Solution (The Experimental Question):
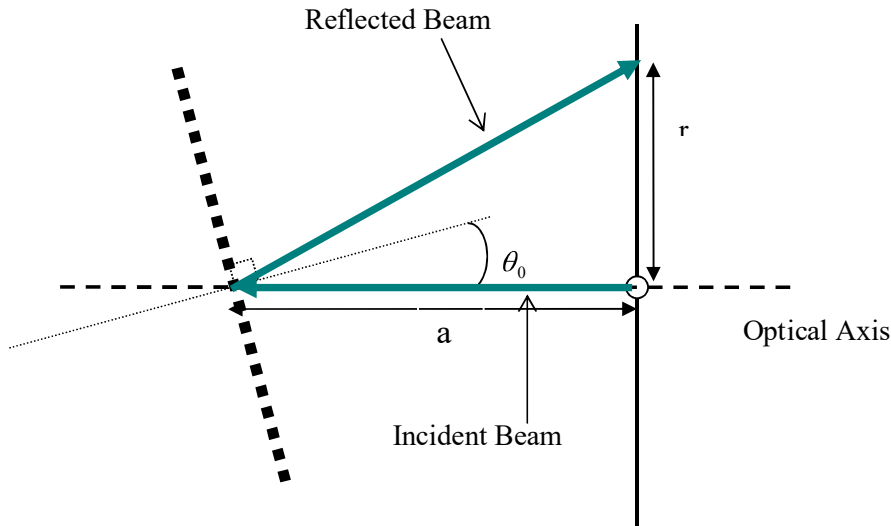
Task 1

1a.

$\Delta\theta_{nominal}=5'=0.08°$

$\Delta\theta_{nominal}$ (degree)	0.08

1b.

If "a" is the distance between card and the grating and "r" is the distance between the hole and the light spot so we have

$$\Delta f(x_1, x_2, ...) = \sqrt{\left(\frac{\partial f}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\Delta x_2\right)^2 + ...}$$

$$\tan(2\theta_0) = \frac{r}{a}, \ If \ \theta_0 << 1 \Rightarrow \theta_0 = \frac{r}{2a} \Rightarrow \Delta\theta_0 = \sqrt{\left(\frac{\Delta r}{2a}\right)^2 + \left(\frac{r\,\Delta a}{2a^2}\right)^2}$$

We want θ_0 to be zero i.e. $r = 0 \Rightarrow \Delta\theta_0 = \frac{\Delta r}{2a}$

$\Delta r = 1mm$, $a = (70\pm 1)mm \Rightarrow \theta_0 = \frac{\Delta r}{2a} \ rad = 0.007 \ rad = 0.4°$

$\Delta\theta_0$	0.4°
θ range of visible light (degree)	$13°\leq \theta \leq 26°$

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1c.

$R_{min}^{(0)}$	(21.6±0.1) kΩ
$\Delta\varphi_0$	5′ = 0.08˚
$R_{min}^{(1)}$	R=(192±1) kΩ

$\Delta\varphi_0$=5′ because

θ = 5′ => R= (21.9±0.1) kΩ

θ =-5′ => R= (21.9±0.1) kΩ

1d.

Table 1d. The measured parameters

θ (degree)	R_{glass}(MΩ)	ΔR_{glass}(MΩ)	R_{film}(MΩ)	ΔR_{film}(MΩ)
15.00	3.77	0.03	183	3
15.50	2.58	0.02	132	2
16.00	1.88	0.01	87	1
16.50	1.19	0.01	51.5	0.5
17.00	0.89	0.01	33.4	0.3
17.50	0.68	0.01	19.4	0.1
18.00	0.486	0.005	10.4	0.1
18.50	0.365	0.005	5.40	0.03
19.00	0.274	0.003	2.66	0.02
19.50	0.225	0.002	1.42	0.01
20.00	0.200	0.002	0.880	0.005
20.50	0.227	0.002	0.822	0.005
21.00	0.368	0.003	1.123	0.007
21.50	0.600	0.005	1.61	0.01
22.00	0.775	0.005	1.85	0.01
22.50	0.83	0.01	1.87	0.01
23.00	0.88	0.01	1.93	0.02
23.50	1.01	0.01	2.14	0.02
24.00	1.21	0.01	2.58	0.02
24.50	1.54	0.01	3.27	0.02
25.00	1.91	0.01	4.13	0.02
16.25	1.38	0.01	66.5	0.5
16.75	1.00	0.01	40.0	0.3
17.25	0.72	0.01	23.4	0.2
17.75	0.535	0.005	12.8	0.1
18.25	0.391	0.003	6.83	0.05
18.75	0.293	0.003	3.46	0.02
19.25	0.235	0.003	1.76	0.01
19.75	0.195	0.002	0.988	0.005
20.25	0.201	0.002	0.776	0.005
20.75	0.273	0.003	0.89	0.01

1e.

In $\theta=-20°$ => $R_{glass}= (132\pm 2)$ kΩ , $R_{film}= (518\pm5)$ kΩ

θ	T_{film}	θ	T_{film}
$\theta = -20°$	0.255	19.25	0.134
		19.50	0.158
		19.75	0.197
		20.00	0.227
		20.25	0.259
		20.50	0.276
		20.75	0.307

Graphics

$T_{film}|\theta=-20°$
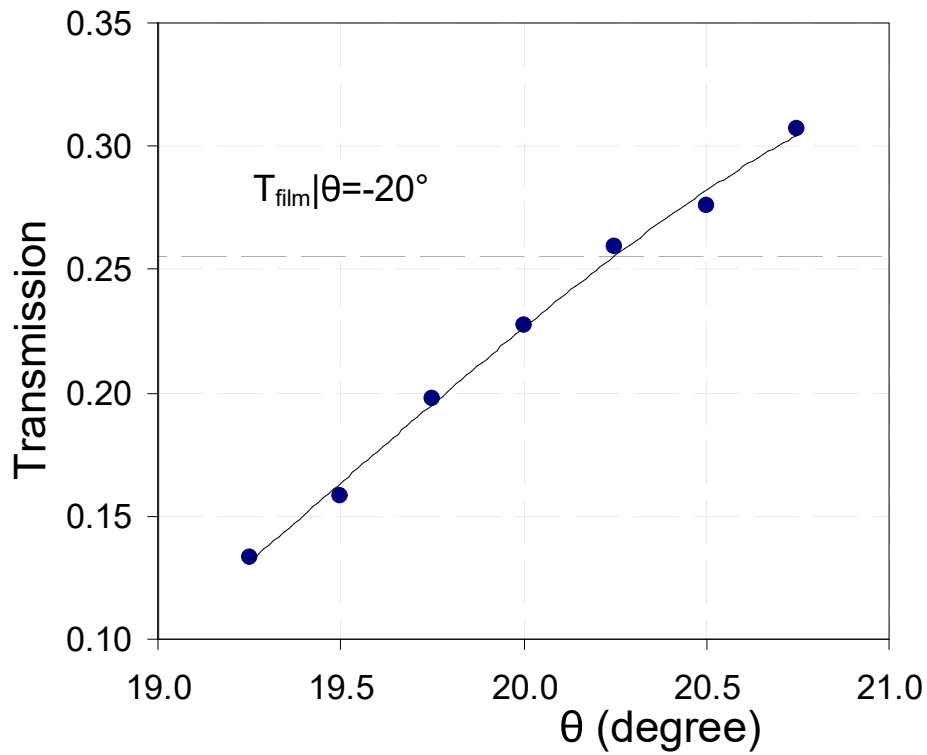
We see that: $T(\theta= 20.25°) = T(\theta= -20°)$

δ (degree)	0.25 ± 0.08

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Task 2.

2a.

$$\lambda = d\,\sin\left(\theta - \frac{\delta}{2}\right) \Rightarrow \Delta\lambda = \lambda\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \cot^2\left(\theta - \frac{\delta}{2}\right)\left(\Delta\theta^2 + \frac{\Delta\delta^2}{4}\right)} \approx d\,\cos(\theta)\left(\frac{0.1\pi}{180}\right)$$

where $\Delta\theta = \Delta\delta = 5' = 0.08$ degree

and $d = \dfrac{1}{600}$ mm

$$\boxed{\Delta\lambda = 2.9\,\cos(\theta)\ (\text{nm})}$$

$$T_{film} = \frac{R_{glass}}{R_{film}} \Rightarrow \Delta T = T_{film}\sqrt{\left(\frac{\Delta R_{film}}{R_{film}}\right)^2 + \left(\frac{\Delta R_{glass}}{R_{glass}}\right)^2}$$

$$\boxed{\Delta T = \frac{R_{glass}}{R_{film}}\sqrt{\left(\frac{\Delta R_{film}}{R_{film}}\right)^2 + \left(\frac{\Delta R_{glass}}{R_{glass}}\right)^2}}$$

2b.

$$13 \le \theta \le 26$$

$$\boxed{2.6 \le \Delta\lambda \le 2.8\ \text{nm}}$$

2c.

Table 2c. The calculated parameters using the measured parameters

θ (degree)	λ (nm)	$I_g/C(\lambda)$ $(M\Omega^{-1})$	$I_s/C(\lambda)$ $(M\Omega^{-1})$	T_{film}	αt
15.0	428	0.265	0.00546	0.0206	3.88
15.5	442	0.388	0.00758	0.0195	3.94
16.0	456	0.532	0.0115	0.0216	3.83
16.25	463	0.725	0.0150	0.0208	3.88
16.5	470	0.840	0.0194	0.0231	3.77
16.75	477	1.00	0.0250	0.0250	3.69
17.0	484	1.12	0.0299	0.0266	3.63
17.25	491	1.39	0.0427	0.0308	3.48
17.5	498	1.47	0.0515	0.0351	3.35
17.75	505	1.87	0.0781	0.0418	3.17
18.0	512	2.06	0.096	0.0467	3.06
18.25	518	2.56	0.146	0.0572	2.86
18.5	525	2.74	0.185	0.0676	2.69
18.75	532	3.41	0.289	0.0847	2.47
19.0	539	3.65	0.376	0.103	2.27
19.25	546	4.26	0.568	0.134	2.01
19.5	553	4.44	0.704	0.158	1.84
19.75	560	5.13	1.01	0.197	1.62
20.0	567	5.00	1.14	0.227	1.48
20.25	573	4.98	1.29	0.259	1.35
20.5	580	4.41	1.22	0.276	1.29
20.75	587	3.66	1.12	0.307	1.18
21.0	594	2.72	0.890	0.328	1.12
21.5	607	1.67	0.621	0.373	0.99
22.0	621	1.29	0.541	0.419	0.87
22.5	634	1.20	0.535	0.444	0.81
23.0	648	1.14	0.518	0.456	0.79
23.5	661	0.99	0.467	0.472	0.75
24.0	675	0.826	0.388	0.469	0.76
24.5	688	0.649	0.306	0.471	0.75
25.0	701	0.524	0.242	0.462	0.77

2d.

Graphics
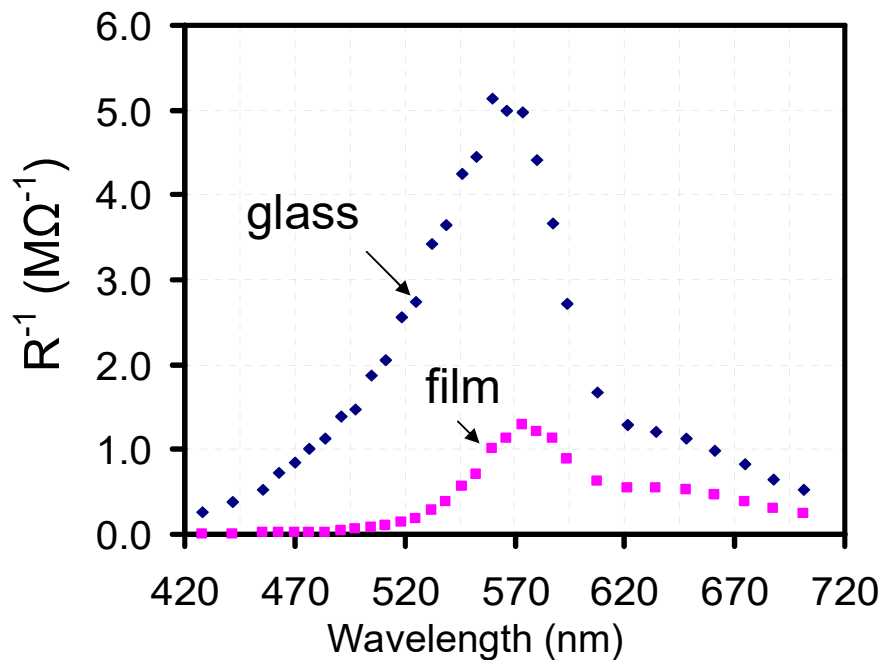
$\lambda_{max}(I_{glass})$	564±5 (nm)
$\lambda_{max}(I_{film})$	573±5 (nm)

2e. Graphics
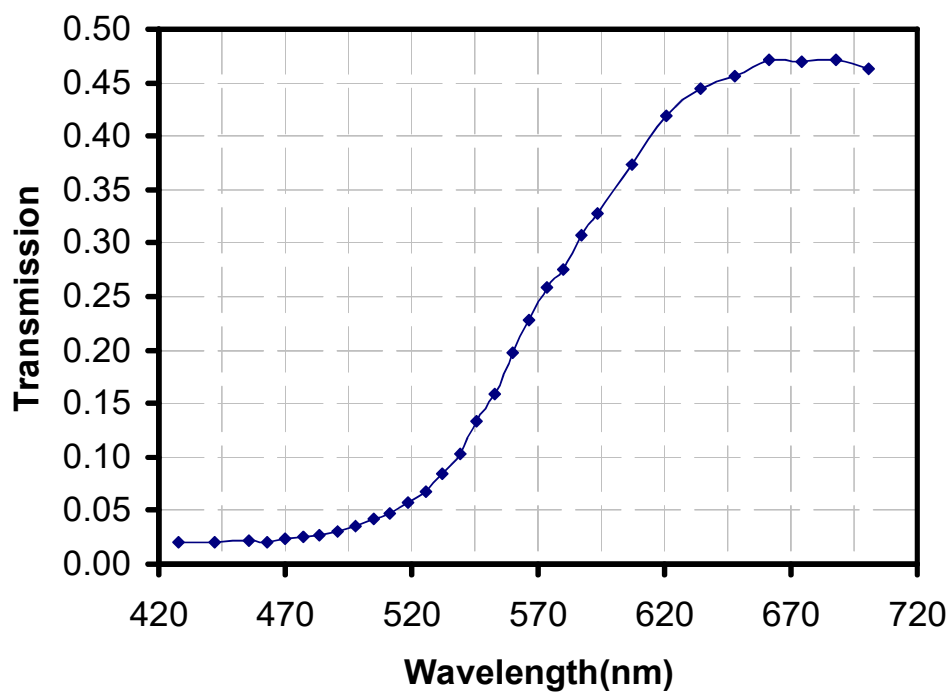
Task 3.

3a.

Table 3a. The calculated parameters for each measured data point

θ (degree)	x (eV)	y (eV^2)
15.00	2.898	126.6
15.50	2.806	121.9
16.00	2.720	108.8
16.25	2.679	107.8
16.50	2.639	98.9
16.75	2.600	92.0
17.00	2.563	86.3
17.25	2.527	77.4
17.50	2.491	69.7
17.75	2.457	60.9
18.00	2.424	55.1
18.25	2.392	46.8
18.50	2.360	40.4
18.75	2.330	33.1
19.00	2.300	27.3
19.25	2.271	20.91
19.50	2.243	17.07
19.75	2.215	12.92
20.00	2.188	10.51
20.25	2.162	8.53
20.50	2.137	7.56
20.75	2.112	6.23
21.00	2.088	5.43
21.50	2.041	4.06
22.00	1.997	3.02
22.50	1.954	2.52
23.00	1.914	2.26
23.50	1.875	1.98
24.00	1.838	1.94
24.50	1.803	1.84
25.00	1.769	1.86

3b.

Graphics
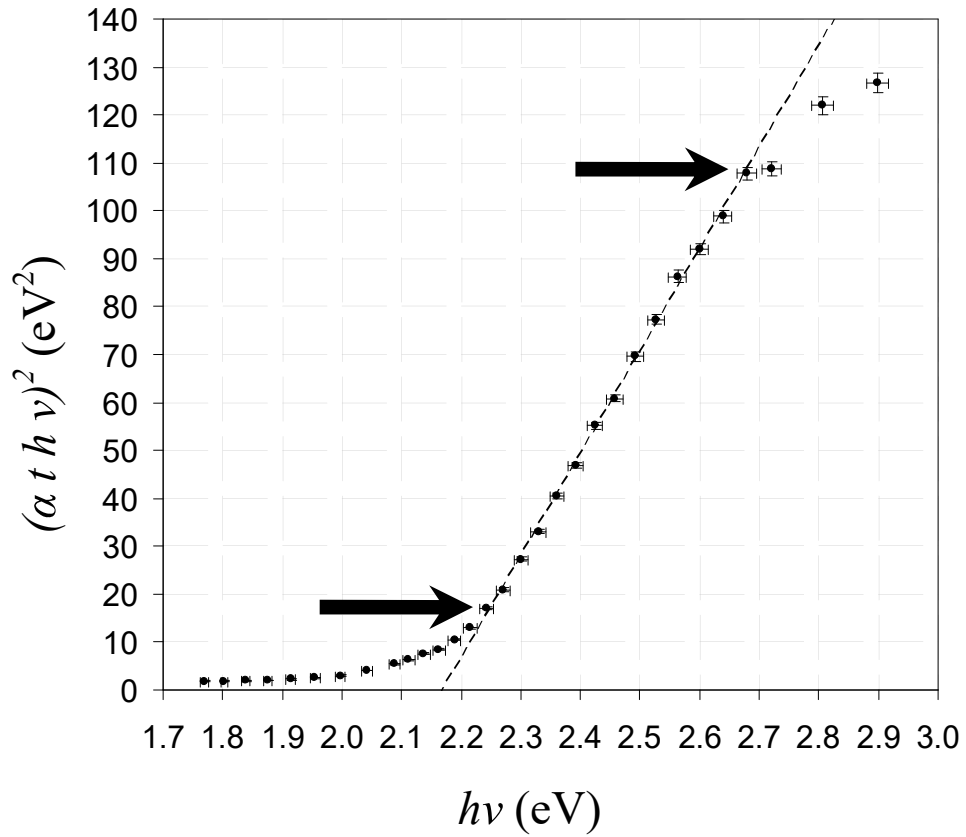
$x_{min} = 2.24 \, (\text{eV})$	$x_{max} = 2.68 \, (\text{eV})$

3c.

$$\alpha h\nu = A\left(h\nu - E_g\right)^{\frac{1}{2}} \Rightarrow \left(\alpha t\, h\nu\right)^2 = \left(A\,t\right)^2\left(h\nu - E_g\right)$$

$$\Rightarrow y = \left(A\,t\right)^2\left(x - E_g\right) \Rightarrow m = \left(A\,t\right)^2 \Rightarrow t = \frac{\sqrt{m}}{A}$$

$$\Rightarrow \frac{\Delta t}{t} = \frac{\Delta m}{2\,m}$$

$t = \dfrac{\sqrt{m}}{A}$
$\Delta t = \dfrac{\Delta m}{2A\sqrt{m}}$

In linear range we have, m=213 (eV), r^2= 0.9986, E_g=2.17 (eV)
and we have A = 0.071 $\left(eV^{1/2}/nm\right)$ so we find t= 206 (nm)

$$\Delta m = \sqrt{\frac{(\delta y)^2 + \dfrac{m^2}{R^2}(\delta x)^2}{\sum_i x_i^2 - N\bar{x}^2}} \approx \sqrt{\frac{(\delta y)^2 + (m\,\delta x)^2}{\sum_i x_i^2 - N\bar{x}^2}} = \sqrt{\frac{(\delta\,xy)^2}{\sum_i x_i^2 - N\bar{x}^2}},\ (\delta\,xy)^2 = (\delta\,y)^2 + (m\,\delta\,x)^2$$

where δx & δy are the mean of error range of x & y

$$\delta x \approx \sqrt{\frac{\sum_i \delta x_i^2}{N}} \ \&\ \delta y = \sqrt{\frac{\sum_i \delta y_i^2}{N}} \quad \text{So} \quad \delta x \approx 0.014\ (eV),\ \delta y \approx 0.9\,(eV)^2$$

→ Δm ≈ 10 (eV) → Δt = t × Δm/(2 m) ≈ 5 (nm)

$$\Delta E_g = \frac{1}{m}\sqrt{\left(\left(\frac{m^2\,\delta x^2 + \delta y^2}{N}\right) + \left(\frac{\bar{y}}{m}\right)^2 \Delta m^2\right)} = \frac{1}{m}\sqrt{\left(\left(\frac{\delta xy^2}{N}\right) + \left(\frac{\bar{y}}{m}\right)^2 \Delta m^2\right)}$$

$$\Delta E_g \approx 0.02\,(eV)$$

Table 3d. The calculated values of E_g and t using Fig. 3

E_g (eV)	ΔE_g (eV)	t (nm)	Δt (nm)
2.17	0.02	206	5