

Recursion & Backtracking - II

Find the sum of digits in a number using Recursion.

$$N = 2314 \quad \text{Sum} \rightarrow 10 \quad \xrightarrow{\text{for } C}$$

$$N=0 \\ \text{ans}=0$$

$$2314 \% 10 = 4$$

$$\downarrow \\ 2314 / 10$$

$$\downarrow \\ 231 \% 10 = 1$$

$$\xrightarrow{231 / 10}$$

$$23 \% 10 = 3$$

$$\stackrel{0}{\cancel{q}} 210$$

$$2 \% 10 = 2$$

$$\xrightarrow{23 / 10} 3$$

Problem



Subproblem (Substructure)

10

(23)¹⁴

4 + (231)

1 + (23)

3 + (2)

2 + (0)

N

p
p1
p2

O(log N) → Space

O(log N) → Time

10 → 2

$$\log_{10} 10 + 1 = 2$$

$$\boxed{\log_{10} n + 1}$$

to calculate number
of digits



[Recursive]
Stack

Recursive Leap of faith

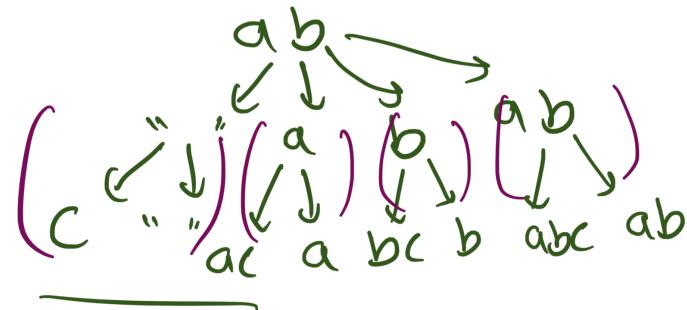
$$\begin{array}{c} (2 \ 3 \ 1 \ 4) \\ \downarrow \\ 4 + (\underline{2 \ 3 \ 1}) \end{array}$$

Generate all Subsets/Subsequence of a String

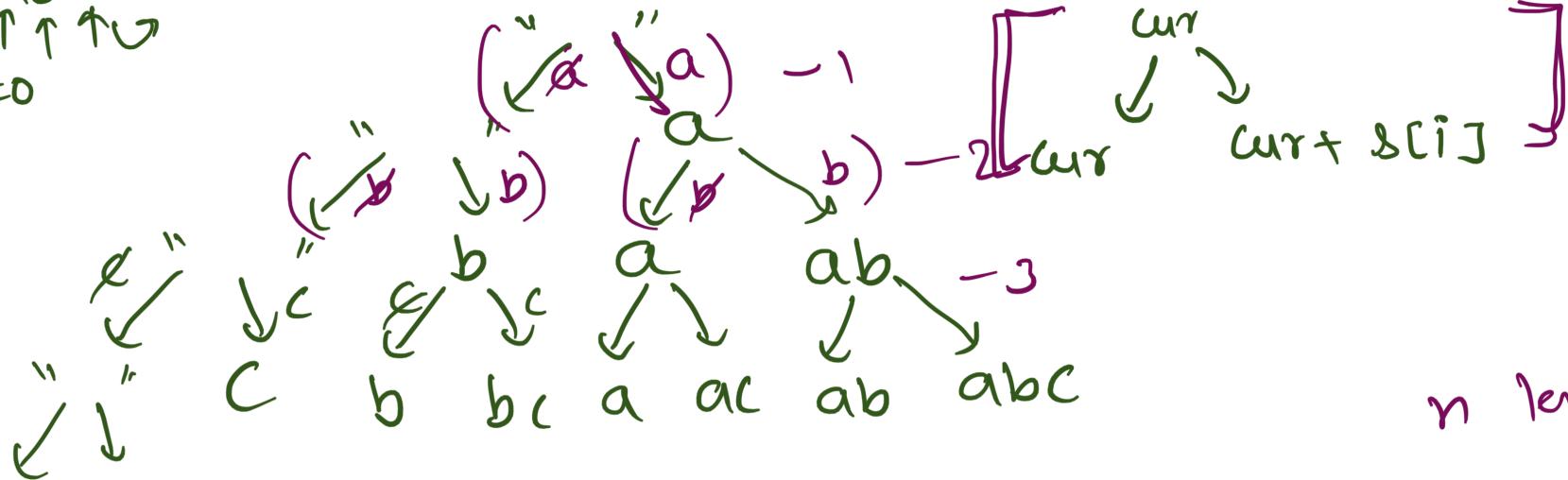
abc
↓
" "
a
b
c
ab
bc
ac
abc

(a b c)
↓
(ab)

→ a string formed by the original string by removing zero or more characters.

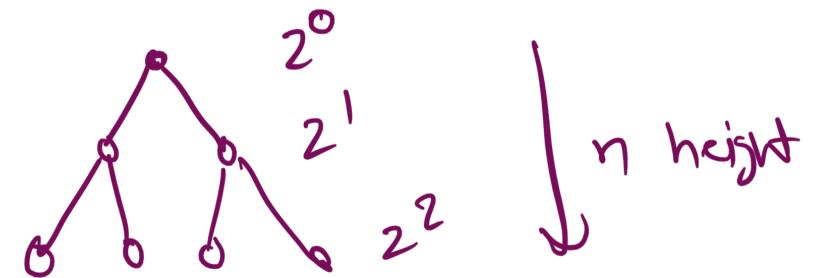


$\begin{matrix} abc \\ \uparrow \uparrow \uparrow \uparrow \\ i=0 \end{matrix}$



base case

$\begin{cases} i = s.length() \\ print(s) \end{cases}$

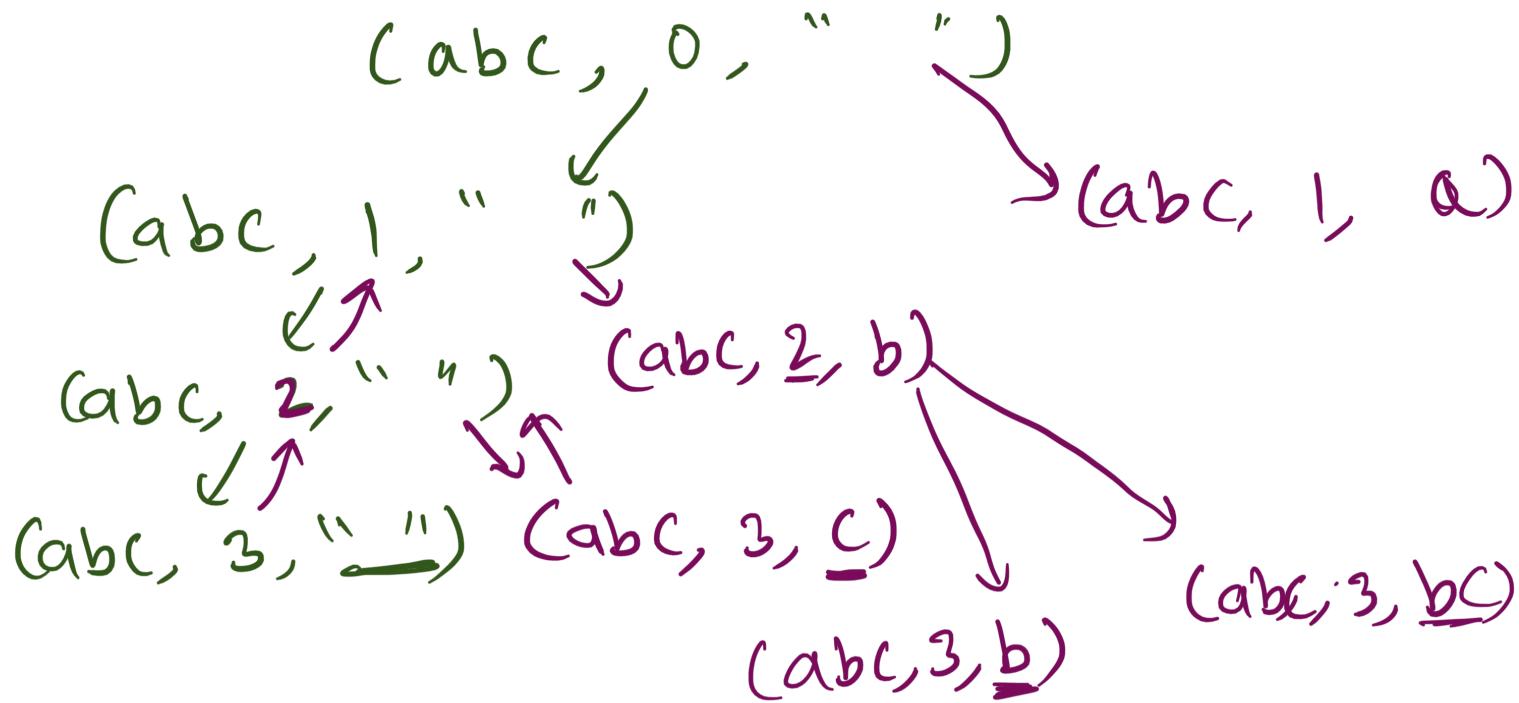


$$\text{Sum} = \frac{a(r^n - 1)}{(r - 1)}$$

$$2^0 + 2^1 + 2^2 + \dots + 2^{n-1} = \boxed{\text{O}(2^n)}$$

Time.

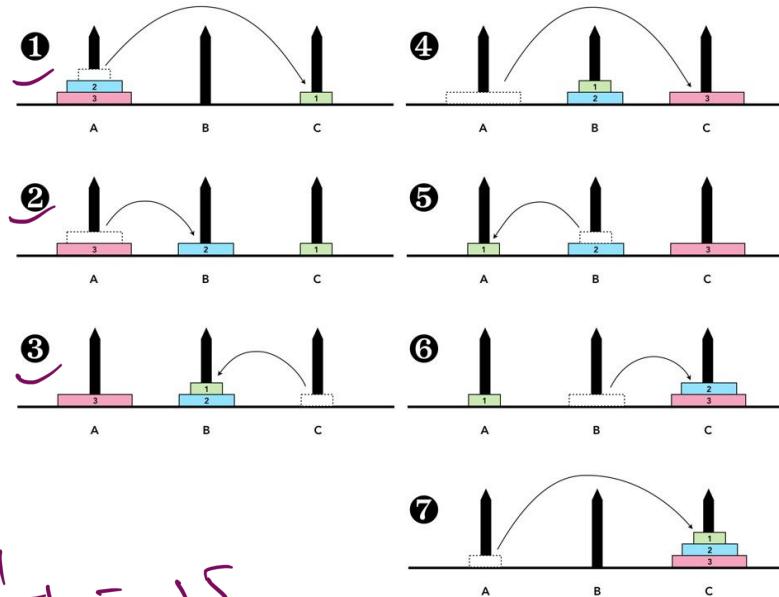
$\text{O}(N)$
Space



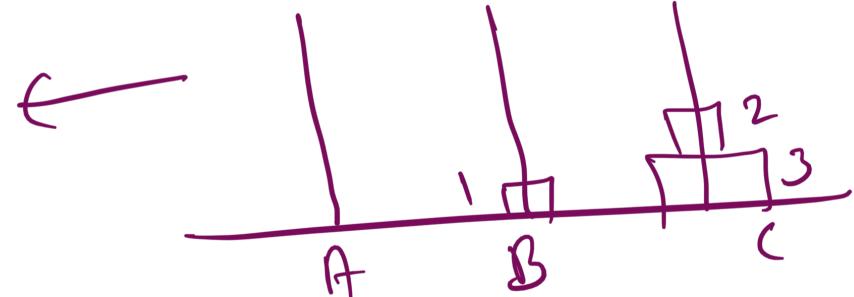
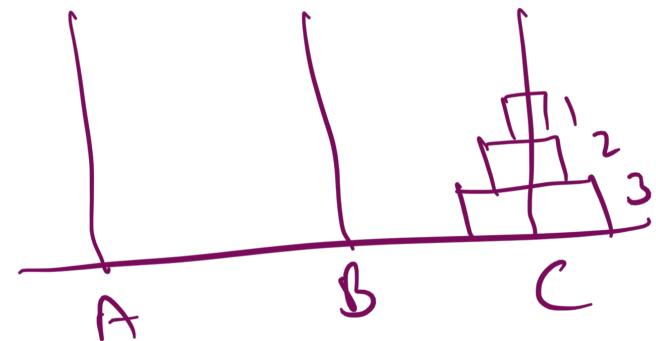
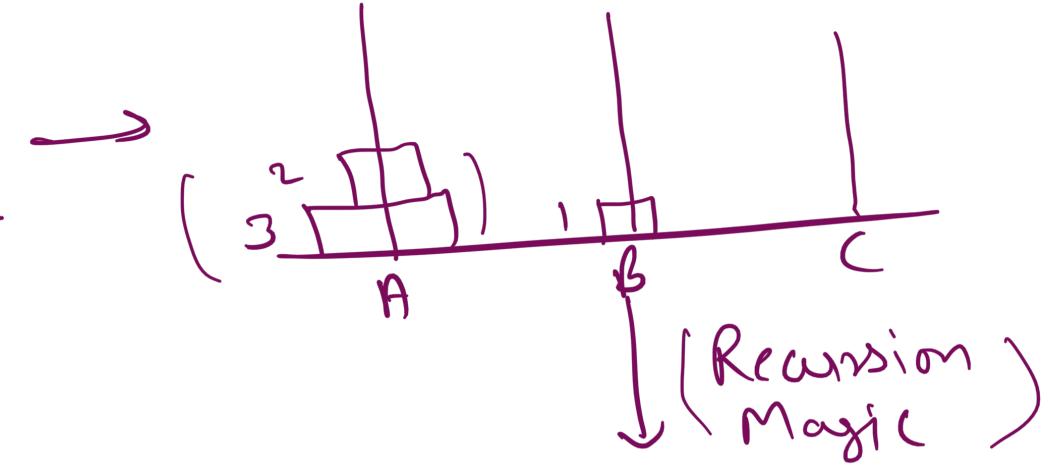
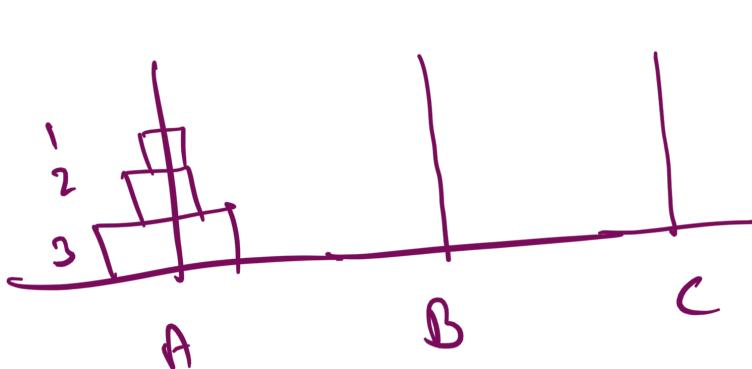
"", c, b, bc

Tower of Hanoi Problem

1. Move 1 from A to C
 2. Move 2 from A to B
 3. Move 1 from C to B
 - 4.
 - 5.
 - 6.
 - 7.
- 3
 $2^n - 1$

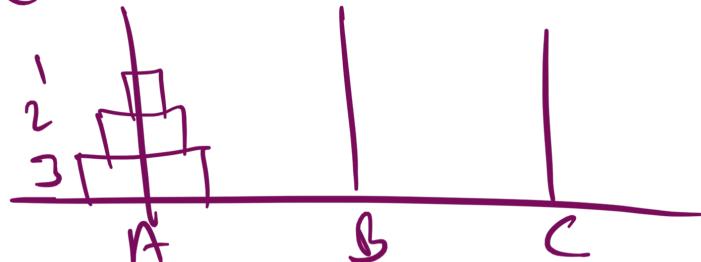


WRONG



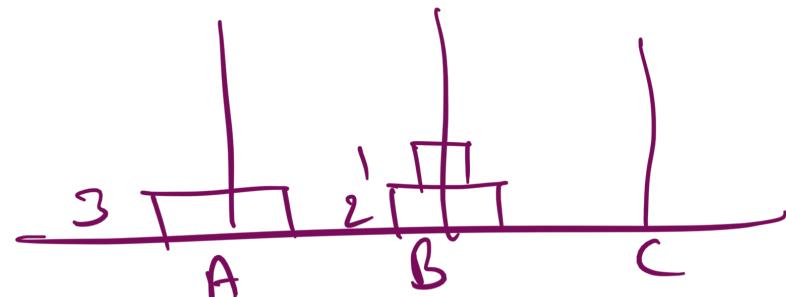
$f(n, A, C, B)$

$f(n, \text{from}, \text{to}, \text{aux})$



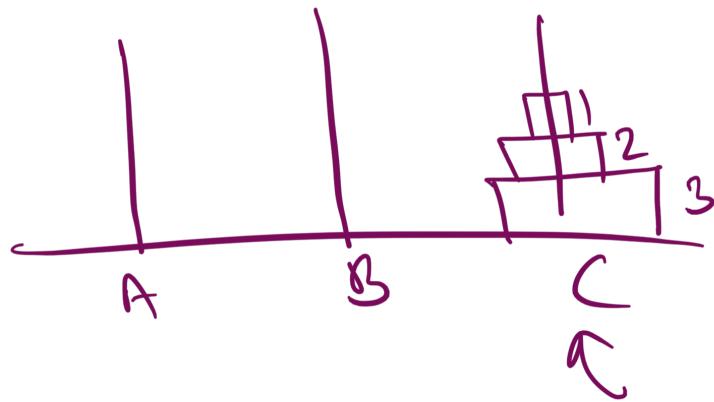
$f(n-1, A, B, C)$

$\xrightarrow{n-1}$



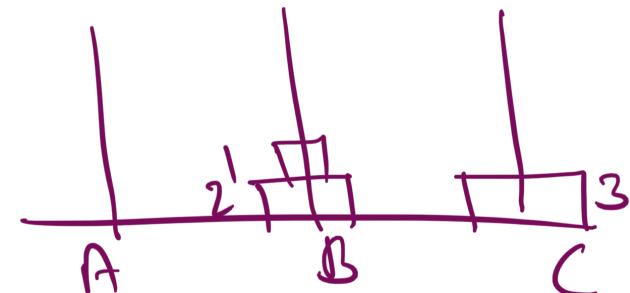
print ("Move n from A to C")

✓

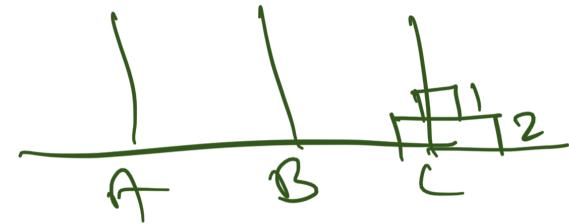
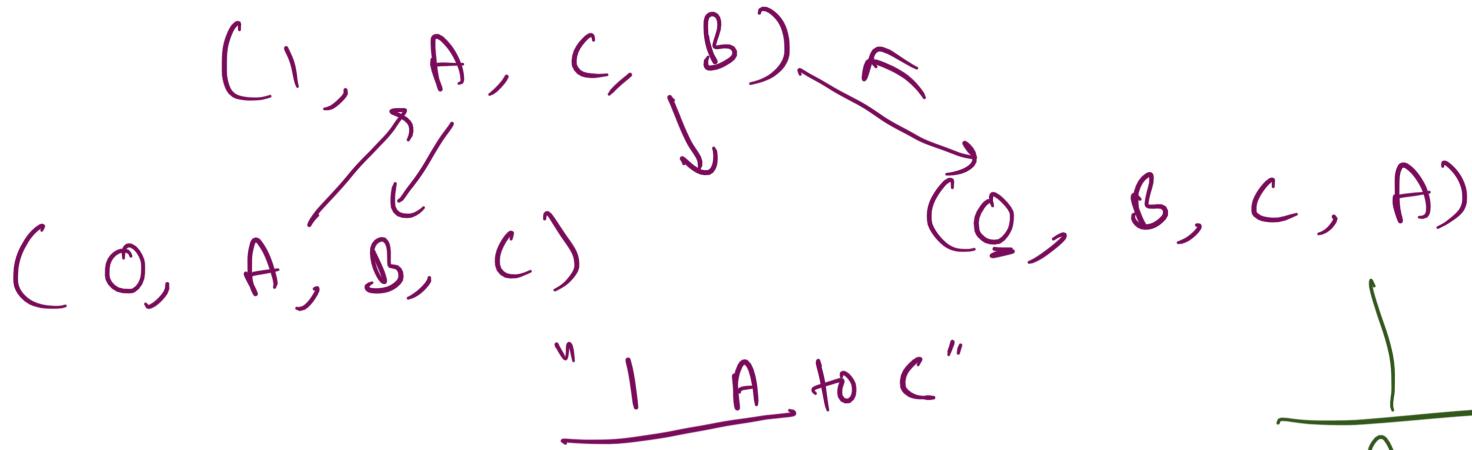


$f(n-1, B, C, A)$

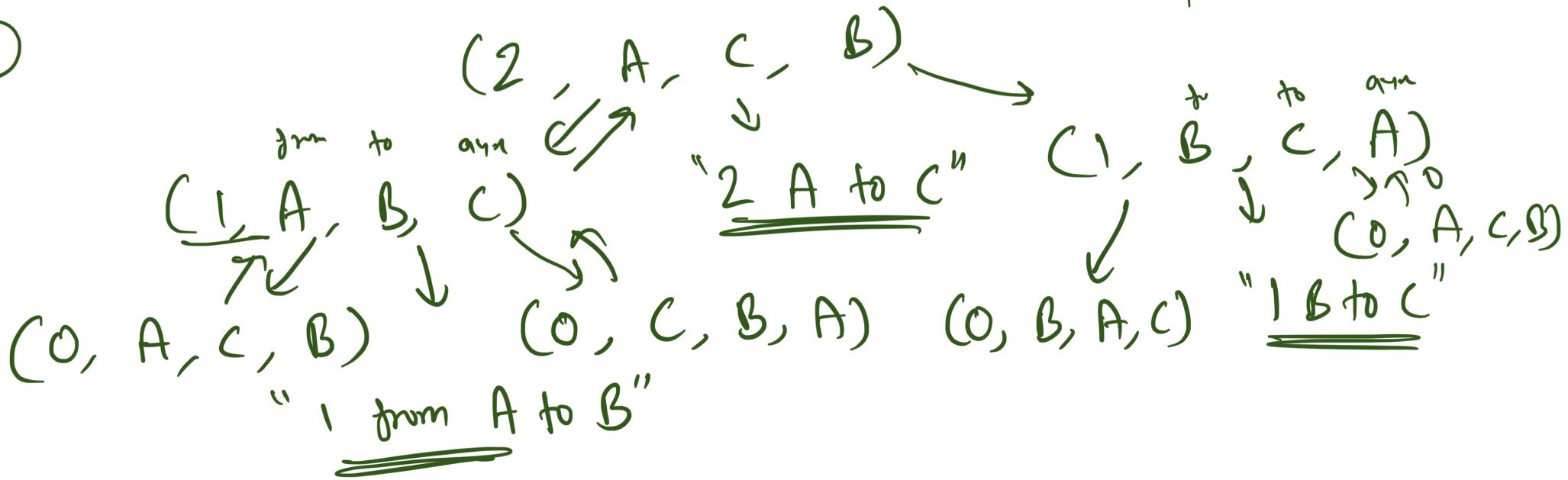
\leftarrow



①



②



Practice Problems

1. Print all permutations of a given String.
2. Rope Cutting Problem - Maximise the number of rods after cutting a rod with three given sizes.
3. Josephus Problem
4. More Recursion Problems:

<https://www.geeksforgeeks.org/recursion-practice-problems-solutions/>