Roll No ....

CS/IT - 302

B.E. III Semester

Examination, June 2014

Discrete Structure

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each question are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max. 50 words) carry 2 marks, part C (Max. 100 words) carry 3 marks, part D (Max. 400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

1. a) For any three sets A, B and C prove that:

$A\times(B\cup C)=(A\times B)\cup(A\times C)$ 2

b) Show that the relation R = {(a, b) : a–b is an even integer} is an equivalence relation on the set of all integers. 2

c) Let X and Y be two sets. If a mapping $f: X\rightarrow Y$ is one-one onto, prove that f^{-1} is also one-one onto. 3

d) Use mathematical induction to prove that $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 for $n\in N$. 7

Or

Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology and 30 do not study any of the subjects. Find the number of students (i) studying all the three subjects and (ii) studying exactly one of the three subjects. 7

2. a) Show that the set $G=\{1,\omega,\omega^2\}$ is a group with respect multiplication, ω being an imagery cube root of unity. 2

b) Prove that every cyclic group is abelian. 2

c) Prove that the intersection of any two normal subgroups of a group G is a normal subgroup of G. 3

d) Prove that the order of each subgroup of a finite group is a divisor of the order of group. 7

Or

Define ring. If a, b, c are arbitrary elements of a ring R, prove that:

i) a0 = 0a = 0

ii) a(–b) = –(ab) = (–a)b 7

3. a) Prove that the proposition $(p\vee\sim q)\wedge(\sim p\vee\sim q)\vee q$ is a tautology. 2

b) Prove that the propositions $p\vee\sim(q\wedge r)$ and $(p\vee\sim q)\vee\sim r$ are equivalent. 2

c) Obtain Conjunctive normal form of $(\sim p\rightarrow r)\wedge(q\leftrightarrow p)$ 3

d) Show that the set $L=\{a^{i^2}:i\geq 1\}$ is not regular. 7

Or

Construct a finite state accepter that will accept the set of natural numbers x which are divisible by 3. 7

4. a) Define Euler's graph and write its properties. 2

b) Explain isomorphism and homomorphism of graphs. 2

c) Show that every tree with two or more vertices is 2-chromatic. 3

d) Write an algorithm to find the shortest path in a weighted graph. 7

Or

Prove that in a graph G with n-vertices always has a Hamiltonian path if the sum of degrees of every pairs of vertices v_i, v_j in G satisfies the condition $d(v_i)+d(v_j)\geq n-1$. 7

5. a) Define lattice with example. 2

b) Prove that the complement of an element in a bounded complemented distributed lattice is unique. 2

c) In a complemented distributed lattice L, prove that:

$(a\wedge b)'=a'\vee b',\quad a,b\in L$ 3

d) Determine the discrete numeric function corresponding to the generating function

$$A(z)=\frac{(1+z)^2}{(1-z)^4}$$ 7

Or

Solve the recurrence relation

$$a_r-5a_{r-1}+6a_{r-2}=r^2-r,\ r\geq 2$$ 7