Measurement of Elementary charge by Millikan oil drop method

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The Millikan oil drop experiment was performed to determine the elementary charge and demonstrate charge quantization. Using both the dynamic and balancing methods, the measured values of the charge were found to be $e = (3.995 \pm 0.349) \times 10^{-18}$ C and $e = (2.599 \pm 0.049) \times 10^{-18}$ C, respectively. Although these results differ from the accepted value of 1.602×10^{-19} C, the observed charges occurred in integral multiples of a fundamental unit, confirming the quantized nature of electric charge. The deviations arise primarily from experimental limitations such as droplet size estimation, timing errors, and limited data points.

I. OBJECTIVE

1. To demonstrate the quantization electrical charge in discrete multiples of the electronic charge (e)
2. To measure the value of e

II. INTRODUCTION

The Millikan oil drop experiment provides a direct, classical measurement of the elementary electronic charge e and a clear demonstration that electric charge is quantized. Charged oil droplets are suspended between two parallel plates: by measuring their terminal fall and rise velocities (with and without an applied electric field) one obtains the droplet radius via Stokes' law (with the Cunningham correction) and hence the net charge on each droplet. Repeating this for many drops reveals that measured charges occur as integer multiples of a smallest unit, identified with e. This simple yet powerful method thus links macroscopic force balance and fluid dynamics to a fundamental atomic constant and remains a cornerstone experiment in precision laboratory physics.

III. THEORY [1]

Consider an spherical oil droplet of radius r and density ρ falling through air of density ρ_a and viscosity η under gravity g. When the droplet reaches terminal velocity v_f (by stoke's law, $F_\eta = 6\pi\eta r v_f$), viscous drag balances effective weight (see figure 1 (a)):

$$6\pi\eta r v_f = \frac{4}{3}\pi r^3 g(\rho - \rho_a). \tag{1}$$

From Eq. (1), the falling velocity v_f is obtained as

$$v_f = \frac{2}{9}\frac{gr^2}{\eta}(\rho - \rho_a). \tag{2}$$

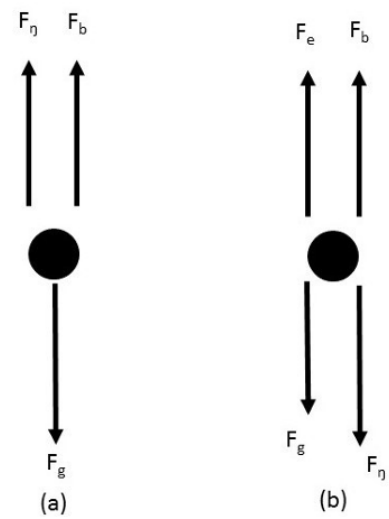

FIG. 1. (a) Forces acting on the oil droplet during the free fall (b) Forces acting during the rise of the droplet

When an electric field $E = V/d$ is applied between the plates, the charged droplet (charge $q = ne$) experiences an upward electric force qE. If v_r is its terminal rise velocity (see figure 1(b)), then:

$$6\pi\eta r v_r = neE - \frac{4}{3}\pi r^3 g(\rho - \rho_a). \tag{3}$$

Combining the two equations 1 and 3, gives the charge on the droplet as:

$$ne = \frac{6\pi\eta r(v_f - v_r)d}{V}. \tag{4}$$

Also, dividing Eq. (4) by Eq. (2), we get (for dynamic method)

$$ne = \frac{4\pi}{3}\frac{gd}{V}(\rho - \rho_a)r^3\left(1 + \frac{v_r}{v_f}\right), \tag{5}$$

Because Stokes' law assumes a continuous medium, a correction factor for small droplet size is applied in equation 2 as:

$$6\pi\eta r v_r = neE - \frac{4}{3}\pi r^3 g(\rho - \rho_a)(1 + \frac{C}{Pr}) \tag{6}$$

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where $C = 6.17 \times 10^{-8}$ m of Hg-m and P (in m of Hg) is the atmospheric pressure.
Equation 6 can be reduced to

$$r^2 + 2\zeta r - \xi = 0 \implies r = -\zeta + \sqrt{\zeta^2 + \xi} \tag{7}$$

where, $\zeta = \frac{C}{2P}$ and $\xi = \frac{9\eta}{2g}\frac{v_f}{(\rho-\rho_a)}$.
Here, positive root is considered.
In the balancing method, the droplet is held stationary by adjusting the applied potential, making the upward velocity zero. Hence, from Eq.5, the charge on the droplet is given by

$$ne = \frac{4\pi}{3}\frac{gd}{V_b}(\rho - \rho_a)r^3, \tag{8}$$

where V_b is the balancing potential.

The charges calculated for many droplets are then divided by the smallest observed charge to obtain integer ratios, confirming that charge occurs in discrete multiples of e.

IV. OBSERVATION AND DATA ANALYSIS

The experiment was performed via two methods: Dynamic method and Balancing method. All required data tables are listed below were generated with the Python code available at [2].

TABLE I. Observational data — Dynamic method

Drop. No	V(V)	t_f (s)	t_r (s)	$t_{f,m}$ (s)	$t_{r,m}$ (s)	$v_f \times 10^{-4}$(m/s)
1	321	8.9	6.3	9.14	6.10	2.19
		8.5	6.4			
		9.1	5.9			
		9.4	6.0			
		9.8	5.9			
2	245	12.5	3.5	13.62	3.52	1.47
		13.9	3.6			
		13.7	3.4			
		14.4	3.6			
3	377	4.9	4.6	4.95	4.53	4.04
		4.9	4.4			
		5.0	4.5			
		4.9	4.4			
		4.9	4.5			
		5.1	4.8			
4	211	6.5	3.1	6.29	3.17	3.18
		6.4	3.1			
		6.4	3.2			
		6.0	3.3			
		6.2	3.0			
		6.4	3.1			
		6.4	3.4			
		6.0	3.2			
5	264	9.8	3.8	10.05	3.72	1.99
		10.4	3.8			
		10.1	3.8			
		10.6	3.5			
		9.6	3.6			
		9.8	3.8			

TABLE II. Observational data — Balancing method

Drop. No	t_f **(s)**	$t_{f,m}$ **(s)**	$v_{f,m} \times 10^{-4}$ (m s^{-1})	V_b (V)
1	10.3	10.73	1.86	145
	10.8			160
	10.8			146
	10.6			141
	10.7			160
	11.2			157
2	2.2	2.22	9.01	208
	2.2			214
	2.2			210
	2.1			208
	2.4			205
	2.2			203
3	4.3	4.38	4.57	274
	4.3			274
	4.2			277
	4.7			273
	4.4			279
4	3.6	3.53	5.67	141
	3.6			141
	3.6			141
	3.5			143
	3.4			141
	3.5			141
5	2.5	2.50	8.00	206
	2.5			204
	2.6			205
	2.5			212
	2.5			208
	2.4			215

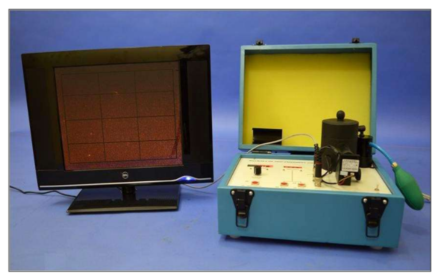

FIG. 2. Millikan oil drop experiment set up

A. Calculations

Common constants and numeric parameters used throughout the calculations are:

$$d = 5 \times 10^{-3} \text{ m}, \qquad L = 1 \times 10^{-3} \text{ m},$$
$$\rho = 929 \text{ kg m}^{-3}, \qquad \rho_a = 1 \text{ kg m}^{-3},$$
$$T = 27.8^\circ\text{C}, \qquad P = 0.76 \text{ m of Hg},$$
$$\eta = 1.8432 \times 10^{-5} \text{ kg m}^{-1}\text{s}^{-1}$$

c = 6.17$\times$ 10^{-8} (Cunningham constant)
At room temperature (27.8 $°C$), the value of η is 1.8610$\times$

$10^{-5}\ \mathrm{kg\,m^{-1}s^{-1}}$ (this value of η is calculated from equation available at [3]) .
Using these values we obtain the common constants:

$$C = \frac{4\pi dg(\rho-\rho_a)}{3} = \frac{4\pi(5\times10^{-3})(9.81)(928)}{3} = 190.13, \tag{9}$$

$$D = \frac{9\eta}{2g(\rho-\rho_a)} = \frac{9(1.8610\times10^{-5})}{2(9.81)(929-1)} = 9.199\times10^{-9}, \tag{10}$$

$$\zeta = \frac{c}{2P} = \frac{6.17\times10^{-8}}{2\times0.76} = 4.06\times10^{-8}. \tag{11}$$

TABLE III. Calculated parameters for the Dynamic method

Drop	ξ (10^{-12})	r (10^{-6}m)	r^3 ($10^{-18}m^3$)	T	ne (10^{-18}C)
1	2.015	1.379	2.624	2.498	3.883
2	1.352	1.123	1.416	4.869	5.351
3	3.716	1.888	6.726	2.093	7.098
4	2.925	1.670	4.659	2.984	12.529
5	1.831	1.313	2.264	3.702	6.034

TABLE IV. Charge analysis for the Dynamic Method

Drop	ne ($10^{-18}C$)	ne/lowest	n_{eff}	e = ne/n_{eff}($10^{-18}C$)
1	3.883	1	1	3.883
2	5.351	1.378	1	5.351
3	7.098	1.828	2	3.549
4	12.529	3.227	3	4.176
5	6.034	1.554	2	3.017

TABLE V. Calculated parameters for the Balancing method

Drop	ξ (10^{-12})	r (10^{-6}m)	r^3 ($10^{-18}m^3$)	ne (10^{-18}C)
1	1.711	1.268	2.039	2.559
2	8.288	2.839	22.87	20.908
3	4.204	2.010	8.122	5.607
4	5.216	2.244	11.29	15.193
5	7.359	2.672	19.09	17.420

TABLE VI. Charge analysis for the Balancing Method

Drop	ne ($10^{-18}C$)	ne/lowest	n_{eff}	e = ne/n_{eff}($10^{-18}C$)
1	2.559	1	1	2.559
2	20.908	8.170	8	2.613
3	5.607	2.191	2	2.803
4	15.193	5.937	6	2.532
5	17.420	6.807	7	2.489

B. Error Analysis

From the calculated values of the electronic charge e_i for individual droplets, the mean and standard deviation were evaluated using:

$$\bar{e} = \frac{1}{N}\sum_{i=1}^{N} e_i, \qquad \sigma_e = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(e_i-\bar{e})^2}. \tag{12}$$

From the individual estimates e_i (last column of each table VI and IV) we obtain :

a. Dynamic method

$$\sigma_{\mathrm{dyn}} = \frac{0.780\times10^{-18}\ \mathrm{C}}{\sqrt{5}} = 0.349\times10^{-18}\ \mathrm{C}$$

$$\bar{e}_{\mathrm{dyn}} = \frac{1}{5}\sum_{i=1}^{5} e_i = (3.995\pm0.349)\times10^{-18}\ \mathrm{C}$$

Equivalently,

$$\bar{e}_{\mathrm{dyn}} = (39.95\pm3.49)\times10^{-19}\ \mathrm{C}.$$

Percentage deviation from the literature value $e_{\mathrm{lit}} = 1.602\times10^{-19}$ C:

$$\mathrm{Deviation_{dyn}} = \frac{\bar{e}_{\mathrm{dyn}}-e_{\mathrm{lit}}}{e_{\mathrm{lit}}}\times100\% \approx 2395\%.$$

b. Balancing method

$$\sigma_{\mathrm{bal}} = \frac{0.109\times10^{-18}\ \mathrm{C}}{\sqrt{5}} = 0.049\times10^{-18}\ \mathrm{C}$$

$$\bar{e}_{\mathrm{bal}} = (2.599\pm0.049)\times10^{-18}\ \mathrm{C}$$

Equivalently,

$$\bar{e}_{\mathrm{bal}} = (25.99\pm0.49)\times10^{-19}\ \mathrm{C}.$$

Percentage deviation:

$$\mathrm{Deviation_{bal}} \approx 1523\%.$$

Remarks. Both methods give mean charges much larger (order-of-magnitude) than the accepted elementary charge. The statistical uncertainties (SEM) are small compared to these systematic discrepancies, indicating the presence of large systematic errors (radius estimation, incorrect assignment of n_{eff}, timing errors, calibration of V_b.

V. RESULTS AND DISCUSSION

From the charge estimates in Tables IV and VI we obtain the following statistical results.

a. Dynamic method

$$\bar{e}_{\text{dyn}} = (39.95 \pm 3.49) \times 10^{-19}\ \text{C},$$

Compared to the literature value $e_{\text{lit}} = 1.602 \times 10^{-19}$ C, the dynamic result shows a percentage deviation

$$\text{Deviation}_{\text{dyn}} = \frac{\bar{e}_{\text{dyn}} - e_{\text{lit}}}{e_{\text{lit}}} \times 100\% \approx 2395\%.$$

b. Balancing method

$$\bar{e}_{\text{bal}} = (25.99 \pm 0.49) \times 10^{-19}\ \text{C},$$

$$\text{Deviation}_{\text{bal}} \approx 1523\%.$$

Both methods yield mean charges that are *one order of magnitude larger* than the accepted value. The quoted statistical uncertainties (SEM ∼1–2% of the mean) are small compared to these very large systematic deviations. This indicates that random scatter among the five droplets is not the dominant source of error; rather, one or more systematic errors strongly bias the results.

The measured charge values show clear multiples of a smallest unit, confirming charge quantization. However, the experimental values deviate significantly from the accepted value of the elementary charge due to limited data, timing inaccuracies, and uncertainties in droplet size estimation. Small sample size and manual observation also contributed to systematic errors. Despite these deviations, the results qualitatively validate Millikan's method and the discrete nature of electric charge.

VI. CONCLUSION

The Millikan oil drop experiment was successfully performed using both the Dynamic and Balancing methods. The mean charge values obtained were higher than the accepted elementary charge, indicating significant systematic errors despite good internal consistency. Possible sources include inaccuracies in droplet radius estimation, voltage calibration or assumptions in the theory. Overall, the experiment effectively demonstrated the quantization of electric charge, even though precise determination of e requires improved calibration and larger data sets.

[1] School of Physical Sciences, National Institute of Science Education and Research (NISER), *Millikan Oil Drop Experiment – Lab Manual*, NISER (2025), accessed: 2025-11-08.

[2] A. Arya, Codify, `https://github.com/Anuj-Arya1/Codify.git` (2025).

[3] B. Hogan, The millikan oil drop experiment: A simulation suitable for introductory physics labs, `https://digitalcommons.gaacademy.org/cgi/viewcontent.cgi?article=1003\&context=gjs` (2016), accessed: 2025-11-08.