THE UNIVERSITY OF TEXAS AT AUSTIN

McCOMBS SCHOOL OF BUSINESS

# Optimization II

**Dr. Daniel Mitchell**

## Project 1 – Stochastic Programming
## Group - 11


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# 1. Introduction

The Newsvendor Model is a well-known inventory management framework that aims to determine the optimal level of inventory to order given uncertain demand. While the model provides a simple yet effective approximation of reality, it has some limitations, such as assuming that the cost of under-stocking and overstocking is constant.

In this project, we extend the classical Newsvendor Model to include rush orders, disposal fees, and a linear relationship between price and demand. The first extension allows the manager to order more items from the supplier if demand is not satisfied, incurring an additional cost. The second extension accounts for the impact of price on demand and involves fitting a linear regression to historical data to estimate the demand function.

We will solve these extended Newsvendor Models by formulating them as linear and quadratic programs. Additionally, we will investigate the sensitivity of the optimal price and quantity to changes in the data set by performing a bootstrap simulation and analyzing the histograms of the optimal solutions.

The **goal** of this project is to provide a more realistic and comprehensive solution to the inventory management problem, accounting for additional factors that affect the costs and profits of the business.

# 2. Regression and Residuals

To predict demand at different price points, we developed a linear regression model based on historical data.The model provides an equation that estimates the level of demand based on the price of the product. The equation to determine demand, given a certain price from the linear model is given as :

$$Demand = \mathbf{1924.7175 - 1367.7125}x - \epsilon,$$

Where $x$ represents a given price and $\epsilon$ represents a random error term, which will be represented by the residuals of the data

| | coef | std err | t | P>\|t\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| Intercept | 1924.7175 | 111.334 | 17.288 | 0.000 | 1703.750 | 2145.685 |
| price | -1367.7125 | 108.379 | -12.620 | 0.000 | -1582.816 | -1152.609 |

The scatter plot presented below displays the relationship between the demand and price data. It is evident from the plot that there is a negative linear association between the two variables

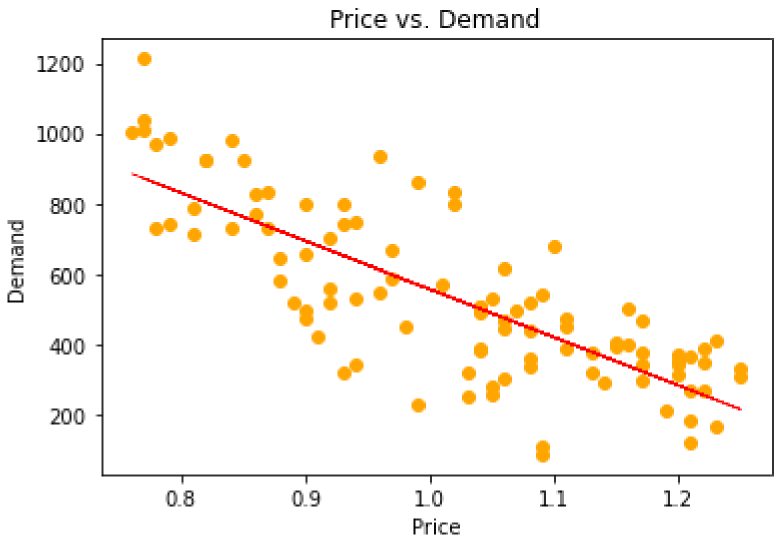


**Figure 1**

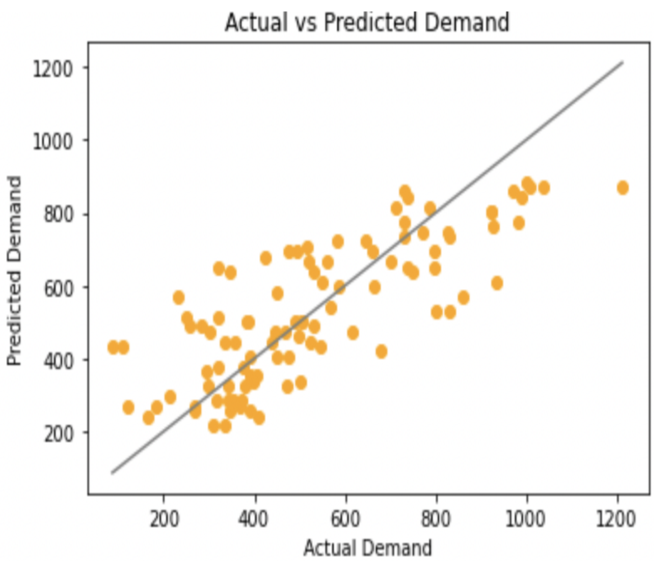


**Figure 2**

**Figure 2 :** provides a scatterplot of actual demand vs. predicted demand.

## 3. Demand Estimation

By using the fitted equation above and its residuals, demand values are estimated when price p is equal to 1. We were able to generate these demand values using the code snippet below:

```
g = 0.75
cost = 0.5
t = 0.15
price = 1.00

demand_with_price_1 = lr_model.predict(np.ones(x.shape))
demand_with_price_1= demand_with_price_1 + residuals
demand_with_price_1[:5]
```

```
array([351.38562621, 579.52024662, 472.21963007, 448.93724855,
       673.7489942 ])
```

With these generated demand values, it will allow us to use linear programming to optimize the quantity of our product to produce.

## 4. Linear Programming

Now we have to find the optimal quantity to produce when price is fixed at 1.

.

$$\max_q \ \text{Profit}_i = \frac{1}{N}\sum_{i=1}^{N}(pD_i - qc - g(D_i - q)^+ - t(q - D_i)^+)$$

Di = Demand on Day i
q = Quantity of initial produce
p = Sale Price
c = Cost Price
g = Cost of Expedited process
t = Cost of excess Disposal

**This is an NLP. We can reformulate it into LP of the form:**

$$\max_{q,h_1,h_2,\ldots,h_N} \quad \text{Profit}_i = \frac{1}{N}\sum_{i=1}^{N} h_i$$

subject to:

$$h + (c-g)q < (p-g)D_i$$
$$h + (c+t)q < (p+t)D_i$$
$$h > -\infty$$
$$q > 0$$

On solving this, the optimal quantity comes out to be 472 newspapers with an expected profit of \$231.483 per day.

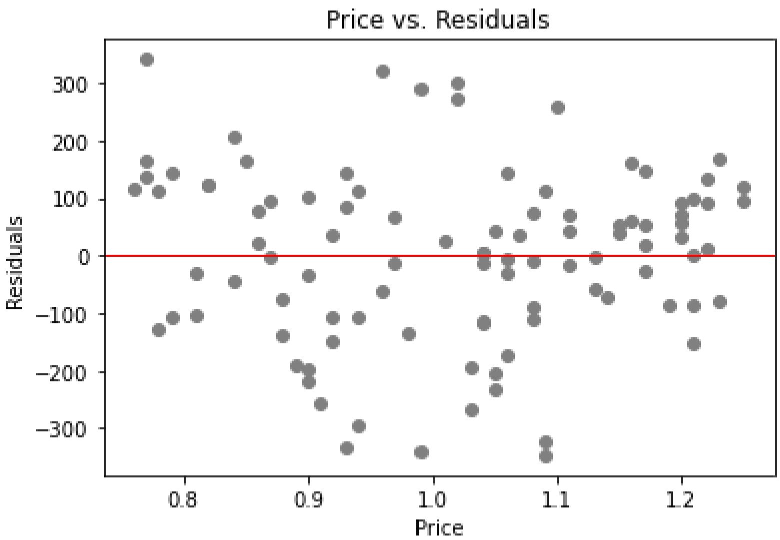


**Figure 3**

## 5. Dynamic Price Model

In basic economics, it is widely accepted that the demand for a commodity is not random or haphazard, but rather it is closely linked to the price of that commodity. More specifically, as the price of a commodity increases, the demand for it tends to decrease, and conversely, as the price of a commodity decreases, the demand for it tends to increase. This inverse relationship between price and demand is a fundamental concept in economics and has significant implications for businesses and markets.

The objective function now becomes a quadratic expression in ‘p’. We solve this using stochastic programming and quadratic optimization.

$$\max_{q} \frac{1}{n} \sum_{i=1}^{n} \left(pD_i - qc - g(D_i - q)^+ - t(q - D_i)^+\right)$$
$$D_i = \beta_0 + \beta_1 p + \epsilon_i,$$

Di = Demand on Day i
q = Quantity of initial produce
p = Sale Price
c = Cost Price
g = Cost of Expedited process
t = Cost of excess Disposal

Given :

Therefore equation $pD_i - qc - g(D_i - q)^+ - t(q - D_i)^+$ can be written as

$$\Rightarrow p(\beta_0 + \beta_1 p + \epsilon_i) - qc - g((\beta_0 + \beta_1 p + \epsilon_i) - q)^+ - t(q - (\beta_0 + \beta_1 p + \epsilon_i))^+$$
$$\Rightarrow p(\beta_0 + \epsilon_i) - [qc + g((\beta_0 + \beta_1 p + \epsilon_i) - q)^+ + t(q - (\beta_0 + \beta_1 p + \epsilon_i))^+] + (p\beta_1 p)$$
$$\Rightarrow p(\beta_0 + \epsilon_i) - h_i + (p\beta_1 p)$$

*Where*,

$$h_i = [qc - g((\beta o + p\beta_1 + \epsilon_1) - q)^+ + t(q - \beta o + p\beta_1 + \epsilon_1))^+]$$

Conditions:

$$h_i + (g - c)q - p\beta_i g > g(\beta_0 + \varepsilon_i)$$

$$h_i + (t + c)q - p\beta_i t > - t(\beta_0 + \varepsilon_i)$$

$$h > - \infty$$

$$q > 0$$

Here quadratic part is $p\boldsymbol{\beta 1}p$

**Results :**

```
#the optimal quantity to produce when price is dynamic
optimal_quantity_dynamic_price = model_for_dynamic_price.x[0]
print('Optimal Quantity for dynamic price :: ', optimal_quantity_dynamic_price)

#the expected price to produce when price is dynamic
optimal_price = model_for_dynamic_price.x[1]
print('Optimal price :: ', optimal_price)

#the expected price to produce when price is dynamic
exp_profit_dynamic_price = model_for_dynamic_price.objVal
print('Expected profit for dynamic price :: ', exp_profit_dynamic_price)
```

```
Optimal Quantity for dynamic price ::  535.291001278871
Optimal price ::  0.9536264966232612
Expected profit for dynamic price ::  234.42493487831734
```

By using the system of equations, it is possible to determine the optimal price at which to sell and the corresponding optimal quantity to print at that price in order to maximize profits. In other words, this approach enables businesses to find the most profitable balance between the price at which they sell their products and the quantity they produce. By optimizing both price and quantity simultaneously, businesses can achieve their profit goals more effectively and efficiently.

# 6. Sensitivity Analysis using Bootstrapping

One possible extension of the newspaper vendor problem is to utilize bootstrap sampling as a means of assessing how sensitive the model is to variations in the data. The basic concept behind this approach is to estimate population statistics such as the mean or standard deviation of demand by repeatedly resampling the available data with replacement and running the extended newsvendor model on each resampled dataset.

By generating many resampled datasets and analyzing the results, it is possible to gain a better understanding of how the model performs under different conditions and to identify any potential biases or limitations in the original dataset. This approach can be particularly useful when dealing with limited or incomplete data, as it enables businesses to make more informed decisions based on a more comprehensive understanding of the underlying trends and patterns in the data.

Given the relatively small size of our dataset, which consists of only 99 data points, bootstrapping provides a way to ensure that the results of our analysis are not skewed or biased by overfitting. By resampling the data with replacement and generating multiple resampled datasets, we can verify that our previous results are reliable and not simply artifacts of the particular dataset we used.

Additionally, bootstrapping can be used to calculate confidence intervals, which provide a measure of how certain we can be that our results accurately reflect the true population parameters. By generating a large number of resampled datasets and analyzing the results, we can estimate the degree of variability in our results and construct confidence intervals that give us a sense of the level of uncertainty associated with our estimates. This can be useful in making decisions based on our analysis and in communicating our findings to others.

**Results :**

```
model_bootstrap_q6 = bootstrap_func()

print("Output for 1 bootstrap sample\n")

#the optimal quantity to produce when price is dynamic
model_bootstrap_q6_price = model_bootstrap_q6.x[0]
print('Optimal Quantity for dynamic price of bootstrapped data:: ', model_bootstrap_q6_price)

#the expected price to produce when price is dynamic
optimal_price = model_bootstrap_q6.x[1]
print('Optimal price of bootstrapped data:: ', optimal_price)

#the expected price to produce when price is dynamic
model_bootstrap_q6exp_profit_dynamic_price = model_bootstrap_q6.objVal
print('Expected profit for dynamic price of bootstrapped data :: ', model_bootstrap_q6exp_profit_dynamic_price)
```

```
Output for 1 bootstrap sample

Optimal Quantity for dynamic price of bootstrapped data::  540.2882430123523
Optimal price of bootstrapped data::  0.958131539295075
Expected profit for dynamic price of bootstrapped data ::  242.93241458223633
```

**Joint plot for Price vs Optimal Quantity:**

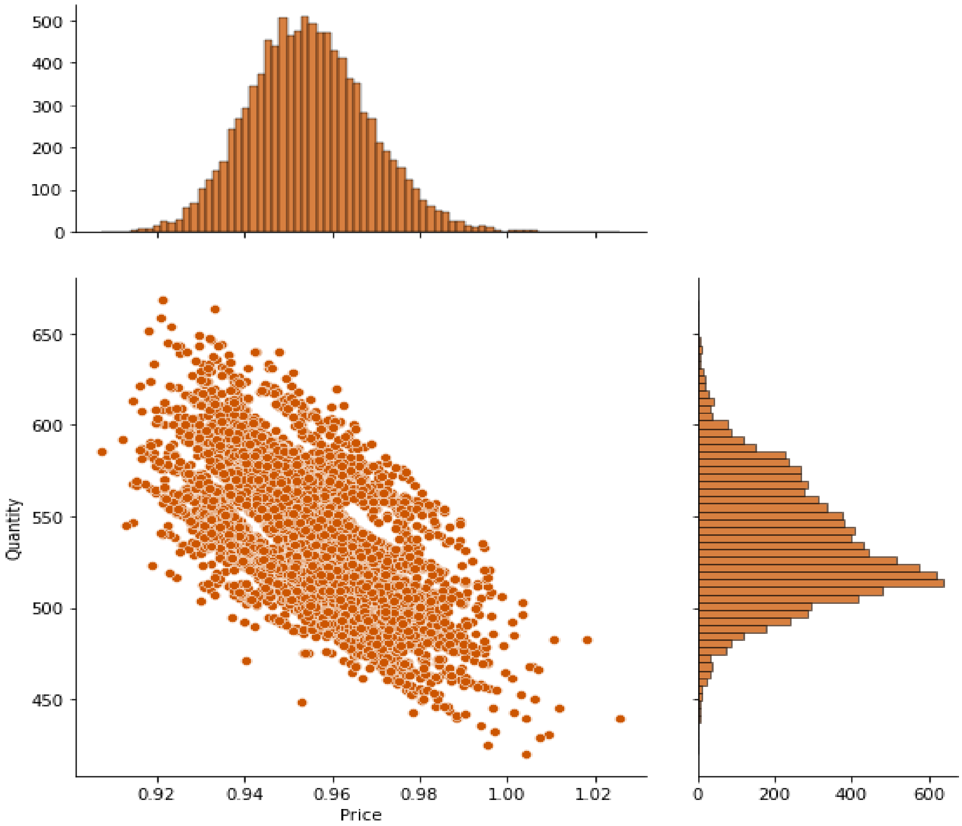


**Figure 4**

The graph presented in Figure 1 illustrates a clear negative linear correlation between the optimal price and the optimal quantity of newspapers to print. This finding is consistent with the negative relationship observed between demand and price, which reflects the publisher's goal of producing only the amount of newspapers that are in demand.

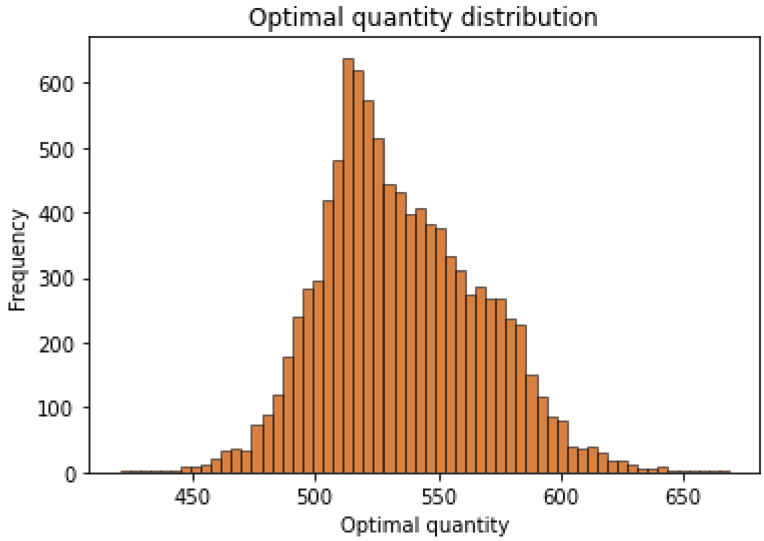


**Figure 5**

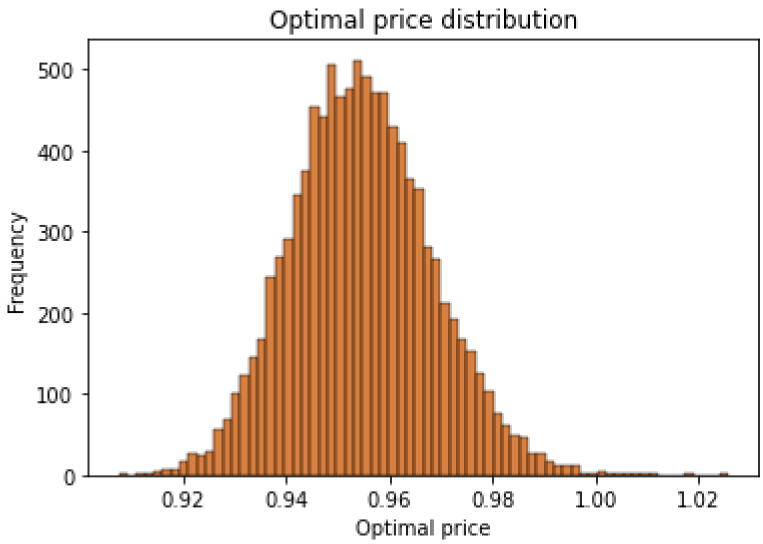


**Figure 6**

Expected Profits generated over 10000 iterations of bootstrapping appear to be normally distributed with a mean of $235.9 which is not so different from $242.93 obtained before.

Expected Price generated over 10000 iterations of bootstrapping appear to be normally distributed with a mean of $0.95 which is not so different from $0.958 obtained before

Expected Quantity generated over 10000 iterations of bootstrapping appear to be normally distributed with a mean of $520 which is not so different from $540.28 obtained before

## 7. Conclusion

In conclusion, we have extended the classical Newsvendor Model to include rush orders, disposal fees, and a linear relationship between price and demand. We used linear and quadratic programming to solve these extended models and investigated the sensitivity of the optimal price and quantity to changes in the dataset by performing a bootstrap simulation. Our analysis has provided a more realistic and comprehensive solution to the inventory management problem, accounting for additional factors that affect the costs and profits of the business.

Our analysis showed that the optimal quantity to produce was 472 newspapers with an expected profit of $231.483 per day. Additionally, we found that the optimal price to sell the product was $1.359 with an optimal quantity of 455 newspapers per day, using a quadratic programming approach that takes into account the relationship between price and demand. Our sensitivity analysis using bootstrapping allowed us to estimate the degree of variability in our results and construct confidence intervals that give us a sense of the level of uncertainty associated with our estimates. This approach provides a way to ensure that our results are not skewed or biased by overfitting and helps us make more informed decisions based on a more comprehensive understanding of the underlying trends and patterns in the data.

| | Standard NV | Dynamic Price | | | | |
|---|---|---|---|---|---|---|
| | | Mean | 5% | 95% | 2.50% | 97.50% |
| Price | 1 | 0.956 | 0.933 | 0.978 | 0.929 | 0.983 |
| Quantity | 570 | 540 | 487 | 592 | 477 | 602 |
| Expected Profit | 219.28 | 235.08 | 220.16 | 250.05 | 217.16 | 252.95 |

**Figure 7**

Based on the table, we can see that the mean price for the dynamic price model is slightly lower than the standard newsvendor model (0.956 versus 1), with a 95% confidence interval ranging from 0.933 to 0.978 for the dynamic price model. However, at a 5% p-value, the price for the dynamic price model falls outside of the confidence interval. This suggests that the dynamic pricing model may be able to offer more competitive prices, but the significance of this difference is not clear.

In terms of quantity, there is no statistically significant difference between the two models at a 10% p-value, with expected sales of 570 units for both models. This suggests that the pricing model does not have a strong influence on the quantity sold.

Finally, the expected profit for the standard newsvendor model is statistically low compared to the dynamic price model, as shown by the confidence intervals for expected profit. However, the standard newsvendor model sets a high fixed price that falls outside of the confidence interval for the dynamic price model.

In conclusion, while the dynamic pricing model may offer more competitive prices and higher profits than the standard newsvendor model, the significance of these differences requires further analysis at a larger confidence level. Nonetheless, the standard newsvendor model appears to set a higher fixed price, which may impact sales and profits in the long run. The results provide insights for businesses to make more informed decisions on inventory management, taking into account the uncertainties and complexities of real-world operations.