Interactions

David Puelz

Two categorical variables

A categorical and a quantitative variable

Two quantitative variables

What is an interaction?

Definition

An interaction is an additional term in a regression model that allows the slope of one variable to depend on the value of another.

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Two quantitative variables

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- But does the effect of tenure seem to be different for men than for women?
- In other words, does the effect of one variable (i.e., its slope coefficient) depend on the value of another?
- This is what interactions let us model

The algebra of interactions

The idea is to add a term that is the product of the two variables:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1(\text{male}) + \hat{\beta}_2(\text{tenured}) + \hat{\beta}_3(\text{male})(\text{tenured})$$

The algebra of interactions

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For female professors, male = 0, so the β_1 and β_3 terms cancel out:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_2(\text{tenured})$$

For male professors, male = 1, so we get both a different intercept and a different slope for `tenured`:

$$\hat{Y} = (\hat{\beta}_0 + \hat{\beta}_1) + (\hat{\beta}_2 + \hat{\beta}_3)(\text{tenured})$$

```
model1 <- lm(eval ~ gender * tenure, data=profs)
summary(model1)


Call:
lm(formula = eval ~ gender * tenure, data = profs)

Residuals:
     Min       1Q   Median       3Q      Max
-1.89028 -0.36000  0.00972  0.40972  1.00972

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)           3.86000    0.07585  50.890  < 2e-16 ***
gendermale            0.53615    0.10623   5.047 6.48e-07 ***
tenureyes             0.05517    0.08796   0.627 0.530813
gendermale:tenureyes -0.46105    0.12083  -3.816 0.000154 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5363 on 459 degrees of freedom
Multiple R-squared:  0.07173,Adjusted R-squared:  0.06567
F-statistic: 11.82 on 3 and 459 DF,  p-value: 1.795e-07
```

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- In a model with an interaction term X_1X_2, you must also keep the main effects: the variables that are being interacted together
- The main effect of X_1 represents the predicted increase in Y for a 1-unit change in X_1, holding X_2 constant at zero
- In other words, the main effect `gendermale` represents the predicted advantage for men in student evaluation scores, among professors without tenure
- You can also include other variables in the model that are not being interacted

Two categorical variables

A categorical and a quantitative variable

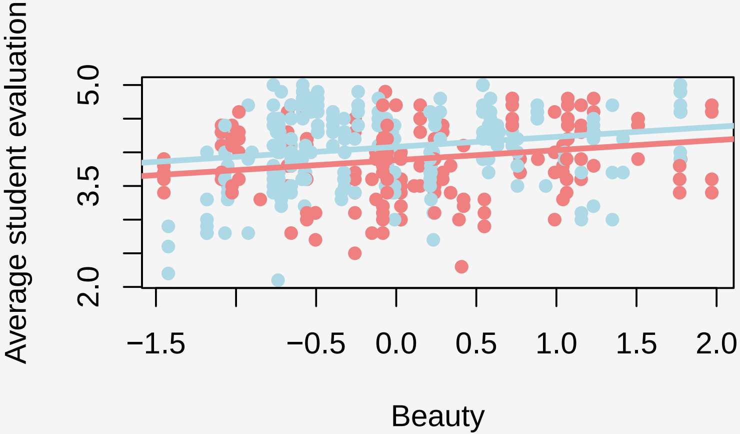
Two quantitative variables

Does beauty matter more for men, or for women?

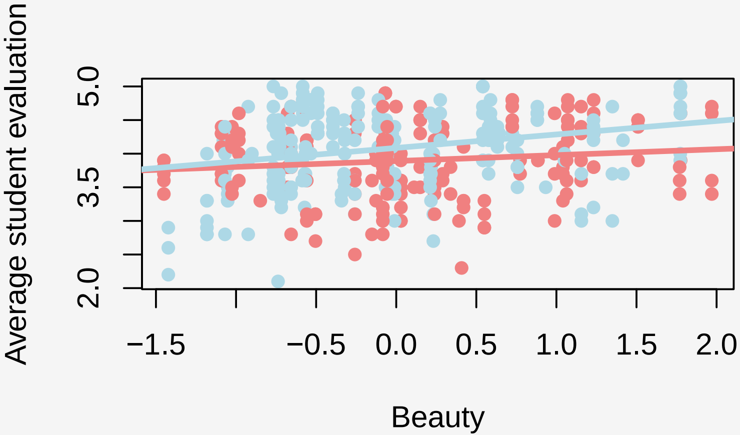
- We found that for the same level of attractiveness, male professors tend to get higher evaluation scores than female professors
- But what if the effect of beauty depend on gender?

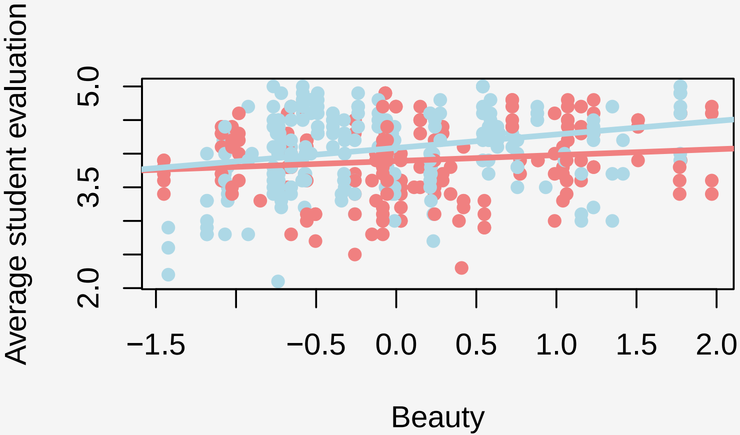
Does beauty matter more for men, or for women?

Another way to think about it—what if these regression lines didn't have to be parallel?



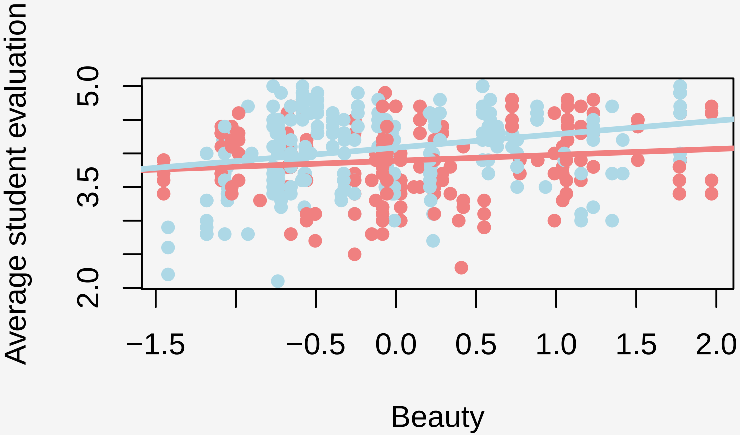
```
model2 <- lm(eval ~ beauty * gender, data=profs)
summary(model2)


Call:
lm(formula = eval ~ beauty * gender, data = profs)

Residuals:
     Min       1Q   Median       3Q      Max
-1.83820 -0.37387  0.04551  0.39876  1.06764

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        3.89085    0.03878 100.337  < 2e-16 ***
beauty             0.08762    0.04706   1.862 0.063294 .
gendermale         0.19510    0.05089   3.834 0.000144 ***
beauty:gendermale  0.11266    0.06398   1.761 0.078910 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5361 on 459 degrees of freedom
Multiple R-squared:  0.07256,Adjusted R-squared:  0.0665
F-statistic: 11.97 on 3 and 459 DF,  p-value: 1.47e-07
```


Average student evaluation
5.0
3.5
2.0
−1.5
−0.5
0.0
0.5
1.0
1.5
2.0
Beauty



Two takeaways:

- Beauty seems to matter more for men than for women!



Two takeaways:

- Beauty seems to matter more for men than for women!
- The gender gap is largest for good-looking professors

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NBA data

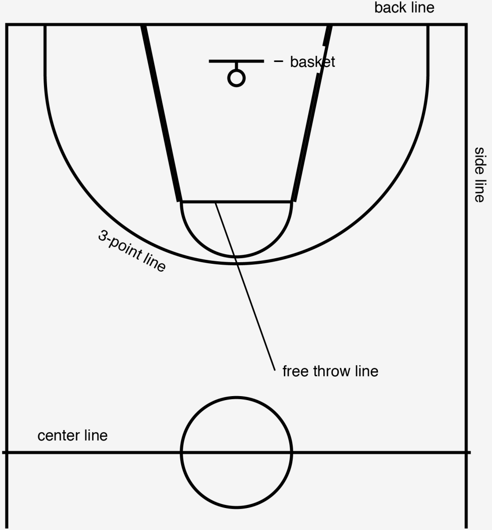
Basketball-Reference.com provides detailed data on NBA teams and players. We'll look at team data for 4 seasons ending in 2016; each of these metrics is the average across the season:

- **PTS**: Total points
- **PCT3P**: Percentage of 3-point shots made
- **N3PA**: Number of 3-point shots attempted

There are 30 NBA teams $\times$ 4 seasons = 120 cases in this file.

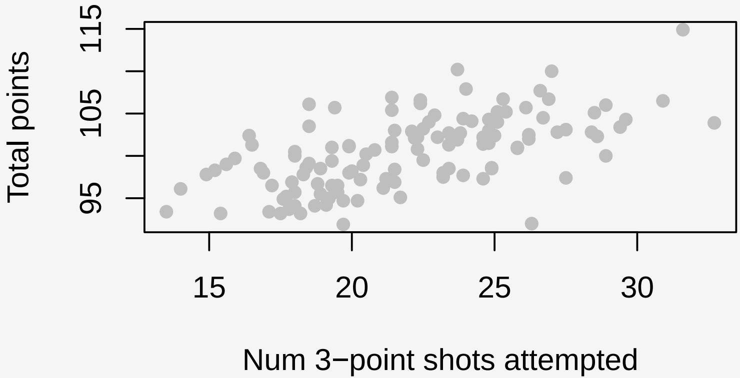
NBA data

In basketball, there are three ways to score:



- **1 point** for free throws made after a foul by the other team
- **2 points** for shots made inside the 3-point line
- **3 points** for shots made outside the 3-point line

The more 3-pointers you attempt, the more you tend to score:



```
model1 <- lm(PTS ~ N3PA, data=nba)
summary(model1)
```

```
Call:
lm(formula = PTS ~ N3PA, data = nba)

Residuals:
     Min       1Q   Median       3Q      Max
-11.2454  -2.5114   0.0549   2.2252   8.6405

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 86.19204    1.77464  48.569  < 2e-16 ***
N3PA         0.64842    0.07935   8.171 3.89e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.496 on 118 degrees of freedom
Multiple R-squared:  0.3614,Adjusted R-squared:  0.356
F-statistic: 66.77 on 1 and 118 DF,  p-value: 3.889e-13
```

Can we do better?

- $R^2 = 36\%$, so we can explain 36% of the variance in total points based only on knowing the number of 3-point attempts.

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- $R^2 = 36\%$, so we can explain 36% of the variance in total points based only on knowing the number of 3-point attempts.
- This means that **most** of the variance (64%) in total points is **not** explained by the number of 3-point attempts.
- Let's add another variable to our model — why might 3-point percentage be useful as another predictor?

Can we do better?

```
model2 <- lm(PTS ~ N3PA + PCT3P, data=nba)
summary(model2)


Call:
lm(formula = PTS ~ N3PA + PCT3P, data = nba)

Residuals:
    Min      1Q  Median      3Q     Max
-8.3487 -2.1392 -0.0791  1.8691  9.1904

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 62.00493    5.61396  11.045  < 2e-16 ***
N3PA         0.56467    0.07587   7.442 1.82e-11 ***
PCT3P        0.73415    0.16292   4.506 1.57e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.241 on 117 degrees of freedom
Multiple R-squared:  0.4558,Adjusted R-squared:  0.4465
F-statistic:    49 on 2 and 117 DF,  p-value: 3.478e-16
```

Can we do even better?

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It would make sense that the **impact** of the number of 3-pointers taken on total points would **depend on** how well the team shoots the 3!

This sounds like an interaction — let's make a model with an interaction between the two predictors!

```r
model3 <- lm(PTS ~ N3PA * PCT3P, data=nba)
summary(model3)
```

```

Call:
lm(formula = PTS ~ N3PA * PCT3P, data = nba)

Residuals:
    Min      1Q  Median      3Q     Max
-7.2629 -2.2757  0.1148  1.9698  9.3756

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 122.84903   30.58937   4.016 0.000105 ***
N3PA         -2.11904    1.32903  -1.594 0.113561
PCT3P        -0.98410    0.86465  -1.138 0.257400
N3PA:PCT3P    0.07561    0.03739   2.023 0.045423 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.199 on 116 degrees of freedom
Multiple R-squared:  0.4743,Adjusted R-squared:  0.4608
F-statistic: 34.89 on 3 and 116 DF,  p-value: 3.798e-16
```

Model 3 corresponds to the regression equation

$$\widehat{\text{PTS}} = 122.85 - 2.12 \cdot \text{N3PA} - 0.98 \cdot \text{PCT3P} + 0.08 \cdot \text{N3PA} \cdot \text{PCT3P}.$$

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- **Intercept** (122.85) is our prediction of total points when $\text{N3PA} = \text{PCT3P} = 0$. (Meaningless in this context!)

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- **PCT3P** (−0.98) is the predicted increase in total points for each additional percentage point of 3-point shooting accuracy, when N3PA = 0.
- **N3PA · PCT3P** (0.08) can be interpreted in two ways:
    - the increase in the *slope coefficient* for N3PA for each 1-unit increase of PCT3P.

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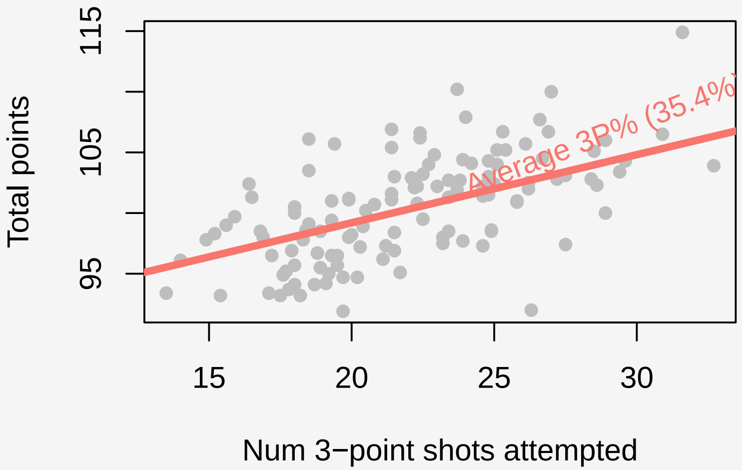
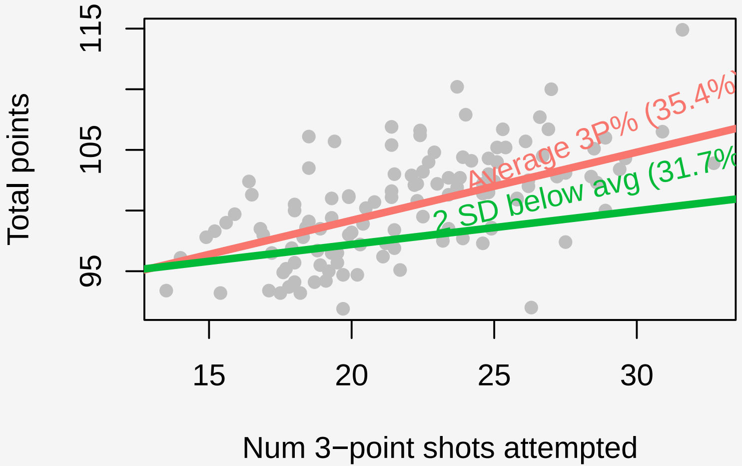


Average 3P% (35.4%)
Total points
115
105
95
15
20
25
30
Num 3−point shots attempted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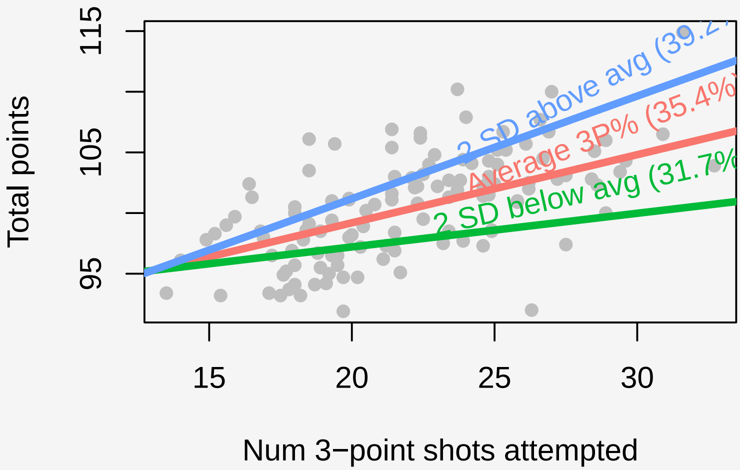
105
95
Total points
15
20
25
30
Num 3−point shots attempted
Average 3P% (35.4%)
2 SD below avg (31.7%)


Total points
115
105
95
15
20
25
30
Num 3−point shots attempted
2 SD above avg (39.2
Average 3P% (35.4%)
2 SD below avg (31.7%

$$\widehat{\text{PTS}} = 122.85 - 2.12 \cdot \text{N3PA} - 0.98 \cdot \text{PCT3P} + 0.08 \cdot \text{N3PA} \cdot \text{PCT3P}.$$

- How many points per game do you predict for a team that shoots 3-pointers at the NBA average rate (35.4) and that takes 30 3-pointers per game?

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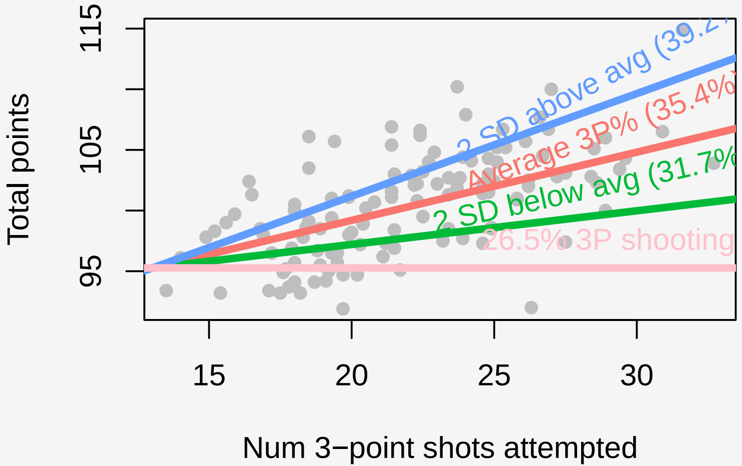
- How many points per game do you predict for a team that shoots 3-pointers at the NBA average rate (35.4) and that takes 30 3-pointers per game?
- How bad would a team have to shoot the 3 before taking 3-point shots start to have a negative impact on total points?


115
105
95
Total points
15
20
25
30
Num 3−point shots attempted
2 SD above avg (39.2
Average 3P% (35.4%
2 SD below avg (31.7%
26.5% 3P shooting

When should you use interactions in a model?

- Choose interactions by thinking about what you are trying to model: if you suspect that the impact of one variable depends on the value of another, try an interaction term between them!
- But remember: Interactions make a model more complex to analyze and explain, so unless you have a good substantive reason it's usually only worth doing so when you get a substantial bump in R^2/decrease in residual standard error by including the interaction