

BE - 401**B.E. IV Semester Examination, December 2014****Mathematics - III****(Common for all Branches)****Time : Three Hours****Maximum Marks :70**

- Note:** i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.
 ii) All parts of each question are to be attempted at one place.
 iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.
 iv) Except numericals, Derivation, Design and Drawing etc.

1. a) Show that $w=e^z$ is an analytic function and determine $f'(z)$.
- b) Evaluate $\int_C \frac{z^2 - z + 1}{z - 1} dz$, where C is the circle $|z|=1$.
- c) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z|=2$.
- d) If $f(z)$ is a regular function of z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$.

OR

If $f(\xi) = \int_C \frac{3z^2 + 7z + 1}{z - \xi} dz$, where C is the circle $x^2 + y^2 = 4$, find the values of $f(3)$, $f'(1-i)$ and $f''(1-i)$.

2. a) Define algebraic and transcendental equations.
- b) Find the smallest positive root of the equation $x^3 - 2x + 0.5 = 0$ by Newton-Raphson method.
- c) The equation $x^6 - x^4 - x^3 - 1 = 0$, has one real root between 1.4 and 1.5. Find the route to four decimal places by the method of False-Position.
- d) Solve the following system of equations $8x - y + z = 18$; $2x + 5y - 2z = 3$; $x + 2y - 3z = -6$ using Gauss-Seidel iterative method.

OR

Apply Gauss-Jordan method to find the solution of the following system of equations:
 $10x + y + z = 12$; $2x + 10y + z = 13$, $x + y + 5z = 7$.

3. a) Define interpolation and write the Newton's forward and Backward interpolation formula.
- b) Find the cubic polynomial which takes the following values:

$x :$	0	1	2	3
$f(x) :$	1	2	1	10

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's rule.

d) The following table gives the normal weights of babies during the first 12 months of life.

Age in months: 0 2 5 8 10 12

Weights in lbs: $7\frac{1}{2}$ $10\frac{1}{4}$ 15 16 18 21

Estimate the weight of the baby at the age of 7 months.

OR

Find $f'(x)$ and $f''(x)$ at $x = 6$, given that

x : 4.5 5.0 5.5 6.0 6.5 7.0 7.5

$y = f(x)$: 9.69 12.90 16.71 21.18 26.37 32.34 39.15

4. a) Use Picards method to approximate y when $x = 0.1$ given that $y = 1$, when $x = 0$ and $\frac{dy}{dx} = \frac{y-x}{y+x}$.

b) Write the steps of Runge-Kutta method.

c) From the following data, obtain the value of the correlation coefficient: $n = 10$, $\Sigma x = 140$, $\Sigma y = 150$, $\Sigma (x-10)^2 = 180$, $\Sigma (y-15)^2 = 215$ and $\Sigma (x-10)(y-15) = 60$.

d) By the method of least squares, find the curve $y = ax + bx^2$, that best fits the following data:

x : 1 2 3 4 5

y : 1.8 5.1 8.9 14.1 19.8

OR

Given $\frac{dy}{dx} = 1 + \frac{y}{x}$, $y = 2$ at $x = 1$. Find approximate value of y at $x = 1.4$ by taking step size $h = 0.2$, apply modified Euler's method.

5. a) Find the mean of the binomial distribution.

b) Given A and B two events with $P(A \cup B) = \frac{7}{8}$, $P(A \cap B) = \frac{1}{4}$ and $P(A \cap \bar{B}) = \frac{5}{8}$. Find $P(A)$ and $P(B)$.

c) If the probability of a bad reaction from certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get a bad reaction.

d) Fit a Poisson distribution for the following data and test the goodness of fit, given that $\chi^2 = 0.05 = 7.815$ for 3 d.f.

x : 0 1 2 3 4

f : 122 60 15 2 1

OR

The life time of a certain kind of battery is a random variable, which as an exponential distribution with a mean of 200 hrs. Find the probability that such a battery will last. (i) At most 100 hrs. and (ii) last any-where from 400 to 600 hrs.
