Ensemble Optimization to Tackle Uncertainty and Variability in Demand in Inventory Management

Abstract—**This study performs an empirical analysis to explore an ensemble optimization approach for inventory management (IM) under demand uncertainty. The objective is to identify the best course of action that reduces inventory costs under stochastic demand. Total costs include purchase cost, order cost, holding cost, and stockout cost. The approach combines a continuous review of IM policies with a Monte Carlo Simulation (MCS). To find the optimal solution, the study focuses on meta-heuristic approaches and compares six optimization algorithms, including the Grey Wolf Optimizer, Simulated Annealing, Whale Optimization, MCS with Bayesian Optimization, and Differential Evolution (DE). The findings suggest that DE outperforms other optimization algorithms in this specific context. The parameters of the selected approach are calibrated using the Latin Hypercube Sampling (LHS) statistical method. To select the best result as the final solution, an ensemble approach is used, which combines multiple independent DE optimizations with different random initial conditions. The use of ensemble optimization is a novel approach in IM and offers a promising avenue for improving performance under stochastic demands.**

Keywords— differential evolution; ensemble optimization; genetic algorithm; inventory management; non-linear optimization; stochastic demand.

I. Introduction

Inventory management is a crucial component of supply chain management and efficient management can significantly alter business profitability. Considering the current business trend, where uncertainties and complexities are constantly rising, more complex models and sophisticated methodologies are required to drive data-driven decision-making under uncertainty. In real-world settings, demand and lead times may differ, resulting in issues such as stockouts, excess inventory, and significant revenue losses. To overcome these issues, businesses frequently use mathematical optimization approaches and make data-driven decisions under uncertainty.

In the past, Koumanakos (2008), Simchi-Levi et al. (2003) and Chen et al. (2000) have made significant contributions to IM and highlighted the importance of sophisticated computational techniques to optimize inventory decisions to manage demand variations. Building on their arguments, more recent studies by Yao and Chen (2018) and Fathi et al. (2020) further highlighted the growing complexities in IM driven by demand uncertainties, which have led to the development of computation-intensive simulation and optimization in supply chain management (SCM) methods. Several recent studies have emphasized the relevance of optimization throughout the value chain (Moons et al., 2019; Modgil et al., 2022; Fonseca & Azevedo, 2020; Bag et al., 2020).

Despite several available works, with the advancement of technology, globalization, and evolving customer expectations, IM has become a complex task and an active research area. Researchers are constantly exploring innovative approaches and methodologies to handle this complexity effectively. We address the questions of how to effectively manage inventory with stochastic demand, focusing on a continuous review policy approach, and how to optimize the total cost through this work. Meta-heuristic optimization techniques such as Grey Wolf Optimizer (GWO), Whale Optimization Algorithm (WOA), Metaheuristics (MH) with Simulated Annealing (SA), Monte Carlo Simulation (MCS) with Bayesian Algorithm (BA), and Differential Evolution (DE) have been explored in this work. The findings reveal that DE is the most effective and simple heuristic optimization to deal with stochastic demands.

We introduced ensemble optimization, which combines multiple independent optimizations to obtain the best result. The ensemble helps to mitigate the risk of local optima and enhance the optimization process. The goal is to explore different regions of the parameter space to find a robust and reliable solution. The efficacy of the optimized policy may be sensitive to demand distribution. Therefore, this work performed a sensitivity analysis to assess the robustness of the policy under various scenarios.

The major contribution of this study is the development of a simulation-optimization model that can be applied with DE to select a nearly ideal IM policy under stochastic demand. We have shown DE technique to ensemble optimization to efficiently solve inventory policies by using the structure of the objective function rather than an exhaustive approach.

II. Previous Work

Several authors have emphasized the importance of optimization (for example Moons et al. 2019; Modgil et al., 2022; Fonseca & Azevedo, 2020; Bag et al., 2020 etc.). Optimization approaches, as highlighted by Silva et al. (2022) and Thevenin et al. (2021), offer a systematic method to further enhance the IM. Studies (e.g., Simchi-Levi et al., 2000; Singh & Verma, 2018; Muller, 2019) and industry reports suggest that IM costs can range a sizeable portion which is approximately from 20–40% of the total SCM costs. Moreover, Franco & Alfonso-Lizarazo (2020) studied on optimization under uncertainty and proposed a simulation-optimization approach showing a 16% reduction in supply and managing costs by implementing the optimal policy.

Several studies have discussed different optimization techniques (Gruler et al., 2018; Azadi et al., 2019; Wu & Frazier, 2019). However, Jackson et al. (2020) concluded that none of the works guaranteed an optimal solution if the original assumptions and considerations were violated. On the same note, Qiu et al. (2021) emphasized that when using optimization techniques to solve IM problems, it is important to carefully consider the assumptions and constraints underlying the model. Their work highlighted the importance of incorporating uncertainty and variability in the model, because real-world inventory systems are often subject to these factors. Fallahi et al. (2022) and Khalilpourazari et al. (2016) have pointed out that IM under uncertainty is challeging to solve due to the non-linearity of the model and several local optimum solutions. As a result, metaheuristic algorithms are frequently employed as powerful solutions for IM (Goodarzian et al., 2021; Fahimnia et al., 2018). A growing body of work on meta-heuristic optimization is noticeable (e.g., Abdi et al., 2021; Faramarzi-Oghani et al., 2022; Wang et al., 2022; Fahimnia et al., 2018; Soleimani & Kannan, 2015; etc.).

In their recent work, Sadeghi et al. (2023) employed meta-heuristic algorithms for inventory optimization by presenting GWO and WOA as two novel solution approaches. Owing to their ability to effectively search for the solution space of complicated problems, meta-heuristic algorithms have received considerable attention in recent years. Mirjalili et al. (2014) introduced GWO to solve optimization problems, and Lu et al. (2018); Tu et al. (2019) criticized the fact that GWO mostly suffers from a lack of population diversity. To overcome this limitation, Nadimi-Shahraki et al. (2021) presented an improved version of GWO. Vahdani et al. (2017) and Meisheri et al. (2022) have emphasised the application of SA in the context of IM and the efficiency of SA in resolving the difficulties brought on by the unpredictability of demand and the requirement to optimise inventory policies under uncertainty. Storn and Price (1997) introduced DE for optimization. They compared different optimization approaches and found that the DE method outperformed all other approaches in terms of the required number of function evaluations necessary to locate a global minimum of the test functions. Xue et al., (2019) studied meta-heuristic approaches for inventory forecasting and found superior performance of DE compared to CNN-LSTM. An exhaustive literature review by Ahmad et al. (2022) revealed that 158 out of 192 papers were published between 2016 and 2021, showing that academics have improved DE to increase its effectiveness and efficiency in handling a variety of optimization challenges.

Moreover, the MCS method is commonly used to propagate the uncertainties of random inputs in the case of stochastic demand (for example, Janssen et al. 2018; Gruler et al. 2018; Shokouhifar et al. 2021). This establishes that simulation is an integral part of IM during stochastic demands. MCS allows the incorporation of stochastic variability in demand patterns. The growing body of work in metaheuristic optimization indicates ongoing research efforts to improve the effectiveness of these techniques and their application in solving various optimization challenges in inventory management.

III. METHODOLOGY

A three-stage approach employing a comprehensive methodology was adopted in this study to analyze and optimize the IM policy. Fig. 1 displays the methodological framework applied in this study, with shaded areas for various stages. First, the demand for products was collected over 365 days. The data were simulated to estimate the probability of experiencing various demand levels. These simulations allowed us to create multiple scenarios and observe the potential outcomes. The policies considered herein are continuous reviews and cross-docking. The performance of these policies was compared based on their ability to minimize total costs while ensuring an acceptable level of service.

TotalCost = PurchaseCost + OrderCost + HoldingCost + StockoutCost.

By considering all of these costs, we aim to develop an inventory policy that minimizes costs and maximizes profits.

The results of the simulations were analyzed and interpreted to provide insights into the effectiveness of each policy. Once the optimal policy is identified, the next goal is to determine the optimal inventory levels that balance the totalCosts. With both goals in place, in stage three (blue shaded area), the various optimization techniques (e.g., GWO, MH + SA, MCS, WO, MCS + BO, and DE) are employed.

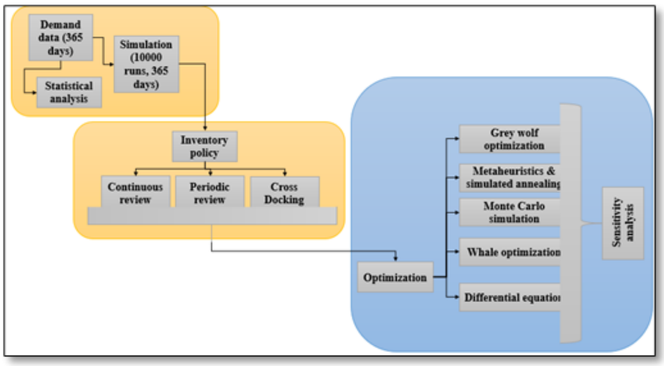

Fig. 1. Methodological framework (Source: Authors)

By employing the optimization technique and running simulations for a full year (365 days), we aim to fine-tune the inventory policy to minimize the totalCosts while considering the uncertainties in demand. In the final stage, sensitivity analysis is performed to identify different convergence rates, quality of solutions, and computational efficiency.

IV. DATA ANALYSIS

The business case selected here examines the sale of four distinct products and considers the adoption of a suitable IM policy. The goal is to minimize the total cost associated with purchasing, ordering, and holding inventory by optimizing inventory levels. We use historical demand data to calculate the central tendency of the data. Table I displays the statistics related to the four products.

- Pr A, Pr B, Pr C, and Pr D are four distinct products.
- PurchaseCost is the cost of purchasing one unit of the item from the supplier.
- LeadTime is the time it takes for the supplier to deliver the item after placing an order.
- Size is the size or quantity of each item.
- SellingPrice is the price at which each item is sold to the customers.
- StartingStock is the initial stock level of each item in the inventory.

- Mean is the average demand for each item over a given period.
- StdDev is the stdDev of demand for each item over a given period.
- OrderCost is the cost of placing an order with the supplier.
- HoldingCost is the cost of holding one unit of inventory for a given period of time.
- Probability is the probability of a stock-out event occurring, i.e., the probability of demand exceeding the available inventory level.
- DemandLead: This is the lead time demand for each item, i.e., the demand that is expected to occur during the lead time.

TABLE I. SUMMARY STATISTICS

	Pr A	Pr B	Pr C	Pr D
PurchaseCost	€ 12	€ 7	€ 6	€ 37
LeadTime	9	6	15	12
Size	0.57	0.05	0.53	1.05
SellingPrice	€ 16.10	€ 8.60	€ 10.20	€ 68
StartingStock	2750	22500	5200	1400
Mean	103.50	648.55	201.68	150.06
StdDev	37.32	26.45	31.08	3.21
OrderCost	€ 1000	€ 1200	€ 1000	€ 1200
HoldingCost	€ 20	€ 20	€ 20	€ 20
Probability	0.76	1.00	0.70	0.23
DemandLead	705	3891	2266	785

Fig. 2 displays the KDE plots of the demand distribution of the products over 365 days. The shapes of the curves provide insight into the underlying stochastic distribution of the data. The isolated peaks in the curves show potential outliers in the demand data.

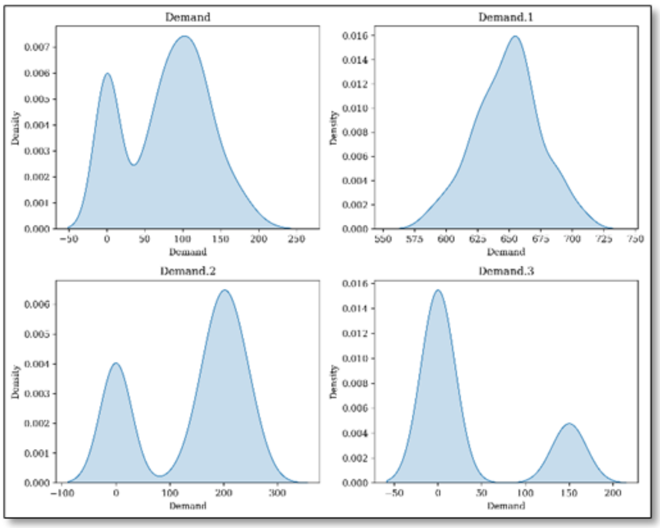

Fig. 2. KDE plots of demand distribution

A. ABC analysis

ABC analysis was used to categorize items based on their initial IM values. The analysis follows the Pareto Principle for the annual consumption value of each product. Considering CV (Consumption Value) = consumption value, then,

$$CV_{Annual} = Demand * Selling_{Price} \quad \text{.... (1)}$$

$$CVcumulative = \sum(CV_{Annual}) \quad \text{.... (2)}$$

$$Cumulative\% = \sum(CV_{Annual}/CV_{cumulative}) \quad \text{.... (3)}$$

Product A: CV_{Annual} = $Demand_{Annual}$ * $Selling_{PriceAnnual}$ = 705 * €16.10 = €11,335.50; likewise, Product B: CV_{Annual} = 3891 * €8.60 = €33,516.60, Product C: CV_{Annual} = 2266 * €10.20 = €23,099.20 and Product D: CV_{Annual} = €53,380.00.

$CV_{cumulative}$ = CV_A + CV_B + CV_C + CV_D = €11,335.50 + €33,516.60 + €23,099.20 + €53,380.00 = €121,331.30.

$Cumulative\%_A$ = CV_A / $CV_{cumulative}$ = € 11,335.50 / € 121,331.30 ≈ 0.0934; likewise $Cumulative\%_B$ = CV_B / $CV_{cumulative}$ = € 33,516.60 / € 121,331.30 ≈ 0.2763; $Cumulative\%_C$ = CV_C / $CV_{cumulative}$ = € 23,099.20 / € 121,331.30 ≈ 0.1903 and $Cumulative\%_D$ = CV_D / $CV_{cumulative}$ = € 53,380.00 / € 121,331.30 ≈ 0.4399.

ABC categories (A, B, and C) were assigned based on predefined cutoff points. Here, cutoff A was set at 0.8, and cutoff B was set at 0.95. Products with cumulative % below cutoff A were assigned category A, products with cumulative % between cutoff A and cutoff B were assigned category B, and products with cumulative % above cutoff B were assigned category C. Table II lists the metrics used in the analysis.

ABC analysis classifies products according to their consumption value, with Category A being the most critical products, Category B representing products of moderate importance, and Category C representing products of lower relevance.

B. Latin Hypercube Sampling

The LHS is used to sample parameter combinations in a more evenly distributed manner. Table II lists the parameter values associated with the lowest average costs. The objective was to identify combinations that resulted in low total costs, indicating an efficient IM. The parameter space for calibration was taken as reorderPoint = {100, 200, 300}, safetyStock = {50, 100, 50}, leadTime factor = {0.8, 1.0, 1.2}, and orderQuantity factor = {0.8, 1.0, 1.2}. For experimentation and calibration, we treated treorderPoint and safetyStock as separate entities. This enabled a comprehensive exploration of different inventory control strategies.

TABLE II. LATIN HYPERCUBE SAMPLING REPORT

	Pr A	Pr B	Pr C	Pr C	Lead time	Order Qty	Average cost
Reorder point	753	6164	1425	383	0.8	0.8	77,540

Safety stock	377	3082	712	192			

The number of experiments was set at 27 because, based on orthogonal arrays, it represents the total number of unique combinations for the specified parameter space.

V. SIMULATION & INVENTORY SYSTEMS

CR is more suitable for managing inventory with stochastic demand (Melo ter, 2009; Axsäter, 2015; Sridhar et al., 2021). Perera and Sethi (2023) conducted an extensive literature review of over seven decades and found that continuous policy is the most employed policy in stochastic inventory literature. Taking a clue from their work, we examined both CR and CD in our empirical analysis. MCS was used to simulate and observe the cost and inventory levels over multiple simulation runs (10,000) for a period of 365 days.

Table III depicts the output; the reorder points and safety stocks for the analysis have been taken from Table II.

TABLE III. INVENTORY REVIEW SYSTEMS

System definition	Average cost	Average inventory level
Continuous review	515,262	21,251
Cross docking	515,268	21,253

Average cost and average inventory levels are calculated as: $average_{cost} = total_{cost} / (num_{simulations} * num_{periods})$ and $average_{inventory\_level} = total_{inventory\_level} / (num_{simulations} * num_{periods})$. The CD strategy involves minimizing the need for inventory storage by transferring products directly from the supplier to the customer. We implemented CD as additional logic within the IM simulation.

A. Optimization

The importance of optimization cannot be ignored in the IM. It enables businesses to minimize costs, improve customer service, efficiently utilize resources, mitigate risks, and make strategic decisions. Multiple optimization algorithms (GWO, SA, MCS, MCS with BO, WO, and DE) were tested to get the optimal cost.

1) Cost breakdown:

The total cost is computed based on the following parameters:

- Purchase Cost is computed as unit $purchase_{cost}$ * max ($order_{quantities}$, 0).
- Order Cost is only applied when the order quantity is greater than zero and is computed as unit $order_{cost}$ * ($order_{quantities} > 0$).
- Holding Cost is computed as unit $holding_{cost}$ * max (inventory - demand, 0).
- Stockout Cost is inferred from the equation unit $holding_{cost}$ * max (demand - inventory, 0). This equation calculates the cost of stockouts where demand exceeds the inventory level.

These costs can be subtracted from the revenue to give the corresponding profit for that one realization of the year. Annual profit, which is the future direction of this work, can be formulated as:

$$SP_i \sum_{t=1}^{365} S_{i,t} - \left\{ \left(\frac{20V_i}{365}\right) \sum_{t=1}^{365} I_{i,t} + N_i C_{o,i} + \sum_{t=1}^{365} c_i P_{i,t} \right\} \ldots (4)$$

Our goal is to minimize costs. Table IV presents a summary of all the algorithms tested on the given parameters and MCS to simulate the data for 365 days. We chose meta-heuristic techniques that are designed to tackle complex and non-linear optimization issues where typical optimization techniques struggle to find the global optimum.

TABLE IV. OPTIMAL POLICY FOR 365 DAYS (ABOUT 12 MONTHS)

Optimization	Stock				Total Cost
	Pr A	*Pr B*	*Pr C*	*Pr D*	
GWO	110	1836	0	21	17,391,348
SA	2,600	21,843	4,984	1,268	6,179,739
MCS	1,527	455	4,768	599	631,398
WO	1,070	10,865	3,787	150	504,939
MCS with BO	2,750	14,724	4,465	1,350	254,137
DE (best1bin)	1220	13204	3359	1317	250,774

Based on the totalCost, the optimization method with the lowest total cost appears to be DE (best1bin), with the corresponding cost of 250,774. Although Storn and Price (1997) introduced DE, which showed promising results in the optimization space, they also indicated that DE could be further improved. Our next approach is to check if DE can be further optimized.

B. Ensemble optimization

In this approach, we performed optimization using multiple optimizers (five optimizers) in parallel to determine the best parameters and cost. By using an ensemble approach, we aim to mitigate the impact of random variations in the MCS and increase the likelihood of finding a robust and optimal solution for IM.

Considering, the below parameters:

- purchaseCost = [Pc_1, Pc_2, Pc_3…., Pc_n], where Pc_i is the purchaseCost of product i; leadTimes = [L_1, L_2, ..., L_n], where L_i is the leadTime of product i; sizes = [s_1, s_2, ..., s_n], where s_i is the sizes of product i; sellingPrice = [SP_1, SP_2, ..., SP_n], where SP_i is the sellingPrice of product i; startingStock = [ss_1, ss_2, ..., ss_n], where ss_i is the initial inventory level of product i; means = [μ_1, μ_2, ..., μ_n], where μ_i is the mean demand of product i; standard deviation = [σ_1, σ_2, ..., σ_n], where σ_i is the standard deviation of demand of product i; order cost = [C_1, C_2, ..., C_n], where C_i is the order cost of product i; holdingCost = [V_1, V_2, ..., V_n], where V_i is the order cost of product i; probabilities = [p_1, p_2, ..., p_n], where p_i is the probability of demand for product i; demand lead = [D_1, D_2, ..., D_n], where D_i is the demand lead time of product i.
- Parameters space of optimization: bounds = [(0, ss_1), (0, ss_2) …. (0, ss_n)], where each tuple represents the

lower and upper bounds of inventory levels of the respective products.

- MCS and objective function (x), where $x = x_1, x_2, x_3, \ldots, x_n$ representing inventory levels; reorder levels = $[\text{means}_1 * L_1 + \sqrt{L_1} * \sigma_1, \ldots, \text{means}_n * \sqrt{L_n} * \sigma_n]$, orderQuantities = [max (reorderLevel$_1 - x_1$, 0), …., max (reorderLevel$_n - x_n$, 0)].

totalCost = for each day and product: if $x_i <$ reorderLevel, orderQuantity = orderQuantities$_i$ (this checks if the current inventory level is below the reorder level). If it is, the order quantity is set to the predetermined value for that product (orderQuantities$_i$), indicating that an order should be placed to replenish the inventory.

- increaseInventory (x_i) = x_i + orderQuantity
- increaseTotalCost = totalCost + $\begin{cases} orderCosts_i, if\ orderQuantity > 0 \\ 0, otherwise \end{cases}$
- decreaseInventory (x_i) = x_i – dailyDemand, if $x_i < 0$, $x_i = 0$ and totalCost = totalCost + $\frac{holdingCost}{2}$
- if (d+1) % leadTimes$_i$ = 0, decrease inventory again: $x_i = x_i$ – dailyDemand$_i$ and totalCost = totalCost + holdingCost$_i$ * x_i

meanCost = $\frac{1}{numSamples} \sum_{s=1}^{numSamples} totalCost_s$

Finally, the ensembleOptimization = [result$_1$, result$_2$, …., result$_{\text{numEnsemble}}$ where, each result represents the optimization result of one ensemble member.

Table V presents the output.

TABLE V. ENSEMBLE OPTIMIZATION

Optimization	Stock				Total Cost
	Pr A	*Pr B*	*Pr C*	*Pr D*	
Ensemble	2567	9063	4277	1322	249,128

The total cost has been marginally reduced from 250,774 (Table VI) to 249,128 (Table VIII). The stocks are optimized from 19,100 (1220, 13204, 3359, 1317) to 17,229 (2,567, 9,063, 4,277, 1,322). Sensitivity analysis is employed on this to ensure the robustness of the ensemble model under different scenarios.

C. Sensitivity analysis

Sensitivity analysis was performed on the population size parameters of the DE algorithm. The goal was to evaluate the effects of different population sizes on the optimization results. Different values of population size, for example, 10, 20, 50, and 100, were tested to observe how they affected the optimization results. By exploring different population sizes, we have assessed their impact on convergence behavior and the quality of the obtained solutions.

TABLE VI. Sensitivity analysis

Analysis	Stock				Total Cost
	Pr A	*Pr B*	*Pr C*	*Pr D*	
Population size 10	2,271	4,736	4,146	1,321	251,238
Population size 20	1,901	20,134	3,355	1,325	249,780
Population size 50	1,525	12,753	4,992	1,326	246,251
Population size 100	1,552	9,667	3,695	1,326	246,745

By varying the population size in the DE, we can observe how it affects the optimization results. In this case, the observed differences in total cost are small, indicating that the model's performance is stable and not heavily influenced by changes in population size.

D. Critical findings

This study emphasizes the importance of optimization in IM, specifically in the context of stochastic demand and supply disruptions. DE is a successful method for establishing near-optimal inventory policies when combined with best/1/bin mutation strategy, LHS, and ensemble optimization. Sensitivity analysis with varying population sizes confirmed the stability of the optimization model.

VI. CONCLUSION

This study aimed to analyze and optimize inventory management (IM) policy by considering different policies, simulation experiments, and the application of various optimization techniques. An empirical analysis was conducted using stochastic demand data covering 365 days, and the results of the optimization process using several algorithms (GWO, SA, MCS, MCS with BO, WO, and DE) were presented. This study reported the optimal policies identified and their corresponding total costs. In addition, the utilization of LHS to achieve even and efficient sampling of parameter combinations is discussed. Furthermore, the study applied ABC analysis to categorize items based on their value in inventory management, and the Pareto principle was employed to assign ABC classes to products. The optimal policy and corresponding inventory levels were determined by analyzing the outcomes of the simulations and optimization techniques. To assess the convergence rate, solution quality, and computational efficiency, a sensitivity analysis was performed. This analysis provided insights into the performance and robustness of the proposed method. This study contributes to IM by providing a comprehensive approach that integrates different policies, simulation experiments, optimization techniques, and analytical tools. The findings offer valuable insights for decision-makers seeking to improve the efficiency and cost-effectiveness of inventory management while addressing demand uncertainties.

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