5. a) Find the directional derivative of $\phi = xy + yz + zx$ in the direction of $2i + j + k$ at the point (1, 1, 2). Also find the maximum value of the directional derivative at the point.

b) Show that the vector $F = \dfrac{\overline{r}}{r^3}$ is irrotational. Find the scalar potential.

OR

a) Evaluate $\iint_s A \cdot \hat{n} ds$, where $A = (x + y^2)\, i - 2xj + 2yzk$ and s is the surface of the plane $2x + y + 2z = 6$ in the first octant.

b) Using Stoke's theorem, evaluate

$$\int_C \left[(x+y)dx + (2x-z)dy + (y+z)dz\right]$$

Where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6)

******

Roll No ....

BE-301

B.E. III Semester

Examination, December 2016

Mathematics - II

(Common for all Branches)

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt all questions. Each question has an internal choice.

ii) All questions carry equal marks.

iii) All parts of each question are to be attempted at one place.

1. a) Expand in Fourier series $f(x) = x + x^2, -\pi < x < \pi$

b) Obtain half-range Fourier cosine series for $f(x) = x$ in $0 < x < 2$

OR

a) Find the Fourier series for the even function

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi}, & 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + ... = \pi^2/8$$

b) Find the Fourier sine transform of

$$f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$$

2. a) Find the L.T. of

i) $f(t)=\begin{cases} t, & 0\le t\le 1 \\ 1, & t>0 \end{cases}$

ii) $f(t)=t^2\sin 2t$

b) Use convolution theorem, to find

$$L^{-1}\left\{\frac{1}{s^3\left(s^2+1\right)}\right\}$$

OR

a) Using L.T. techniques, solve the following initial value problem:

$$\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=4e^{2t},\ y(0)=-3,\ y'(0)=5$$

b) Find the L.T. of the triangular wave function of period 2k given by:

$$f(t)=\begin{cases} t, & 0<t<k \\ 2k-t, & k<t<2k \end{cases}$$

3. a) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$, given that $y=x+\frac{1}{x}$ is one integral.

b) Find the general solution of $\frac{d^2y}{dx^2}+(x-3)\frac{dy}{dx}+y=0$ in series.

OR

a) Solve

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\cos\log(1+x)$$

b) Solve by method of variation of parameters:

$$\frac{d^2y}{dx^2}+a^2y=\sec ax$$

4. a) Solve $\left(x^2-y^2-z^2\right)p+2xy\,q=2xz$

b) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$

OR

a) Solve the following partial differential equation by Charpit's method:

$pxy+pq+qy=yz$

b) A tightly stretched string of length L is fixed at both ends. Find the displacement $u(x,t)$, if the string is given an initial displacement $f(x)$ and an initial velocity $g(x)$.