Exercise Class Solutions 2

13 Modeling Computation

13.1 Languages and Grammars

13.1.1

Use the set of productions to show that each of these sentences is a valid sentence.

sentence → **noun phrase transitive verb phrase noun phrase**
sentence → **noun phrase intransitive verb phrase noun phrase**
noun phrase → **article adjective noun**
noun phrase → **article noun**
transitive verb phrase → **transitive verb**
intransitive verb phrase → **intransitive verb adverb**
intransitive verb phrase → **intransitive verb**
article → *the*
adjective → *sleepy*
adjective → *happy*
noun → *tortoise*
noun → *hare*
transitive verb → *passes*
intransitive verb → *runs*
adverb → *quickly*
adverb → *slowly*

Solution

a)

sentence ⇒ **noun phrase intransitive verb phrase**
⇒ **article adjective noun intransitive verb phrase**
⇒ **article adjective noun intransitive verb adverb**
⇒ *the* **adjective noun intransitive verb adverb**
⇒ *the sleepy* **noun intransitive verb adverb**
⇒ *the sleepy hare* **intransitive verb adverb**
⇒ *the sleepy hare runs* **adverb**
⇒ *the sleepy hare runs quickly*

13.1.7

Construct a derivation of $0^3 1^3$ using the grammar given in Example 5.

$S \to 0S1$
$S \to \lambda$

Solution

$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000S111 \Rightarrow 000111$

13.1.9

a) Construct a derivation of $0^2 1^4$ using the grammar G_1 in Example 6.
$G_1 = \{V_1, T_1, S_1, P_1\}$
$V_1 = \{S, 0, 1\}$
$T_1 = \{0, 1\}$
$S_1 = S$
$P_1 = \{S \to 0S, S \to S1, S \to \lambda\}$

b) Construct a derivation of $0^2 1^4$ using the grammar G_2 in Example 6.
$G_2 = \{V_2, T_2, S_2, P_2\}$
$V_2 = \{S, A, 0, 1\}$
$T_2 = \{0, 1\}$
$S_2 = S$
$P_2 = \{S \to 0S, S \to 1A, S \to 1, A \to 1A, A \to 1, S \to \lambda\}$

Solution

a) $S \Rightarrow 0S \Rightarrow 00S \Rightarrow 00S1 \Rightarrow 00S11 \Rightarrow 00S111 \Rightarrow 00S1111 \Rightarrow 001111$

b) $S \Rightarrow 0S \Rightarrow 00S \Rightarrow 001A \Rightarrow 0011A \Rightarrow 00111A \Rightarrow 001111A \Rightarrow 001111$

13.1.15

Find a phrase-structure grammar for each of these languages.

a) the set of all bit strings containing an even number of 0s and no 1s.

b) the set of all bit strings made up of a 1 followed by an odd number of 0s.

d) the set of all strings containing 10 or more 0s and no 1s.

f) the set of all strings containing an equal number of 0s and 1s.

Solution

a) $S \to 00S$, $S \to \lambda$

b) $S \to 10A$, $A \to 00A$, $A \to \lambda$

d) $S \to 0000000000A$, $A \to 0A$, $A \to \lambda$

f) $S \to ABS$, $S \to \lambda$, $AB \to BA$, $BA \to AB$, $A \to 0$, $B \to 1$

13.1.21

Let G_1 and G_2 be context-free grammars, generating the languages $L(G_1)$ and $L(G_2)$, respectively. Show thtat there is a context-free grammar generating each of these sets.

a) $L(G_1) \cup L(G_2)$

b) $L(G_1)L(G_2)$

c) $L(G_1)^*$

Solution

a) Let S_1 be the initial nonterminal in G_1 and S_2 be the initial nonterminal in G_2. We construct a new grammar G that has an initial nonterminal S and the product $S \to S_1 \mid S_2$. Then $L(G) = L(G_1) \cup L(G_2)$.

b) Let S_1 be the initial nonterminal in G_1 and S_2 be the initial nonterminal in G_2. We construct a new grammar G that has an initial nonterminal S and the product $S \to S_1 S_2$. Then $L(G) = L(G_1)L(G_2)$.

c) Let S_1 be the initial nonterminal in G_1. We construct a new grammar G that has an initial nonterminal S and the product $S \to S_1 S \mid \epsilon$. Then $L(G) = L(G_1)^*$.

13.1.27

Construct a derivation tree for -109 using the grammar given in Example 15.

$\langle \textit{signed integer} \rangle$ ::= $\langle \textit{sign} \rangle \langle \textit{integer} \rangle$

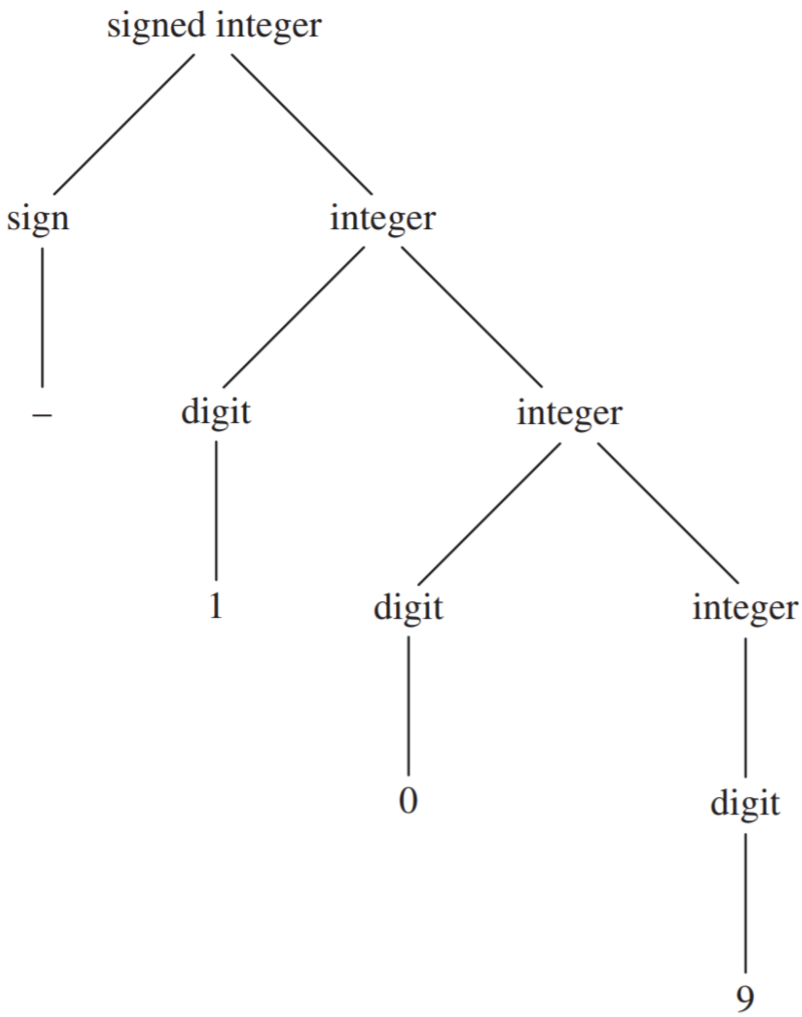
$\langle \textit{sign} \rangle$ ::= +|-

$\langle \textit{integer} \rangle$ ::= $\langle \textit{digit} \rangle | \langle \textit{digit} \rangle \langle \textit{integer} \rangle$

$\langle \textit{digit} \rangle$ ::= 0|1|2|3|4|5|6|7|8|9

Solution

27.



13.1.29

a) Construct a phrase-structure grammar that generates all signed decimal numbers, consisting of a sign, either $+$ or $-$; a nonnegative integer; and a decimal fraction that is either the empty string or a decimal point followed by a positive integer, where initial zeros in an integer are allowed.

b) Give the Backus-Naur form of this grammar.

c) Construct a derivation tree for -31.4 in this grammar.

Solution

a)
$S \rightarrow \langle sign\rangle\langle integer\rangle$
$S \rightarrow \langle sign\rangle\langle integer\rangle\ .\ \langle positive\ integer\rangle$
$\langle sign\rangle \rightarrow +$
$\langle sign\rangle \rightarrow -$
$\langle integer\rangle \rightarrow \langle digit\rangle$
$\langle integer\rangle \rightarrow \langle integer\rangle\langle digit\rangle$
$\langle digit\rangle \rightarrow i,\ i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 0$
$\langle positive\ integer\rangle \rightarrow \langle integer\rangle\langle nonzero\ digit\rangle$
$\langle positive\ integer\rangle \rightarrow \langle nonzero\ digit\rangle\langle integer\rangle$
$\langle positive\ integer\rangle \rightarrow \langle integer\rangle\langle nonzero\ digit\rangle$
$\quad\langle integer\rangle$
$\langle positive\ integer\rangle \rightarrow \langle nonzero\ digit\rangle$
$\langle nonzero\ digit\rangle \rightarrow i,\ i = 1, 2, 3, 4, 5, 6, 7, 8, 9$

b) $\langle signed\ decimal\ number\rangle ::= \langle sign\rangle\langle integer\rangle$
$\quad | \quad \langle sign\rangle\langle integer\rangle.\langle positive\ integer\rangle$

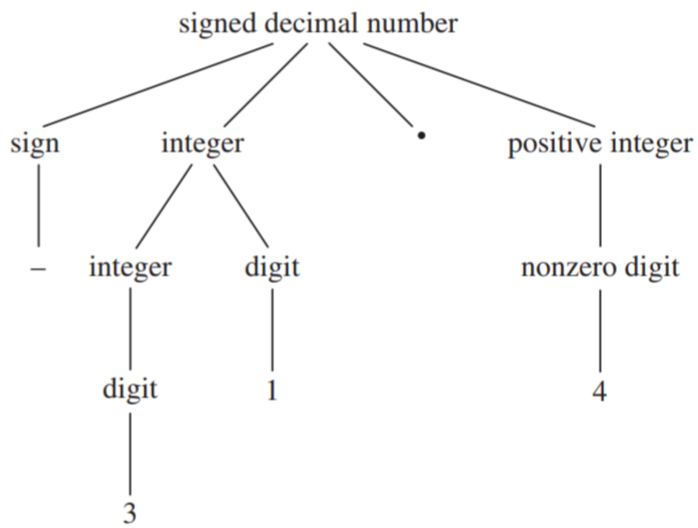
$\langle sign\rangle ::= +|\text{-}$

$\langle integer\rangle ::= \langle digit\rangle|\langle integer\rangle\langle digit\rangle$

$\langle digit\rangle ::= 0|1|2|3|4|5|6|7|8|9$

$\langle nonzero\ digit\rangle ::= 1|2|3|4|5|6|7|8|9$

$\langle positive\ integer\rangle ::= \langle integer\rangle\langle nonzero\ digit\rangle$
$\quad | \quad \langle nonzero\ digit\rangle\langle integer\rangle$
$\quad | \quad \langle integer\rangle\langle nonzero\ integer\rangle\langle integer\rangle$
$\quad | \quad \langle nonzero\ digit\rangle$


signed decimal number
sign
integer
.
positive integer
–
integer
digit
nonzero digit
digit
1
4
3

c)

13.2 Finite-State Machines with Output

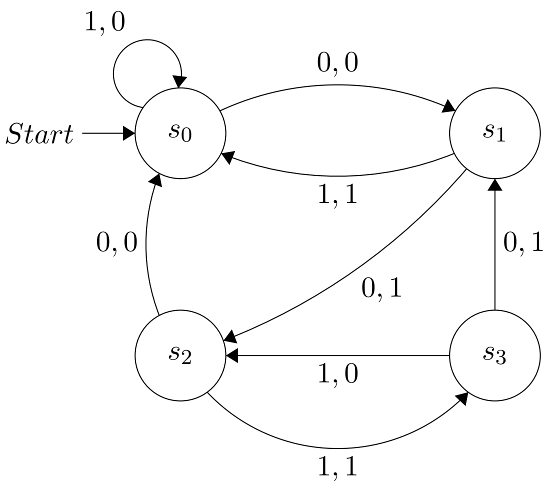
13.2.1

Draw the state diagrams for the finite-state machine with these state tables

	f		*g*	
	Input		*Input*	
State	**0**	**1**	**0**	**1**
s_0	s_1	s_0	0	0
s_1	s_2	s_0	1	1
s_2	s_0	s_3	0	1
s_3	s_1	s_2	1	0

b)

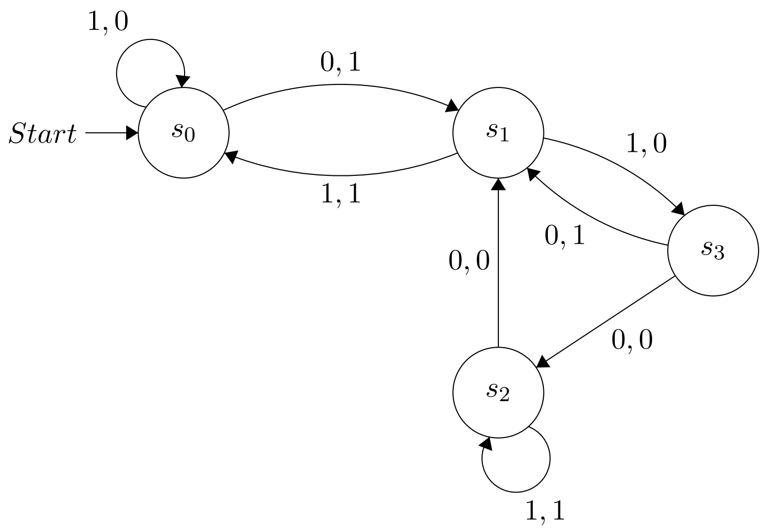
Solution



b)

13.2.5

Find the output for each of these input strings when given as input to the finite-state machine in Example 2.



a) 0111

Solution

a) 1100

13.2.13

Construct a finite-state machine for a toll machine that opens a gate after 25 cents, in nickels, dimes, or quarters, has been deposited. No change is given for overpayment, and no credit is given to the next driver when more than 25 cents has been deposited.

Solution