The Math:

Goal:

$$x_d(t) = \frac{W}{2} \sin\left(\frac{2\pi t}{T}\right)$$

$$y_d(t) = \frac{H}{2} \sin\left(\frac{4\pi t}{T}\right)$$

Robot kinematics:

$$\dot{x} = v\cos\theta$$

$$\dot{y} = v\sin\theta.$$

$$\dot{\theta} = \omega.$$

→ So, we have:

$$\dot{x}_d(t) = \frac{W\pi}{T} \cos\left(\frac{2\pi t}{T}\right). = V_d\cos\theta.$$

$$\dot{y}_d(t) = \frac{2\pi H}{T} \cos\left(\frac{4\pi t}{T}\right) = V_d\sin\theta.$$

$$V_d = \sqrt{(\dot{x}_d)^2 + (\dot{y}_d)^2}.$$

&.

$$\theta = \tan^{-1}\left(\frac{\dot{y}_d}{\dot{x}_d}\right).$$

$$\Rightarrow. \ \dot{\theta} = \omega = \frac{1}{1 + \left(\frac{\dot{y}_d}{\dot{x}_d}\right)} \times \left[\frac{\dot{x}_d \times \ddot{y}_d - \dot{y}_d \times \ddot{x}_d}{[\dot{x}_d]^2}\right]$$

Differentiating $\dot{x}_d$ & $\dot{y}_d$ wrt (t) we have:

$$\ddot{x}_d(t) = \frac{-2\pi^2}{T^2} \sin\left(\frac{2\pi t}{T}\right).$$

$$\ddot{y}_d(t) = -\frac{8\pi^2 H}{T^2} \sin\left(\frac{4\pi t}{T}\right).$$

→ Substituting above values of $\dot{x}_d(t)$, $\dot{y}_d(t)$ & $\ddot{x}_d(t)$ & $\ddot{y}_d(t)$, we can get values of v_d & $\dot{\theta}$ [ω] at any instant 't'.

The above equations have been implemented in code in 'homework2.py'.