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BE - 301

B.E. III Semester

Examination, June 2014

Engineering Mathematics - II

(Common for all Branches)

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each question are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max. 50 words) carry 2 marks, part C (Max. 100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and drawing etc.

1. a) Write Dirichlet's conditions for Fourier series.

b) Write linear property and change of scale property four Fourier transform.

c) Find the Fourier sine transform of $e^{-|x|}$. Hence show that

$$\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi e^{-m}}{2}, m>0$$

d) Expand the function $f(x)=x\sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. Deduce that

$$\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\frac{1}{7.9}+\ldots\ldots=\frac{\pi-2}{4}$$

OR

Obtain a half-range cosine series for

$$f(x)=\begin{cases} kx \ \text{for}\ 0\le x\le \dfrac{l}{2} \\ k(l-x)\ \text{for}\ \dfrac{l}{2}\le x\le l \end{cases}$$

Deduce the sum of the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$.

2. a) If $L\{f(t)\}=\overline{f}(s)$ then prove that $L\{e^{at}f(t)\}=\overline{f}(s-a)$.

b) Find the inverse L.T of $\overline{f}(s)=\log\frac{s+2}{s+3}$.

c) Using convolution theorem, find the inverse Laplace transform of $\overline{f}(s)=\frac{1}{(s+1)(s^2+1)}$.

d) Solve the following differential equation by Laplace transform: $\frac{d^2y}{dt^2}+\frac{5dy}{dt}+6y=5e^t$, *given* $y(0)=2, y'(0)=1$.

OR

Find the Laplace transform of

(i) $f(t)=\int_0^t e^{-t}\frac{\sin t}{t}dt$ (ii) $f(t)=t^2e^{-2t}\cos 3t$

3\. a) In the differential equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$, satisfies the equation $1-P+Q=0$, then find the one part of complimentary function of the differential equation.

b) Write a part of C.F of the differential equation

$$(3-x)\frac{d^2y}{dx^2}-(9-4x)\frac{dy}{dx}+(6-3x)y=0$$

c) Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$

d) Solve in series the differential equation

$$(1-x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+y=0.$$

OR

Solve the differential equation

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin x^2.$$

4\. a) Solve the p.d.e. $xp+yq=3z$.

b) Find the particular integral of the p.d.e.

$$\frac{\partial^2 z}{\partial x^3}-7\frac{\partial^3 z}{\partial x\partial y^2}-6\frac{\partial^3 z}{\partial y^3}=\sin(x+2y)$$

c) Solve $p^2-q^2=x-y$.

d) Solve the p.d.e $(D-D'-1)(D-D'-2)z=e^{3x-y}+x$.

OR

Use the method of separation of variables to solve the

equation $\frac{\partial^2 u}{\partial x^2}-2\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$.

5\. a) Find the grad ϕ when ϕ is given by $\phi=3x^2y-y^3z^2$ at the point $(1, -2, -1)$.

b) Write the statement of Gauss divergence theorem.

c) A vector field is given by $\overline{A}=(x^2+xy^2)i+(y^2+x^2y)j$. Show that the field is irrotational.

d) Find the work done when a force $\overline{F}=(x^2-y^2+x)i-(2xy+y)j$ moves a particle in the xy - plane from (0,0) to (1,1) along the parabola $y^2=x$. Is the work done different when the path is the straight line $y=x$?

OR

Verify stoke's theorem for $\overline{F}=(x^2-y^2)i+2xyj$ over the box bounded by the planes $x=0, x=a; y=0, y=b; z=0, z=c$; if the face $z=0$ is cut.