Tutorial - 2.

Maximum likelihood equation.

$$\ln p(t|x, w, \beta) = \frac{N}{2}\ln\beta - \frac{\beta}{2}\sum_{n=1}^{N}\left(y(x_n, w) - t_n\right)^2 - \frac{N}{2}\ln \quad — ①$$

(Differentiate) Maximizing the above ① eqn. w.r.t w. & equating to zero.

$$\frac{-\beta}{\cancel{2}} \times \cancel{2}\sum_{n=1}^{N}\left(y(x_n, w) - t_n\right) \times \frac{d}{dw}y(x_n, w) = 0$$

$$= \sum_{n=1}^{N}\left(y(x_n, w) - t_n\right) = 0 \quad — ⊛⊛ \qquad \Big| \qquad \frac{d}{dw}y(x_n, w) = 0$$

let $y = w_1 x + w_0 \rightarrow ⊛$ | $\rightarrow w_1 x + w_0 - y = 0$

Given dataset :- (1, 1.2), (2, 1.9); (3, 3.2)

substituting ⊛ in ⊛⊛

$$⊛ \quad \sum_{n=1}^{3}\left(w_1 x + w_0 - t_n\right) = 0$$

$$\Rightarrow (w_1 \times 1 + w_0 - 1.2) + (w_1 \times 2 + w_0 - 1.9) + (w_1 \times 3 + w_0 - 3.2)$$

$$6w_1 + 3w_0 - (1.2 + 1.9 + 3.2) = 0$$

$$6w_1 + 3w_0 = 6.3$$

$$2w_1 + w_0 = 2.1 \rightarrow w_0 = 2.1 - 2w_1$$

②

$III^{ly}$ Now Differentiation eqn (1) with respect to $\beta$

$$\frac{N}{2\beta} = \frac{1}{2}\sum_{n=1}^{N}(y(x_n,\omega)-t_n)^2.$$

$$= \frac{1}{\beta} = \frac{1}{N}\sum_{n=1}^{N}(y(x_n,\omega)-t_n)^2$$

Given that : $\beta = 1$ ; $N = 3$.

$$3 = \sum_{n=1}^{3}(\omega_1 x + \omega_0 - t_n)^2.$$

$$3 = \sum_{n=1}^{3}(\omega_1 x + 2.1 - 2\omega_1 - t_n)^2$$

$$3 = ((\omega_1 + 2.1 - 2\omega_1 - 1.2)^2 + (2\omega_1 + 2.1 - 2\omega_1 - 1.9)^2 + (3\omega_1 + 2.1 - 2\omega_1 - 3.2$$

$$3 = ((0.9-\omega_1)^2 + (0.2)^2 + (\omega_1 - 1.1)^2)$$

$$3 = 0.81 + \omega_1^2 - 1.8\omega_1 + 0.04 + \omega_1^2 + 1.21 - 2.2\omega_1$$

$$3 = 2\omega_1^2 - 4\omega_1 + 2.06.$$

$$2\omega_1^2 - 4\omega_1 - 0.94 = 0.$$

$$\omega_1^2 - 2\omega_1 - 0.47 = 0.$$

$$\frac{b \pm \sqrt{\cancel{b}^2 + 4a\ell}}{2}$$

$$\omega_1 = \frac{2 \pm \sqrt{4 + 4\times 0.47}}{2}$$

$$\omega_1 = \frac{2 \pm \sqrt{5.88}}{2}$$

$$\omega_1 = \frac{2 \pm 2.42}{2}$$

$$\omega = 2.21 \;/\; -0.21$$

For $\omega_1 = 2.21$

$y = 2.21x - 2.32$

For $x_1 = 1$ $y_1 = -0.1$

$x_2 = 2$ $y_2 = 2.1$

$x_3 = 3$ $y_3 = 4.31$

for $\omega_1 = -0.21$

$y = 0.21x + 2.6x$

For $x_1 = 1$ $y_1 = 2.42$

$x_2 = 2$ $y_2 = 2.21$

$x_3 = 3$ $y_3 = 2.00$

$\omega_1 = 2.21$ //

~~eqn~~ $\cancel{y = 2.2x}$

② $= \omega_0 = 2.1 - 2\omega_1$

$= 2.1 - 4.42$

$\omega_0 = -2.32$ //

reqd equation $- \; y = 2.21x - 2.32$