Assignment 01
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1. * Reflexive : A relation is reflexive xRx for all $x \in \mathbb{Z}$.

$x - x = 0$ (since $0 = 0 \cdot k$)
$\therefore xRx$

* Symmetric : A relation is symmetric if whenever xRy, then yRx.

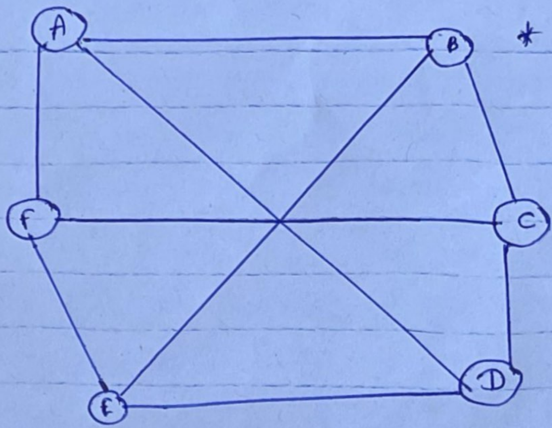
$xRy = x - y$
$x - y = mk$
then $y - x = -(x-y) = -(mk) = (-m)k$.
since $-m$ is also an integer, $y - x$ is divisible by k.
$\therefore yRx$.

* Transitive : A relation is transitive if whenever xRy and yRz, then xRz.

$xRy = x - y = mk$
$yRz = y - z = nk$ (m and n is integer)

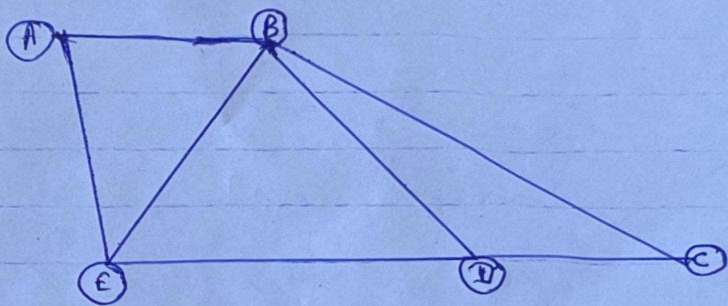
adding the two equations :
$(x-y) + (y-z) = mk + nk$
$x - z = (m+n)k$
Since $m+n$ is also an integer,
$x - z$ is divisible by k.
$\therefore xRz$.

2 (a)



* The sum of degrees: 6 x 3 = 18
since is an even number, a graph with these properties can exist.

(b)



* The sum of the degrees is 4 + 3 + 3 + 2 + 2 = 14.
since 14 is an even number.
∴ a graph with these properties can exist

(c) The sum of the degrees is 3 + 3 + 3 + 2 + 2 = 13.
since 13 is an odd number, no such grap.
can exist.

3 (a)

~~$e_1 = (u_1, u_2)$, e_2~~

graph G the edge based on the adjacency matrix A_G are :

$$\{ (u_1, u_2), (u_1, u_4), (u_2, u_2), (u_2, u_3), (u_2, u_4), (u_2, u_5), (u_3, u_4), (u_3, u_6), (u_4, u_5), (u_5, u_5), (u_5, u_6) \}$$

(b) $e_1 = (u_1, u_2)$, $e_2 = (u_1, u_4)$, $e_3 = (u_2, u_3)$, $e_4 = (u_2, u_6)$, $e_5 = (u_3, u_4)$, $e_6 = (u_4, u_5)$, $e_7 = (u_5, u_6)$

$$M_G = \begin{array}{ccccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & \\ \end{array}$$
$$M_G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}^T$$

$$M_G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$ //

(c) $M_H = A_G \times M_G$

$$M_H = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}^T$$

$$M_H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

* M_H does not represent the incidence matrix of a graph H. because an incidence matrix of an undirected graph must have exactly two non-zero entries in each column while some columns in M_H have more than two 1's.