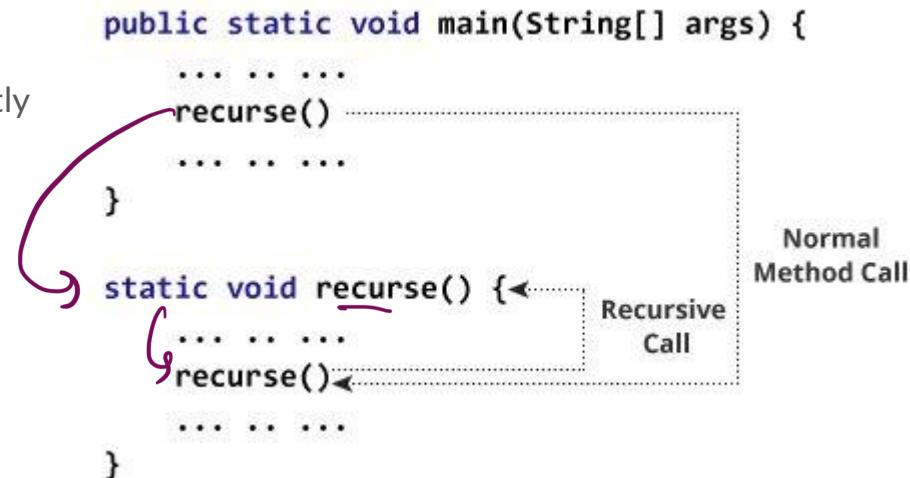


Recursion & Backtracking - I

Recursion

The process in which a function calls itself directly or indirectly is called recursion and the corresponding function is called a recursive function. Using a recursive algorithm, certain problems can be solved quite easily.



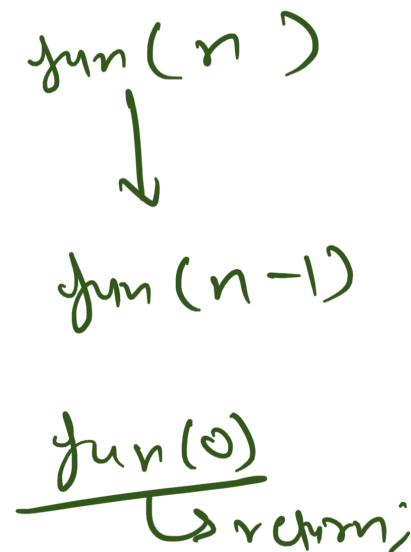
3 Steps for Recursion

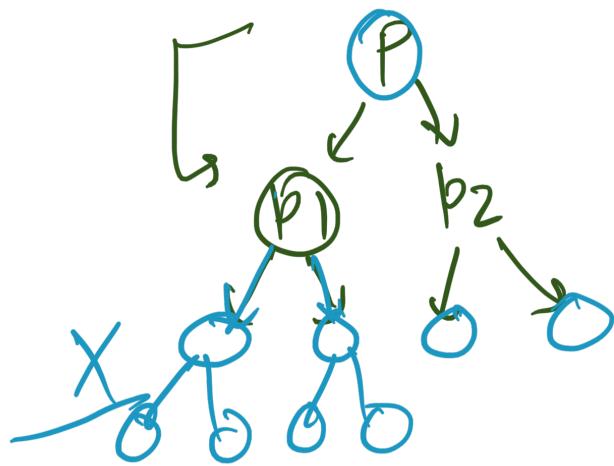
1. Find the base case.
- 2. find the relation b/w problem and the subproblem.
3. Generalise this relation.

a. Print N natural numbers.

1. $\begin{cases} \text{if } (N == 0) \\ \quad \text{return } n; \end{cases}$

2.





1. Is there a subproblem.
2. Do these subproblems have a sub-structure.

$$\begin{aligned} \text{fun}(S) &= "S" + \text{fun}(V) \\ &\downarrow \\ \boxed{\text{fun}(n)} &= "n" + \boxed{\text{fun}(n-1)} \end{aligned}$$

Recursive
Leap of
FAITH

```

    → fun( N ) {
        print( N )
        fun( N-1 )
    }

```

$N=1$

[Tail Recursion]

```

    fun( 5 )
    ↗
    if( n == 0 ) return
    ↗
    fun( n-1 );
    ↗
    print( n );
    ↗
}

```

$N=2$

$O(N)$

Space $O(N)$

\downarrow
 fun(5)
 \downarrow
 fun(4)
 \downarrow
 fun(3)
 \downarrow
 fun(2)
 \downarrow
 fun(1)
 \downarrow
 fun(0)

1
2
3
4
5
5
5
5
5

Sum of n Natural Numbers using Recursion

$$N = 5$$

$$1 + 2 + 3 + 4 + 5 \rightarrow 15$$

1.

$$\text{fun}(0)$$

$\rightarrow N = 0$ return 0;

$\rightarrow \text{if } (N <= 1)$ return N;

2. $\underbrace{\text{fun}(4)}_{1+2+3+4} = \underbrace{\text{fun}(3)}_{1+2+3} + 4$

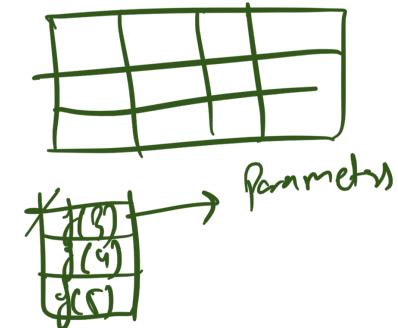
3. $\text{fun}(n) = \text{fun}(n-1) + n;$

Advantages and Disadvantages of Recursion

- When a recursive call is made, new storage locations for variables are allocated on the stack. As, each recursive call returns, the old variables and parameters are removed from the stack. Hence, recursion generally uses more memory and is generally slow.
- On the other hand, a recursive solution is much simpler and takes less time to write, debug and maintain.

Competitive
Programming

$O(N)$
Recursive : 3 rec.
Iterative : 2 rec



Math. Pow (3, 4)

Find the Power of a Number using Recursion

$\text{pow}(a, b) \rightarrow a^b$
 $(3, 4) \rightarrow 3^4 \rightarrow 81$
 $(3, 0) \rightarrow 3^0 \rightarrow 1$

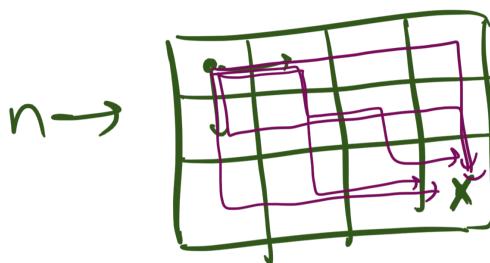
```

if (b == 0)
    return 1;
else
    if (b == 1)
        return a;
    else
        return a * pow(a, b - 1);
}

```

$$3^4 = \underline{3^3} * \underline{3} \quad \begin{matrix} O(b) \\ O(n) \end{matrix} \rightarrow \boxed{O(\log n)}$$

Find the Number of paths in an \overbrace{nxm} Matrix

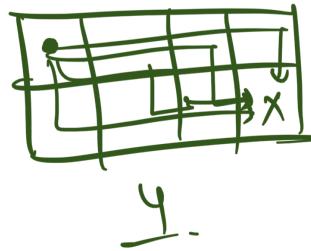


Total number
of paths.

Rule: Right or Down



start $(0, 0)$ top left
end $(n-1, m-1)$ bottom right

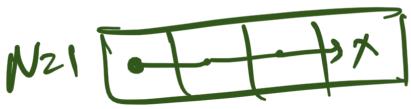


① Base Case:

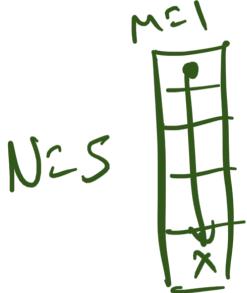
$$N=1$$

$$M=1$$

return 1;

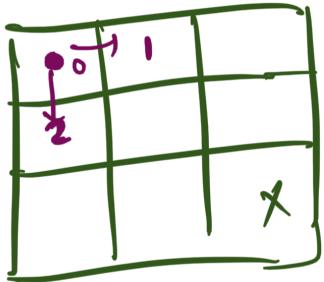


$N=1$ | Path

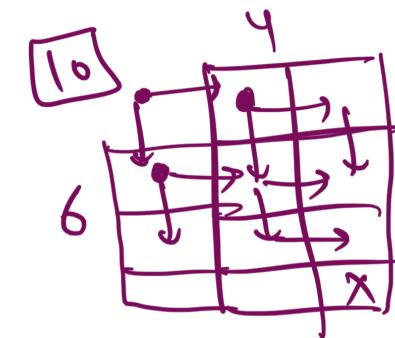


$M=1$ | Path

②



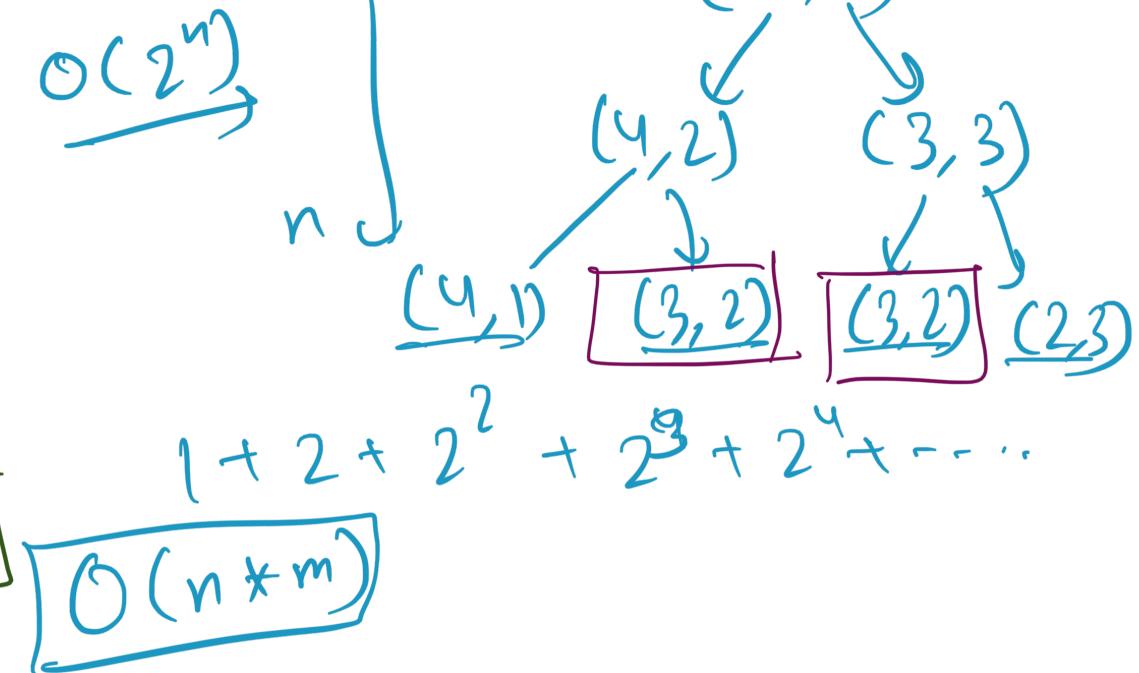
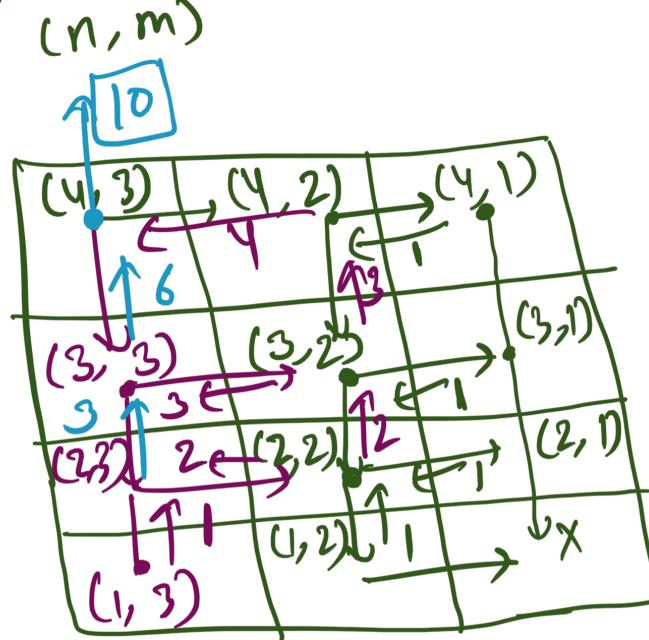
ans(1)
+
ans(2)



③

$$f(n, m) = f(n-1, m) + f(n, m-1)$$

$$\begin{aligned} n &= 4 \\ m &= 3 \end{aligned}$$



Practice Problems

1. Check if an array is a Palindrome using Recursion.
2. Factorial of a Number using Recursion
3. Find the sum of square of N Natural Numbers using Recursion.
4. Find Greatest common divisor of two numbers (GCD using Euclid Formula)
5. More Recursion Problems:

<https://www.geeksforgeeks.org/recursion-practice-problems-solutions/>

Easy .