
$C \to 1C$
$C \to 0$
$C \to 1$

Additional Exercises

1

Construct a state diagram for a DFA ("deterministic finite-state automaton") that recognizes the set of all bit strings that contain an odd number of 0-bits and end with two 1-bits.

Solution



2

We have the following language over the alphabet $\Sigma = \{a, b\}$

$$A = \{w | w \text{ ends with } \mathbf{ba}\}$$

a) Draw a state diagram for a DFA which recognizes A.

b) Express A using a regular expression.

c) Find a regular grammar that generates A.

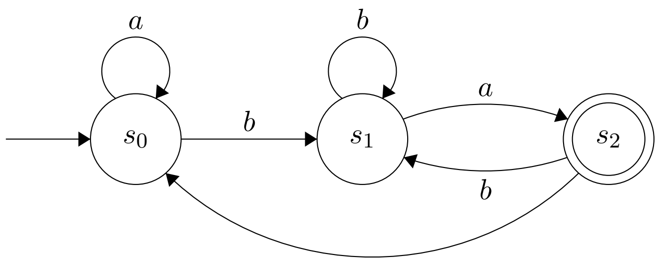
d) We have the following language over the alphabet $\Sigma = \{a, b\}$

$$B = \{w | w \text{ contains an odd number of } \mathbf{b} \text{ or more than one } \mathbf{a}\}$$

Draw a state diagram for a DFA which recognizes B.

Solution

a)



b) $(a \cup b)^* ba$

c) $S \to aS$
$S \to bS$
$S \to bA$
$A \to a$

d)