Emission Spectrum of Hydrogen (Balmer Series) and Determination of Rydberg's Constant

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(Dated: September 14, 2025)

The visible lines of the Balmer series of atomic hydrogen were observed using a diffraction grating spectrometer. Initially, the unknown grating element g was determined by calibrating with known mercury spectral lines and then the wavelengths of hydrogen emission lines were measured and compared with literature values. After fitting the obtained data, the Rydberg constant(R) was determined using a linearized form of the Balmer relation, i.e.,$R = (1.0877 \pm 0.0177) \times 10^7 \ m^{-1}$. This Rydberg constant obtained R is in good agreement with theory within experimental uncertainties.

I. OBJECTIVE

1. To measure the wavelengths of visible spectral lines in Balmer series of atomic hydrogen

2. To determine the value of *Rydberg's* constant

II. EQUIPMENT

1. Spectrometer with telescope and collimator

2. Unknown diffraction grating

3. Mercury spectral tube (for calibration)

4. Hydrogen spectral tube

5. Power supplies for spectral tubes

III. INTRODUCTION

Spectroscopy studies discrete wavelengths emitted or absorbed by atoms and molecules. Among atomic spectra, hydrogen is the simplest and most fundamental, as it consists of only a single electron bound to a nucleus. Also, its spectrum consists Balmer series which lie in visible region, makes the experiment easy to perform. the Balmer series, arises from electronic transitions terminating at the $n = 2$ level, with the initial (upper) levels being $m = 3, 4, 5, \ldots$ (as shown in Fig. 1). In this experiment, a diffraction grating spectrometer is used to measure the diffraction angles θ for spectral lines and compute wavelengths using the grating equation.

IV. THEORY

Diffraction Grating and Calibration[1]

The experiment relies on the principle of diffraction from a grating. If a is the width of the slit (transparent
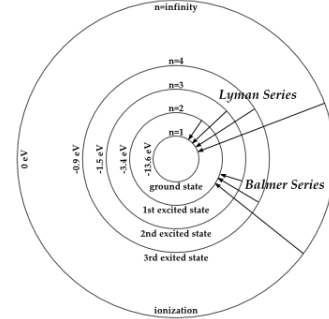
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FIG. 1. Hydrogen energy levels transitions(Balmer series)

spacing) and b is the width of the opaque part, then the grating element is $g = a + b$. For a grating with N lines over a length L, the grating element is

$$g = \frac{L}{N} \tag{1}$$

When monochromatic light of wavelength λ is incident normally on a diffraction grating with a line spacing (grating constant) g, constructive interference occurs at specific angles determined by the grating equation

$$p\lambda = g \sin \theta \tag{2}$$

where λ is the wavelength, θ is the diffraction angle, and p is the order of diffraction ($p = 1$ in this experiment). Calibration of the grating constant g is performed using mercury emission lines of known wavelength.

Bohr Model and Rydberg Formula[2]

According to Bohr's theory, a hydrogen atom consists of a nucleus with a positive charge Ze, and a single electron of charge $-e$, which revolves around it in a circular orbit of radius r(shown in figure 2). Here Z is the atomic number and for hydrogen $Z = 1$. The electrostatic force of attraction between the nucleus and the electron is
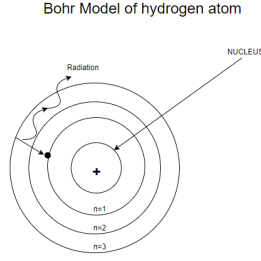
FIG. 2. Bohr model diagram for H-atom

$$F = \frac{kZe \cdot e}{r^2} = \frac{ke^2}{r^2}\,(\text{for Z=1}) \qquad (3)$$

where $k = \frac{1}{4\pi\epsilon_0}$, e the electronic charge and ϵ_0 the permittivity of free space.

To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic attraction. Therefore,

$$\frac{m_e v^2}{r} = \frac{ke^2}{r^2} \qquad (4)$$

where m_e is the mass of the electron.
Bohr postulated that the angular momentum is quantized as

$$m_e v r = n\hbar, \qquad n = 1, 2, 3, \ldots \qquad (5)$$

where $\hbar = h/2\pi$.

Using above two equations 4 and 5, the allowed orbital radii can be calculated as

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} n^2 \equiv a_0 n^2 \qquad (6)$$

where a_0 is the Bohr radius.

Since the total energy of the electron in the nth orbit is the sum of kinetic energy(KE) and potential energy(PE). So, total energy of an orbital in hydrogen atom is

$$E_n = KE + PE = \frac{1}{2}m_e v_e^2 + \left(-\frac{ke^2}{r_n}\right) = -\frac{m_e e^4}{8\epsilon_0^2 h^2}\frac{1}{n^2} \qquad (7)$$

When an electron transitions from an orbit m to n ($m > n$), the emitted photon has energy

$$\Delta E = E_m - E_n = \frac{m_e e^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad (8)$$

Since $\Delta E = hc/\lambda$, this yields

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad (9)$$

where the Rydberg constant(R) is

$$R = \frac{m_e e^4}{8\epsilon_0^2 h^3 c} \qquad (10)$$

PROCEDURE

A. Spectrometer adjustment

1. Level the spectrometer and adjust the telescope focus on a distant object. Determine the vernier constants of the spectrometer as per the instrument manual.

2. Mount the diffraction grating on the prism table and fix the slit and collimator. Do not disturb the spectrometer once aligned.

B. Determination of grating constant g (using Hg lamp)

1. Bring the Hg spectral tube and allow it to warm up for stable emission.

2. With the telescope centered on the direct (zero-order) image of the slit, rotate the telescope(left and right) to observe first-order spectral lines of Hg.

3. Identify well-known Hg lines (shown in table I).

TABLE I. Spectral lines of the Hg lamp

Sl. No.	Colour	Wavelength (nm)
1	Violet	413
2	Indigo	448
3	Green-1	501
4	Green-2	560
5	Yellow	588
6	Orange	614
7	Red-1	627
8	Red-2	637

4. Record the vernier readings for the left and right diffracted positions of each selected Hg line. Compute 2θ and then θ (average left and right).

5. Plot a graph between λ and $\sin\theta$ and determine g from its slope(as in equation 2).

C. Measurement of hydrogen Balmer lines

1. Replace the Hg tube with the hydrogen spectral tube and allow it to warm up.

2. Using the same spectrometer alignment, observe and record the diffraction angles for the hydrogen visible lines.

3. For each line, measure left and right positions and compute the average θ.

4. Compute $\lambda = g\sin\theta$ using the calibrated g.

5. Plot $1/\lambda$ v/s $\frac{1}{4} - \frac{1}{m^2}$ for $m = 3, 4, 5$ and determine R from its slope.

V. OBSERVATION

The experiment is done in two parts. In first part, calibration is done using Hg lamp and data collected to determine the value of g and in the second part, data is taken for hydrogen spectral lines and to find R.

TABLE II. Processed table showing Vernier1 and Vernier2 double-angle values, calculated diffraction angle θ, and corresponding $\sin\theta$ for different spectral lines of grating element.

Colour/λ (nm)	Vernier1 (2θ)	Vernier2 (2θ)	θ (°)	$\sin\theta$
Violet (413)	28.25	28.35	14.15	0.2440
Indigo (448)	30.42	30.50	15.229	0.2627
Green-1 (501)	34.62	34.45	17.27	0.2968
Green-2 (560)	38.43	38.53	19.24	0.3295
Yellow (588)	41	40.80	20.45	0.3493
Orange (614)	41.95	42.43	21.10	0.3599
Red-1 (627)	43.43	43.67	21.77	0.3710
Red-2 (637)	43.95	44	21.99	0.3744

TABLE III. Processed table for the determination of spectrals lines of H

Colour/λ (nm)	Vernier1 (2θ)	Vernier2 (2θ)	θ (°)	λ (in nm)
Red (670 nm)	46.450	46.466	23.299	670.01
Green (490 nm)	34.000	34.066	17.016	497.13
Violet (440 nm)	30.167	30.000	15.042	440.89

VI. CALCULATION

Using the data of table II, λ vs $\sin\theta$ graph is plotted and least square fitted to determine the value of g bu using equation 2. The slope of the fitted curve is the value of grating element g, which come out to be 1699.267 nm with an error 23.36 nm.
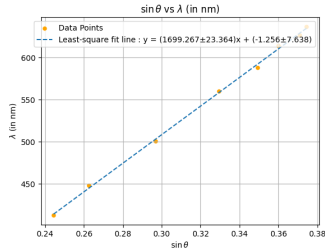

FIG. 3. Least square fit for the determination of g

FIG. 4. Least square fit parameters of Figure 3

Here, λ is calculated from the equaion 2 where g is 1699.267 nm (which determined earlier). To determine the experimental value of rydberg constant(R), $1/\lambda$ vs $\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$ for m = 3,4,5 graph is plotted (from table III) and its least square fit is done. The slope of the plot will give the Rydberg's Constant for hydrogen atom i.e., R = $1.0877 \times 10^7 \ m^{-1}$.

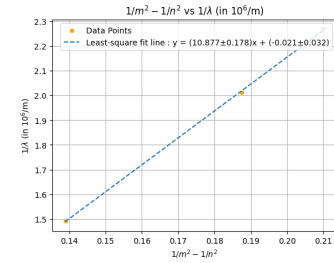FIG. 5. Least square fit to determine Rydberg constant

FIG. 6. Least square fit parameters of Figure 5

All these plots and error analysis is done using python (full code is provided here [3]).

A. Error Analysis

For the least-squares fitted plots — both the fitting of wavelength vs. $\sin\theta$ and the plot of $1/\lambda$ vs. $\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ — the errors in slope and intercept can be estimated from the following relations:

$$\sigma_m = \sigma_y \sqrt{\frac{N}{\Delta}}, \qquad (11)$$

$$\sigma_c = \sigma_y \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{\Delta}}, \qquad (12)$$

where σ_y is the variance of the residuals, N is the total number of data points, and Δ is defined below.

The variance of residuals is

$$\sigma_y^2 = \frac{1}{N-2} \sum_{i=1}^{N} \left(y_i - (mx_i + c) \right). \qquad (13)$$

The quantity Δ is given by

$$\Delta = N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i \right)^2. \qquad (14)$$

After doing propagation error analysis, Δg (error in g) is coming out to be 23.3638 nm, and error in R, i.e., ΔR is $0.0177 \times 10^7 \, m^{-1}$.

VII.   RESULTS AND DISCUSSION

From the calibration of mercury spectral lines, the grating constant was obtained as

$$g = (1699.267 \pm 23.364) \, \text{nm},$$

corresponding to an uncertainty of about 1.38%. This relatively small error confirms that the least-squares fit of λ vs. $\sin\theta$ provided a reliable calibration of the diffraction grating.

Using this calibrated grating constant, the visible Balmer lines of hydrogen were measured. A plot of $1/\lambda$ against $\left(\frac{1}{n^2} - \frac{1}{m^2} \right)$ for $n = 2$ and $m = 3, 4, 5$ yielded a straight line in accordance with the Rydberg relation. From the slope of the fitted line, the experimental Rydberg constant was determined to be

$$R = (1.0877 \pm 0.0177) \times 10^7 \, \text{m}^{-1}.$$

This result is in very good agreement with the accepted value

$$R_{lit.} = 1.09737 \times 10^7 \, \text{m}^{-1},$$

differing by less than 0.88%. Such close agreement validates both the experimental method and the theoretical predictions of the Bohr model of hydrogen. The residual discrepancy can be attributed to several systematic effects:

1. **Instrumental limitations:** The finite least count of the spectrometer and possible parallax errors in reading the vernier scale affect the precision of θ measurements and hence $\sin\theta$ values.

2. **Alignment errors:** Small misalignments of the collimator, grating, or telescope introduce systematic shifts in the measured angles.

3. **Spectral line broadening:** Natural linewidth, Doppler broadening due to atomic motion, and the finite resolution of the spectrometer make it difficult to determine the exact line center.

4. **Calibration uncertainties:** Any inaccuracies in identifying or measuring the mercury calibrion lines directly propagate into the hydrogen line measurements.

5. **Diffraction order effects:** The analysis assumes first-order diffraction ($m = 1$). Overlap with higher orders, if present, could distort the results.

Overall, the experiment demonstrates that while systematic uncertainties dominate, statistical errors from the least-squares analysis are comparatively small. The experimentally determined Rydberg constant thus provides strong confirmation of the quantum structure of hydrogen and highlights the power of optical spectroscopy in measuring fundamental constants.

VIII.   CONCLUSION

Through this experiment, the quantized emission spectrum of hydrogen is successfully demonstrated and a clear verification of the Balmer relation is provided. Calibration of the diffraction grating using known mercury lines allowed the grating element to be determined with good precision. Subsequent measurements of the hydrogen Balmer lines, followed by least-squares fitting, yielded an experimental value of the Rydberg constant,

$$\boxed{R = (1.0877 \pm 0.0177) \times 10^7, \text{m}^{-1}}$$

with slight deviation from the accepted value, i.e., R $= 1.09737 \times 10^7 \, m^{-1}$. This result validates both the Bohr model of hydrogen and the Rydberg formula, reinforcing the quantum nature of atomic spectra. Overall, the experiment illustrates the role of optical spectroscopy as an effective tool in the measurement of nature's fundamental constants, and also emphasizes how precise calibration and systematic error assessment are necessary for bridging theoretical calculations with experimental results.

IX.   PRECAUTIONS

1. Do not touch the grating surface with fingers; handle by edges only.

2. Allow spectral tubes enough warm-up time to reach stable emission.

3. Keep the spectrometer fixed once aligned; avoid moving the optical bench.

4. Avoid parallax errors during taking readings.

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[1] *Emission Spectra of Hydrogen (Balmer Series) Lab Manual*, National Institute of Science Education and Research (NISER) (2017), updated version.

[2] Online resource from google share, `https://share.google/MWKwVqxZSk1ghGMpW`, accessed: 2025-09-14.

[3] A. Arya, Codify, `https://github.com/Anuj-Arya1/Codify.git` (2025).