A bar with insulated sides is initially at a temperature 0°C throughout. The end $x = 0$ is kept at 0°C, and heat is suddenly applied at the end $x = l$ so that $\frac{\partial u}{\partial x} = A$ for $x = l$, where A is a constant. Find the temperature $u(x, t)$.

5. a) Find the directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $P(1, 1, 1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$

b) Prove that vector $f(r)\vec{r}$ is irrotational.

c) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2, z = 0$.

d) Verify Stoke's theorem for the vector $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the xy - plane.

OR

Evaluate $\iint_s \vec{A} \bullet \hat{n}\, dS$ where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2 z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

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Roll No ...

BE - 301

B.E. III Semester

Examination, December 2015

Mathematics - II

(Common for all Branches)

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each question are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max. 50 words) carry 2 marks, part C (Max. 100 words) carry 3 marks, part D (Max. 400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

1. a) Expand $\pi x - x^2$ in a half-range sine series in the interval $(0, \pi)$ upto the first three terms.

b) Find the Fourier transform of Dirac Delta Function $\delta(t-a)$.

c) Find the function whose sine transform is $\frac{e^{-s}}{s}$.

d) Find the Fourier series to represent the function $f(x)$ given by

$$f(x) = \begin{cases} 0 & for \quad -\pi \le x \le 0 \\ \sin x & for \quad 0 \le x \le \pi \end{cases}$$

Deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi - 2}{4}$.

OR

Develop $\sin\left(\frac{\pi x}{l}\right)$ in half-range cosine series in the range $0 < x < l$.

2. a) Find $L\{F(t)\}$ if $F(t)=\begin{cases}\sin\left(t-\frac{\pi}{3}\right), & t>\frac{\pi}{3}\\ 0 & , t<\frac{\pi}{3}\end{cases}$

b) Define Unit Step Function and Find its Laplace Transform.

c) Prove that : $L^{-1}\left(\frac{s}{s^4+s^2+1}\right)=\frac{2}{\sqrt{3}}\sin h\frac{t}{2}\sin\frac{\sqrt{3}}{2}t$.

d) State convolution theorem and hence evaluate

$$L^{-1}\left\{\frac{p}{(p^2+1)(p^2+4)}\right\}.$$

OR

Solve the simultaneous equations using Laplace Transform:

$$\frac{dx}{dt}-y=e^t, \frac{dy}{dt}+x=\sin t, \text{ given } x(0)=1, y(0)=0.$$

3. a) Solve the differential equation by Removal of first derivative method.

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$$

b) Solve by changing the independent variable

$$(1+x^2)^2\frac{d^2y}{dx^2}+2x(1+x^2)\frac{dy}{dx}+4y=0$$

c) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$$

d) Solve in series the equation

$$2x(1-x)\frac{d^2y}{dx^2}+(5-7x)\frac{dy}{dx}-3y=0$$

OR

Solve in series the equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

4. a) Solve the differential equation $(z^2-2yz-y^2)\,p+(xy+zx)\,q=xy-zx.$

b) Solve $p^2-q^2=x-y.$

c) Solve $(D^2+5DD'+6D'^2)z=\frac{1}{y-2x}$.

d) If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity

$$\left(\frac{dy}{dt}\right)_{t=0}=b\sin^3\frac{\pi x}{l}, \text{ find the displacement } y(x, t).$$

OR