Numerical Analysis of the Rössler Attractor: Comparison of ODE Solvers and Variation of parameters

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This project represents a numerical investigation of the Rössler attractor, a classic example of deterministic chaos in three-dimensional dynamical systems. The work is divided into two parts: first, a systematic comparison of six numerical methods for solving ordinary differential equations (Euler, RK2, RK4, odeint, RK45, and DOP853) with respect to their accuracy and computational efficiency, and second, an investigation of the Rössler system for various parameter regimes using 3D phase space plots, time series analysis, and power spectrum analysis. Our results confirm that the adaptive approach, particularly provided by the application of the odeint method, strikes the optimal balance between speed and accuracy for chaotic systems. Analysis of the power spectrum corroborates the broadband frequency distribution typical for chaotic attractors; variation of the parameters reveals distinct dynamical regimes ranging from periodic to strongly chaotic behavior.

I. INTRODUCTION

A. Motivation

Chaos theory is an interdisciplinary area of scientific study and branch of mathematics. Specifically, it focuses on the underlying patterns and deterministic laws of dynamical systems that are highly sensitive to initial conditions. The Rössler attractor, first proposed by Otto Rössler in 1976, is one of the simplest three-dimensional continuous dynamical systems exhibiting chaotic behavior [1]. In comparison with the more complicated Lorenz system, the Rössler attractor has a rather simple algebraic form and this makes it an ideal system for investigations into chaos and testing numerical methods.

B. The Rössler System [5]

The Rössler attractor is described by three coupled first-order ordinary differential equations:

$$\begin{aligned} \frac{dx}{dt} &= -y - z \\ \frac{dy}{dt} &= x + ay \\ \frac{dz}{dt} &= b + z(x - c) \end{aligned} \tag{1}$$

where x, y, and z are state variables representing the system's position in three-dimensional phase space, and a, b, and c are control parameters. For the standard chaotic regime, typical parameter values are $a = 0.2$, $b = 0.2$, and $c = 5.7$.

C. Objectives

This project has two main objectives:

1. **Method Comparison**: Evaluate the performance of various numerical ODE solvers for integrating the Rössler system, comparing their accuracy (using DOP853 as reference) and computational speed.
2. **Dynamical Analysis**: Investigate how changes in system parameters affect the attractor's geometry, temporal dynamics, and spectral properties.

II. NUMERICAL METHODS

Solving the Rössler system requires reliable ODE integrators. We implemented and compared six integrators that span simple fixed-step schemes and modern adaptive solvers: Euler (1st order), RK2 (midpoint, 2nd order), RK4 (classic, 4th order), odeint (LSODA, adaptive), RK45 (Dormand–Prince, adaptive) and DOP853 (8th order, adaptive). The Rössler system is solved using SciPy's ODE solvers (`odeint`, RK45, DOP853) [4].
In chaotic systems truncation error and exponential error growth make adaptive, higher-order methods generally preferable; DOP853 was used as the benchmark reference, while `odeint` and RK45 provided the best practical balance of speed and accuracy.
All plots and analysis are done with python code (also, scipy library is used), available at[6].

III. PART 1: COMPARISON OF NUMERICAL METHODS

A. Rossler Parameters for plotting

All methods were tested with identical conditions:

1. Initial conditions: $(x_0, y_0, z_0) = (1.0, 1.0, 1.0)$
2. Parameters: $(a, b, c) = (0.2, 0.2, 5.7)$ (chaotic regime)
3. Time span: $t \in [0, 100]$
4. Fixed step size (for Euler, RK2, RK4): $h = 0.01$

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5. Adaptive methods use default tolerances

B. Results

1. *Computational Performance*

TABLE I: Comparison of numerical methods for solving the Rössler system.

Method	Time (s)	Order	Type	Adaptive
Euler	0.8051	1	Explicit	No
RK2	2.5987	2	Explicit	No
RK4	1.8737	4	Explicit	No
odeint	**0.0353**	Variable	Automatic	Yes
RK45	0.1034	4-5	Adaptive	Yes
DOP853	0.2120	8	Adaptive	Yes

Key Observations: The results show that the adaptive solvers clearly outperform fixed-step schemes: `odeint` was the fastest (0.0353 s) and achieved comparable or better accuracy than the others, running roughly 23× faster than RK2 and 53× faster than RK4. In general, adaptive methods (odeint, RK45, DOP853) provided a superior balance of speed and accuracy compared to fixed-step integrators, while RK2 was unexpectedly slow (about 2.6 s) despite being only a second-order method.

Figure 1 depicts the reconstructed Rössler attractors, as seen in three-dimensional phase space trajectories. It's clear that Euler's work displays some distortion in the spiral structure, indicating significant numerical error. In contrast, other methods yield attractors that are qualitatively comparable, maintaining the unique spiral shape. Adaptive approaches, such as odeint, RK45, and DOP853, preserve the attractor structure while utilizing fewer function evaluations.

2. *Accuracy Analysis*

Taking DOP853 as our reference, the root-mean-square (RMS) error is calculated for each method:

$$\text{RMS Error} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \|\mathbf{y}_i^{\text{method}} - \mathbf{y}_i^{\text{DOP853}}\|^2} \qquad (2)$$

Figure 2 shows the error analysis: Accuracy analysis shows distinct variations among the numerical methods. Euler's method has the biggest error (of order 10^1) and hence is not suitable for chaotic systems. The slow RK2 gives amazingly good accuracy and shows the least error among the non-reference solvers. On the other hand, RK4, `odeint`, and RK45 have moderate errors within the range 10^{-1} to 10^0 and therefore provide a better trade-off between accuracy and computational cost.

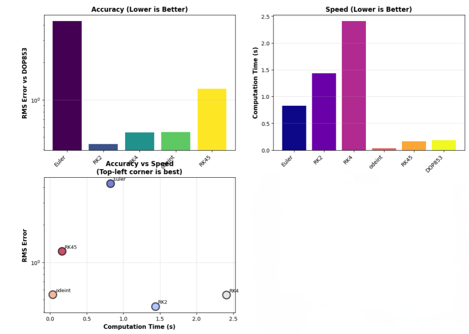

FIG. 2: Performance comparison of numerical methods.Top left: RMS error relative to DOP853 (logarithmic scale). Top right: Computation time in seconds.Bottom: Accuracy vs speed scatter plot.

IV. PART 2: PARAMETER STUDY AND DYNAMICAL ANALYSIS

A. Parameter Regimes

We investigated three different parameter sets to explore how the Rössler system's behavior changes:

1. $(a, b, c) = (0.2, 0.2, 5.7)$ – Standard chaotic regime
2. $(a, b, c) = (0.3, 0.2, 5.7)$ – Increased a parameter (stronger feedback in y equation)
3. $(a, b, c) = (0.2, 0.1, 5.7)$ – Decreased b parameter (weaker z growth)

For each case, we analyzed: (1) 3D phase space structure, (2) temporal dynamics via time series, and (3) frequency content via power spectrum.

a. Case 1: Standard chaotic regime $(a, b, c) = (0.2, 0.2, 5.7)$. The system yields Rössler attractor: a tightly wound spiral in the xy plane with irregular large upward-path in z. Trajectories remain bounded but aperiodic; $x(t)$ and $y(t)$ show irregular oscillations while $z(t)$ shows sharp, irregular spikes. The frequency is broadband with a dominant low-frequency peak corresponding to the timescale, consistent with chaotic dynamics.

b. Case 2: Increased feedback $(a, b, c) = (0.3, 0.2, 5.7)$. Raising a amplifies the y–feedback and increases overall instability: the attractor expands (much larger z excursions) and develops more complex folding. Time series amplitudes increase and spike timings become more irregular, with the power spectrum broadening further, which represents stronger chaotic behavior with richer high-frequency content.

c. Case 3: Reduced growth $(a, b, c) = (0.2, 0.1, 5.7)$. Lowering b reduces the autonomous z growth and yields a cleaner, more regular spiral with smaller z amplitude. Spikes in $z(t)$ become more evenly spaced and the temporal dynamics appear closer to quasi-periodic than in the other cases, although the spectrum still retains a broadband component characteristic of chaotic motion.

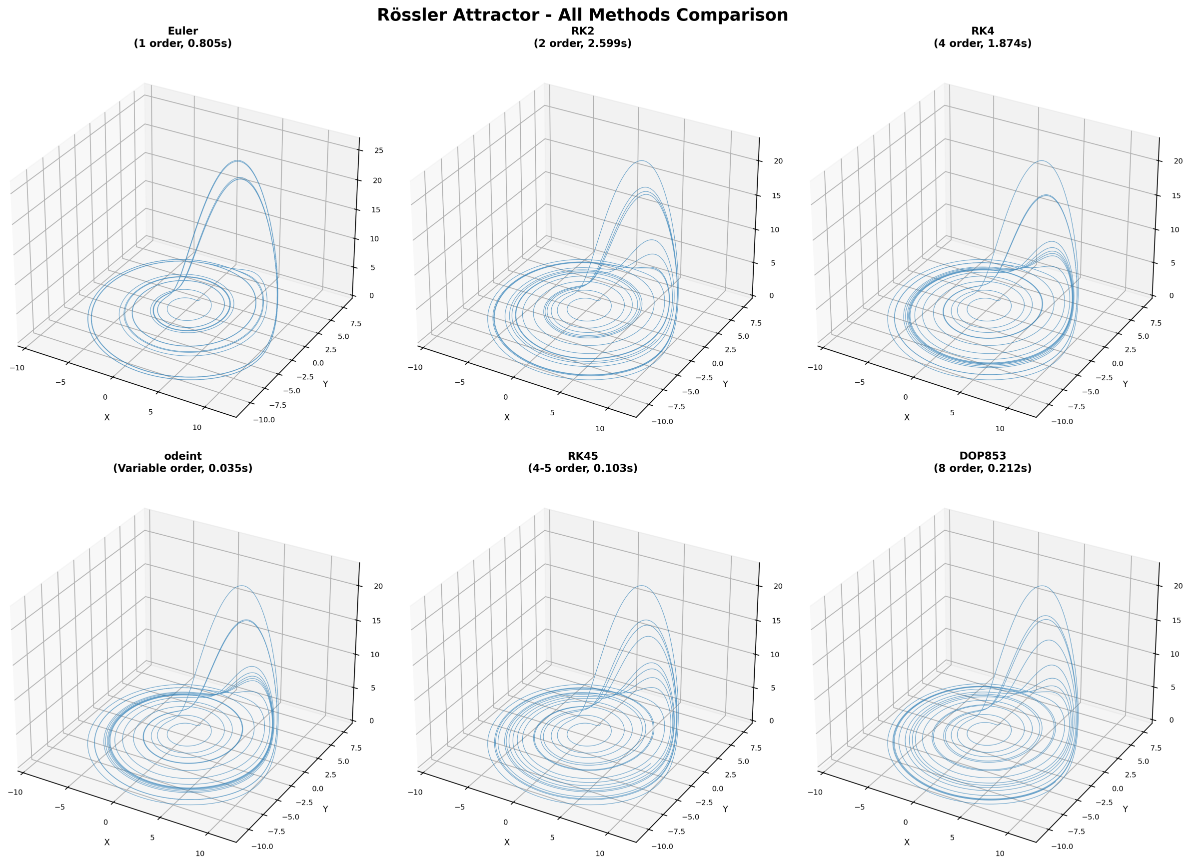

FIG. 1: 3D phase space trajectories of the Rössler attractor computed using six different numerical methods. Each subplot shows the (x, y, z) trajectory along with computation time and method order. Note the visible distortion in Euler's result compared to higher-order methods.

V. DISCUSSION

A. Part 1: Different numerical methods comparison

Among the six methods, `odeint` (LSODA) shows the best overall behaviour between execution time and accuracy: it ran an order of magnitude faster than common fixed-step schemes while matching or exceeding their numerical accuracy. The principal advantages of `odeint` are automatic step-size control, method switching for stiff/non-stiff regimes, and robust error handling; these features remove most manual tuning and make it a reliable default for exploratory work.

High-order schemes like DOP853 are great as reference solutions when very high accuracy is required. Fixed-step algorithms (RK4) have good value and also enable reproducible, controlled experiments where step invariance is important; low-order schemes (Euler, RK2), though, are not suitable for chaotic long-time integrations since their truncation error grows exponentially (due to the positive Lyapunov exponents).

B. Part 2: Variation of parameters: Rossler attractor

In figure 3, the time series plot shows how x,y, and z coordinates vary with time. It can be seen that x oscillates with irregular amplitude and irregular period, and y has similar behaviour but with a shifted phase. While the time series plot of z is quite interesting, spikes are appearing at irregular intervals; these are the peaks which is present in the 3D plot. Because the system is chaotic, the rotation speed varies, and the spiral trajectory stretches and contracts unpredictably. In addition, on varying the parameters, the height of the peak also varies significantly.
In the power spectrum, low-frequency dominance is present, which corresponds to the average rotational frequency of the attractor around its spiral trajectory. The amplitude and period vary irregularly due to the sensitive dependence on initial conditions. So, the power is spread across many frequencies, producing a noise-like plot.

Varying the Rössler parameters produced systematic and interpretable changes in geometry and spectral content. Increasing the feedback coefficient a inflates the attractor and enhances high-frequency content, reflect-

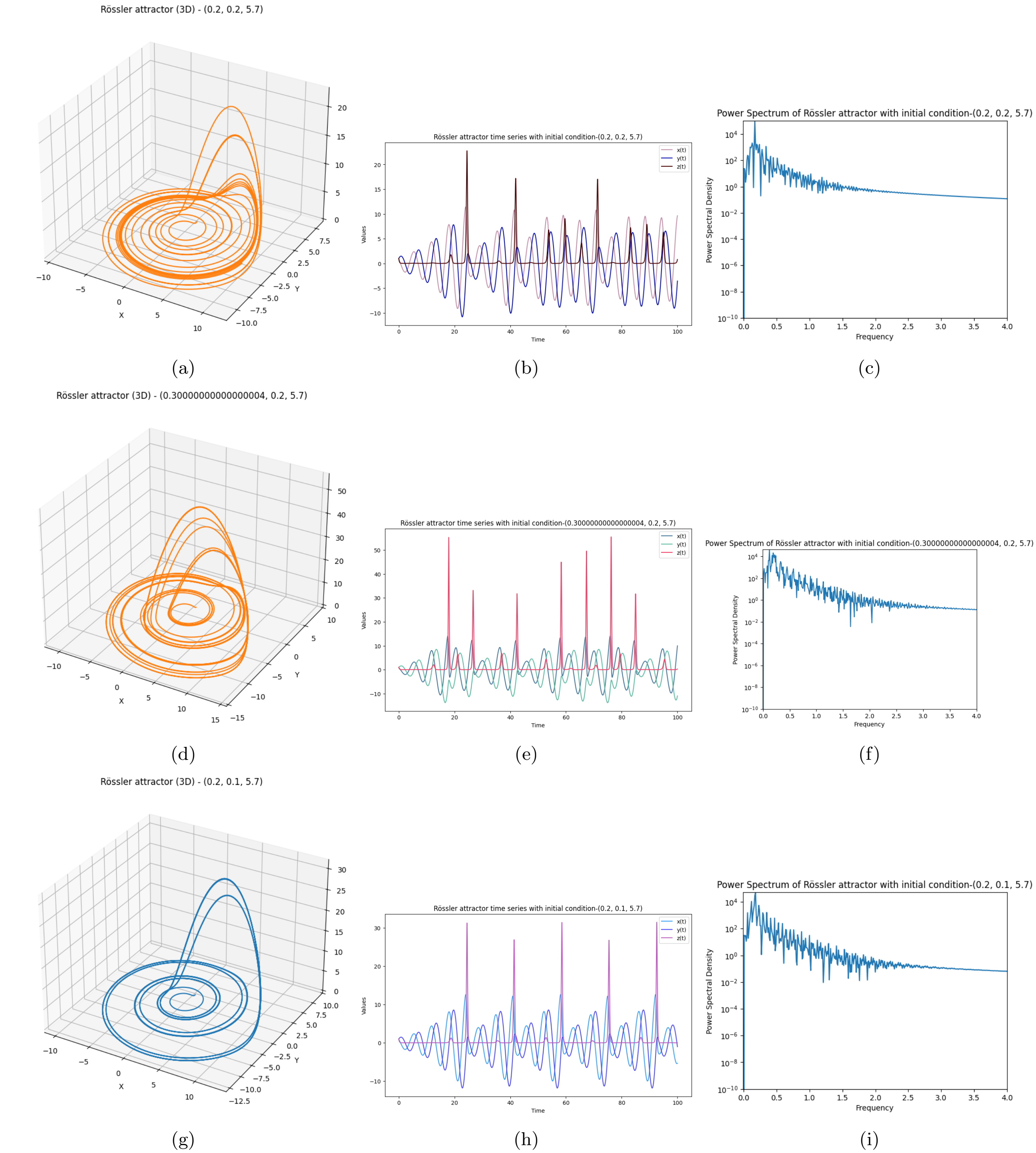

FIG. 3: Nine-panel comparison of the Rössler system for three parameter sets. Each row shows the 3D attractor, time series, and power spectrum for one case. The first row represents the standard chaotic regime $(0.2, 0.2, 5.7)$, the second row shows enhanced chaos for $(0.3, 0.2, 5.7)$, and the third row illustrates a weaker chaotic state for $(0.2, 0.1, 5.7)$.

ing stronger local instability and more complex folding. Reducing the growth parameter b produces a tighter, more regularly spiralling trajectory with reduced spike amplitude—behavior closer to quasi-periodicity while retaining a broadband spectrum. Across all tested regimes the core signatures of chaos (bounded aperiodic motion, broadband power spectra, and sensitive dependence on initial conditions) persist, showing that the Rössler system exhibits robust chaotic dynamics over a wide parameter range and that qualitative transitions (e.g. period doubling) can be revealed by systematic parameter sweeps.

VI. CONCLUSION

This project successfully accomplished two main objectives: comparing numerical methods for chaotic ODE systems and characterizing the Rössler attractor's behavior under parameter variations.

A. Key Findings

1. *Numerical methods*

Adaptive, high-order integrators give the best practical performance for the Rössler system. In our benchmarks `odeint` (LSODA) offered the most favourable speed–accuracy tradeoff, while DOP853 served reliably as a high-precision reference. Fixed-step schemes (Euler, RK2, RK4) require much smaller steps to match accuracy and therefore incur substantial cost; in particular, first-order methods are unsuitable for long-time chaotic integration because truncation errors are exponentially amplified by positive Lyapunov exponents.

2. *Dynamical behaviour*

Parameter changes produce systematic and interpretable changes in the attractor: the standard set $(0.2, 0.2, 5.7)$ yields the canonical spiral with broadband spectrum; increasing a inflates the attractor and broadens the spectrum (stronger chaos); decreasing b tightens the spiral and moves the dynamics closer to quasi-periodicity while retaining chaotic signatures. These trends illustrate robust chaotic behaviour across a wide parameter range and point to clear routes (e.g., period-doubling) between regimes.

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