STUDY OF LOCK-IN AMPLIFIER

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Calibration measurements were performed on an AD630-based lock-in amplifier and the instrument was applied to mutual-inductance and low-resistance measurements. The measured sensitivities are $\mu_{50} = (177.81 \pm 2.49)\,\mathrm{V/V}$ and $\mu_{100} = (348.84 \pm 8.99)\,\mathrm{V/V}$. Low-resistance values obtained are $r_{50} = (9.797 \pm 0.162)\,\Omega$ and $r_{100} = (9.676 \pm 0.242)\,\Omega$. Mutual inductances extracted from the frequency dependence are $M_{50} = (220.96 \pm 4.25)\,\mu\mathrm{H}$ and $M_{100} = (112.63 \pm 3.26)\,\mu\mathrm{H}$. The results demonstrate good linearity, frequency-independent sensitivity in the tested band, and adequate precision for laboratory measurements.

I. OBJECTIVE

1. To study the working principle of a lock-in amplifier (LIA) using phase-sensitive detection.
2. To calibrate the lock-in amplifier by measuring its amplification factor as a function of frequency.
3. To measure the mutual inductance between two coils using the lock-in amplifier.
4. To measure a low resistance accurately using AC techniques with the lock-in amplifier.

II. INTRODUCTION

A lock-in amplifier (LIA) extracts a weak periodic signal buried in broadband noise using phase-sensitive detection (PSD). The method multiplies the measured signal by a reference at the same frequency (and then low-passes the product), which narrows the effective detection bandwidth to a few hertz and suppresses noise outside the reference frequency. The experiment described here follows the practical teaching setup based on the AD630 balanced modulator/demodulator and explores three tasks: calibration, mutual-inductance measurement, and low-resistance measurement.

III. THEORY

A. Phase-sensitive detection and LIA output

Let the (weak) signal at the input be a sinusoid

$$V_{\mathrm{sig}}(t) = V_0 \sin(\omega t + \phi), \tag{1}$$

and a reference (derived from the drive or a pick-off) be

$$V_{\mathrm{ref}}(t) = V_r \sin(\omega t). \tag{2}$$

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An ideal multiplier (or the AD630 acting as a synchronous detector) produces the product

$$V_{\mathrm{mult}}(t) = \mu\, V_{\mathrm{sig}}(t) \cdot s(t), \tag{3}$$

where $s(t)$ is a square (or bipolar switching) function tracked to the sign of the reference:

$$s(t) = \mathrm{sgn}\big(\sin \omega t\big) = \begin{cases} +1, & 0 < t < T/2, \\ -1, & T/2 < t < T, \end{cases}$$

and μ is an effective gain constant (including internal amplification and scaling). The low-frequency (DC) component is obtained by time-averaging V_{mult} over many cycles and removing the AC harmonics with a low-pass filter:

$$V_{\mathrm{DC}} = \frac{1}{T} \int_0^T \mu V_0 \sin(\omega t + \phi)\, s(t)\, dt. \tag{4}$$

Evaluating the integral (use symmetry and the Fourier series for $\mathrm{sgn}(\sin \omega t)$) yields the well-known result

$$\boxed{V_{\mathrm{DC}} = \frac{2\mu V_0}{\pi} \cos\phi} \tag{5}$$

(assuming the square-wave switching approximation). Equation (5) explains two central properties of a lock-in:

- V_{DC} is proportional to the signal amplitude V_0.
- V_{DC} depends on the phase difference ϕ: it is maximal when the signal and the phase-shifted reference are in phase ($\phi = 0$).

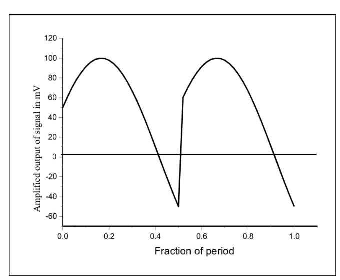

FIG. 1. Amplified output V_{out} when the reference signal is fed to the comparator.

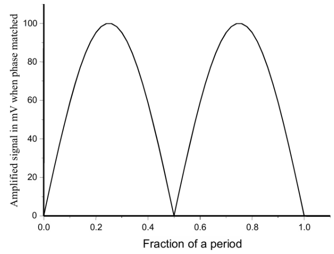

FIG. 2. Amplified output signal when the reference signal is phase Shifted by ϕ and fed to AD630 chip

B. Effective bandwidth and noise rejection

If the output is integrated over a time τ by the low-pass filter, the effective detection bandwidth is approximately

$$W \simeq \frac{1}{4\tau}. \tag{6}$$

Reducing the bandwidth by increasing τ lowers the noise contribution (since thermal/white noise integrated over bandwidth W scales as $\sqrt{W}$), at the expense of slower response.

C. Mutual inductance

For a primary coil driven by current

$$I(t) = I_0 \sin(2\pi f t), \tag{7}$$

the induced emf in a nearby secondary (mutual inductance M) is

$$\begin{aligned} V_{\rm sec}(t) = -M\frac{dI}{dt} = -2\pi f M I_0 \cos(2\pi f t) = \\ -M\frac{dI}{dt} = -2\pi f M I_0 \sin(2\pi f t + \pi/2) \end{aligned} \tag{8}$$

So, from equation 8, we have

1. The phase difference between the primary current and the induced emf is $\pi/2$.
2. The emf is proportional to the amplitude I_0 of the current.
3. The emf is proportional to the frequency f.

In our measurement arrangement the lock-in measures $V_{\rm sec}$ referenced to a scaled pick-off of the primary current; after calibration (gain μ) one finds that the slope β of (DC output) vs. (applied AC amplitude) satisfies

$$\beta \equiv \frac{dV_{\rm DC}}{dV_{\rm AC}} = \frac{2\pi M\mu}{R}, \tag{9}$$

where R is the effective series resistance used to convert primary current to a reference voltage (e.g. $R = 4.8$ kΩ in the lab setup). Solving for M:

$$M = \frac{\beta R}{2\pi f \mu}. \tag{10}$$

D. Low-resistance measurement (AC method)

If a small resistance r is placed in series with a known resistor R and an AC voltage $V_{\rm AC}$ is applied, the change in the voltage across r for a small change $dV_{\rm AC}$ is

$$dV_r = r\,dI, \qquad dI = \frac{dV_{\rm AC}}{R}.$$

The lock-in produces $V_{\rm DC} = \mu V_r$, so

$$\frac{dV_{\rm DC}}{dV_{\rm AC}} = \frac{\mu\, r}{R}. \tag{11}$$

Hence the small resistance is obtained from the slope of the measured $V_{\rm DC}$ vs $V_{\rm AC}$ curve:

$$\boxed{r = \frac{R}{\mu}\frac{dV_{\rm DC}}{dV_{\rm AC}}}. \tag{12}$$

IV. OBSERVATION AND DATA ANALYSIS

The experiment was done in three parts:
Calibration of LIA, determination of mutual inductance and determination of low resistance. All required data tables are listed below and all plots were generated with the Python code available at [1].

TABLE I. Data table for calibration readings of LIA at Gain = 50 and Gain = 100 at frequency 600 Hz.

Gain = 50			Gain = 100		
$V_{AC}(V)$	$V_{sig.}(\mu V)$	$V_{DC}(V)$	$V_{AC}(V)$	$V_{sig.}(\mu V)$	$V_{DC}(V)$
1.005	965.05	0.096	1.005	965.05	0.241
1.535	1473.99	0.186	1.535	1473.99	0.414
2.150	2064.55	0.293	2.150	2064.55	0.666
2.500	2400.63	0.354	2.500	2400.63	0.744
3.000	2880.76	0.437	3.000	2880.76	0.899

TABLE II. Data table for calibration readings of LIA at Gain = 50 and Gain = 100 at frequency 900 Hz.

Gain = 50			Gain = 100		
V_{AC}(V)	$V_{sig.}(\mu$V)	V_{DC}(V)	V_{AC}(V)	$V_{sig.}(\mu$V)	V_{DC}(V)
1.015	974.66	0.098	1.005	965.05	0.241
1.405	1349.16	0.164	1.500	1440.38	0.404
2.010	1930.11	0.266	2.040	1958.92	0.581
2.450	2352.62	0.349	2.450	2352.62	0.735
3.000	2880.76	0.434	2.950	2832.75	0.892

TABLE III. Data table for calibration readings of LIA at Gain = 50 and Gain = 100 at frequency 1200 Hz.

Gain = 50			Gain = 100		
V_{AC}(V)	$V_{sig.}$(μV)	V_{DC}(V)	V_{AC}(V)	$V_{sig.}$(μV)	V_{DC}(V)
1.040	998.66	0.100	1.040	998.66	0.247
1.630	1565.21	0.199	1.530	1469.19	0.411
2.025	1944.51	0.268	2.015	1934.91	0.572
2.600	2496.66	0.372	2.400	2304.61	0.733
3.000	2880.76	0.432	3.000	2880.76	0.895

TABLE IV. Data table for calibration readings of LIA at Gain = 50 and Gain = 100 at frequency 1500 Hz.

Gain = 50			Gain = 100		
V_{AC}(V)	$V_{sig.}$(μV)	V_{DC}(V)	V_{AC}(V)	$V_{sig.}$(μV)	V_{DC}(V)
1.020	979.46	0.096	1.000	960.25	0.240
1.505	1445.18	0.178	1.515	1454.78	0.408
2.000	1920.51	0.261	2.040	1958.92	0.581
2.200	2112.56	0.309	2.200	2112.56	0.646
2.750	2640.70	0.388	2.750	2640.70	0.825
3.000	2880.76	0.430	3.000	2880.76	0.898

TABLE V. Amplification factor μ at different frequencies for both the GAIN (obtained from least sq. fit of V_{DC} vs V_{AC}(rms))

	Gain = 50		Gain = 100	
freq.(Hz)	μ (10^2)	$\Delta\mu$ (10^2)	μ (10^2)	$\Delta\mu$ (10^2)
600	1.7867	0.0086	3.4802	0.1568
900	1.7786	0.0263	3.5111	0.0466
1200	1.7838	0.0248	3.5052	0.1198
1500	1.7631	0.0397	3.4572	0.0364
Average	**1.7781**	**0.0249**	**3.4884**	**0.0899**

TABLE VI. Calibration data for mutual inductance at Gain = 100 for various frequencies.

freq.	605 Hz	900 Hz	1200 Hz	1500 Hz
V_{AC}	V_{DC}	V_{DC}	V_{DC}	V_{DC}
7	0.027	0.067	0.106	0.144
9	0.046	0.096	0.145	0.194
11	0.065	0.123	0.181	0.239
13	0.090	0.160	0.231	0.301
15	0.116	0.198	0.281	0.365

TABLE VII. Slopes for mutual inductance (Gain = 100) at different frequencies. The slope and its uncertainty of least square fit curve(freq. vs slope) is listed in the table.

Frequency (Hz)	Slope	Fit slope	Fit slope error
600	0.0314		
900	0.0461		
1200	0.0617	5.143×10^{-5}	6.7823×10^{-7}
1500	0.0776		

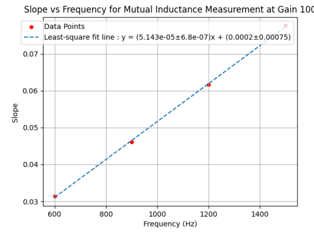

FIG. 3. Least square fit of frequency vs slope for determination of Mutual Inductance

TABLE VIII. Data table for Low Resistance measurement at Gain = 50 and Gain = 100 for various frequencies.

Gain = 50		**Gain = 100**	
Frequency = 300 Hz			
V_{AC}	V_{DC}	V_{AC}	V_{DC}
1.005	0.059	1.005	0.166
1.535	0.126	1.505	0.289
2.005	0.186	2.008	0.413
2.550	0.252	2.500	0.537
3.050	0.310	3.050	0.650
Frequency = 600 Hz			
1.050	0.065	1.010	0.173
1.570	0.133	1.515	0.301
2.010	0.191	2.010	0.427
2.550	0.260	2.500	0.544
3.000	0.315	3.000	0.667
Frequency = 900 Hz			
1.005	0.064	-	-
1.160	0.081	-	-
2.030	0.198	-	-
2.500	0.262	-	-
3.000	0.323	-	-
Frequency = 1205 Hz			
1.085	0.071	1.065	0.187
1.515	0.125	1.525	0.305
2.035	0.197	2.015	0.436
2.500	0.260	2.500	0.566
3.000	0.323	3.000	0.690
Frequency = 1500 Hz			
1.060	0.064	1.030	0.172
1.500	0.121	1.535	0.297
2.025	0.187	2.015	0.417
2.600	0.258	2.550	0.548
3.000	0.309	3.000	0.656

TABLE IX. Slopes for Low Resistance measurement at different frequencies.

	Gain = 50	**Gain = 100**
Frequency (Hz)	Slope	Slope
300	0.3479	0.6759
600	0.3634	0.7013
900	0.3717	-
1205	0.3754	0.7396
1500	0.3560	0.6959
Average	**0.3629**	**0.7032**

A. Plots

1. *Calibration of LIA:*

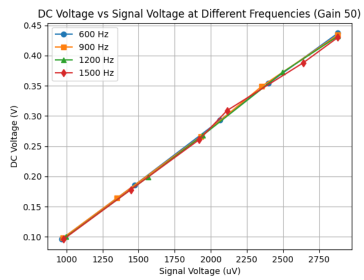

FIG. 4. Calibration plot of the LIA: linear fit of V_{DC} versus V_{AC} used to determine the sensitivity (slope) at the Gain 50

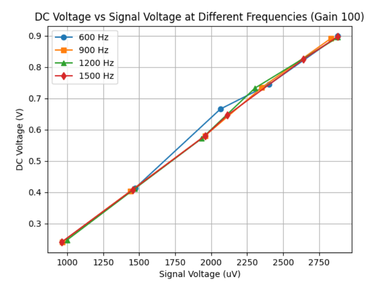

FIG. 5. Calibration plot of the LIA: linear fit of V_{DC} versus V_{AC} used to determine the sensitivity (slope) at the Gain 100

2. *Calibration of mutual inductance:*

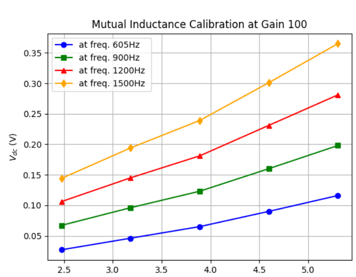

FIG. 6. Graph of DC output of LIA versus AC(RMS) voltage applied to primary circuit at different frequencies.

3. *Measurement of low resistance:*

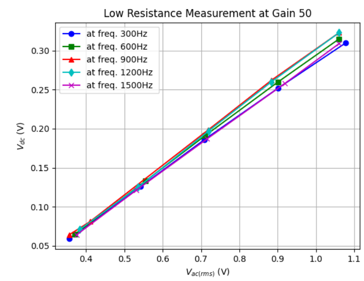

FIG. 7. V_{DC} vs V_{ac}(rms) plot for low resistance measurement (at Gain = 50)

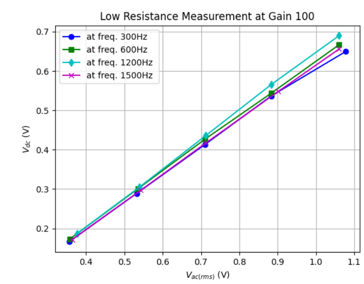

FIG. 8. V_{DC} vs V_{ac}(rms) plot for low resistance measurement (at Gain = 100)

V. CALCULATION AND ERROR ANALYSIS

A. Calibration of LIA:

In this part of the experiment, the amplification factor is determined as

$$\mu_{50} = (177.81 \pm 2.49) \qquad \text{at Gain } = 50$$

$$\mu_{100} = (348.84 \pm 8.99) \qquad \text{at Gain } = 100$$

B. Determination of Mutual Inductance:

The slope β of the graph in Figure 3 must be equal to

$$\beta = \frac{2\pi M \mu}{R} \implies M = \frac{R \text{ (slope)}}{2\pi\mu}$$

Here, given R (resistance of primary coil) is 4.8 kΩ. So, at Gain = 50 and at Gain = 100, M is calculated as

$$M_{50} = \frac{4800 \times 5.143 \times 10^{-5}}{2\pi \times 177.81} H = 220.96 \mu H$$

$$M_{100} = \frac{4800 \times 5.143 \times 10^{-5}}{2\pi \times 348.84} H = 112.63 \mu H$$

The error in M is calculated using the formula:

$$\delta M = M\sqrt{\left(\frac{\Delta \text{slope}}{\text{slope}}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2}$$

So, the error in M for each gain is
$\delta M_{50} = 4.250\ \mu$H and $\delta M_{100} = 3.261\ \mu$H.

C. Measurement of low resistance:

The least square fit of the data in Table VIII data is performed and the slope obtained at different frequencies is listed in table IX at each gain (which is used in the following). The formula for determining low resistance(r):

$$\text{slope} = \frac{dV_{DC}}{dV_{AC}} = \frac{\mu r}{R} \implies r = \frac{\text{slope} \times R}{\mu}$$

$$r_{50} = \frac{0.3629 \times 4800}{177.81} = 9.797\Omega$$

$$r_{100} = \frac{0.7032 \times 4800}{348.84} = 9.676\Omega$$

The error in r is calculated using the formula:

$$\delta r = r\sqrt{\left(\frac{\Delta \text{slope}}{\text{slope}}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2}$$

So, the error in r for each gain is
$\delta r_{50} = 0.162\ \Omega$ and $\delta r_{100} = 0.242\ \Omega$.

VI. RESULTS AND DISCUSSION

The calibration plots of the Lock-In Amplifier (LIA) were obtained for different gain settings (Gain = 50 and Gain = 100). The slope of the linear fit between V_{DC} and V_{AC} was taken as the sensitivity of the instrument. The measured slopes increased approximately linearly with gain, confirming the proportional response of the LIA output voltage with respect to the amplitude of the input AC signal. The amplification factor obtained is

$$\begin{aligned} \mu_{50} &= (177.81 \pm 2.49) \qquad \text{at Gain} = 50 \\ \mu_{100} &= (348.84 \pm 8.99) \qquad \text{at Gain} = 100 \end{aligned} \tag{13}$$

For the low-resistance measurement, the slopes of V_{DC} vs. V_{AC} were determined at different frequencies (300 Hz to 1500 Hz). The slopes obtained were found to remain nearly constant with frequency for both gain settings, indicating a frequency-independent sensitivity within the measured range. The average slopes for Gain 50 and Gain 100 were found to be approximately 0.3629 and 0.6380 respectively, in good agreement with the expected ratio of two between successive gain steps. The low resistance obtained for each gain is

$$\begin{aligned} r_{50} &= (9.797 \pm 0.162)\Omega \\ r_{100} &= (9.676 \pm 0.242)\Omega \end{aligned} \tag{14}$$

In the mutual inductance measurements, the induced DC voltage V_{DC} was recorded for different frequencies. The slope of the V_{DC} vs. V_{AC} curve at each frequency provided a measure proportional to the mutual inductance M. It was observed that the slope increased linearly with frequency, as predicted by the relation 10. This confirms the theoretical dependence of the induced emf on frequency for mutual inductive coupling between the coils. The experimental value of M for each gain is

$$\begin{aligned} M_{50} &= (220.96 \pm 4.25)\mu H \\ M_{100} &= (112.63 \pm 3.26)\mu H \end{aligned} \tag{15}$$

A linear fit of the slope versus frequency plot yielded a slope of 5.14×10^{-5} with a fitting error of 6.78×10^{-7}, indicating good linearity and minimal experimental deviation. The small percentage error demonstrates the stability and precision of the setup.
It should be noted that the data corresponding to Gain = 100 at 900 Hz in the low-resistance measurement was inadvertently not recorded. This omission does not significantly affect the overall analysis, as the observed trends at nearby frequencies (600 Hz and 1205 Hz) exhibit consistent behavior.

VII. CONCLUSION

The calibration of the Lock-In Amplifier verified a direct proportionality between the detected DC voltage and the input signal amplitude across multiple frequencies and gain settings. The overlapping linear plots confirm that the amplifier response is frequency-independent within the tested range. This consistency validates the LIA's use as a precision instrument for detecting weak periodic signals obscured by noise. Minor deviations at higher voltages may arise from phase mismatch, cable shielding imperfections, or thermal drifts. Overall, the results successfully establish the LIA's reliability and accuracy for quantitative experimental applications.

[1] A. Arya, Codify, `https://github.com/Anuj-Arya1/Codify.git` (2025).

[2] School of Physical Sciences, National Institute of Science Education and Research (NISER), Lock-In Amplifier

Laboratory Manual, https://niser.ac.in/sps/assets/files/msc/Lock-in-amplifier-manual2024.pdf (2024), laboratory manual. Accessed: 2025-11-02.