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Determination of Optimum Inventory Model for Minimizing Total Inventory Cost

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Abstract

In most of the medium scale industries, demand is uncertain and difficult to forecast. Hence Ordering in right quantities at right time is always a crucial issue. In this paper; the authors present a model for determining the ordering policy which will minimize the total inventory cost. This paper takes into consideration various models such as lot by lot size, economic order quantity, periodic order quantity, least unit cost, least total cost, least period cost, Wagner-Whitin algorithm etc. Total annual inventory costs for various items are calculated by each method. The results obtained by applying each model for different items are summarized which shows that Wagner-Whitin algorithm gives optimum cost in each case.

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1. Introduction

Inventory is the stock of any item or resource used in an organization. An inventory system is the set of policies that controls and maintains inventory levels [1]. It decides when stock should be replenished, and how large orders should be. The main concern of any manufacturing organization is to minimize the overall cost and thus increasing profit. The inventory cost comprises of four costs-purchase cost, ordering cost, inventory carrying cost and shortage costs. [2] Establishing the correct quantity to order from vendors or the size of the lots submitted to the firm's productive facilities involves a search for the minimum total cost resulting from the combined effect of the four individual costs [1] Often the inventory managers have to take a crucial decision regarding the balance between ordering cost and carrying cost. If order quantity per unit time is small, the number of orders increases resulting in higher ordering cost, though carrying cost is very less. This sometimes may lead to stock outs, market loss.

Alternately, if a bigger quantity is ordered, number of orders and hence ordering cost reduces considerably. But there is increase in carrying cost. Also more storage space, store staff is required. Storage for longer time may lead to defects in inventory items. Thus a proper balance between carrying costs and ordering cost is very necessary. This leads to development of an effective inventory model that will decide a lot size with minimum total inventory cost.

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Inventory modelling deals with determining the level of a commodity that must be maintained to ensure smooth operation. The basis for the decision is a model that balances the cost of capital resulting from holding too much inventory against the penalty cost resulting from inventory shortage. The principal factor affecting the solution is the nature of the demand: deterministic or probabilistic. The uncertainty in demand makes it hard to maintain and control inventories. Many demand histories behave like random walks that evolve over time with frequent changes in their directions and rates of growth or decline. [3]

1.1. General guidelines for the deciding nature of demand.

In general, the analytic complexity of inventory models depends on whether the demand for an item is deterministic or probabilistic. Within either category, the demand may or may not vary with time.

In practical situations, the demand pattern in an inventory model may assume one of four types: [4]

1. Deterministic and static with time.
2. Deterministic and dynamic with time.
3. Probabilistic and static with time.
4. Probabilistic and dynamic with time.

To determine the nature of the demand, the coefficient of variation V can be used with following guidelines:

1. If the average monthly demand is approximately constant for all months and V is reasonably small (<20%), then the demand may be considered as deterministic and constant.
2. If the average monthly demand is varies appreciably for different months but V is reasonably small (<20%), then the demand may be considered as deterministic but variable.
3. If, in case 1, V is high (>20%), but approximately constant, then the demand is probabilistic and stationary.
4. The only remaining case is the probabilistic non-stationary demand which occurs when the means and coefficients of variation vary appreciably over time.

The above criteria are tabulated as follows:

Table 1: Criteria for deciding nature of demand

Demand	Coefficient of variation	Demand Nature
Constant over periods	<20%	Deterministic & Constant
Varies over periods	<20%	Deterministic but Variable
Constant over periods	>20%	Probabilistic & Stationary
Varies over periods	>20%	Probabilistic & Non-stationary

1.2. Details of current study

The current study is carried out in a medium scale organization which manufactures a large variety of products. Each product consists of 20 to 50 components. Thus the variety of inventory items which the organization has to control is very large. The demand for each item varies greatly over time. Out of the large variety of components, only a few are considered here.

The following table shows the varying nature of demand for these selected items for one year. The mean, standard deviation and coefficient of variation are also calculated.

Table 2: Nature of demand, Standard Deviation and Coefficient of Variation

Month	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
April	4	22	1	936	1716	1
May	18	49	1	2382	2574	0
June	10	27	0	1326	3354	3
July	18	62	0	3120	2730	2

August	14	22	1	1170	3258	0
September	20	46	1	2010	3630	4
October	16	54	1	2652	3024	2
November	16	45	0	2886	3336	1
December	16	40	1	2730	3438	0
January	22	48	0	3042	2478	2
February	16	59	0	2772	3942	1
March	34	61	1	3432	1230	6
Mean	17	44.58	0.58	2371.5	2892.5	1.83
Std. Dev.	6.8	13.7	0.49	790.3	761.1	1.72
CoV	40.04	30.73	84.52	33.32	26.31	94.04

An examination of the mean and the coefficient of variation in the above table, reveals two results:

- Average consumption is dynamic i.e. not approximately constant per month.
- The coefficient of variation is also far greater than 20%.

Thus, comparison of the standard deviation and coefficient of variation with the criteria mentioned above, leads to the development of an inventory model in which the monthly demand is probabilistic and variable.

2. Methodology

2.1. Selection of items for study

The inventory of an industrial organization consists of hundreds of items having widely varying costs, usage and lead time together with procurement and technical problems. It is neither desirable nor possible to exercise the same degree of control over all these items. The management should pay more attention to items whose usage value is high. [2]

The organisation, in which the current study has been carried out, requires 288 different items from external agencies. To simplify the controlling action over the inventory, ABC analysis is done in this case.

ABC (Always Better Control) analysis, contemplates to classify all the inventory items into three categories based on their usage values. Items of high usage value but small in number are classified as A items and would be under strict control of top level management. C items are large in number but require little capital and would be under simple control. Items of moderate value and size are classified as B items and would attract reasonable attention of the middle level management.

Normal inventory items in most organisations show following distribution patterns:
A: 5 to 10% of the total number of items accounting for 70 to 80% of the annual usage value.
B: 10 to 20% of the total number of items accounting for 15 to 20% of the annual usage value
C: 70 to 80% of the total number of items accounting for 5 to 15% of the annual usage value

After carrying out the procedure for ABC classification, it is found that,

1. Only 48 items out of 288 items lay in category A i.e. only 48 items contribute to 80% of the annual usage value.
2. 64 items belong to category B i.e. they contribute to 15% of annual usage value.
3. Remaining 175 items are C category items.

Thus it will be useful for the management to devote much time and effort in the control of these 48 items

All A category items need close control and careful attention. Study of all the 48 items is a tedious task. Hence the selection is further narrowed by selecting some items from A class.

- Item 1: Item with the highest annual usage value.
- Item 2: Item with the lowest annual usage value.
- Item 3: Item with the highest unit cost.
- Item 4: Item with the lowest unit cost.
- Item 5: Item with the highest annual consumption.
- Item 6: Item with the lowest annual consumption

2.2. Inventory Models

Various inventory models are considered here with the aim of minimization of total inventory cost.

Model 1: Lot for Lot: Order exactly what is required for each period.

Here items are purchased in the exact quantities required for each period. It sets planned orders to exactly match the net requirements. It produces exactly what is needed for each period with none carried over into further periods. It does not take into account setup costs or capacity limitations.[1]The ending and beginning inventory is zero as only required quantity is ordered. Thus the carrying cost is zero. The only costs involved are purchase cost and ordering cost. But the ordering activity has to be carried out whenever there is demand which results in higher ordering costs. Thus, this model is suitable in case of large holding costs and low ordering costs.

Model 2: Economic order Quantity:

Economic order quantity is the order quantity that minimizes total inventory holding costs and ordering costs. It is the optimal replenishment order size of inventory item that achieves the optimum total inventory cost during the given period of time. [5] EOQ applies only when demand for a product is constant over the period and each new order is delivered in full when inventory reaches zero. There is a fixed cost for each order placed, regardless of the number of units ordered. There is also a cost for each unit held in storage, sometimes expressed as a percentage of the purchase cost of the item.

Model 3: Period order quantity:

This is the EOQ expressed in terms of periods. POQ=EOQ divided by the average demand per period. The economic-order quantity attempts to minimize the total cost of ordering and carrying inventory and is based on the assumption that demand is uniform. Often demand is not uniform, particularly in material requirements planning, and using the EOQ does not produce a minimum cost.

The period-order quantity lot-size rule is based on the same theory as the economic-order quantity. It uses the EOQ formula to calculate an economic time between orders. This is calculated by dividing the EOQ by the demand rate. This produces a time interval for which orders are placed. Instead of ordering the same quantity (EOQ), orders are placed to satisfy requirements for the calculated time interval. The number of orders placed in a year is the same as for an economic-order quantity, but the amount ordered each time varies. Thus, the ordering cost is the same but, because the order quantities are determined by actual demand, the carrying cost is reduced.

Model 4: Least unit cost:

The goal of least unit cost method is to minimize the average cost per unit. This method does not perform well if the ordering costs are constant. [1] Whenever the net requirements are positive, we find the order size that will cover the next "n" periods, where "n" is set to minimize the average cost per unit.

The least unit cost method is a dynamic lot-sizing technique that adds ordering and inventory carrying cost for each trial lot size and divides by the number of units in each lot size, picking the lot size with the lowest unit cost. The advantage of the least unit cost method is that it is a more complete analysis and it would take into account ordering or setup costs that might change as the order size increases. [1]

Model 5: Least total cost:

The least total cost method is a dynamic lot sizing technique that calculates the order quantity by comparing the carrying cost and the ordering cost for various lot sizes and selects the lot in which these are most nearly equal. [1]Whenever the net requirements are positive, we find the order size that will cover the next "n" periods, where "n" is the period where the carrying cost and ordering cost are the closest.

When lot size is increased, the number of orders and thus ordering cost reduces, but the inventory carrying cost increases considerably. On the other hand, if lot size is small, carrying cost decreases with increase in number of orders and ordering cost. This model finds out the number of components to be ordered with nearly same ordering cost and carrying cost.

Model 6: Least period cost:

Whenever the net requirements are positive, we find the order size that will cover the next "n" periods, where "n" is set to minimize the average cost per unit time. This method tries to minimize the cost per unit time.

Model 7: Wagner-Whitin Algorithm [[6]

It is accurate method for determining the optimal batch size for a product with dynamic demand with single-stage production without consideration of capacity constraints.

As with the classical lot size formula assumes an infinite production rate and of a uniform consumption over the period, in W-W process in a forward calculation possible alternatives are identified and then selected in a backward calculation, the optimal strategy. The Wagner-Whitin method produces the optimum even if fixed costs vary from period to period. The condition is that the applicable fixed costs are used in the periods.

An important implication of the Wagner-Whitin- Algorithm is the zero-inventory property: a production takes place only when the store is empty. Then the optimum of the complete needs of a period is either fully covered from stock or from the production of this period. A situation that is, when the demand is partially satisfied out of production and partly from the camp, so that may arise in a period of storage and setup costs, may not be cost-minimal, because the set-up costs can be saved by advancing the production. An optimal lot includes always a sum of complete period requirements. This general finding has been considered in the heuristic method to determine the lot size as a basis.

To find out the best models in current situation, these models are applied to the demand data for one year of selected products.

2.3. Assumptions

The current study is based on following assumptions:

1. Lead time in each case is zero.
2. The procurement cost is the cost of purchasing the item.
3. The holding cost is per unit per period cost, not a carrying charge.
4. The ordering cost for any item is constant

3. Data

The data involved in current analysis is as follows:

3.1. Demand Data

1. Demand data: The following table shows the demand for the items under consideration for one year.

Table 3: Demand per month

Month	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
April	4	22	1	936	1716	1
May	18	49	1	2382	2574	0
June	10	27	0	1326	3354	3
July	18	62	0	3120	2730	2
August	14	22	1	1170	3258	0

September	20	46	1	2010	3630	4
October	16	54	1	2652	3024	2
November	16	45	0	2886	3336	1
December	16	40	1	2730	3438	0
January	22	48	0	3042	2478	2
February	16	59	0	2772	3942	1
March	34	61	1	3432	1230	6

There is a great variation in the unit price of the items selected. The monthly consumption of each item also varies drastically. The following table shows unit prices and annual consumption of the selected items.

Table 4: Unit Price & Annual Consumption of each selected item

Item	Unit Price	Annual Consumption
1 (Highest Usage Value)	Rs. 1460	204
2 (Lowest Usage Value)	Rs. 11	535
3 (Highest Unit Cost)	Rs. 3400	7
4 (Lowest Unit Cost)	Rs. 0.47	28458
5 (Highest Annual Consumption)	Rs. 0.47	34710
6 (Lowest Annual Consumption)	Rs. 2489	22

4. Analysis

4.1. Total Inventory Costs

The values of total inventory cost (in Rupees) obtained by each one of the above said models are tabulated as follows:

Table 5: Total Inventory Costs by Different Models

Model	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
LFL	Rs. 299640	Rs. 7685	Rs. 24850	Rs. 15175	Rs. 18113	Rs. 56108
EOQ	Rs.330460	Rs. 8244	Rs. 27977	Rs. 17654	Rs. 18670	Rs. 63778
POQ	Rs. 299640	Rs. 6360	Rs. 24497	Rs. 14081	Rs. 17277	Rs. 56083
LUC	Rs. 317730	Rs. 6393	Rs. 25189	Rs. 14424	Rs. 18392	Rs. 58666
LTC	Rs. 300210	Rs. 6360	Rs. 24568	Rs. 14113	Rs. 17275	Rs. 56057
LPC	Rs. 314880	Rs. 6487	Rs. 25189	Rs. 14437	Rs. 18058	Rs. 57727
W-W	Rs. 299640	Rs. 6340	Rs. 24497	Rs. 14049	Rs. 17269	Rs. 55689

The table shows considerable variations in annual total cost on application of different inventory models. In all cases, Wagner-Whitin model gives the minimum total annual inventory cost. In each case, a different model gives minimum cost along with Wagner-Whitin model, though no model except Wagner-Whitin model proves to be consistent. However, period order quantity model gives inventory costs which are nearer to the Wagner-Whitin model in almost every case i.e. it shows least percentage variation from the Wagner-Whitin model. The most common model i.e. economic order quantity model gives the highest cost.

4.2. % Saving in Total Annual Inventory Cost

To emphasis this point, the total annual inventory cost obtained by Wagner-Whitin model is considered as a base and the

% saving in total annual inventory cost it gives in comparison with each model is calculated. The table below gives the values of % saving in total annual inventory cost.

Table 6: % Saving in Total annual Inventory Costs

Model	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
LFL	**0.0**	21.2	1.44	8.01	4.89	0.75
EOQ	10.3	30.3	14.21	25.66	8.11	14.53
POQ	**0.0**	**0.32**	**0.0**	**0.23**	**0.05**	**0.71**
LUC	6.0	0.84	2.82	2.67	6.50	5.35
LTC	0.19	**0.32**	0.29	0.46	**0.03**	**0.66**
LPC	5.1	2.32	2.82	2.76	4.57	3.66

Above table shows that the % variation in total inventory cost is very large in case of economic order quantity model, where as periodic order quantity model shows the least variation from Wagner-Whitin model.

5. Conclusion

From the above discussion, it can be concluded that the Wagner-Whitin model gives the least total annual inventory cost in all the cases. The theory has been applied to variety of items, i.e. items with highest and lowest annual usage value, highest and lowest unit cost, highest and lowest annual consumption. But in each case Wagner-Whitin model shows optimum results. The periodic order quantity model also proves to be competent as it shows very slight variations from the Wagner-Whitin model.

The study was carried out considering only 6 items out of 48 A class items i.e. only 12.5% items were taken into consideration. If the organization is purchasing using economic order quantity model, then it can save around 18% of the cost using Wagner-Whitin model. These values may increase by a great amount if all 48 A class items are studied.

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