DATA SCIENCE & ARTIFICIAL INTELLIGENCE

Permutations and Combinations

Lecture No. 01

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Recap of previous lecture

Topic X X X X X X X

1. Linear Algebra
2. Calculus
3. ~~Prob~~ & Stats

CS/IT — 7-8 Marks

DA — 36

Class Notes
DPP
WT
PYQ
QTS
1500 Q

Topics to be Covered

Topic

BASICS of P & C

(& Counting Principle)

Counting Principle

① **Fundamental Principle of Addition** → If we have to perform only one of the job at a time then use this Principle. keywords "Either or / only one / Any one"

② **Fundamental Principle of Multiplication** → If we have to perform all the jobs at a time then use this Principle. keywords "AND / All / Both"

Eg: There are 10 Boys & 8 Girls in a class then in how many ways we can select

(1) A boy and A Girl = ? $= 10 \text{ ways} \times 8 \text{ ways} = 80 \text{ ways}$.

(2) Either a Boy or a Girl = ? $= 10 \text{ ways} + 8 \text{ ways} = 18 \text{ ways}$

Eg: There are (3) students appearing for Maths scholarship test, (4) for Physics and (5) for Chemistry scholarship test then in how many ways.

(1) these Scholarships all can be awarded = ? $= 3 \text{ ways} \times 4 \text{ ways} \times 5 \text{ ways} = 60 \text{ ways}$

(2) one of these " " " " = ? $= 3 \text{ ways} + 4 \text{ ways} + 5 \text{ ways} = 12 \text{ ways}$

Room A.

Eg. There are two friends living in a hostel. They have 5 Jeans & 4 shirts in their Almirah. Then in how many ways they can dress up for party?

Sol: Total ways $= \left(\frac{5}{J} \times \frac{4}{S}\right) \times \left(\frac{4}{J} \times \frac{3}{S}\right) = 240$ ways

Eg. There are 4 Letters (f_1) & 5 Letter Boxes (f_2) then in how many ways we can post these letters

Sol: 4×5, ${}^5P_4$, ${}^5C_4$, $5!$, $4!$, 5^4, 4^5, $5+4$

Total ways of posting letters $= \underbrace{5 \text{ ways}}_{L_1} \times \underbrace{5 \text{ ways}}_{L_2} \times \underbrace{5 \text{ ways}}_{L_3} \times \underbrace{5 \text{ ways}}_{L_4} = 5^4 = 625$ ways

RA

Wrong App: Total ways $= \frac{4}{B_1} \times \frac{4}{B_2} \times \frac{4}{B_3} \times \frac{4}{B_4} \times \frac{4}{B_5} = 4^5$ ✗

In this case we are moving Letter Boxes which is not possible.

Q How many three letter words (with or w/o) meaning can be formed using Vowels

a, e, i, o, u

① If R.A = ?

$= \frac{5 \text{ ways}}{P_1} \times \frac{5 \text{ ways}}{P_2} \times \frac{5 \text{ ways}}{P_3} = 5^3 = 125$ words

② If R.N.A = ?

$= \frac{5 \text{ ways}}{P_1} \times \frac{4 \text{ ways}}{P_2} \times \frac{3 \text{ ways}}{P_3} = {}^5C_3 \times 3! = {}^5P_3$

Eg How many 3 digit Number can be formed using odd digits if (1, 3, 5, 7, 9)

① RA $= \underline{5} \times \underline{5} \times \underline{5} = 125$ Numbers

② RNA $= \underline{5} \times \underline{4} \times \underline{3} = {}^5C_3 \times 3! = {}^5P_3$

③ How many 5 digit nos can be formed using odd digits if a digit can be used only once = ? (RNA) $= \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5! = {}^5P_5$

Q: How many 4 digit Numbers can be formed if Repitition of digits is not allowed

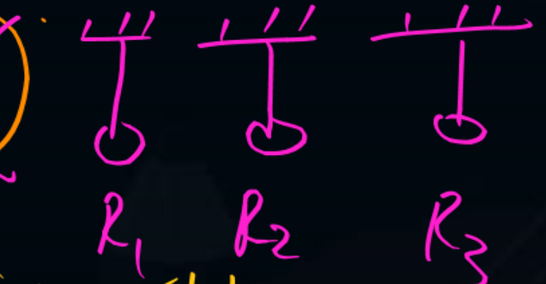
Total 4 digit Nos (RNA) = $\underline{9 \text{ ways}}_{P_1} \times \underline{9 \text{ ways}}_{P_2} \times \underline{8 \text{ ways}}_{P_3} \times \underline{7 \text{ ways}}_{P_4}$

(0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Q. There are (3) Rings & (4) fingers then in how many ways we can worn these Rings

if (1) there is No Restriction = ? (RA) = $\underset{R_1}{4 \text{ ways}} \times \underset{R_2}{4 \text{ ways}} \times \underset{R_3}{4 \text{ ways}} = 4^3 = 64$

Wrong Approach: Total ways = $\underset{f_1}{3} \times \underset{f_2}{3} \times \underset{f_3}{3} \times \underset{f_4}{3} = 3^4$



in this Method, we are Cracking fingers which is not possible.

(2) with at most one Ring in a Single finger = ? (RNA) = $\underset{R_1}{4} \times \underset{R_2}{3} \times \underset{R_3}{2} = {}^4(3 \times 3!) = {}^4P_3$

Q: (1) There are (5 Persons) & (8 Chairs) then How many seating arrangements are possible?

Total seating arrangements (RNA) $= \underset{P_1}{8} \times \underset{P_2}{7} \times \underset{P_3}{6} \times \underset{P_4}{5} \times \underset{P_5}{4} = {}^{8}C_5 \times 5! = {}^{8}P_5$

(2) if there are (8) Persons and (8) chairs then T.S.A = ?

Total seating Arrangements (RNA) $= \underset{P_1}{8} \times \underset{P_2}{7} \times \underset{P_3}{6} \times \underset{P_4}{5} \times \underset{P_5}{4} \times \underset{P_6}{3} \times \underset{P_7}{2} \times \underset{P_8}{1} = 8! = {}^{8}P_8$

(3) If there are (8) Persons and (5) chairs then T.S.A = ?

= Senseless Question

$= 8 \times 7 \times 6 \times 5 \times 4$

Note n = Number of Places, r = No. of persons.

(1) If $n = r$ & RNA then (Multi Rule = Perm. Rule = Factorial Rule)

(2) if $n > r$ & RNA then Multi Rule = Perm. Rule

(3) if (RA) then only use Multi Rule

(4) Combination → (when Counting is Based on only selection then use this Rule)

$${}^{n}C_r = \frac{n!}{r!\,(n-r)!} \quad \& \quad \boxed{{}^{n}C_r = {}^{n}C_{n-r}}, \quad {}^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}, \quad {}^{22}C_{18} = ? = {}^{22}C_4 = \frac{22\times21\times20\times19}{4\times3\times2\times1}$$

(5) Permutation (when Counting is Based on Selection & Arrangement both then use this Rule)

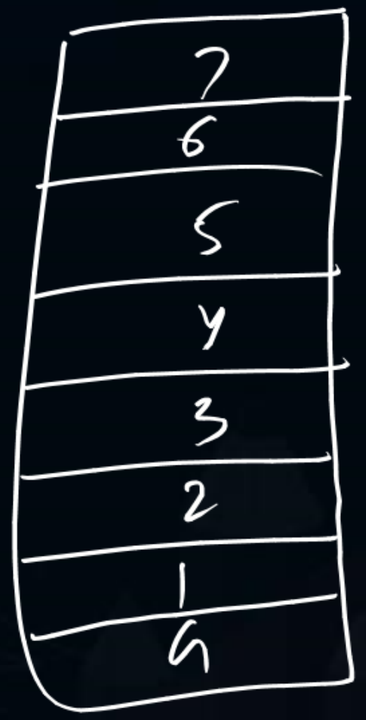
$$\boxed{{}^{n}P_r = \frac{n!}{(n-r)!} = {}^{n}C_r \times r!}, \quad {}^{11}P_3 = 11\times10\times9, \quad {}^{22}P_4 = 22\times21\times20\times19$$

Q: 5 persons entered in a lift at Ground floor in an 8 floor house then in how many ways they can leave the lift ?

① At any floor = ? (RA) $= \frac{7}{P_1}\frac{7}{P_2}\frac{7}{P_3}\frac{7}{P_4}\frac{7}{P_5} = 7^5$ ways

② At different floor = ? (RNA) $= 7 \times 6 \times 5 \times 4 \times 3$

$= {}^7C_5 \times 5! = {}^7P_5$



Q. Salads are made from one or more eatables then how many different salads can be made using onion, Tomato, Reddish, Cucumber, Beetroot

(a) $5!$

(b) 5^5

(c) 31

(d) 32

TN-I Number of ways to deal to with each vegitable = 2 ways (Either S or R)

So Total Number of ways to deal with all vegies $= \underset{O}{2} \times \underset{T}{2} \times \underset{R}{2} \times \underset{C}{2} \times \underset{B}{2} = 2^5 = 32$

But Imaginary Salad is not possible so $Ans = 32 - 1 = 31$

Various possibilities are;

(SSSSS) or (SSSSR) or (SSSRR) or (SSRRR) or (SRRRR) or (RRRRR) ??

$= {}^5C_5 + {}^5C_4 + {}^5C_3 + {}^5C_2 + {}^5C_1 + {}^5C_0$

$= 1 + 5 + 10 + 10 + 5 + 1 = 32$

(M-II) we can make salad either by selecting

$= 1V$ or $2V$ or $3V$ or $4V$ or $5V$

$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= 5 + 10 + 10 + 5 + 1 = 31$

(ii) How many salads can be made using exactly 3 vegies at a time $\Rightarrow = {}^5C_3 = 10$

(iii) " " " " " " " at least 3 " " " $= ?$

$= {}^5C_3 + {}^5C_4 + {}^5C_5 = 10 + 5 + 1 = 16$

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THANK - YOU