

**CS/IT - 302****B.E. III Semester** Examination, December 2014**Discrete Structure****Time : Three Hours****Maximum Marks : 70**

- Note:** i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.
- ii) All parts of each question are to be attempted at one place.
- iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.
- iv) Except numericals, Derivation, Design and Drawing etc.

**Unit - I**

1. a) Prove that intersection of sets is distributive w.r.t. union of sets i.e  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- b) Define subset of a set with an example. Also define proper subset and improper subset.
- c) Prove by mathematical induction-

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

- d) Let  $X = \{a, b, c\}$ . Define  $f : X \rightarrow X$  such that  $f = \{(a,b), (b,a), (c,c)\}$

Find (i)  $f^{-1}$  (ii)  $f^2$  (iii)  $f^3$  (iv)  $f^4$

OR

Among the first 500 positive integers:

- i) Determine the integers which are not divisible by 2, nor by 3, nor by 5.
- ii) Determine the integers which are exactly divisible by one of them.

**Unit - II**

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2. a) Define monoid with an example.
- b) Determine whether a semigroup with more than one idempotent element can be group.
- c) Explain Homomorphism and Isomorphism of groups with an example.
- d) Consider an algebraic system  $(\theta, *)$  where  $\theta$  is the set of rational numbers and  $*$  is a binary operation defined by

$$a * b = a + b - ab \quad a, b \in \theta$$

Determine whether  $(\theta, +)$  is a group.

OR

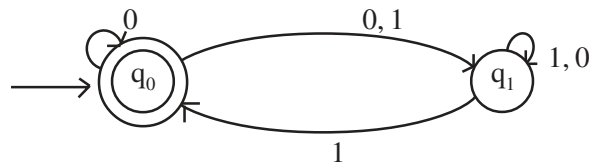
[2]

Let  $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$  and multiplication modulo 8, that is  $x \oplus y = (xy) \bmod 8$ .

- i) Prove that  $(\{0, 1\}, \oplus)$  is not a group.
- ii) Write three distinct groups  $(G, \otimes)$  where  $G$  has 2 elements.

### Unit - III

3. a) Write down the principle of Duality.
- b) Show that  $(p \oplus q) \vee (p \downarrow q)$  is equivalent to  $p \uparrow q$ .
- c) Convert the following NFA to DFA



- d) Determine whether the following are equivalent using biconditional statement.
  - i)  $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
  - ii)  $(p \rightarrow q) \rightarrow t \equiv (p \wedge \sim q) \rightarrow t$

OR

Show that the rule of hypothetical syllogism is valid

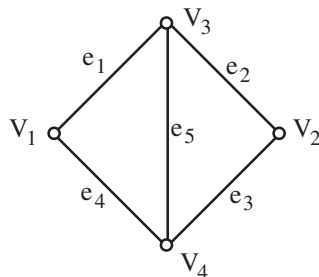
$$p \rightarrow q$$

$$q \rightarrow r$$

$$p \rightarrow r$$

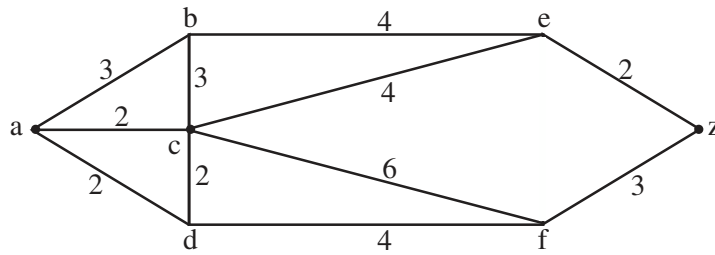
### Unit - IV

4. a) Prove that in any graph, there are an even number of vertices of odd degree.
- b) Consider the undirected graph  $G$  as shown in fig below. Find its incidence matrix  $M_I$ .



- c) Prove that every Planar graph has at least one vertex of degree 5 or less than 5.
- d) Find the shortest path between  $a$  and  $z$  in the graph shown below.

[3]



OR

Explain briefly:

- i) Euler Path                      ii) Hamiltonian Path
- iii) Graph Coloring            iv) Chromatic Number

**Unit - V**

5. a) Prove that if  $L$  be a lattice then  $a \wedge b = a$  if and only if  $a \vee b = b$ .
- b) There are  $n$  objects out of which  $r$  objects are to be arranged. Find total number of permutations when
- i) Four particular objects always occur
  - ii) Four particular objects never occur.
- c) Solve the difference equation  $Y_K - Y_{K-1} - Y_{K-2} = 0$ .
- d) Solve the recurrence relation  $a_{r+2} - 2a_{r+1} + a_r = r^2 \cdot 2^r$ .

OR

Write short notes : (any three)

- i) Bounded Lattice
- ii) Isomorphic Lattice
- iii) Hasse Diagram
- iv) Binomial Theorem

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