6. The value of $\log_{(.01)} (1000)$ is :

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

7. The logarithm of 0.0625 to the base 2 is :

(a) -4 (b) -2 (c) 0.25 (d) 0.5

8. If $\log_3 x = -2$, then x is equal to :

(a) -9 (b) -6 (c) -8 (d) $\frac{1}{9}$

9. If $\log_8 x = \frac{2}{3}$, then the value of x is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 3 (d) 4

10. If $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2}$, then x is equal to :

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{81}{256}$ (d) $\frac{256}{81}$

11. If $\log_x 4 = 0.4$, then the value of x is : **(Asstt. Grade, 1998)**

(a) 1 (b) 4 (c) 16 (d) 32

12. If $\log_{10000} x = -\frac{1}{4}$, then x is equal to :

(a) $\frac{1}{10}$ (b) $\frac{1}{100}$ (c) $\frac{1}{1000}$ (d) $\frac{1}{10000}$

13. If $\log_x 4 = \frac{1}{4}$, then x is equal to :

(a) 16 (b) 64 (c) 128 (d) 256

14. If $\log_x (0.1) = -\frac{1}{3}$, then the value of x is :

(a) 10 (b) 100 (c) 1000 (d) $\frac{1}{1000}$

15. If $\log_{32} x = 0.8$, then x is equal to :

(a) 25.6 (b) 16 (c) 10 (d) 12.8

16. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is : **(S.S.C. 1999)**

(a) 2^{10} (b) 2^{100} (c) 2^{1000} (d) 2^{10000}

17. The value of $\log_{(-1/3)} 81$ is equal to :

(a) -27 (b) -4 (c) 4 (d) 27

18. The value of $\log_{2\sqrt{3}} (1728)$ is :

(a) 3 (b) 5 (c) 6 (d) 9

19. $\frac{\log \sqrt{8}}{\log 8}$ is equal to : **(I.A.F. 2002)**

(a) $\frac{1}{\sqrt{8}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

20. Which of the following statements is not correct ? **(M.B.A. 2003)**

(a) $\log_{10} 10 = 1$ (b) $\log (2 + 3) = \log (2 \times 3)$

(c) $\log_{10} 1 = 0$ (d) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

21. The value of $\log_2 (\log_5 625)$ is :
(a) 2 (b) 5 (c) 10 (d) 15

22. If $\log_2 [\log_3 (\log_2 x)] = 1$, then x is equal to :
(a) 0 (b) 12 (c) 128 (d) 512

23. The value of $\log_2 \log_2 \log_3 \log_3 27^3$ is :
(a) 0 (b) 1 (c) 2 (d) 3

24. If $a^x = b^y$, then : **(Hotel Management, 2001)**
(a) $\log \frac{a}{b} = \frac{x}{y}$ (b) $\frac{\log a}{\log b} = \frac{x}{y}$ (c) $\frac{\log a}{\log b} = \frac{y}{x}$ (d) None of these

25. log 360 is equal to :
(a) 2 log 2 + 3 log 3 (b) 3 log 2 + 2 log 3
(c) 3 log 2 + 2 log 3 − log 5 (d) 3 log 2 + 2 log 3 + log 5

26. The value of $\left(\frac{1}{3} \log_{10} 125 - 2 \log_{10} 4 + \log_{10} 32\right)$ is :
(a) 0 (b) $\frac{4}{5}$ (c) 1 (d) 2

27. $2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4 = ?$ **(M.B.A. 2002)**
(a) 2 (b) 4 (c) $2 + 2 \log_{10} 2$ (d) $4 - 4 \log_{10} 2$

28. If $\log_a (ab) = x$, then $\log_b (ab)$ is : **(M.A.T. 2002)**
(a) $\frac{1}{x}$ (b) $\frac{x}{x+1}$ (c) $\frac{x}{1-x}$ (d) $\frac{x}{x-1}$

29. If log 2 = x, log 3 = y and log 7 = z, then the value of $\log (4 \cdot \sqrt[3]{63})$ is :
(a) $2x + \frac{2}{3}y - \frac{1}{3}z$ (b) $2x + \frac{2}{3}y + \frac{1}{3}z$
(c) $2x - \frac{2}{3}y + \frac{1}{3}z$ (d) $-2x + \frac{2}{3}y + \frac{1}{3}z$ **(S.S.C. 1998)**

30. If $\log_4 x + \log_2 x = 6$, then x is equal to :
(a) 2 (b) 4 (c) 8 (d) 16

31. If $\log_8 x + \log_8 \frac{1}{6} = \frac{1}{3}$, then the value of x is :
(a) 12 (b) 16 (c) 18 (d) 24

32. If $\log_{10} 125 + \log_{10} 8 = x$, then x is equal to :
(a) $\frac{1}{3}$ (b) .064 (c) − 3 (d) 3

33. The value of $(\log_9 27 + \log_8 32)$ is :
(a) $\frac{7}{2}$ (b) $\frac{19}{6}$ (c) 4 (d) 7

34. $(\log_5 3) \times (\log_3 625)$ equals :
(a) 1 (b) 2 (c) 3 (d) 4

35. $(\log_5 5)\ (\log_4 9)\ (\log_3 2)$ is equal to :
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 5

36. If $\log_{12} 27 = a$, then $\log_6 16$ is : **(Assistant Grade, 1998)**
(a) $\frac{3-a}{4(3+a)}$ (b) $\frac{3+a}{4(3-a)}$ (c) $\frac{4(3+a)}{(3-a)}$ (d) $\frac{4(3-a)}{(3+a)}$

37. If $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$, then x is equal to : **(C.D.S. 2003)**
(a) 1 (b) 3 (c) 5 (d) 10

38. If $\log_5 (x^2 + x) - \log_5 (x + 1) = 2$, then the value of x is :
(a) 5 (b) 10 (c) 25 (d) 32

39. The value of $\left(\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}\right)$ is :
(a) 0 (b) 1 (c) 5 (d) 60

40. The value of $(\log_3 4)\,(\log_4 5)\,(\log_5 6)\,(\log_6 7)\,(\log_7 8)\,(\log_8 9)$ is :
(a) 2 (b) 7 (c) 8 (d) 33

41. The value of $16^{\log_4 5}$ is :
(a) $\frac{5}{64}$ (b) 5 (c) 16 (d) 25

42. If $\log x + \log y = \log (x + y)$, then :
(a) $x = y$ (b) $xy = 1$ (c) $y = \frac{x-1}{x}$ (d) $y = \frac{x}{x-1}$

43. If $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$, then :
(a) $a + b = 1$ (b) $a - b = 1$ (c) $a = b$ (d) $a^2 - b^2 = 1$

44. $\left[\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right)\right]$ is equal to :
(a) 0 (b) 1 (c) 2 (d) abc

45. $(\log_b a \times \log_c b \times \log_a c)$ is equal to :
(a) 0 (b) 1 (c) abc (d) $a + b + c$

46. $\left[\frac{1}{(\log_a bc) + 1} + \frac{1}{(\log_b ca) + 1} + \frac{1}{(\log_c ab) + 1}\right]$ is equal to :
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3

47. The value of $\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x}\right]$ is :
(a) 0 (b) 1 (c) 2 (d) 3

48. If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ is equal to : **(C.D.S. 2003)**
(a) $-(1 + a)$ (b) $(1 + a)^{-1}$ (c) $\frac{a}{10}$ (d) $\frac{1}{10a}$

49. If $a = b^x$, $b = c^y$ and $c = a^z$, then the value of xyz is equal to :
(a) -1 (b) 0 (c) 1 (d) abc

50. If $\log 27 = 1.431$, then the value of $\log 9$ is : **(Section Officers', 2001)**
(a) 0.934 (b) 0.945 (c) 0.954 (d) 0.958

51. If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is equal to : **(S.S.C. 2000)**
(a) $\frac{699}{301}$ (b) $\frac{1000}{301}$ (c) 0.3010 (d) 0.6990

52. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 5$ is : **(S.S.C. 2001)**
(a) 0.3241 (b) 0.6911 (c) 0.6990 (d) 0.7525

53. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is :
(a) 1.6020 (b) 1.9030 (c) 3.9030 (d) None of these

54. If $\log 3 = 0.477$ and $(1000)^x = 3$, then x equals : **(S.S.C. 2000)**
(a) 0.0159 (b) 0.0477 (c) 0.159 (d) 10

55. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 25$ is :
(a) 0.6020 (b) 1.2040 (c) 1.3980 (d) 1.5050

56. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, the value of $\log_5 512$ is : **(M.A.T. 2002)**
(a) 2.870 (b) 2.967 (c) 3.876 (d) 3.912

57. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{10} 1.5$ is :
(a) 0.1761 (b) 0.7116 (c) 0.7161 (d) 0.7611

58. If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is :
(a) 0.4471 (b) 1.4471 (c) 2.4471 (d) None of these
(S.S.C. 1999)

59. If $\log (0.57) = \bar{1}.756$, then the value of $\log 57 + \log (0.57)^3 + \log \sqrt{0.57}$ is :
(a) 0.902 (b) $\bar{2}.146$ (c) 1.902 (d) $\bar{1}.146$
(Section Officers', 2003)

60. If $\log 2 = 0.30103$, the number of digits in 2^{64} is : **(C.B.I. 1997)**
(a) 18 (b) 19 (c) 20 (d) 21

61. If $\log 2 = 0.30103$, the number of digits in 4^{50} is :
(a) 30 (b) 31 (c) 100 (d) 200

62. If $\log 2 = 0.30103$, then the number of digits in 5^{20} is :
(a) 14 (b) 16 (c) 18 (d) 25

ANSWERS

1. (b)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (d)
9. (d)	**10.** (d)	**11.** (d)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (c)
17. (b)	**18.** (c)	**19.** (c)	**20.** (b)	**21.** (a)	**22.** (d)	**23.** (a)	**24.** (c)
25. (d)	**26.** (c)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (d)	**31.** (a)	**32.** (d)
33. (b)	**34.** (d)	**35.** (a)	**36.** (d)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (a)
41. (d)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (a)
49. (c)	**50.** (c)	**51.** (b)	**52.** (c)	**53.** (b)	**54.** (c)	**55.** (c)	**56.** (c)
57. (a)	**58.** (a)	**59.** (a)	**60.** (c)	**61.** (b)	**62.** (a)		

SOLUTIONS

1. Let $\log_2 16 = n$. Then, $2^n = 16 = 2^4 \Rightarrow n = 4$.
$\therefore \log_2 16 = n$.

2. Let $\log_{343} 7 = n$. Then, $(343)^n = 7 \Leftrightarrow (7^3)^n = 7 \Leftrightarrow 3n = 1 \Leftrightarrow n = \frac{1}{3}$.
$\therefore \log_{343} 7 = \frac{1}{3}$.

3. Let $\log_5 \left(\frac{1}{125}\right) = n$. Then, $5^n = \frac{1}{125} \Leftrightarrow 5^n = 5^{-3} \Leftrightarrow n = -3$.
$\therefore \log_5 \left(\frac{1}{125}\right) = -3$

4. Let $\log_{\sqrt{2}} 32 = n$. Then, $(\sqrt{2})^n = 32 \Leftrightarrow (2)^{n/2} = 2^5 \Leftrightarrow \frac{n}{2} = 5 \Leftrightarrow n = 10$.
$\therefore \log_{\sqrt{2}} 32 = 10$.

5. Let $\log_{10} (.0001) = n$.
Then, $10^n = .0001 \Leftrightarrow 10^n = \frac{1}{10000} = \frac{1}{10^4} \Leftrightarrow 10^n = 10^{-4} \Leftrightarrow n = -4$.
$\therefore \log_{10} (.0001) = -4$.

6. Let $\log_{(.01)} (1000) = n$.
Then, $(.01)^n = 1000 \Leftrightarrow \left(\frac{1}{100}\right)^n = 10^3 \Leftrightarrow \left(10^{-2}\right)^n = 10^3 \Leftrightarrow -2n = 3 \Leftrightarrow n = -\frac{3}{2}$.

7. Let $\log_2 0.0625 = n$.
Then, $2^n = 0.0625 = \frac{625}{10000} \Leftrightarrow 2^n = \frac{1}{16} \Leftrightarrow 2^n = 2^{-4} \Leftrightarrow n = -4$.
$\therefore \log_2 0.0625 = -4$.

8. $\log_3 x = -2 \Leftrightarrow x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

9. $\log_8 x = \frac{2}{3} \Leftrightarrow x = 8^{2/3} = (2^3)^{2/3} = 2^{\left(3 \times \frac{2}{3}\right)} = 2^2 = 4$.

10. $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2} \Leftrightarrow x^{-1/2} = \frac{9}{16} \Leftrightarrow \frac{1}{\sqrt{x}} = \frac{9}{16} \Leftrightarrow \sqrt{x} = \frac{16}{9}$
$\Leftrightarrow x = \left(\frac{16}{9}\right)^2 = \frac{256}{81}$.

11. $\log_x 4 = 0.4 \Leftrightarrow \log_x 4 = \frac{4}{10} = \frac{2}{5} \Leftrightarrow x^{2/5} = 4 \Leftrightarrow x = 4^{5/2} = (2^2)^{5/2}$
$\Leftrightarrow x = 2^{\left(2 \times \frac{5}{2}\right)} = 2^5 \Leftrightarrow x = 32$.

12. $\log_{10000} x = -\frac{1}{4} \Leftrightarrow x = (10000)^{-1/4} = (10^4)^{-1/4} = 10^{-1} = \frac{1}{10}$.

13. $\log_x 4 = \frac{1}{4} \Leftrightarrow x^{1/4} = 4 \Leftrightarrow x = 4^4 = 256$.

14. $\log_x (0.1) = -\frac{1}{3} \Leftrightarrow x^{-1/3} = 0.1 \Leftrightarrow \frac{1}{x^{1/3}} = 0.1 \Leftrightarrow x^{1/3} = \frac{1}{0.1} = 10$
$\Leftrightarrow x = (10)^3 = 1000$.

15. $\log_{32} x = 0.8 \Leftrightarrow x = (32)^{0.8} = (2^5)^{4/5} = 2^4 = 16$.

16. $\log_2 x = 10 \Rightarrow x = 2^{10}$.
$\therefore \log_x y = 100 \Rightarrow y = x^{100} = (2^{10})^{100} \Rightarrow y = 2^{1000}$.

17. Let $\log_{(-1/3)} 81 = x$. Then, $\left(-\frac{1}{3}\right)^x = 81 = 3^4 = (-3)^4 = \left(-\frac{1}{3}\right)^{-4}$
$\therefore x = -4$ *i.e.*, $\log_{(-1/3)} 81 = -4$.

18. Let $\log_{2\sqrt{3}} (1728) = x$.
Then, $(2\sqrt{3})^x = 1728 = (12)^3 = [(2\sqrt{3})^2]^3 = (2\sqrt{3})^6$.
$\therefore x = 6$, *i.e.*, $\log_{2\sqrt{3}} (1728) = 6$.

19. $\dfrac{\log\sqrt{8}}{\log 8} = \dfrac{\log (8)^{1/2}}{\log 8} = \dfrac{\frac{1}{2}\log 8}{\log 8} = \dfrac{1}{2}$.

20. (*a*) Since $\log_a a = 1$, so $\log_{10} 10 = 1$.

(*b*) $\log (2 + 3) = 5$ and $\log (2 \times 3) = \log 6 = \log 2 + \log 3$

$\therefore \quad \log (2 + 3) \neq \log (2 \times 3)$.

(*c*) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$.

(*d*) $\log (1 + 2 + 3) = \log 6 = \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$.

So, (*b*) is incorrect.

21. Let $\log_5 625 = x$. Then, $5^x = 625 = 5^4$ or $x = 4$.

Let $\log_2 (\log_5 625) = y$. Then, $\log_2 4 = y$ or $2^y = 4 = 2^2$ or $y = 2$.

$\therefore \quad \log_2 (\log_5 625) = 2$.

22. $\log_2 [\log_3 (\log_2 x)] = 1 = \log_2 2$

$\Leftrightarrow \log_3 (\log_2 x) = 2 \Leftrightarrow \log_2 x = 3^2 = 9 \Leftrightarrow x = 2^9 = 512$.

23. $\log_2 \log_2 \log_3 (\log_3 27^3) = \log_2 \log_2 \log_3 [\log_3 (3^3)^3] = \log_2 \log_2 \log_3 [\log_3 (3)^9]$

$= \log_2 \log_2 \log_3 (9 \log_3 3) = \log_2 \log_2 \log_3 9 \quad [\because \log_3 3 = 1]$

$= \log_2 \log_2 [\log_3 (3)^2] = \log_2 \log_2 (2 \log_3 3)$

$= \log_2 \log_2 2 = \log_2 1 = 0$.

24. $a^x = b^y \Rightarrow \log a^x = \log b^y \Rightarrow x \log a = y \log b \Rightarrow \dfrac{\log a}{\log b} = \dfrac{y}{x}$.

25. $360 = (2 \times 2 \times 2) \times (3 \times 3) \times 5$.

So, $\log 360 = \log (2^3 \times 3^2 \times 5) = \log 2^3 + \log 3^2 + \log 5 = 3 \log 2 + 2 \log 3 + \log 5$.

26. $\dfrac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32$

$= \log_{10} (125)^{1/3} - \log_{10} (4)^2 + \log_{10} 32 = \log_{10} 5 - \log_{10} 16 + \log_{10} 32$

$= \log_{10}\left(\dfrac{5 \times 32}{16}\right) = \log_{10} 10 = 1$.

27. $2\log_{10} 5 + \log_{10} 8 - \dfrac{1}{2}\log_{10} 4 = \log_{10} (5^2) + \log_{10} 8 - \log_{10} (4^{1/2})$

$= \log_{10} 25 + \log_{10} 8 - \log_{10} 2 = \log_{10}\left(\dfrac{25 \times 8}{2}\right) = \log_{10} 100 = 2$.

28. $\log_a (ab) = x \quad \Leftrightarrow \quad \dfrac{\log ab}{\log a} = x \quad \Leftrightarrow \quad \dfrac{\log a + \log b}{\log a} = x$

$\Leftrightarrow \quad 1 + \dfrac{\log b}{\log a} = x \quad \Leftrightarrow \quad \dfrac{\log b}{\log a} = x - 1$

$\Leftrightarrow \quad \dfrac{\log a}{\log b} = \dfrac{1}{x-1} \quad \Leftrightarrow \quad 1 + \dfrac{\log a}{\log b} = 1 + \dfrac{1}{x-1}$

$\Leftrightarrow \quad \dfrac{\log b}{\log b} + \dfrac{\log a}{\log b} = \dfrac{x}{x-1} \quad \Leftrightarrow \quad \dfrac{\log b + \log a}{\log b} = \dfrac{x}{x-1}$

$\Leftrightarrow \quad \dfrac{\log (ab)}{\log b} = \dfrac{x}{x-1} \quad \Leftrightarrow \quad \log_b (ab) = \dfrac{x}{x-1}$.

29. $\log (4 \cdot \sqrt[3]{63}) = \log 4 + \log (\sqrt[3]{63}) = \log 4 + \log (63)^{1/3} = \log (2^2) + \log (7 \times 3^2)^{1/3}$

$= 2 \log 2 + \dfrac{1}{3}\log 7 + \dfrac{2}{3}\log 3 = 2x + \dfrac{1}{3}z + \dfrac{2}{3}y$.

30. $\log_4 x + \log_2 x = 6 \Leftrightarrow \dfrac{\log x}{\log 4} + \dfrac{\log x}{\log 2} = 6$

$\Leftrightarrow \dfrac{\log x}{2\log 2} + \dfrac{\log x}{\log 2} = 6 \Leftrightarrow 3\log x = 12\log 2$

$\Leftrightarrow \log x = 4\log 2 \Leftrightarrow \log x = \log(2^4) = \log 16 \Leftrightarrow x = 16.$

31. $\log_8 x + \log_8\left(\dfrac{1}{6}\right) = \dfrac{1}{3} \Leftrightarrow \dfrac{\log x}{\log 8} + \dfrac{\log\frac{1}{6}}{\log 8} = \dfrac{1}{3}$

$\Leftrightarrow \log x + \log\dfrac{1}{6} = \dfrac{1}{3}\log 8 \Leftrightarrow \log x + \log\dfrac{1}{6} = \log(8^{1/3}) = \log 2$

$\Leftrightarrow \log x = \log 2 - \log\dfrac{1}{6} = \log\left(2\times\dfrac{6}{1}\right) = \log 12$

$\therefore \quad x = 12.$

32. $\log_{10} 125 + \log_{10} 8 = x \Rightarrow \log_{10}(125\times 8) = x$

$\Rightarrow x = \log_{10}(1000) = \log_{10}(10)^3 = 3\log_{10} 10 = 3.$

33. Let $\log_9 27 = x$. Then, $9^x = 27 \Leftrightarrow (3^2)^x = 3^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \dfrac{3}{2}.$

Let $\log_8 32 = y$. Then, $8^y = 32 \Leftrightarrow (2^3)^y = 2^5 \Leftrightarrow 3y = 5 \Leftrightarrow y = \dfrac{5}{3}.$

$\therefore \quad \log_9 27 + \log_8 32 = \left(\dfrac{3}{2} + \dfrac{5}{3}\right) = \dfrac{19}{6}.$

34. Given expression $= \left(\dfrac{\log 3}{\log 5}\times\dfrac{\log 625}{\log 3}\right) = \dfrac{\log 625}{\log 5} = \dfrac{\log(5^4)}{\log 5} = \dfrac{4\log 5}{\log 5} = 4.$

35. Given expression $= \dfrac{\log 9}{\log 4}\times\dfrac{\log 2}{\log 3}$ $\quad [\because \log_5 5 = 1]$

$= \dfrac{\log 3^2}{\log 2^2}\times\dfrac{\log 2}{\log 3} = \dfrac{2\log 3}{2\log 2}\times\dfrac{\log 2}{\log 3} = 1.$

36. $\log_{12} 27 = a \Rightarrow \dfrac{\log 27}{\log 12} = a \Rightarrow \dfrac{\log 3^3}{\log(3\times 2^2)} = a$

$\Rightarrow \dfrac{3\log 3}{\log 3 + 2\log 2} = a \Rightarrow \dfrac{\log 3 + 2\log 2}{3\log 3} = \dfrac{1}{a}$

$\Rightarrow \dfrac{\log 3}{3\log 3} + \dfrac{2\log 2}{3\log 3} = \dfrac{1}{a} \Rightarrow \dfrac{2}{3}\dfrac{\log 2}{\log 3} = \dfrac{1}{a} - \dfrac{1}{3} = \left(\dfrac{3-a}{3a}\right)$

$\Rightarrow \dfrac{\log 2}{\log 3} = \left(\dfrac{3-a}{2a}\right) \Rightarrow \log 3 = \left(\dfrac{2a}{3-a}\right)\log 2.$

$\log_6 16 = \dfrac{\log 16}{\log 6} = \dfrac{\log 2^4}{\log(2\times 3)} = \dfrac{4\log 2}{\log 2 + \log 3} = \dfrac{4\log 2}{\log 2\left[1 + \left(\dfrac{2a}{3-a}\right)\right]}$

$= \dfrac{4}{\left(\dfrac{3+a}{3-a}\right)} = \dfrac{4(3-a)}{(3+a)}.$

37. $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

$\Rightarrow \log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + \log_{10} 10$

$\Rightarrow \log_{10} [5 (5x + 1)] = \log_{10} [10 (x + 5)] \Rightarrow 5 (5x + 1) = 10 (x + 5)$

$\Rightarrow 5x + 1 = 2x + 10 \Rightarrow 3x = 9 \Rightarrow x = 3.$

38. $\log_5 (x^2 + x) - \log_5 (x + 1) = 2 \Rightarrow \log_5 \left(\frac{x^2 + x}{x + 1}\right) = 2$

$\Rightarrow \log_5 \left[\frac{x (x + 1)}{x + 1}\right] = 2 \Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25.$

39. Given expression $= \log_{60} 3 + \log_{60} 4 + \log_{60} 5 = \log_{60} (3 \times 4 \times 5) = \log_{60} 60 = 1.$

40. Given expression $= \left(\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}\right)$

$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2 \log 3}{\log 3} = 2$

41. We know that : $a^{\log_a x} = x.$

$\therefore \quad 16^{\log_4 5} = (4^2)^{\log_4 5} = 4^{2 \log_4 5} = 4^{\log_4 (5^2)} = 4^{\log_4 25} = 25.$

42. $\log x + \log y = \log (x + y) \Rightarrow \log (x + y) = \log (xy)$

$\Rightarrow x + y = xy \Rightarrow y (x - 1) = x \Rightarrow y = \frac{x}{x - 1}.$

43. $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b) \Rightarrow \log (a + b) = \log \left(\frac{a}{b} \times \frac{b}{a}\right) = \log 1.$

So, $a + b = 1.$

44. Given expression $= \log \left(\frac{a^2}{bc} \times \frac{b^2}{ac} \times \frac{c^2}{ab}\right) = \log 1 = 0.$

45. Given expression $= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

46. Given expression $= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} (abc) = 1.$

47. Given expression $= \log_x \left(\frac{p}{q}\right) + \log_x \left(\frac{q}{r}\right) + \log_x \left(\frac{r}{p}\right) = \log_x \left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p}\right) = \log_x 1 = 0.$

48. $\log_{10} \left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70 = -\log_{10} (7 \times 10) = -(\log_{10} 7 + \log_{10} 10) = -(a + 1).$

49. $a = b^x, b = c^y, c = a^z \Rightarrow x = \log_b a, y = \log_c b, z = \log_a c$

$\Rightarrow xyz = (\log_b a) \times (\log_c b) \times (\log_a c) \Rightarrow xyz = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

50. $\log 27 = 1.431 \Rightarrow \log (3^3) = 1.431 \Rightarrow 3 \log 3 = 1.431$

$\Rightarrow \log 3 = 0.477$

$\therefore \quad \log 9 = \log (3^2) = 2 \log 3 = (2 \times 0.477) = 0.954.$

51. $\log_2 10 = \dfrac{1}{\log_{10} 2} = \dfrac{1}{0.3010} = \dfrac{10000}{3010} = \dfrac{1000}{301}$.

52. $\log_{10} 5 = \log_{10}\left(\dfrac{10}{2}\right) = \log_{10} 10 - \log_{10} 2 = 1 - \log_{10} 2 = (1 - 0.3010) = 0.6990.$

53. $\log_{10} 80 = \log_{10} (8 \times 10) = \log_{10} 8 + \log_{10} 10 = \log_{10} (2^3) + 1 = 3 \log_{10} 2 + 1$
$= (3 \times 0.3010) + 1 = 1.9030.$

54. $(1000)^x = 3 \Rightarrow \log [(1000)^x] = \log 3 \Rightarrow x \log 1000 = \log 3$
$\Rightarrow x \log (10^3) = \log 3 \Rightarrow 3x \log 10 = \log 3$
$\Rightarrow 3x = \log 3 \Rightarrow x = \dfrac{0.477}{3} = 0.159.$

55. $\log_{10} 25 = \log_{10}\left(\dfrac{100}{4}\right) = \log_{10} 100 - \log_{10} 4 = 2 - 2 \log_{10} 2 = (2 - 2 \times 0.3010)$
$= (2 - 0.6020) = 1.3980.$

56. $\log_5 512 = \dfrac{\log 512}{\log 5} = \dfrac{\log 2^9}{\log\left(\dfrac{10}{2}\right)} = \dfrac{9 \log 2}{\log 10 - \log 2}$

$= \dfrac{(9 \times 0.3010)}{1 - 0.3010} = \dfrac{2.709}{0.699} = \dfrac{2709}{699} = 3.876.$

57. $\log_{10} (1.5) = \log_{10}\left(\dfrac{3}{2}\right) = \log_{10} 3 - \log_{10} 2 = (0.4771 - 0.3010) = 0.1761.$

58. $\log_{10} (2.8) = \log_{10}\left(\dfrac{28}{10}\right) = \log_{10} 28 - \log_{10} 10$
$= \log_{10} (7 \times 2^2) - 1 = \log_{10} 7 + 2 \log_{10} 2 - 1$
$= 0.8451 + 2 \times 0.3010 - 1 = 0.8451 + 0.602 - 1 = 0.4471.$

59. $\log (0.57) = \bar{1}.756 \Rightarrow \log 57 = 1.756$ [∵ mantissa will remain the same]

$\therefore \log 57 + \log (0.57)^3 + \log \sqrt{0.57}$

$= \log 57 + 3 \log\left(\dfrac{57}{100}\right) + \log\left(\dfrac{57}{100}\right)^{1/2}$

$= \log 57 + 3 \log 57 - 3 \log 100 + \dfrac{1}{2} \log 57 - \dfrac{1}{2} \log 100$

$= \dfrac{9}{2} \log 57 - \dfrac{7}{2} \log 100 = \dfrac{9}{2} \times 1.756 - \dfrac{7}{2} \times 2 = 7.902 - 7 = 0.902.$

60. $\log (2^{64}) = 64 \times \log 2 = (64 \times 0.30103) = 19.26592.$
Its characteristic is 19. Hence, the number of digits in 2^{64} is 20.

61. $\log 4^{50} = 50 \log 4 = 50 \log 2^2 = (50 \times 2) \log 2 = 100 \times \log 2 = (100 \times 0.30103) = 30.103.$
$\therefore$ Characteristic = 30. Hence, the number of digits in $4^{50} = 31$.

62. $\log 5^{20} = 20 \log 5 = 20 \times \left[\log\left(\dfrac{10}{2}\right)\right] = 20 (\log 10 - \log 2)$
$= 20 (1 - 0.3010) = 20 \times 0.6990 = 13.9800.$
$\therefore$ Characteristic = 13. Hence, the number of digits in 5^{20} is 14.

24. AREA

FUNDAMENTAL CONCEPTS

I. **Results on Triangles :**

1. Sum of the angles of a triangle is 180°.
2. The sum of any two sides of a triangle is greater than the third side.
3. **Pythagoras Theorem** : In a right-angled triangle,
$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2.$$
4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the ***median***.
5. The point where the three medians of a triangle meet, is called ***centroid***. The centroid divides each of the medians in the ratio 2 : 1.
6. In an isosceles triangle, the altitude from the vertex bisects the base.
7. The median of a triangle divides it into two triangles of the same area.
8. The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-fourth of the area of the given triangle.

II. **Results on Quadrilaterals :**

1. The diagonals of a parallelogram bisect each other.
2. Each diagonal of a parallelogram divides it into two triangles of the same area.
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

I. 1. Area of a rectangle = (Length × Breadth).

$$\therefore \quad \text{Length} = \left(\frac{\text{Area}}{\text{Breadth}}\right) \text{ and Breadth} = \left(\frac{\text{Area}}{\text{Length}}\right).$$

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square = $(\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) × Height.

IV. 1. Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a+b+c)$.

3. Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.

5. Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.

6. Radius of incircle of a triangle of area Δ and semi-perimeter $s = \frac{\Delta}{s}$.

V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus = $\frac{1}{2} \times$ (Product of diagonals).

3. Area of a trapezium = $\frac{1}{2} \times$ (sum of parallel sides) × distance between them.

VI. 1. Area of a circle = πR^2, where R is the radius.

2. Circumference of a circle = $2\pi R$.

3. Length of an arc = $\frac{2\pi R\theta}{360}$, where θ is the central angle.

4. Area of a sector = $\frac{1}{2}$(arc × R) = $\frac{\pi R^2\theta}{360}$.

VII. 1. Area of a semi-circle = $\frac{\pi R^2}{2}$.

2. Circumference of a semi-circle = πR.

SOLVED EXAMPLES

Ex. 1. ***One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.***

Sol. Other side = $\sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8$ m.

$\therefore$ Area = (15×8) m^2 = 120 m^2.

Ex. 2. ***A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.***

Sol. Let length = $2x$ metres and breadth = $3x$ metres.

Now, area = $\left(\frac{1}{6} \times 1000\right)$ m^2 = $\left(\frac{5000}{3}\right)$ m^2.

So, $2x \times 3x = \frac{5000}{3} \Leftrightarrow x^2 = \frac{2500}{9} \Leftrightarrow x = \left(\frac{50}{3}\right)$.

$\therefore$ Length = $2x = \frac{100}{3}$ m = $33\frac{1}{3}$ m and Breadth = $3x = \left(3 \times \frac{50}{3}\right)$ m = 50 m.

Ex. 3. ***Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.***

Sol. Area of the carpet = Area of the room = (13×9) m^2 = 117 m^2.

Length of the carpet = $\left(\frac{\text{Area}}{\text{Width}}\right) = \left(117 \times \frac{4}{3}\right)$ m = 156 m.

$\therefore$ Cost of carpeting = Rs. (156×12.40) = Rs. 1934.40.

Ex. 4. ***If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle.***

Sol. Let length = x and breadth = y. Then,

$2(x + y) = 46$ or $x + y = 23$ and $x^2 + y^2 = (17)^2 = 289$.

Now, $(x + y)^2 = (23)^2 \Leftrightarrow (x^2 + y^2) + 2xy = 529 \Leftrightarrow 289 + 2xy = 529 \Leftrightarrow xy = 120$.

$\therefore$ Area = $xy = 120$ cm^2.

Ex. 5. ***The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.***

Sol. Let breadth = x. Then, length = $2x$. Then,

$(2x - 5)(x + 5) - 2x \times x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.

$\therefore$ Length of the rectangle = 20 cm.

Ex. 6. ***In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements.*** **(M.B.A. 2003)**

Sol. Let x and y be the sides of the rectangle. Then, Correct area = xy.

Calculated area = $\left(\frac{105}{100}x\right) \times \left(\frac{96}{100}y\right) = \frac{504}{500}xy$.

Error in measurement = $\left(\frac{504}{500}xy\right) - xy = \frac{4}{500}xy$.

$\therefore$ Error % = $\left[\frac{4}{500}xy \times \frac{1}{xy} \times 100\right]\% = \frac{4}{5}\% = 0.8\%$.

Ex. 7. ***A rectangular grassy plot 110 m by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.***

Sol. Area of the plot = (110×65) m^2 = 7150 m^2.

Area of the plot excluding the path = $[(110 - 5) \times (65 - 5)]$ m^2 = 6300 m^2.

$\therefore$ Area of the path = $(7150 - 6300)$ m^2 = 850 m^2.

Cost of gravelling the path = Rs. $\left(850 \times \frac{80}{100}\right)$ = Rs. 680.

Ex. 8. ***The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares.*** **(S.S.C. 2003)**

Sol. Side of first square = $\left(\frac{40}{4}\right)$ cm = 10 cm;

Side of second square = $\left(\frac{32}{4}\right)$ cm = 8 cm.

Area of third square = $[(10)^2 - (8)^2]$ cm^2 = $(100 - 64)$ cm^2 = 36 cm^2.

Side of third square = $\sqrt{36}$ cm = 6 cm.

$\therefore$ Required perimeter = (6×4) cm = 24 cm.

Ex. 9. ***A room 5m 55 cm long and 3m 74 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor.***

Sol. Area of the room = (544×374) cm^2.

Size of largest square tile = H.C.F. of 544 cm and 374 cm = 34 cm.

Area of 1 tile = (34×34) cm^2.

$\therefore$ Number of tiles required = $\left(\frac{544 \times 374}{34 \times 34}\right) = 176$.

Ex. 10. ***Find the area of a square, one of whose diagonals is 3.8 m long.***

Sol. Area of the square $= \frac{1}{2} \times (\text{diagonal})^2 = \left(\frac{1}{2} \times 3.8 \times 3.8\right) \text{m}^2 = 7.22 \text{ m}^2$.

Ex. 11. ***The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas.*** **(Section Officers', 2003)**

Sol. Let the diagonals of the squares be $2x$ and $5x$ respectively.

$\therefore$ Ratio of their areas $= \frac{1}{2} \times (2x)^2 : \frac{1}{2} \times (5x)^2 = 4x^2 : 25x^2 = 4 : 25$.

Ex. 12. ***If each side of a square is increased by 25%, find the percentage change in its area.***

Sol. Let each side of the square be a. Then, area $= a^2$.

New side $= \frac{125a}{100} = \frac{5a}{4}$. New area $= \left(\frac{5a}{4}\right)^2 = \frac{25a^2}{16}$.

Increase in area $= \left(\frac{25a^2}{16} - a^2\right) = \frac{9a^2}{16}$.

$\therefore$ Increase % $= \left(\frac{9a^2}{16} \times \frac{1}{a^2} \times 100\right)\% = 56.25\%$.

Ex. 13. ***If the length of a certain rectangle is decreased by 4 cm and the width is increased by 3 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle.***

Sol. Let x and y be the length and breadth of the rectangle respectively.

Then, $x - 4 = y + 3$ or $x - y = 7$ ...(*i*)

Area of the rectangle $= xy$; Area of the square $= (x - 4)(y + 3)$

$\therefore$ $(x - 4)(y + 3) = xy \Leftrightarrow 3x - 4y = 12$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 16$ and $y = 9$.

$\therefore$ Perimeter of the rectangle $= 2(x + y) = [2(16 + 9)]$ cm $= 50$ cm.

Ex. 14. ***A room is half as long again as it is broad. The cost of carpeting the room at Rs. 5 per sq. m is Rs. 270 and the cost of papering the four walls at Rs. 10 per m² is Rs. 1720. If a door and 2 windows occupy 8 sq. m, find the dimensions of the room.***

Sol. Let breadth $= x$ metres, length $= \frac{3x}{2}$ metres, height $= H$ metres.

Area of the floor $= \left(\frac{\text{Total cost of carpeting}}{\text{Rate / m}^2}\right) \text{m}^2 = \left(\frac{270}{5}\right) \text{m}^2 = 54 \text{ m}^2$.

$\therefore$ $x \times \frac{3x}{2} = 54 \Leftrightarrow x^2 = \left(54 \times \frac{2}{3}\right) = 36 \Leftrightarrow x = 6$.

So, breadth $= 6$ m and length $= \left(\frac{3}{2} \times 6\right)$ m $= 9$ m.

Now, papered area $= \left(\frac{1720}{10}\right) \text{m}^2 = 172 \text{ m}^2$.

Area of 1 door and 2 windows $= 8 \text{ m}^2$.

Total area of 4 walls $= (172 + 8) \text{ m}^2 = 180 \text{ m}^2$.

$\therefore$ $2(9 + 6) \times H = 180 \Leftrightarrow H = \left(\frac{180}{30}\right) = 6$ m.

Ex. 15. ***Find the area of a triangle whose sides measure 13 cm, 14 cm and 15 cm.***

Sol. Let $a = 13$, $b = 14$ and $c = 15$. Then, $s = \frac{1}{2}(a+b+c) = 21$.

$\therefore$ $(s-a) = 8$, $(s-b) = 7$ and $(s-c) = 6$.

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21\times 8\times 7\times 6} = 84 \text{ cm}^2$.

Ex. 16. ***Find the area of a right-angled triangle whose base is 12 cm and hypotenuse 13 cm.***

Sol. Height of the triangle $= \sqrt{(13)^2-(12)^2} \text{ cm} = \sqrt{25} \text{ cm} = 5 \text{ cm}$.

$\therefore$ Its area $= \frac{1}{2}\times \text{Base}\times \text{Height} = \left(\frac{1}{2}\times 12\times 5\right) \text{cm}^2 = 30 \text{ cm}^2$.

Ex. 17. ***The base of a triangular field is three times its altitude. If the cost of cultivating the field at Rs. 24.68 per hectare be Rs. 333.18, find its base and height.***

Sol. Area of the field $= \frac{\text{Total cost}}{\text{Rate}} = \left(\frac{333.18}{24.68}\right)$ hectares $= 13.5$ hectares

$= (13.5\times 10000) \text{ m}^2 = 135000 \text{ m}^2$.

Let altitude $= x$ metres and base $= 3x$ metres.

Then, $\frac{1}{2}\times 3x\times x = 135000 \Leftrightarrow x^2 = 90000 \Leftrightarrow x = 300$.

$\therefore$ Base $= 900$ m and Altitude $= 300$ m.

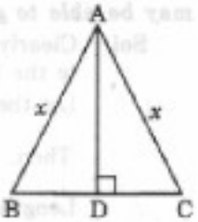
Ex. 18. ***The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. Find the area of the triangle.***

Sol. Let ABC be the isosceles triangle and AD be the altitude.

Let AB $=$ AC $= x$. Then, BC $= (32-2x)$.

Since, in an isosceles triangle, the altitude bisects the base,

so BD $=$ DC $= (16-x)$.



In Δ ADC, $AC^2 = AD^2 + DC^2 \Rightarrow x^2 = (8)^2 + (16-x)^2$

$\Rightarrow 32x = 320 \Rightarrow x = 10$.

$\therefore$ BC $= (32-2x) = (32-20) \text{ cm} = 12 \text{ cm}$.

Hence, required area $= \left(\frac{1}{2}\times \text{BC}\times \text{AD}\right) = \left(\frac{1}{2}\times 12\times 10\right) \text{cm}^2 = 60 \text{ cm}^2$.

Ex. 19. ***Find the length of the altitude of an equilateral triangle of side $3\sqrt{3}$ cm.***

Sol. Area of the triangle $= \frac{\sqrt{3}}{4}\times (3\sqrt{3})^2 = \frac{27\sqrt{3}}{4}$. Let the height be h.

Then, $\frac{1}{2}\times 3\sqrt{3}\times h = \frac{27\sqrt{3}}{4} \Leftrightarrow h = \frac{27\sqrt{3}}{4}\times \frac{2}{3\sqrt{3}} = \frac{9}{2} = 4.5 \text{ cm}$.

Ex. 20. ***In two triangles, the ratio of the areas is 4 : 3 and the ratio of their heights is 3 : 4. Find the ratio of their bases.***

Sol. Let the bases of the two triangles be x and y and their heights be $3h$ and $4h$ respectively. Then,

$$\frac{\frac{1}{2}\times x\times 3h}{\frac{1}{2}\times y\times 4h} = \frac{4}{3} \Leftrightarrow \frac{x}{y} = \left(\frac{4}{3}\times \frac{4}{3}\right) = \frac{16}{9}.$$

$\therefore$ Required ratio $= 16 : 9$.

Ex. 21. ***The base of a parallelogram is twice its height. If the area of the parallelogram is 72 sq. cm, find its height.***

Sol. Let the height of the parallelogram be x cm. Then, base = $(2x)$ cm.

$\therefore \quad 2x \times x = 72 \Leftrightarrow 2x^2 = 72 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6.$

Hence, height of the parallelogram = 6 cm.

Ex. 22. ***Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.***

Sol. Let other diagonal = $2x$ cm.

Since diagonals of a rhombus bisect each other at right angles, we have :

$(20)^2 = (12)^2 + x^2 \Leftrightarrow x = \sqrt{(20)^2 - (12)^2} = \sqrt{256} = 16$ cm.

So, other diagonal = 32 cm.

$\therefore$ Area of rhombus $= \frac{1}{2} \times$ (Product of diagonals) $= \left(\frac{1}{2} \times 24 \times 32\right)$ cm^2 = 384 cm^2.

Ex. 23. ***The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm^2, find the lengths of the parallel sides.*** **(R.R.B. 2002)**

Sol. Let the two parallel sides of the trapezium be a cm and b cm.

Then, $a - b = 4$ ...(*i*)

And, $\frac{1}{2} \times (a + b) \times 19 = 475 \Leftrightarrow (a + b) = \left(\frac{475 \times 2}{19}\right) \Leftrightarrow a + b = 50$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $a = 27$, $b = 23$.

So, the two parallel sides are 27 cm and 23 cm.

Ex. 24. ***Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 sq. metres.*** **(M.A.T. 2003)**

Sol. Clearly, the cow will graze a circular field of area 9856 sq. metres and radius equal to the length of the rope.

Let the length of the rope be R metres.

Then, $\pi R^2 = 9856 \Leftrightarrow R^2 = \left(9856 \times \frac{7}{22}\right) = 3136 \Leftrightarrow R = 56.$

$\therefore$ Length of the rope = 56 m.

Ex. 25. ***The area of a circular field is 13.86 hectares. Find the cost of fencing it at the rate of Rs. 4.40 per metre.***

Sol. Area = (13.86×10000) m^2 = 138600 m^2.

$\pi R^2 = 138600 \Leftrightarrow R^2 = \left(138600 \times \frac{7}{22}\right) \Leftrightarrow R = 210$ m.

Circumference = $2\pi R = \left(2 \times \frac{22}{7} \times 210\right)$ m = 1320 m.

$\therefore$ Cost of fencing = Rs. (1320×4.40) = Rs. 5808.

Ex. 26. ***The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 kmph ?***

Sol. Distance to be covered in 1 min. $= \left(\frac{66 \times 1000}{60}\right)$ m = 1100 m.

Circumference of the wheel $= \left(2 \times \frac{22}{7} \times 0.70\right)$ m = 4.4 m.

$\therefore$ Number of revolutions per min. $= \left(\frac{1100}{4.4}\right) = 250.$

Ex. 27. ***A wheel makes 1000 revolutions in covering a distance of 88 km. Find the radius of the wheel.***

Sol. Distance covered in one revolution $= \left(\frac{88 \times 1000}{1000}\right)$ m $= 88$ m.

$\therefore \quad 2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88 \Leftrightarrow R = \left(88 \times \frac{7}{44}\right) = 14$ m.

Ex. 28. ***The inner circumference of a circular race track, 14 m wide, is 440 m. Find the radius of the outer circle.***

Sol. Let inner radius be r metres. Then, $2\pi r = 440 \Rightarrow r = \left(440 \times \frac{7}{44}\right) = 70$ m.

$\therefore$ Radius of outer circle $= (70 + 14)$ m $= 84$ m.

Ex. 29. ***Two concentric circles form a ring. The inner and outer circumferences of the ring are $50\frac{2}{7}$ m and $75\frac{3}{7}$ m respectively. Find the width of the ring.***

Sol. Let the inner and outer radii be r and R metres.

Then, $2\pi r = \frac{352}{7} \Rightarrow r = \left(\frac{352}{7} \times \frac{7}{22} \times \frac{1}{2}\right) = 8$ m.

$2\pi R = \frac{528}{7} \Rightarrow R = \left(\frac{528}{7} \times \frac{7}{22} \times \frac{1}{2}\right) = 12$ m.

$\therefore$ Width of the ring $= (R - r) = (12 - 8)$ m $= 4$ m.

Ex. 30. ***A sector of 120°, cut out from a circle, has an area of $9\frac{3}{7}$ sq. cm. Find the radius of the circle.*** (C.B.I. 1997)

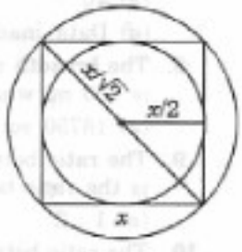
Sol. Let the radius of the circle be r cm. Then,

$\frac{\pi r^2 \theta}{360} = \frac{66}{7} \Leftrightarrow \frac{22}{7} \times r^2 \times \frac{120}{360} = \frac{66}{7} \Leftrightarrow r^2 = \left(\frac{66}{7} \times \frac{7}{22} \times 3\right) = 9 \Leftrightarrow r = 3.$

Hence, radius $= 3$ cm.

Ex. 31. ***Find the ratio of the areas of the incircle and circumcircle of a square.***

Sol. Let the side of the square be x. Then, its diagonal $= \sqrt{2}\,x$.



Radius of incircle $= \frac{x}{2}$ and

radius of circumcircle $= \frac{\sqrt{2}\,x}{2} = \frac{x}{\sqrt{2}}$.

$\therefore$ Required ratio $= \left(\frac{\pi x^2}{4} : \frac{\pi x^2}{2}\right) = \frac{1}{4} : \frac{1}{2} = 1 : 2.$

Ex. 32. ***If the radius of a circle is decreased by 50%, find the percentage decrease in its area.***

Sol. Let original radius $= R$. New radius $= \frac{50}{100} R = \frac{R}{2}$.

Original area $= \pi R^2$ and New area $= \pi \left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$.

$\therefore$ Decrease in area $= \left(\frac{3\pi R^2}{4} \times \frac{1}{\pi R^2} \times 100\right)\% = 75\%$.

EXERCISE 24A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The length of a room is 5.5 m and width is 3.75 m. Find the cost of paving the floor by slabs at the rate of Rs. 800 per sq. metre. **(IGNOU, 2003)**
   (*a*) Rs. 15,000 (*b*) Rs. 15,550 (*c*) Rs. 15,600 (*d*) Rs. 16,500

2. The length of a rectangle is 18 cm and its breadth is 10 cm. When the length is increased to 25 cm, what will be the breadth of the rectangle if the area remains the same ?
   (*a*) 7 cm (*b*) 7.1 cm (*c*) 7.2 cm (*d*) 7.3 cm

3. A rectangular plot measuring 90 metres by 50 metres is to be enclosed by wire fencing. If the poles of the fence are kept 5 metres apart, how many poles will be needed ?
   (*a*) 55 (*b*) 56 (*c*) 57 (*d*) 58

4. The length of a rectangular plot is 60% more than its breadth. If the difference between the length and the breadth of that rectangle is 24 cm, what is the area of that rectangle ? **(Bank P.O. 1998)**
   (*a*) 2400 sq. cm (*b*) 2480 sq. cm (*c*) 2560 sq. cm
   (*d*) Data inadequate (*e*) None of these

5. A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet ? **(M.A.T. 2003)**
   (*a*) 46 (*b*) 81 (*c*) 126 (*d*) 252

6. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is : **(Section Officers', 2003)**
   (*a*) 1520 m^2 (*b*) 2420 m^2 (*c*) 2480 m^2 (*d*) 2520 m^2

7. The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot @ Rs. 26.50 per metre is Rs. 5300, what is the length of the plot in metres ? **(Bank P.O. 1999)**
   (*a*) 40 (*b*) 50 (*c*) 120
   (*d*) Data inadequate (*e*) None of these

8. The breadth of a rectangular field is 60% of its length. If the perimeter of the field is 800 m, what is the area of the field ?
   (*a*) 18750 sq. m (*b*) 37500 sq. m (*c*) 40000 sq. m (*d*) 48000 sq. m

9. The ratio between the length and the perimeter of a rectangular plot is 1 : 3. What is the ratio between the length and breadth of the plot ?
   (*a*) 1 : 2 (*b*) 2 : 1 (*c*) 3 : 2 (*d*) Data inadequate

10. The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km / hr completes one round in 8 minutes, then the area of the park (in sq. m) is : **(S.S.C. 2003)**
    (*a*) 15360 (*b*) 153600 (*c*) 30720 (*d*) 307200

11. The length of a rectangular hall is 5 m more than its breadth. The area of the hall is 750 m^2. The length of the hall is : **(S.S.C. 2004)**
    (*a*) 15 m (*b*) 22.5 m (*c*) 25 m (*d*) 30 m

12. The area of a rectangle is 460 square metres. If the length is 15% more than the breadth, what is the breadth of the rectangular field ? **(Bank P.O. 2003)**
    (*a*) 15 metres (*b*) 26 metres (*c*) 34.5 metres
    (*d*) Cannot be determined (*e*) None of these

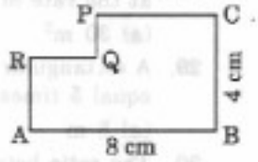
13. A rectangular field is to be fenced on three sides leaving a side of 20 feet uncovered. If the area of the field is 680 sq. feet, how many feet of fencing will be required ?
(a) 34 (b) 40 (c) 68 (d) 88
(R.R.B. 2002)

14. The ratio between the perimeter and the breadth of a rectangle is 5 : 1. If the area of the rectangle is 216 sq. cm, what is the length of the rectangle ?
(a) 16 cm (b) 18 cm (c) 24 cm
(d) Data inadequate (e) None of these **(B.S.R.B. 1998)**

15. A farmer wishes to start a 100 sq. m rectangular vegetable garden. Since he has only 30 m barbed wire, he fences three sides of the garden letting his house compound wall act as the fourth side fencing. The dimension of the garden is : **(R.R.B. 2003)**
(a) 15 m × 6.67 m (b) 20 m × 5 m (c) 30 m × 3.33 m (d) 40 m × 2.5 m

16. The sides of a rectangular field are in the ratio 3 : 4. If the area of the field is 7500 sq. m, the cost of fencing the field @ 25 paise per metre is : **(R.R.B. 2004)**
(a) Rs. 55.50 (b) Rs. 67.50 (c) Rs. 86.50 (d) Rs. 87.50

17. A rectangle of certain dimensions is chopped off from one corner of a larger rectangle as shown. AB = 8 cm and BC = 4 cm. The perimeter of the figure ABCPQRA (in cm) is : **(Asstt. Grade, 1998)**



(a) 24 (b) 28 (c) 36 (d) 48

18. A large field of 700 hectares is divided into two parts. The difference of the areas of the two parts is one-fifth of the average of the two areas. What is the area of the smaller part in hectares ?
(a) 225 (b) 280 (c) 300 (d) 315

19. A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper ? **(S.S.C. 2000)**
(a) 140 cm^2 (b) 240 cm^2 (c) 560 cm^2 (d) None of these

20. A rectangular plot is half as long again as it is broad and its area is $\frac{2}{3}$ hectares. Then, its length is :
(a) 100 m (b) 33.33 m (c) 66.66 m (d) $\frac{100\sqrt{3}}{3}$ m

21. A courtyard 25 m long and 16 m broad is to be paved with bricks of dimensions 20 cm by 10 cm. The total number of bricks required is :
(a) 18000 (b) 20000 (c) 25000 (d) None of these

22. The cost of carpeting a room 18 m long with a carpet 75 cm wide at Rs. 4.50 per metre is Rs. 810. The breadth of the room is :
(a) 7 m (b) 7.5 m (c) 8 m (d) 8.5 m

23. The diagonal of the floor of a rectangular closet is $7\frac{1}{2}$ feet. The shorter side of the closet is $4\frac{1}{2}$ feet. What is the area of the closet in square feet ? **(M.B.A. 2003)**
(a) $5\frac{1}{4}$ (b) $13\frac{1}{2}$ (c) 27 (d) 37

24. The length of a rectangle is three times of its width. If the length of the diagonal is $8\sqrt{10}$ cm, then the perimeter of the rectangle is : **(S.S.C. 2000)**

(*a*) $15\sqrt{10}$ cm (*b*) $16\sqrt{10}$ cm (*c*) $24\sqrt{10}$ cm (*d*) 64 cm

25. The diagonal of a rectangle is thrice its smaller side. The ratio of the length to the breadth of the rectangle is :

(*a*) 3 : 1 (*b*) $\sqrt{3}$: 1 (*c*) $\sqrt{2}$: 1 (*d*) $2\sqrt{2}$: 1

26. A rectangular carpet has an area of 120 sq. metres and a perimeter of 46 metres. The length of its diagonal is :

(*a*) 15 m (*b*) 16 m (*c*) 17 m (*d*) 20 m

27. The diagonal of a rectangle is $\sqrt{41}$ cm and its area is 20 sq. cm. The perimeter of the rectangle must be : **(Hotel Management, 2002)**

(*a*) 9 cm (*b*) 18 cm (*c*) 20 cm (*d*) 41 cm

28. A took 15 seconds to cross a rectangular field diagonally walking at the rate of 52 m / min and B took the same time to cross the same field along its sides walking at the rate of 68 m / min. The area of the field is : **(S.S.C. 2003)**

(*a*) 30 m^2 (*b*) 40 m^2 (*c*) 50 m^2 (*d*) 60 m^2

29. A rectangular carpet has an area of 60 sq. m. If its diagonal and longer side together equal 5 times the shorter side, the length of the carpet is :

(*a*) 5 m (*b*) 12 m (*c*) 13 m (*d*) 14.5 m

30. The ratio between the length and the breadth of a rectangular field is 3 : 2. If only the length is increased by 5 metres, the new area of the field will be 2600 sq. metres. What is the breadth of the rectangular field ?

(*a*) 40 metres (*b*) 60 metres (*c*) 65 metres

(*d*) Cannot be determined (*e*) None of these

31. The length of a blackboard is 8 cm more than its breadth. If the length is increased by 7 cm and breadth is decreased by 4 cm, the area remains the same. The length and breadth of the blackboard (in cm) will be :

(*a*) 28, 20 (*b*) 34, 26 (*c*) 40, 32 (*d*) 56, 48

32. If the length and breadth of a rectangular room are each increased by 1 m, then the area of floor is increased by 21 sq. m. If the length is increased by 1 m and breadth is decreased by 1 m, then the area is decreased by 5 sq. m. The perimeter of the floor is : **(M.B.A. 2002)**

(*a*) 30 m (*b*) 32 m (*c*) 36 m (*d*) 40 m

33. The percentage increase in the area of a rectangle, if each of its sides is increased by 20%, is : **(M.A.T. 2004)**

(*a*) 40% (*b*) 42% (*c*) 44% (*d*) 46%

34. A rectangle has width *a* and length *b*. If the width is decreased by 20% and the length is increased by 10%, then what is the area of the new rectangle in percentage compared to '*ab*' ? **(R.R.B. 2002)**

(*a*) 80% (*b*) 88% (*c*) 110% (*d*) 120%

35. If the length and breadth of a rectangular plot be increased by 50% and 20% respectively, then how many times will its area be increased ? **(Bank P.O. 2003)**

(*a*) $1\frac{1}{3}$ (*b*) 2 (*c*) $3\frac{2}{5}$ (*d*) $4\frac{1}{5}$ (*e*) None of these

36. A towel, when bleached, was found to have lost 20% of its length and 10% of its breadth. The percentage of decrease in area is : **(N.I.F.T. 1997)**

(*a*) 10% (*b*) 10.08% (*c*) 20% (*d*) 28%

37. The length of a rectangle is halved, while its breadth is tripled. What is the percentage change in area ? **(S.S.C. 2000)**
(*a*) 25% increase (*b*) 50% increase (*c*) 50% decrease (*d*) 75% decrease

38. The length of a rectangle is decreased by r%, and the breadth is increased by $(r + 5)$%. Find r, if the area of the rectangle is unaltered. **(SCMHRD, 2002)**
(*a*) 5 (*b*) 8 (*c*) 10 (*d*) 15 (*e*) 20

39. The length of a rectangle is increased by 60%. By what percent would the width have to be decreased so as to maintain the same area ? **(M.A.T. 2003)**
(*a*) $37\frac{1}{2}$% (*b*) 60% (*c*) 75% (*d*) 120%

40. If the area of a rectangular plot increases by 30% while its breadth remains same, what will be the ratio of the areas of new and old figures ? **(Bank P.O. 2003)**
(*a*) 1 : 3 (*b*) 3 : 1 (*c*) 4 : 7 (*d*) 10 : 13 (*e*) None of these

41. A typist uses a sheet measuring 20 cm by 30 cm lengthwise. If a margin of 2 cm is left on each side and a 3 cm margin on top and bottom, then percent of the page used for typing is : **(M.A.T. 1998)**
(*a*) 40 (*b*) 60 (*c*) 64 (*d*) 72

42. A room is 15 feet long and 12 feet broad. A mat has to be placed on the floor of this room leaving $1\frac{1}{2}$ feet space from the walls. What will be the cost of the mat at the rate of Rs. 3.50 per square feet ? **(R.R.B. 2002)**
(*a*) Rs. 378 (*b*) Rs. 472.50 (*c*) Rs. 496 (*d*) Rs. 630

43. What will be the cost of gardening 1 metre broad boundary around a rectangular plot having perimeter of 340 metres at the rate of Rs. 10 per square metre ?
(*a*) Rs. 1700 (*b*) Rs. 3400 (*c*) Rs. 3440
(*d*) Cannot be determined (*e*) None of these **(Bank P.O. 2003)**

44. 2 metres broad pathway is to be constructed around a rectangular plot on the inside. The area of the plot is 96 sq. m. The rate of construction is Rs. 50 per square metre. Find the total cost of the construction. **(S.B.I.P.O. 2000)**
(*a*) Rs. 2400 (*b*) Rs. 4000 (*c*) Rs. 4800
(*d*) Data inadequate (*e*) None of these

45. Within a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. How wide should the walk be ?
(*a*) 1 m (*b*) 2 m (*c*) 2.1 m (*d*) 2.5 m

46. A rectangular lawn 55 m by 35 m has two roads each 4 m wide running in the middle of it, one parallel to length and the other parallel to breadth. The cost of gravelling the roads at 75 paise per sq. metre is :
(*a*) Rs. 254.50 (*b*) Rs. 258 (*c*) Rs. 262.50 (*d*) Rs. 270

47. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 sq. m, then what is the width of the road ? **(M.A.T. 1997)**
(*a*) 2.91 m (*b*) 3 m (*c*) 5.82 m (*d*) None of these

48. A housing society has been allotted a square piece of land measuring 2550.25 sq. m. What is the side of the plot ?
(*a*) 50.25 m (*b*) 50.5 m (*c*) 50.65 m (*d*) None of these

49. The cost of cultivating a square field at the rate of Rs. 135 per hectare is Rs. 1215. The cost of putting a fence around it at the rate of 75 paise per metre would be :
(*a*) Rs. 360 (*b*) Rs. 810 (*c*) Rs. 900 (*d*) Rs. 1800

50. The perimeters of five squares are 24 cm, 32 cm, 40 cm, 76 cm and 80 cm respectively. The perimeter of another square equal in area to the sum of the areas of these squares is : **(S.S.C. 2004)**

(*a*) 31 cm (*b*) 62 cm (*c*) 124 cm (*d*) 961 cm

51. The number of marble slabs of size 20 cm × 30 cm required to pave the floor of a square room of side 3 metres, is :

(*a*) 100 (*b*) 150 (*c*) 225 (*d*) 250

52. 50 square stone slabs of equal size were needed to cover a floor area of 72 sq. m. The length of each stone slab is : **(S.S.C. 2003)**

(*a*) 102 cm (*b*) 120 cm (*c*) 201 cm (*d*) 210 cm

53. The length and breadth of the floor of the room are 20 feet and 10 feet respectively. Square tiles of 2 feet length of different colours are to be laid on the floor. Black tiles are laid in the first row on all sides. If white tiles are laid in the one-third of the remaining and blue tiles in the rest, how many blue tiles will be there ?

(*a*) 16 (*b*) 24 (*c*) 32

(*d*) 48 (*e*) None of these **(S.B.I.P.O. 2000)**

54. What is the least number of square tiles required to pave the floor of a room 15 m 17 cm long and 9 m 2 cm broad ? **(S.S.C. 2003)**

(*a*) 814 (*b*) 820 (*c*) 840 (*d*) 844

55. A rectangular room can be partitioned into two equal square rooms by a partition 7 metres long. What is the area of the rectangular room in square metres ?

(*a*) 49 (*b*) 147 (*c*) 196 (*d*) None of these

56. The perimeter of a square is 48 cm. The area of a rectangle is 4 cm^2 less than the area of the square. If the length of the rectangle is 14 cm, then its perimeter is :

(*a*) 24 cm (*b*) 48 cm (*c*) 50 cm (*d*) 54 cm **(S.S.C. 2002)**

57. The area of a rectangle is thrice that of a square. If the length of the rectangle is 40 cm and its breadth is $\frac{3}{2}$ times that of the side of the square, then the side of the square is :

(*a*) 15 cm (*b*) 20 cm (*c*) 30 cm (*d*) 60 cm

58. If the perimeter of a rectangle and a square, each is equal to 80 cm and the difference of their areas is 100 sq. cm, the sides of the rectangle are :

(*a*) 25 cm, 15 cm (*b*) 28 cm, 12 cm (*c*) 30 cm, 10 cm (*d*) 35 cm, 15 cm

59. The cost of fencing a square field @ Rs. 20 per metre is Rs. 10,080. How much will it cost to lay a three metre wide pavement along the fencing inside the field @ Rs. 50 per sq. metre ?

(*a*) Rs. 37,350 (*b*) Rs. 73,800 (*c*) Rs. 77,400 (*d*) None of these

60. A park square in shape has a 3 metre wide road inside it running along its sides. The area occupied by the road is 1764 square metres. What is the perimeter along the outer edge of the road ? **(Bank P.O. 1998)**

(*a*) 576 metres (*b*) 600 metres (*c*) 640 metres

(*d*) Data inadequate (*e*) None of these

61. A man walked diagonally across a square lot. Approximately, what was the percent saved by not walking along the edges ? **(M.B.A. 2003)**

(*a*) 20 (*b*) 24 (*c*) 30 (*d*) 33

62. A man walking at the speed of 4 kmph crosses a square field diagonally in 3 minutes. The area of the field is :

(*a*) 18000 m^2 (*b*) 19000 m^2 (*c*) 20000 m^2 (*d*) 25000 m^2

63. If the length of the diagonal of a square is 20 cm, then its perimeter must be :
(a) $10\sqrt{2}$ cm (b) 40 cm (c) $40\sqrt{2}$ cm (d) 200 cm
(R.R.B. 2003)

64. The area of a square field is 69696 cm^2. Its diagonal will be equal to :
(a) 313.296 m (b) 353.296 m (c) 373.296 m (d) 393.296 m
(S.S.C. 1999)

65. What will be the length of the diagonal of that square plot whose area is equal to the area of a rectangular plot of length 45 metres and breadth 40 metres ?
(a) 42.5 metres (b) 60 metres (c) 75 metres
(d) Data inadequate (e) None of these **(Bank P.O. 1999)**

66. The length of a rectangle is 20% more than its breadth. What will be the ratio of the area of a rectangle to that of a square whose side is equal to the breadth of the rectangle ? **(Bank P.O. 2000)**
(a) 2 : 1 (b) 5 : 6 (c) 6 : 5
(d) Data inadequate (e) None of these

67. A square and a rectangle have equal areas. If their perimeters are p_1 and p_2 respectively, then :
(a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) None of these

68. If the perimeters of a square and a rectangle are the same, then the area A and B enclosed by them would satisfy the condition :
(a) $A < B$ (b) $A \leq B$ (c) $A > B$ (d) $A \geq B$

69. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square whose area is double that of the first square, is : **(S.S.C. 2002)**
(a) 8 cm (b) $8\sqrt{2}$ cm (c) $4\sqrt{2}$ cm (d) 16 cm

70. The ratio of the area of a square to that of the square drawn on its diagonal, is :
(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5
(IGNOU, 2003)

71. The ratio of the areas of two squares, one having its diagonal double than the other, is :
(a) 2 : 1 (b) 2 : 3 (c) 3 : 1 (d) 4 : 1

72. If the ratio of areas of two squares is 225 : 256, then the ratio of their perimeters is :
(a) 225 : 256 (b) 256 : 225 (c) 15 : 16 (d) 16 : 15
(S.S.C. 2004)

73. Of the two square fields, the area of one is 1 hectare while the other one is broader by 1%. The difference in their areas is :
(a) 100 m^2 (b) 101 m^2 (c) 200 m^2 (d) 201 m^2

74. If each side of a square is increased by 50%, the ratio of the area of the resulting square to that of the given square is :
(a) 4 : 5 (b) 5 : 4 (c) 4 : 9 (d) 9 : 4

75. What happens to the area of a square when its side is halved ? Its area will :
(a) remain same (b) become half (c) become one-fourth (d) become double
(R.R.B. 2003)

76. An error of 2% in excess is made while measuring the side of a square. The percentage of error in the calculated area of the square is : **(C.D.S. 2003)**
(a) 2% (b) 2.02% (c) 4% (d) 4.04%

77. If the area of a square increases by 69%, then the side of the square increases by :
(a) 13% (b) 30% (c) 39% (d) 69%
(M.A.T. 1998)

78. If the diagonal of a square is made 1.5 times, then the ratio of the areas of two squares is :
(*a*) 4 : 3 (*b*) 4 : 5 (*c*) 4 : 7 (*d*) 4 : 9

79. The length and breadth of a square are increased by 40% and 30% respectively. The area of the resulting rectangle exceeds the area of the square by :
(*a*) 35% (*b*) 42% (*c*) 62% (*d*) 82%

80. The length of one pair of opposite sides of a square is increased by 5 cm on each side; the ratio of the length and the breadth of the newly formed rectangle becomes 3 : 2. What is the area of the original square ? **(Bank P.O. 1999)**
(*a*) 25 sq. cm (*b*) 81 sq. cm (*c*) 100 sq. cm
(*d*) 225 sq. cm (*e*) None of these

81. If the side of a square is increased by 5 cm, the area increases by 165 sq. cm. The side of the square is :
(*a*) 12 cm (*b*) 13 cm (*c*) 14 cm (*d*) 15 cm

82. The difference of the areas of two squares drawn on two line segments of different lengths is 32 sq. cm. Find the length of the greater line segment if one is longer than the other by 2 cm. **(S.S.C. 2003)**
(*a*) 7 cm (*b*) 9 cm (*c*) 11 cm (*d*) 16 cm

83. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle. **(S.S.C. 2002)**
(*a*) 17 cm (*b*) 26 cm (*c*) 30 cm (*d*) 34 cm

84. A tank is 25 m long, 12 m wide and 6 m deep. The cost of plastering its walls and bottom at 75 paise per sq. m, is : **(C.B.I. 1997)**
(*a*) Rs. 456 (*b*) Rs. 458 (*c*) Rs. 558 (*d*) Rs. 568

85. The dimensions of a room are 10 m × 7 m × 5 m. There are 2 doors and 3 windows in the room. The dimensions of the doors are 1 m × 3 m. One window is of size 2 m × 1.5 m and the other two windows are of size 1 m × 1.5 m. The cost of painting the walls at Rs. 3 per m^2 is :
(*a*) Rs. 474 (*b*) Rs. 578.50 (*c*) Rs. 684 (*d*) Rs. 894

86. The cost of papering the four walls of a room is Rs. 475. Each one of the length, breadth and height of another room is double that of this room. The cost of papering the walls of this new room is :
(*a*) Rs. 712.50 (*b*) Rs. 950 (*c*) Rs. 1425 (*d*) Rs. 1900

87. The ratio of height of a room to its semi-perimeter is 2 : 5. It costs Rs. 260 to paper the walls of the room with paper 50 cm wide at Rs. 2 per metre allowing an area of 15 sq. m for doors and windows. The height of the room is :
(*a*) 2.6 m (*b*) 3.9 m (*c*) 4 m (*d*) 4.2 m

88. The base of a triangle is 15 cm and height is 12 cm. The height of another triangle of double the area having the base 20 cm is : **(S.S.C. 2002)**
(*a*) 8 cm (*b*) 9 cm (*c*) 12.5 cm (*d*) 18 cm

89. ABC is a triangle with base AB. D is a point on AB such that AB = 5 and DB = 3. What is the ratio of the area of Δ ADC to the area of Δ ABC ? **(S.S.C. 2000)**
(*a*) 2 : 3 (*b*) 3 : 2 (*c*) 2 : 5 (*d*) 3 : 5

90. The area of a right-angled triangle is 40 times its base. What is its height ?
(*a*) 45 cm (*b*) 60 cm (*c*) 80 cm
(*d*) Data inadequate (*e*) None of these **(B.S.R.B. 1998)**

91. If the area of a triangle is 1176 cm^2 and base : corresponding altitude is 3 : 4, then the altitude of the triangle is : **(S.S.C. 2000)**
(*a*) 42 cm (*b*) 52 cm (*c*) 54 cm (*d*) 56 cm

92. The three sides of a triangle are 5 cm, 12 cm and 13 cm respectively. Then, its area is :

(*a*) $10\sqrt{3}$ cm^2 (*b*) $10\sqrt{6}$ cm^2 (*c*) 20 cm^2 (*d*) 30 cm^2

93. The sides of a triangle are in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is : **(M.A.T. 2004)**

(*a*) 9 cm (*b*) 10 cm (*c*) 11 cm (*d*) 12 cm

94. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is : **(S.S.C. 2004)**

(*a*) 6 cm (*b*) 12 cm (*c*) 36 cm (*d*) 72 cm

95. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in cm^2) of the triangle formed by joining the mid-points of the sides of this triangle is : **(S.S.C. 2003)**

(*a*) $\frac{3}{4}$ (*b*) $\frac{3}{2}$ (*c*) 3 (*d*) 6

96. One side of a right-angled triangle is twice the other, and the hypotenuse is 10 cm. The area of the triangle is :

(*a*) 20 cm^2 (*b*) $33\frac{1}{3}$ cm^2 (*c*) 40 cm^2 (*d*) 50 cm^2

97. The perimeter of a right-angled triangle is 60 cm. Its hypotenuse is 26 cm. The area of the triangle is : **(M.B.A. 2002)**

(*a*) 120 cm^2 (*b*) 240 cm^2 (*c*) 390 cm^2 (*d*) 780 cm^2

98. If the perimeter of an isosceles right triangle is $(6 + 3\sqrt{2})$ m, then the area of the triangle is : **(M.A.T. 2003)**

(*a*) 4.5 m^2 (*b*) 5.4 m^2 (*c*) 9 m^2 (*d*) 81 m^2

99. The perimeter of a triangle is 30 cm and its area is 30 cm^2. If the largest side measures 13 cm, then what is the length of the smallest side of the triangle ?

(*a*) 3 cm (*b*) 4 cm (*c*) 5 cm (*d*) 6 cm **(S.S.C. 2003)**

100. If the area of an equilateral triangle is $24\sqrt{3}$ sq. cm, then its perimeter is :

(*a*) $2\sqrt{6}$ cm (*b*) $4\sqrt{6}$ cm (*c*) $12\sqrt{6}$ cm (*d*) 96 cm

101. The height of an equilateral triangle is 10 cm. Its area is : **(S.S.C. 2003)**

(*a*) $\frac{100}{3}$ cm^2 (*b*) 30 cm^2 (*c*) 100 cm^2 (*d*) $\frac{100}{\sqrt{3}}$ cm^2

102. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3}$ cm, $2\sqrt{3}$ cm and $5\sqrt{3}$ cm. The perimeter (in cm) of the triangle is :

(*a*) 24 (*b*) 32 (*c*) 48 (*d*) 64

103. If x is the length of a median of an equilateral triangle, then its area is :

(*a*) x^2 (*b*) $\frac{1}{2}x^2$ (*c*) $\frac{\sqrt{3}}{2}x^2$ (*d*) $\frac{\sqrt{3}}{3}x^2$

104. If the area of a square with side a is equal to the area of a triangle with base a, then the altitude of the triangle is : **(B.S.F. 2001)**

(*a*) $\frac{a}{2}$ (*b*) a (*c*) $2a$ (*d*) $4a$

105. An equilateral triangle is described on the diagonal of a square. What is the ratio of the area of the triangle to that of the square ? **(S.S.C. 2002)**

(*a*) 2 : $\sqrt{3}$ (*b*) 4 : $\sqrt{3}$ (*c*) $\sqrt{3}$: 2 (*d*) $\sqrt{3}$: 4

106. What will be the ratio between the area of a rectangle and the area of a triangle with one of the sides of the rectangle as base and a vertex on the opposite side of the rectangle ? **(S.B.I.P.O. 1999)**

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1

(d) Data inadequate (e) None of these

107. If an equilateral triangle of area X and a square of area Y have the same perimeter, then X is : **(C.D.S. 2003)**

(a) equal to Y (b) greater than Y

(c) less than Y (d) less than or equal to Y

108. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, then the area of the triangle is :

(a) $24\sqrt{2}$ cm^2 (b) $24\sqrt{3}$ cm^2 (c) $48\sqrt{3}$ cm^2 (d) $64\sqrt{3}$ cm^2

109. The ratio of bases of two triangles is $x : y$ and that of their areas is $a : b$. Then the ratio of their corresponding altitudes will be : **(S.S.C. 2004)**

(a) $ax : by$ (b) $\frac{a}{x} : \frac{b}{y}$ (c) $ay : bx$ (d) $\frac{x}{a} : \frac{b}{y}$

110. If the side of an equilateral triangle is decreased by 20%, its area is decreased by :

(a) 36% (b) 40% (c) 60% (d) 64%

(C.B.I. 1997)

111. If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area ? **(S.B.I.P.O. 2000)**

(a) No change (b) 8% decrease (c) 16% decrease

(d) 16% increase (e) None of these

112. If every side of a triangle is doubled, the area of the new triangle is K times the area of the old one. K is equal to : **(R.R.B. 2003)**

(a) $\sqrt{2}$ (b) 2 (c) 3 (d) 4

113. One side of a parallelogram is 18 cm and its distance from the opposite side is 8 cm. The area of the parallelogram is :

(a) 48 cm^2 (b) 72 cm^2 (c) 100 cm^2 (d) 144 cm^2

114. A parallelogram has sides 30 m and 14 m and one of its diagonals is 40 m long. Then, its area is :

(a) 168 m^2 (b) 336 m^2 (c) 372 m^2 (d) 480 m^2

115. One diagonal of a parallelogram is 70 cm and the perpendicular distance of this diagonal from either of the outlying vertices is 27 cm. The area of the parallelogram (in sq. cm) is :

(a) 1800 (b) 1836 (c) 1890 (d) 1980

116. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is : **(M.A.T. 2003)**

(a) $10\sqrt{2}$ m (b) 100 m (c) $100\sqrt{2}$ m (d) 200 m

117. If a parallelogram with area P, a rectangle with area R and a triangle with area T are all constructed on the same base and all have the same altitude, then which of the following statements is false ?

(a) P = R (b) P + T = 2R (c) P = 2T (d) T = (1 / 2) R

118. The area of a rhombus is 150 cm^2. The length of one of its diagonals is 10 cm. The length of the other diagonal is : **(S.S.C. 2004)**

(a) 25 cm (b) 30 cm (c) 35 cm (d) 40 cm

119. One of the diagonals of a rhombus is double the other diagonal. Its area is 25 sq. cm. The sum of the diagonals is : **(S.S.C. 2003)**

(*a*) 10 cm (*b*) 12 cm (*c*) 15 cm (*d*) 16 cm

120. The perimeter of a rhombus is 56 m and its height is 5 m. Its area is :

(*a*) 64 sq. m (*b*) 70 sq. m (*c*) 78 sq. m (*d*) 84 sq. m

121. If the diagonals of a rhombus are 24 cm and 10 cm, the area and the perimeter of the rhombus are respectively :

(*a*) 120 cm^2, 52 cm (*b*) 120 cm^2, 64 cm (*c*) 240 cm^2, 52 cm (*d*) 240 cm^2, 64 cm

122. Each side of a rhombus is 26 cm and one of its diagonals is 48 cm long. The area of the rhombus is : **(R.R.B. 2003)**

(*a*) 240 cm^2 (*b*) 300 cm^2 (*c*) 360 cm^2 (*d*) 480 cm^2

123. The length of one diagonal of a rhombus is 80% of the other diagonal. The area of the rhombus is how many times the square of the length of the other diagonal ?

(*a*) $\frac{4}{5}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{1}{4}$

124. If a square and a rhombus stand on the same base, then the ratio of the areas of the square and the rhombus is :

(*a*) greater than 1 (*b*) equal to 1 (*c*) equal to $\frac{1}{2}$ (*d*) equal to $\frac{1}{4}$

125. The two parallel sides of a trapezium are 1.5 m and 2.5 m respectively. If the perpendicular distance between them is 6.5 metres, the area of the trapezium is :

(*a*) 10 m^2 (*b*) 13 m^2 (*c*) 20 m^2 (*d*) 26 m^2

126. The area of a field in the shape of a trapezium measures 1440 m^2. The perpendicular distance between its parallel sides is 24 m. If the ratio of the parallel sides is 5 : 3, the length of the longer parallel side is : **(S.S.C. 2004)**

(*a*) 45 m (*b*) 60 m (*c*) 75 m (*d*) 120 m

127. The cross-section of a canal is trapezium in shape. The canal is 12 m wide at the top and 8 m wide at the bottom. If the area of the cross-section is 840 sq. m, the depth of the canal is :

(*a*) 8.75 m (*b*) 42 m (*c*) 63 m (*d*) 84 m

128. The area of a circle of radius 5 is numerically what percent of its circumference ?

(*a*) 200 (*b*) 225 (*c*) 240 (*d*) 250 **(S.S.C. 2000)**

129. A man runs round a circular field of radius 50 m at the speed of 12 km / hr. What is the time taken by the man to take twenty rounds of the field ? **(M.A.T. 1997)**

(*a*) 30 min. (*b*) 32 min. (*c*) 34 min. (*d*) None of these

130. A cow is tethered in the middle of a field with a 14 feet long rope. If the cow grazes 100 sq. ft. per day, then approximately what time will be taken by the cow to graze the whole field ? **(Bank P.O. 2003)**

(*a*) 2 days (*b*) 6 days (*c*) 18 days
(*d*) 24 days (*e*) None of these

131. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle ? **(Bank P.O. 2004)**

(*a*) 88 cm^2 (*b*) 154 cm^2 (*c*) 1250 cm^2
(*d*) Cannot be determined (*e*) None of these

132. The circumference of a circle, whose area is 24.64 m^2, is : **(R.R.B. 2003)**

(*a*) 14.64 m (*b*) 16.36 m (*c*) 17.60 m (*d*) 18.40 m

133. If the circumference and the area of a circle are numerically equal, then the diameter is equal to : **(S.S.C. 2000)**

(*a*) $\frac{\pi}{2}$ (*b*) 2π (*c*) 2 (*d*) 4

134. The difference between the circumference and the radius of a circle is 37 cm. The area of the circle is : **(Section Officers', 2001)**

(*a*) 111 cm^2 (*b*) 148 cm^2 (*c*) 154 cm^2 (*d*) 259 cm^2

135. The sum of areas of two circles A and B is equal to the area of a third circle C whose diameter is 30 cm. If the diameter of circle A is 18 cm, then the radius of circle B is :

(*a*) 10 cm (*b*) 12 cm (*c*) 15 cm (*d*) 18 cm

136. Between a square of perimeter 44 cm and a circle of circumference 44 cm, which figure has larger area and by how much ? **(S.S.C. 2000)**

(*a*) Both have equal area (*b*) Square, 33 cm^2

(*c*) Circle, 33 cm^2 (*d*) Square, 495 cm^2

137. A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be : **(R.R.B. 2002)**

(*a*) 3520 cm^2 (*b*) 6400 cm^2 (*c*) 7744 cm^2 (*d*) 8800 cm^2

138. A wire when bent in the form of a square encloses an area of 484 sq. cm. What will be the enclosed area when the same wire is bent into the form of a circle ?

(*a*) 462 sq. cm (*b*) 539 sq. cm (*c*) 616 sq. cm (*d*) 693 sq. cm **(S.S.C. 2002)**

139. A circular wire of radius 42 cm is bent in the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is : **(S.S.C. 2004)**

(*a*) 25 cm (*b*) 30 cm (*c*) 36 cm (*d*) 60 cm

140. There is a rectangular tank of length 180 m and breadth 120 m in a circular field. If the area of the land portion of the field is 40000 m^2, what is the radius of the field ?

(*a*) 130 m (*b*) 135 m (*c*) 140 m (*d*) 145 m

141. The areas of two circular fields are in the ratio 16 : 49. If the radius of the latter is 14 m, then what is the radius of the former ? **(IGNOU, 2003)**

(*a*) 4 m (*b*) 8 m (*c*) 18 m (*d*) 32 m

142. If the ratio of areas of two circles is 4 : 9, then the ratio of their circumferences will be : **(R.R.B. 2003)**

(*a*) 2 : 3 (*b*) 3 : 2 (*c*) 4 : 9 (*d*) 9 : 4

143. The perimeter of a circle is equal to the perimeter of a square. Then, their areas are in the ratio :

(*a*) 4 : 1 (*b*) 11 : 7 (*c*) 14 : 11 (*d*) 22 : 7

144. The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions ?

(*a*) 1492 m (*b*) 1980 m (*c*) 2530 m (*d*) 2680 m

145. The number of revolutions a wheel of diameter 40 cm makes in travelling a distance of 176 m, is : **(S.S.C. 2003)**

(*a*) 140 (*b*) 150 (*c*) 160 (*d*) 166

146. The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km will be : **(R.R.B. 2003)**

(*a*) 2800 (*b*) 4000 (*c*) 5500 (*d*) 7000

147. The wheel of an engine, $7\frac{1}{2}$ metres in circumference makes 7 revolutions in 9 seconds. The speed of the train in km per hour is :

(*a*) 130 (*b*) 132 (*c*) 135 (*d*) 150

148. The wheel of a motorcycle, 70 cm in diameter, makes 40 revolutions in every 10 seconds. What is the speed of the motorcycle in km / hr ? **(R.R.B. 2002)**

(*a*) 22.32 (*b*) 27.68 (*c*) 31.68 (*d*) 36.24

149. Wheels of diameters 7 cm and 14 cm start rolling simultaneously from X and Y, which

are 1980 cm apart, towards each other in opposite directions. Both of them make the same number of revolutions per second. If both of them meet after 10 seconds, the speed of the smaller wheel is : **(M.A.T. 2003)**

(a) 22 cm / sec (b) 44 cm / sec (c) 66 cm / sec (d) 132 cm / sec

150. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions ?

(a) 18 (b) 20 (c) 25 (d) 30

151. Find the diameter of a wheel that makes 113 revolutions to go 2 km 26 decametres.

(a) $4\frac{4}{13}$ m (b) $6\frac{4}{11}$ m (c) $12\frac{4}{11}$ m (d) $12\frac{8}{11}$ m

(S.S.C. 2003)

152. The front wheels of a wagon are 2π feet in circumference and the rear wheels are 3π feet in circumference. When the front wheels have made 10 more revolutions than the rear wheels, how many feet has the wagon travelled ? **(M.B.A. 2003)**

(a) 30π (b) 60π (c) 90π (d) 150π

153. A circular ground whose diameter is 35 metres, has a 1.4 m broad garden around it. What is the area of the garden in square metres ? **(S.B.I.P.O. 1999)**

(a) 160.16 (b) 176.16 (c) 196.16

(d) Data inadequate (e) None of these

154. A circular garden has a circumference of 440 m. There is a 7 m wide border inside the garden along its periphery. The area of the border is :

(a) 2918 m^2 (b) 2921 m^2 (c) 2924 m^2 (d) 2926 m^2

155. The areas of two concentric circles forming a ring are 154 sq. cm and 616 sq. cm. The breadth of the ring is :

(a) 7 cm (b) 14 cm (c) 21 cm (d) 28 cm

156. A circular park has a path of uniform width around it. The difference between outer and inner circumferences of the circular path is 132 m. Its width is : **(S.S.C. 2003)**

(a) 20 m (b) 21 m (c) 22 m (d) 24 m

157. A circular swimming pool is surrounded by a concrete wall 4 ft. wide. If the area of the concrete wall surrounding the pool is $\frac{11}{25}$ that of the pool, then the radius of the pool is : **(Assistant Grade, 1998)**

(a) 8 ft (b) 16 ft (c) 20 ft (d) 30 ft

158. The ratio of the outer and the inner perimeters of a circular path is 23 : 22. If the path is 5 metres wide, the diameter of the inner circle is : **(S.S.C. 2004)**

(a) 55 m (b) 110 m (c) 220 m (d) 230 m

159. What will be the area of a semi-circle of 14 m diameter ? **(NABARD, 2002)**

(a) 22 m^2 (b) 77 m^2 (c) 154 m^2 (d) 308 m^2 (e) None of these

160. A semi-circular shaped window has diameter of 63 cm. Its perimeter equals :

(a) 126 cm (b) 162 cm (c) 198 cm (d) 251 cm

(S.S.C. 1999)

161. What will be the area of a semi-circle whose perimeter is 36 cm ?

(a) 154 cm^2 (b) 168 cm^2 (c) 308 cm^2

(d) Data inadequate (e) None of these **(B.S.R.B. 1998)**

162. If a wire is bent into the shape of a square, then the area of the square is 81 sq. cm. When the wire is bent into a semi-circular shape, then the area of the semi-circle will be : **(S.S.C. 2002)**

(a) 22 cm^2 (b) 44 cm^2 (c) 77 cm^2 (d) 154 cm^2

163. The area of a sector of a circle of radius 5 cm, formed by an arc of length 3.5 cm, is :
(a) 7.5 cm^2 (b) 7.75 cm^2 (c) 8.5 cm^2 (d) 8.75 cm^2
(S.S.C. 1999)

164. In a circle of radius 7 cm, an arc subtends an angle of 108° at the centre. The area of the sector is :
(a) 43.2 cm^2 (b) 44.2 cm^2 (c) 45.2 cm^2 (d) 46.2 cm^2

165. The area of the greatest circle which can be inscribed in a square whose perimeter is 120 cm, is : **(S.S.C. 2004)**
(a) $\frac{22}{7}\times\left(\frac{7}{2}\right)^2$ cm^2 (b) $\frac{22}{7}\times\left(\frac{9}{2}\right)^2$ cm^2
(c) $\frac{22}{7}\times\left(\frac{15}{2}\right)^2$ cm^2 (d) $\frac{22}{7}\times(15)^2$ cm^2

166. The area of the largest circle, that can be drawn inside a rectangle with sides 18 cm by 14 cm, is : **(S.S.C. 2000)**
(a) 49 cm^2 (b) 154 cm^2 (c) 378 cm^2 (d) 1078 cm^2

167. The area of a circle is 220 sq. cm. The area of a square inscribed in this circle will be : **(C.B.I. 1997)**
(a) 49 cm^2 (b) 70 cm^2 (c) 140 cm^2 (d) 150 cm^2

168. A square is inscribed in a circle whose radius is 4 cm. The area of the portion between the circle and the square is :
(a) $(8\pi - 16)$ (b) $(8\pi - 32)$ (c) $(16\pi - 16)$ (d) $(16\pi - 32)$

169. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is : **(C.B.I. 2003)**
(a) $50\sqrt{2}$ cm (b) $\frac{100}{\pi}$ cm (c) $\frac{50\sqrt{2}}{\pi}$ cm (d) $\frac{100\sqrt{2}}{\pi}$ cm

170. Four equal sized maximum circular plates are cut off from a square paper sheet of area 784 cm^2. The circumference of each plate is : **(S.S.C. 2003)**
(a) 22 cm (b) 44 cm (c) 66 cm (d) 88 cm

171. There are 4 semi-circular gardens on each side of a square-shaped pond with each side 21 m. The cost of fencing the entire plot at the rate of Rs 12.50 per metre is :
(a) Rs. 1560 (b) Rs. 1650 (c) Rs. 3120 (d) Rs. 3300

172. The ratio of the areas of the incircle and circumcircle of an equilateral triangle is :
(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 9

173. The radius of the circumcircle of an equilateral triangle of side 12 cm is :
(a) $\frac{4\sqrt{2}}{3}$ cm (b) $4\sqrt{2}$ cm (c) $\frac{4\sqrt{3}}{3}$ cm (d) $4\sqrt{3}$ cm

174. The area of the incircle of an equilateral triangle of side 42 cm is : **(S.S.C. 2004)**
(a) $22\sqrt{3}$ cm^2 (b) 231 cm^2 (c) 462 cm^2 (d) 924 cm^2

175. The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle.
(a) 71.5 cm (b) 71.7 cm (c) 72.3 cm (d) 72.7 cm

176. The sides of a triangle are 6 cm, 11 cm and 15 cm. The radius of its incircle is :
(a) $3\sqrt{2}$ cm (b) $\frac{4\sqrt{2}}{5}$ cm (c) $\frac{5\sqrt{2}}{4}$ cm (d) $6\sqrt{2}$ cm
(M.A.T. 2001)

177. The perimeter of a triangle is 30 cm and the circumference of its incircle is 88 cm. The area of the triangle is : **(S.S.C. 2003)**
(a) 70 cm^2 (b) 140 cm^2 (c) 210 cm^2 (d) 420 cm^2

178. If in a triangle, the area is numerically equal to the perimeter, then the radius of the inscribed circle of the triangle is : **(S.S.C. 2000)**
(a) 1 (b) 1.5 (c) 2 (d) 3

179. An equilateral triangle, a square and a circle have equal perimeters. If T denotes the area of the triangle, S, the area of the square and C, the area of the circle, then :
(a) S < T < C (b) T < C < S (c) T < S < C (d) C < S < T
(C.D.S. 2003)

180. If an area enclosed by a circle or a square or an equilateral triangle is the same, then the maximum perimeter is possessed by : **(S.C.R.A. 1997)**
(a) circle (b) square (c) equilateral triangle
(d) triangle and square have equal perimeters greater than that of circle

181. The area of the largest triangle that can be inscribed in a semi-circle of radius r, is :
(a) r^2 (b) $2r^2$ (c) r^3 (d) $2r^3$
(Section Officers', 2001)

182. ABC is a right-angled triangle with right angle at B. If the semi-circle on AB with AB as diameter encloses an area of 81 sq. cm and the semi-circle on BC with BC as diameter encloses an area of 36 sq. cm, then the area of the semi-circle on AC with AC as diameter will be :
(a) 117 cm^2 (b) 121 cm^2 (c) 217 cm^2 (d) 221 cm^2

183. If the radius of a circle is increased by 75%, then its circumference will increase by :
(a) 25% (b) 50% (c) 75% (d) 100%
(C.D.S. 2003)

184. A can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, then the time required by A to go round the new path once, travelling at the same speed as before, is : **(S.S.C. 2000)**
(a) 20 min. (b) 25 min. (c) 50 min. (d) 100 min.

185. If the radius of a circle is increased by 6%, then the area is increased by :
(a) 6% (b) 12% (c) 12.36% (d) 16.64%
(D.M.R.C. 2003)

186. If the radius of a circle is diminished by 10%, then its area is diminished by :
(a) 10% (b) 19% (c) 20% (d) 36%
(Hotel Management, 2003)

187. If the radius of a circle is doubled, its area is increased by : **(C.B.I. 1998)**
(a) 100% (b) 200% (c) 300% (d) 400%.

188. If the circumference of a circle increases from 4π to 8π, what change occurs in its area ?
(a) It is halved. (b) It doubles. (c) It triples. (d) It quadruples.
(S.S.C. 2000)

189. Three circles of radius 3.5 cm are placed in such a way that each circle touches the other two. The area of the portion enclosed by the circles is : **(S.S.C. 2003)**
(a) 1.967 cm^2 (b) 1.975 cm^2 (c) 19.67 cm^2 (d) 21.21 cm^2

190. Four circular cardboard pieces, each of radius 7 cm are placed in such a way that each piece touches two other pieces. The area of the space enclosed by the four pieces is :
(a) 21 cm^2 (b) 42 cm^2 (c) 84 cm^2 (d) 168 cm^2

191. Four horses are tethered at four corners of a square plot of side 63 metres so that they just cannot reach one another. The area left ungrazed is :
(a) 675.5 m^2 (b) 780.6 m^2 (c) 785.8 m^2 (d) 850.5 m^2

ANSWERS

1. (d)	2. (c)	3. (b)	4. (c)	5. (c)	6. (d)	7. (e)	8. (b)	9. (b)	
10. (b)	11. (d)	12. (e)	13. (d)	14. (b)	15. (b)	16. (d)	17. (a)	18. (d)	
19. (a)	20. (a)	21. (b)	22. (b)	23. (c)	24. (d)	25. (d)	26. (c)	27. (b)	
28. (d)	29. (b)	30. (a)	31. (a)	32. (d)	33. (c)	34. (b)	35. (e)	36. (d)	
37. (b)	38. (e)	39. (a)	40. (e)	41. (c)	42. (a)	43. (c)	44. (d)	45. (b)	
46. (b)	47. (b)	48. (b)	49. (e)	50. (c)	51. (c)	52. (b)	53. (a)	54. (a)	
55. (d)	56. (b)	57. (b)	58. (c)	59. (b)	60. (b)	61. (c)	62. (c)	63. (c)	
64. (c)	65. (b)	66. (c)	67. (a)	68. (c)	69. (a)	70. (a)	71. (d)	72. (c)	
73. (d)	74. (d)	75. (c)	76. (d)	77. (b)	78. (d)	79. (d)	80. (c)	81. (c)	
82. (b)	83. (d)	84. (c)	85. (a)	86. (d)	87. (c)	88. (d)	89. (b)	90. (c)	
91. (d)	92. (d)	93. (d)	94. (d)	95. (b)	96. (a)	97. (a)	98. (a)	99. (c)	
100. (c)	101. (d)	102. (c)	103. (d)	104. (c)	105. (c)	106. (b)	107. (c)	108. (d)	
109. (c)	110. (a)	111. (c)	112. (d)	113. (d)	114. (b)	115. (c)	116. (d)	117. (b)	
118. (b)	119. (c)	120. (b)	121. (a)	122. (d)	123. (b)	124. (b)	125. (b)	126. (c)	
127. (d)	128. (d)	129. (d)	130. (b)	131. (e)	132. (c)	133. (d)	134. (c)	135. (b)	
136. (c)	137. (c)	138. (c)	139. (d)	140. (e)	141. (b)	142. (a)	143. (c)	144. (b)	
145. (a)	146. (d)	147. (b)	148. (c)	149. (c)	150. (c)	151. (b)	152. (b)	153. (a)	
154. (d)	155. (a)	156. (b)	157. (c)	158. (c)	159. (b)	160. (b)	161. (e)	162. (c)	
163. (d)	164. (d)	165. (d)	166. (b)	167. (c)	168. (d)	169. (c)	170. (b)	171. (b)	
172. (c)	173. (d)	174. (c)	175. (d)	176. (c)	177. (c)	178. (c)	179. (c)	180. (c)	
181. (a)	182. (a)	183. (c)	184. (c)	185. (c)	186. (b)	187. (c)	188. (d)	189. (a)	
190. (b)	191. (d)								

SOLUTIONS

1. Area of the floor = $(5.5 \times 3.75)\ m^2 = 20.625\ m^2$.
   ∴ Cost of paving = Rs. (800×20.625) = Rs. 16500.

2. Let the breadth be b. Then, $25 \times b = 18 \times 10 \Leftrightarrow b = \left(\frac{18 \times 10}{25}\right)$ cm = 7.2 cm.

3. Perimeter of the plot = 2 (90 + 50) = 280 m.
   ∴ Number of poles = $\left(\frac{280}{5}\right)$ = 56 m.

4. Let breadth = x cm. Then, length = $\left(\frac{160}{100}x\right)$ cm = $\frac{8}{5}x$ cm.
   So, $\frac{8}{5}x - x = 24 \Leftrightarrow \frac{3}{5}x = 24 \Leftrightarrow x = \left(\frac{24 \times 5}{3}\right) = 40$.
   ∴ Length = 64 cm, Breadth = 40 cm.
   Area = $(64 \times 40)\ cm^2 = 2560\ cm^2$.

5. Clearly, we have : $l = 9$ and $l + 2b = 37$ or $b = 14$.
   ∴ Area = $(l \times b) = (9 \times 14)$ sq. ft. = 126 sq. ft.

6. We have : $(l - b) = 23$ and $2(l + b) = 206$ or $(l + b) = 103$.
   Solving the two equations, we get : $l = 63$ and $b = 40$.
   ∴ Area = $(l \times b) = (63 \times 40)\ m^2 = 2520\ m^2$.

7. Let breadth = x metres. Then, length = $(x + 20)$ metres.
   Perimeter = $\left(\frac{5300}{26.50}\right)$ m = 200 m.

$\therefore$ $2\,[(x + 20) + x] = 200 \Leftrightarrow 2x + 20 = 100 \Leftrightarrow 2x = 80 \Leftrightarrow x = 40$.
Hence, length = $x + 20 = 60$ m.

8. Let length = x metres. Then, breadth = $\left(\frac{60}{100}x\right)$ metres = $\left(\frac{3x}{5}\right)$ metres.

Perimeter = $\left[2\left(x + \frac{3x}{5}\right)\right]$ m = $\left(\frac{16x}{5}\right)$ m.

$\therefore$ $\frac{16x}{5} = 800 \Leftrightarrow x = \left(\frac{800 \times 5}{16}\right) = 250$.

So, length = 250 m; breadth = 150 m.
$\therefore$ Area = (250×150) m^2 = 37500 m^2.

9. $\frac{l}{2(l + b)} = \frac{1}{3} \Rightarrow 3l = 2l + 2b \Rightarrow l = 2b \Rightarrow \frac{l}{b} = \frac{2}{1} = 2 : 1$.

10. Perimeter = Distance covered in 8 min. = $\left(\frac{12000}{60} \times 8\right)$ m = 1600 m.

Let length = $3x$ metres and breadth = $2x$ metres.
Then, $2(3x + 2x) = 1600$ or $x = 160$.
$\therefore$ Length = 480 m and Breadth = 320 m.
$\therefore$ Area = (480×320) m^2 = 153600 m^2.

11. Let breadth = x metres. Then, length = $(x + 5)$ metres.
Then, $x(x + 5) = 750 \Leftrightarrow x^2 + 5x - 750 = 0 \Leftrightarrow (x + 30)(x - 25) = 0 \Leftrightarrow x = 25$.
$\therefore$ Length = $(x + 5) = 30$ m.

12. Let breadth = x metres. Then, length = $\left(\frac{115x}{100}\right)$ metres.

$\therefore$ $x \times \frac{115x}{100} = 460 \Leftrightarrow x^2 = \left(\frac{460 \times 100}{115}\right) = 400 \Leftrightarrow x = 20$.

13. We have : $l = 20$ ft and $lb = 680$ sq. ft. So, $b = 34$ ft.
$\therefore$ Length of fencing = $(l + 2b) = (20 + 68)$ ft = 88 ft.

14. $\frac{2(l + b)}{b} = \frac{5}{1} \Rightarrow 2l + 2b = 5b \Rightarrow 3b = 2l \Rightarrow b = \frac{2}{3}l$.

Then, Area = 216 cm^2 $\Rightarrow l \times b = 216 \Rightarrow l \times \frac{2}{3}l = 216 \Rightarrow l^2 = 324 \Rightarrow l = 18$ cm.

15. We have : $2b + l = 30 \Rightarrow l = 30 - 2b$.
Area = 100 m^2 $\Rightarrow l \times b = 100 \Rightarrow b(30 - 2b) = 100 \Rightarrow b^2 - 15b + 50 = 0$
$\Rightarrow (b - 10)(b - 5) = 0 \Rightarrow b = 10$ or $b = 5$.
When $b = 10$, $l = 10$ and when $b = 5$, $l = 20$.
Since the garden is rectangular, so its dimension is 20 m $\times$ 5 m.

16. Let length = $(3x)$ metres and breadth = $(4x)$ metres.
Then, $3x \times 4x = 7500 \Leftrightarrow 12x^2 = 7500 \Leftrightarrow x^2 = 625 \Leftrightarrow x = 25$.
So, length = 75 m and breadth = 100 m.
Perimeter = $[2(75 + 100)]$ m = 350 m.
$\therefore$ Cost of fencing = Rs. (0.25×350) = Rs. 87.50.

17. Required perimeter = (AB + BC + CP + PQ + QR + RA)
= AB + BC + (CP + QR) + (PQ + RA)
= AB + BC + AB + BC = 2 (AB + BC)
= [2 (8 + 4)] cm = 24 cm.

18. Let the areas of the two parts be x and $(700 - x)$ hectares respectively. Then,

$$|x - (700 - x)| = \frac{1}{5} \times \left[\frac{x + (700 - x)}{2}\right] \Leftrightarrow 2x - 700 = 70 \Leftrightarrow x = 385.$$

So, area of smaller part = (700 − 385) hectares = 315 hectares.

19. When folded along breadth, we have : $2\left(\frac{l}{2} + b\right) = 34$ or $l + 2b = 34$ ...(i)

When folded along length, we have : $2\left(l + \frac{b}{2}\right) = 38$ or $2l + b = 38$ ...(ii)

Solving (i) and (ii), we get : $l = 14$ and $b = 10$.

$\therefore$ Area of the paper = (14×10) cm^2 = 140 cm^2.

20. Let breadth = x metres. Then, length = $\left(\frac{3}{2}x\right)$ metres.

Area = $\left(\frac{2}{3} \times 10000\right)$ m^2.

$$\therefore \frac{3}{2}x \times x = \frac{2}{3} \times 10000 \Leftrightarrow x^2 = \frac{4}{9} \times 10000 \Leftrightarrow x = \frac{2}{3} \times 100.$$

$$\therefore \text{Length} = \frac{3}{2}x = \left(\frac{3}{2} \times \frac{2}{3} \times 100\right) \text{m} = 100 \text{ m}.$$

21. Number of bricks = $\left(\frac{\text{Area of courtyard}}{\text{Area of 1 brick}}\right) = \left(\frac{2500 \times 1600}{20 \times 10}\right) = 20000$.

22. Length of the carpet = $\left(\frac{\text{Total cost}}{\text{Rate / m}}\right) = \left(\frac{8100}{45}\right)$ m = 180 m.

Area of the room = Area of the carpet = $\left(180 \times \frac{75}{100}\right)$ m^2 = 135 m^2.

$\therefore$ Breadth of the room = $\left(\frac{\text{Area}}{\text{Length}}\right) = \left(\frac{135}{18}\right)$ m = 7.5 m.

23. Other side = $\sqrt{\left(\frac{15}{2}\right)^2 - \left(\frac{9}{2}\right)^2}$ ft = $\sqrt{\frac{225}{4} - \frac{81}{4}}$ ft = $\sqrt{\frac{144}{4}}$ ft = 6 ft.

$\therefore$ Area of the closet = (6×4.5) sq. ft = 27 sq. ft.

24. Let breadth = x cm. Then, length = $3x$ cm.

$x^2 + (3x)^2 = (8\sqrt{10})^2 \Rightarrow 10x^2 = 640 \Rightarrow x^2 = 64 \Rightarrow x = 8$.

So, length = 24 cm and breadth = 8 cm.

$\therefore$ Perimeter = [2 (24 + 8)] cm = 64 cm.

25. $\sqrt{l^2 + b^2} = 3b \Rightarrow l^2 + b^2 = 9b^2 \Rightarrow l^2 = 8b^2 \Rightarrow \frac{l^2}{b^2} = 8 \Rightarrow \frac{l}{b} = \sqrt{8} = 2\sqrt{2}$.

26. $2(l + b) = 46$ or $l + b = 23$. Also, $lb = 120$.

$\therefore$ Diagonal = $\sqrt{l^2 + b^2} = \sqrt{(l + b)^2 - 2lb} = \sqrt{(23)^2 - 240} = \sqrt{289} = 17$ m.

27. $\sqrt{l^2 + b^2} = \sqrt{41}$ or $l^2 + b^2 = 41$. Also, $lb = 20$.

$(l + b)^2 = (l^2 + b^2) + 2lb = 41 + 40 = 81 \Rightarrow (l + b) = 9$.

$\therefore$ Perimeter = $2(l + b) = 18$ cm.

28. Length of diagonal $= \left(52 \times \frac{15}{60}\right)$ m $= 13$ m.

Sum of length and breadth $= \left(68 \times \frac{15}{60}\right)$ m $= 17$ m.

$\therefore \quad \sqrt{l^2 + b^2} = 13$ or $l^2 + b^2 = 169$ and $l + b = 17$.

Area $= lb = \frac{1}{2}(2lb) = \frac{1}{2}[(l+b)^2 - (l^2 + b^2)] = \frac{1}{2}[(17)^2 - 169] = \frac{1}{2}(289 - 169) = 60 \text{ m}^2$.

29. We have : $lb = 60$ and $\sqrt{l^2 + b^2} + l = 5b$.

Now, $l^2 + b^2 = (5b - l)^2 \Rightarrow 24b^2 - 10lb = 0 \Rightarrow 24b^2 - 600 = 0$

$\Rightarrow b^2 = 25 \Rightarrow b = 5$.

$\therefore \quad l = \left(\frac{60}{5}\right)$ m $= 12$ m. So, length of the carpet $= 12$ m.

30. Let length $= (3x)$ metres and breadth $= (2x)$ metres.

Then, $(3x + 5) \times 2x = 2600 \Leftrightarrow 6x^2 + 10x - 2600 = 0$

$\Leftrightarrow \quad 3x^2 + 5x - 1300 = 0 \Leftrightarrow (3x + 65)(x - 20) = 0 \Leftrightarrow x = 20$.

$\therefore$ Breadth $= 2x = 40$ m.

31. Let breadth $= x$ cm. Then, length $= (x + 8)$ cm.

$\therefore \quad (x + 8)\, x = (x + 15)(x - 4) \Leftrightarrow x^2 + 8x = x^2 + 11x - 60 \Leftrightarrow x = 20$.

So, length $= 28$ cm and breadth $= 20$ cm.

32. Let length $= x$ metres and breadth $= y$ metres. Then,

$(x + 1)(y + 1) - xy = 21 \Rightarrow x + y = 20$ ...(*i*)

And, $xy - [(x + 1)(y - 1)] = 5 \Leftrightarrow x - y = 6$ ...(*ii*)

Solving (*i*) and (*ii*), we get : $x = 13$ and $y = 7$.

So, length $= 13$ m and breadth $= 7$ m.

$\therefore$ Perimeter $= [2(13 + 7)]$ m $= 40$ m.

33. Let original length $= x$ metres and original breadth $= y$ metres.

Original area $= (xy)$ m^2.

New length $= \left(\frac{120}{100}x\right)$ m $= \left(\frac{6}{5}x\right)$ m; New breadth $= \left(\frac{120}{100}y\right)$ m $= \left(\frac{6}{5}y\right)$ m.

New Area $= \left(\frac{6}{5}x \times \frac{6}{5}y\right)$ m^2 $= \left(\frac{36}{25}xy\right)$ m^2.

$\therefore \quad \text{Increase\%} = \left(\frac{11}{25}xy \times \frac{1}{xy} \times 100\right)\% = 44\%$.

34. New area $= \left(\frac{80}{100}a \times \frac{110}{100}b\right) = \left(\frac{4}{5} \times \frac{11}{10}ab\right) = \left(\frac{22}{25}ab\right)$.

$\therefore \quad$ Required percentage $= \left(\frac{22}{25}ab \times \frac{1}{ab} \times 100\right)\% = 88\%$.

35. Let original length $= x$ metres and original breadth $= y$ metres.

Original area $= (xy)$ m^2.

New length $= \left(\frac{150}{100}x\right)$ m $= \left(\frac{3}{2}x\right)$ m; New breadth $= \left(\frac{120}{100}y\right)$ m $= \left(\frac{6}{5}y\right)$ m.

New area $= \left(\frac{3}{2}x \times \frac{6}{5}y\right) m^2 = \left(\frac{9}{5}xy\right) m^2$.

$\therefore$ Increase $= \left(\frac{\frac{4}{5}xy}{xy}\right) = \frac{4}{5}$ times.

36. Let original length $= x$ and original breadth $= y$.

Decrease in area $= xy - \left(\frac{80}{100}x \times \frac{90}{100}y\right) = \left(xy - \frac{18}{25}xy\right) = \frac{7}{25}xy$.

$\therefore$ Decrease% $= \left(\frac{7}{25}xy \times \frac{1}{xy} \times 100\right)\% = 28\%$.

37. Let original length $= x$ and original breadth $= y$.

Original area $= xy$.

New length $= \frac{x}{2}$; New breadth $= 3y$. New Area $= \left(\frac{x}{2} \times 3y\right) = \frac{3}{2}xy$.

$\therefore$ Increase% $= \left(\frac{1}{2}xy \times \frac{1}{xy} \times 100\right)\% = 50\%$.

38. Let original length $= x$ and original breadth $= y$.

Then, original area $= xy$.

New area $= \left[\frac{(100-r)}{100} \times x\right]\left[\frac{(105+r)}{100} \times y\right] = \left[\left(\frac{10500 - 5r - r^2}{10000}\right)xy\right]$

$\therefore \left(\frac{10500 - 5r - r^2}{10000}\right)xy = xy \Leftrightarrow r^2 + 5r - 500 = 0 \Leftrightarrow (r+25)(r-20) = 0 \Leftrightarrow r = 20$.

39. Let original length $= x$ and original breadth $= y$.

Then, original area $= xy$.

New length $= \frac{160x}{100} = \frac{8x}{5}$. Let new breadth $= z$.

Then, $\frac{8x}{5} \times z = xy \Rightarrow z = \frac{5y}{8}$.

$\therefore$ Decrease in breadth $= \left(\frac{3y}{8} \times \frac{1}{y} \times 100\right)\% = 37\frac{1}{2}\%$.

40. Let original length $= x$ and original breadth $= y$.

Then, original area $= xy$.

New length $= \frac{130}{100}x = \frac{13x}{10}$. New breadth $= y$. New area $= \left(\frac{13x}{10} \times y\right) = \frac{13xy}{10}$.

$\therefore$ Required ratio $= \left(\frac{\frac{13xy}{10}}{xy}\right) = \frac{13}{10} = 13 : 10$.

41. Area of the sheet $= (20 \times 30)\ cm^2 = 600\ cm^2$.

Area used for typing $= [(20-4) \times (30-6)]\ cm^2 = 384\ cm^2$.

$\therefore$ Required percentage $= \left(\frac{384}{600} \times 100\right)\% = 64\%$.

42. Area of the mat = $[(15 - 3) \times (12 - 3)]$ sq. ft = 108 sq. ft.

$\therefore$ Cost of the mat = Rs. (108×3.50) = Rs. 378.

43. $2(l + b) = 340$ (Given).

Area of the boundary = $[(l + 2)(b + 2) - lb] = 2(l + b) + 4 = 344$.

$\therefore$ Cost of gardening = Rs. (344×10) = Rs. 3440.

44. $lb = 96$ (Given).

Area of pathway = $[(l - 4)(b - 4) - lb] = 16 - 4(l + b)$, which cannot be determined.
So, data is inadequate.

45. Let the width of walk be x metres. Then,

$(20 - 2x)(10 - 2x) = 96 \Leftrightarrow 4x^2 + 60x - 104 = 0 \Leftrightarrow x^2 + 15x - 26 = 0$

$\Leftrightarrow (x - 13)(x - 2) = 0 \Leftrightarrow x = 2 \quad [\because x \neq 13]$

46. Area of crossroads = $(55 \times 4 + 35 \times 4 - 4 \times 4)\ m^2 = 344\ m^2$.

$\therefore$ Cost of gravelling = Rs. $\left(344 \times \frac{75}{100}\right)$ = Rs. 258.

47. Area of the park = $(60 \times 40)\ m^2 = 2400\ m^2$. Area of the lawn = $2109\ m^2$.

$\therefore$ Area of the crossroads = $(2400 - 2109)\ m^2 = 291\ m^2$.

Let the width of the road be x metres. Then,

$60x + 40x - x^2 = 291 \Leftrightarrow x^2 - 100x + 291 = 0 \Leftrightarrow (x - 97)(x - 3) = 0$

$\Leftrightarrow x = 3 \quad [\because x \neq 97]$.

48. Side = $\sqrt{2550.25} = \sqrt{\frac{255025}{100}} = \frac{505}{10} = 50.5$ m.

49. Area = $\frac{\text{Total cost}}{\text{Rate}} = \left(\frac{1215}{135}\right)$ hectares = (9×10000) sq. m.

$\therefore$ Side of the square = $\sqrt{90000} = 300$ m.

Perimeter of the field = $(300 \times 4) = 1200$ m.

Cost of fencing = Rs. $\left(1200 \times \frac{3}{4}\right)$ = Rs. 900.

50. The sides of the five squares are $\left(\frac{24}{4}\right), \left(\frac{32}{4}\right), \left(\frac{40}{4}\right), \left(\frac{76}{4}\right), \left(\frac{80}{4}\right)$ *i.e.*, 6 cm, 8 cm, 10 cm, 19 cm, 20 cm.

$\therefore$ Area of the new square = $[6^2 + 8^2 + (10)^2 + (19)^2 + (20)^2]$

$= (36 + 64 + 100 + 361 + 400)\ cm^2 = 961\ cm^2$.

Side of the new square = $\sqrt{961}$ cm = 31 cm.

Perimeter of the new square = (4×31) cm = 124 cm.

51. Number of marbles = $\left(\frac{300 \times 300}{20 \times 20}\right) = 225$.

52. Area of each slab = $\left(\frac{72}{50}\right) m^2 = 1.44\ m^2$.

$\therefore$ Length of each slab = $\sqrt{1.44}$ m = 1.2 m = 120 cm.

53. Area left after laying black tiles = $[(20 - 4) \times (10 - 4)]$ sq. ft = 96 sq. ft.

Area under white tiles = $\left(\frac{1}{3} \times 96\right)$ sq. ft = 32 sq. ft.

Area under blue tiles = $(96 - 32)$ sq. ft = 64 sq. ft.

Number of blue tiles = $\frac{64}{(2 \times 2)} = 16$.

54. Length of largest tile = H.C.F. of 1517 cm and 902 cm = 41 cm.
Area of each tile = (41×41) cm^2.

$\therefore$ Required number of tiles = $\left(\frac{1517 \times 902}{41 \times 41}\right) = 814$.

55. Length of the room = $(7 + 7)$ m = 14 m. Breadth of the room = 7 m.
$\therefore$ Area of the room = (14×7) m^2 = 98 m^2.

56. Side of the square = 12 cm. Area of the rectangle = $[(12 \times 12) - 4]$ cm^2 = 140 cm^2.
Now, area = 140 cm^2, length = 14 cm.

$\therefore$ Breadth = $\frac{\text{area}}{\text{length}} = \frac{140}{14}$ cm = 10 cm.

Hence, Perimeter = $2(l + b) = 2(14 + 10)$ cm = 48 cm.

57. Let the side of the square be x cm. Then, its area = x^2 cm^2.
Area of the rectangle = $(3x^2)$ cm^2.

$\therefore 40 \times \frac{3}{2} \times x = 3x^2 \Leftrightarrow x = 20$.

58. Side of the square = $\frac{80}{4}$ cm = 20 cm.

$2(l + b) = 80 \Rightarrow l + b = 40$. Now, $(20 \times 20) - lb = 100 \Leftrightarrow lb = 300$.

$(l - b) = \sqrt{(l+b)^2 - 4lb} = \sqrt{(40 \times 40) - (4 \times 300)} = \sqrt{400} = 20$.

Now, $l + b = 40$ and $l - b = 20 \Rightarrow l = 30$ and $b = 10$.

$\therefore$ Sides of the rectangle are 30 cm and 10 cm.

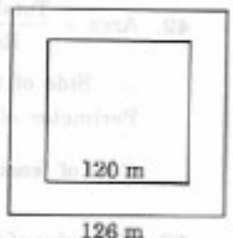
59. Perimeter = $\frac{\text{Total cost}}{\text{Cost per m}} = \frac{10080}{20}$ m = 504 m.

Side of the square = $\frac{504}{4}$ m = 126 m.

Breadth of the pavement = 3 m.

Side of inner square = $(126 - 6)$ m = 120 m.

Area of the pavement = $[(126 \times 126) - (120 \times 120)]$ m^2
= $[(126 + 120)(126 - 120)]$ m^2 = (246×6) m^2.



$\therefore$ Cost of pavement = Rs. $(246 \times 6 \times 50)$ = Rs. 73800.

60. Let the length of the outer edge be x metres. Then, length of the inner edge = $(x - 6)$ m.

$\therefore x^2 - (x - 6)^2 = 1764 \Leftrightarrow x^2 - (x^2 - 12x + 36) = 1764 \Leftrightarrow 12x = 1800 \Leftrightarrow x = 150$.

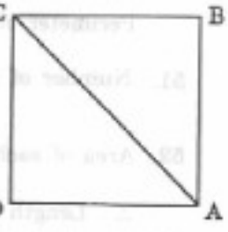
$\therefore$ Required perimeter = $(4x)$ m = (4×150) m = 600 m.

61. Let the side of the square be x metres.
Then, AB + BC = $2x$ metres.

AC = $\sqrt{2}\,x = (1.41x)$ m

Saving on $2x$ metres = $(0.59x)$ m.

Saving% = $\left(\frac{0.59x}{2x} \times 100\right)\% = 30\%$ (approx.).



62. Speed of the man = $\left(4 \times \frac{5}{18}\right)$ m/sec = $\frac{10}{9}$ m/s.

Time taken = (3×60) sec = 180 sec.

Length of diagonal = (speed $\times$ time) = $\left(\frac{10}{9} \times 180\right)$ m = 200 m.

Area of the field = $\frac{1}{2} \times (\text{diagonal})^2 = \left(\frac{1}{2} \times 200 \times 200\right)$ m^2 = 20000 m^2.

63. $d = \sqrt{2} \times l \Rightarrow l = \dfrac{20}{\sqrt{2}}$.

$\therefore$ Perimeter $= (4l)$ cm $= \left(\dfrac{4 \times 20}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}\right)$ cm $= 40\sqrt{2}$ cm.

64. Side $= \sqrt{69696}$ cm $= 264$ cm.

$\therefore$ $d = \sqrt{2} \times$ side $= (264 \times \sqrt{2})$ cm $= (264 \times 1.414)$ cm $= 373.296$ cm.

65. Area $= (45 \times 40)$ m^2 $\Leftrightarrow$ $\dfrac{1}{2} \times (\text{diagonal})^2 = 1800 \Leftrightarrow$ diagonal $= 60$ m.

66. Let breadth be x metres. Then, length $= 120\%$ of $x = \left(\dfrac{120}{100}x\right) = \dfrac{6x}{5}$ m.

Required ratio $= \left(\dfrac{6x}{5} \times x \times \dfrac{1}{x \times x}\right) = 6 : 5$.

67. A square and a rectangle with equal areas will satisfy the relation $p_1 < p_2$.

68. Take a square of side 4 cm and a rectangle having $l = 6$ cm, $b = 2$ cm.
Then, perimeter of square = perimeter of rectangle.
Area of square $= 16$ cm^2, area of rectangle $= 12$ cm^2.
$\therefore$ A > B.

69. $d_1 = 4\sqrt{2}$ cm $\Rightarrow$ area $= \dfrac{1}{2}{d_1}^2 = \dfrac{1}{2} \times (4\sqrt{2})^2 = 16$ cm^2.

Area of new square $= (2 \times 16)$ cm^2 $= 32$ cm^2.

$\therefore$ $\dfrac{1}{2}{d_2}^2 = 32 \Rightarrow {d_2}^2 = 64 \Rightarrow d_2 = 8$ cm.

70. Required ratio $= \dfrac{a^2}{(\sqrt{2}\,a)^2} = \dfrac{a^2}{2a^2} = \dfrac{1}{2} = 1 : 2$.

71. Let the diagonals be $2d$ and d.

Then, ratio of their areas $= \dfrac{\frac{1}{2} \times (2d)^2}{\frac{1}{2} \times d^2} = \dfrac{4d^2}{d^2} = \dfrac{4}{1} = 4 : 1$.

72. $\dfrac{a^2}{b^2} = \dfrac{225}{256} = \dfrac{(15)^2}{(16)^2} \Leftrightarrow \dfrac{a}{b} = \dfrac{15}{16} \Leftrightarrow \dfrac{4a}{4b} = \dfrac{4 \times 15}{4 \times 16} = \dfrac{15}{16}$.

$\therefore$ Ratio of perimeters $= 15 : 16$.

73. Area $= 1$ hect. $= 10000$ sq. m $\Rightarrow$ side $= \sqrt{10000}$ m $= 100$ m.
Side of the other square $= 101$ m.
Difference in their areas $= |(101)^2 - (100)^2|$ m^2
$= |(101 + 100)(101 - 100)|$ m$^2 = 201$ m^2.

74. Let the sides be x cm and $\dfrac{150}{100}x = \dfrac{3x}{2}$ cm.

Required ratio $= \dfrac{\frac{9}{4}x^2}{x^2} = \dfrac{9}{4} = 9 : 4$.

75. $A_1 = x^2$ and $A_2 = \left(\dfrac{1}{2}x\right)^2 = \dfrac{1}{4}x^2 = \dfrac{1}{4}A_1$.

76. 100 cm is read as 102 cm.

$\therefore$ $A_1 = (100 \times 100)\ cm^2$ and $A_2 = (102 \times 102)\ cm^2$.

$(A_2 - A_1) = [(102)^2 - (100)^2] = (102 + 100) \times (102 - 100) = 404\ cm^2$.

$\therefore$ Percentage error $= \left(\frac{404}{100 \times 100} \times 100\right)\% = 4.04\%$.

77. Let original area = 100 cm^2. Then, new area = 169 cm^2.

$\Rightarrow$ Original side = 10 cm, New side = 13 cm.

Increase on 10 cm = 3 cm. Increase% $= \left(\frac{3}{10} \times 100\right)\% = 30\%$.

78. Given diagonal = d. New diagonal $= \frac{3}{2}d$.

Original area $= \frac{1}{2}d^2$, New area $= \frac{1}{2} \times \left(\frac{3}{2}d\right)^2 = \frac{9}{8}d^2$.

$\therefore$ Required ratio $= \frac{1}{2}d^2 : \frac{9}{8}d^2 = \frac{1}{2} : \frac{9}{8} = 4 : 9$.

79. Let length = l metres and breadth = b metres. Then, original area = $(lb)\ m^2$.

New length = (140% of l) m $= \left(\frac{140}{100} \times l\right)$ m $= \frac{7l}{5}$ m.

New breadth = (130% of b) m $= \left(\frac{130}{100} \times b\right)$ m $= \frac{13b}{10}$ m.

New area $= \left(\frac{7l}{5} \times \frac{13b}{10}\right) = \left(\frac{91}{50}lb\right)\ m^2$. Increase $= \left(\frac{91}{50}lb - lb\right) = \frac{41}{50}lb$.

$\therefore$ Increase% $= \left(\frac{41}{50} \times lb \times \frac{1}{lb} \times 100\right)\% = 82\%$.

80. Let original length of each side = x cm. Then, its area = $(x^2)\ cm^2$.

Length of rectangle formed = $(x + 5)$ cm and its breadth = x cm.

$\therefore$ $\frac{x+5}{x} = \frac{3}{2} \Leftrightarrow 2x + 10 = 3x \Leftrightarrow x = 10$.

$\therefore$ Original length of each side = 10 cm and its area = 100 cm^2.

81. Let original side = x cm. Then, new side = $(x + 5)$ cm.

$\therefore$ $(x + 5)^2 - x^2 = 165 \Leftrightarrow x^2 + 10x + 25 - x^2 = 165 \Leftrightarrow 10x = 140 \Leftrightarrow x = 14$.

Hence, the side of the square is 14 cm.

82. Let the lengths of the line segments be x cm and $(x + 2)$ cm.

Then, $(x + 2)^2 - x^2 = 32 \Leftrightarrow x^2 + 4x + 4 - x^2 = 32 \Leftrightarrow 4x = 28 \Leftrightarrow x = 7$.

$\therefore$ Length of longer line segment = (7 + 2) cm = 9 cm.

83. Let the length of each side of the square be x cm.

Then, length of rectangle = $(x + 5)$ cm and its breadth = $(x - 3)$ cm.

$\therefore$ $(x + 5)(x - 3) = x^2 \Leftrightarrow x^2 + 2x - 15 = x^2 \Leftrightarrow x = \frac{15}{2}$.

$\therefore$ Length $= \left(\frac{15}{2} + 5\right)$ cm $= \frac{25}{2}$ cm, breadth $= \left(\frac{15}{2} - 3\right)$ cm $= \frac{9}{2}$ cm.

Hence, perimeter $= 2(l + b) = 2\left(\frac{25}{2} + \frac{9}{2}\right)$ cm $= 34$ cm.

84. Area to be plastered $= [2(l + b) \times h] + (l \times b)$

$= \{[2(25 + 12) \times 6] + (25 \times 12)\}\ m^2$

$= (444 + 300)\ m^2 = 744\ m^2.$

$\therefore$ Cost of plastering $= \text{Rs.}\left(744 \times \frac{75}{100}\right) = \text{Rs. } 558.$

85. Area of 4 walls $= [2(l + b) \times h] = [2(10 + 7) \times 5]\ m^2 = 170\ m^2.$

Area of 2 doors and 3 windows $= [2(1 \times 3) + (2 \times 1.5) + 2(1 \times 1.5)]\ m^2 = 12\ m^2.$

$\therefore$ Area to be painted $= (170 - 12)\ m^2 = 158\ m^2.$

Cost of painting $= \text{Rs. } (158 \times 3) = \text{Rs. } 474.$

86. $A_1 = 2(l + b) \times h$; $A_2 = 2(2l + 2b) \times 2h = 8(l + b) \times h = 4A_1.$

$\therefore$ Required cost $= \text{Rs. } (4 \times 475) = \text{Rs. } 1900.$

87. Let $h = 2x$ metres and $(l + b) = 5x$ metres.

$$\text{Length of the paper} = \frac{\text{Total cost}}{\text{Rate per m}} = \frac{260}{2}\ \text{m} = 130\ \text{m}.$$

$$\text{Area of the paper} = \left(130 \times \frac{50}{100}\right) m^2 = 65\ m^2.$$

Total area of 4 walls $= (65 + 15)\ m^2 = 80\ m^2.$

$\therefore$ $2(l + b) \times h = 80 \Leftrightarrow 2 \times 5x \times 2x = 80 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2.$

Height of the room $= 4$ m.

88. $A_1 = \left(\frac{1}{2} \times 15 \times 12\right) cm^2 = 90\ cm^2.$ $A_2 = 2A_1 = 180\ cm^2.$

$\therefore$ $\frac{1}{2} \times 20 \times h = 180 \Leftrightarrow h = 18$ cm.

89. $a = 5$, $b = 12$ and $c = 13$. So, $s = \frac{1}{2}(5 + 12 + 13)$ cm $= 15$ cm.

$\therefore$ Area $= \sqrt{15 \times 10 \times 3 \times 2} = 30\ cm^2.$

$\frac{1}{2} \times 12 \times \text{Height} = 30 \rightarrow \text{Height} = 5$ cm.

90. $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow 40 \times \text{Base} = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow \text{Height} = 80$ cm.

91. Let Base $= 3x$ cm and Altitude $= 4x$ cm.

Then, $\frac{1}{2} \times 3x \times 4x = 1176 \Leftrightarrow 12x^2 = 2352 \Leftrightarrow x^2 = 196 \Leftrightarrow x = 14$ cm.

$\therefore$ Altitude $= (4 \times 14)$ cm $= 56$ cm.

92. Since $5^2 + (12)^2 = (13)^2$, so, it is a right-angled triangle with

Base $= 12$ cm and Height $= 5$ cm.

$\therefore$ Area $= \left(\frac{1}{2} \times 12 \times 5\right) cm^2 = 30\ cm^2.$

93. Ratio of sides $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3.$

Perimeter $= 52$ cm. So, sides are $\left(52 \times \frac{6}{13}\right)$ cm, $\left(52 \times \frac{4}{13}\right)$ cm and $\left(52 \times \frac{3}{13}\right)$ cm.

$\therefore$ $a = 24$ cm, $b = 16$ cm, $c = 12$ cm.

$\therefore$ Length of smallest side $= 12$ cm.

94. Let $a = 3x$ cm, $b = 4x$ cm and $c = 5x$ cm. Then, $s = 6x$ cm.

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6x \times 3x \times 2x \times x} = (6x^2)$ cm^2.

$\therefore \quad 6x^2 = 216 \Leftrightarrow x^2 = 36 \Leftrightarrow x = 6.$

$\therefore \quad a = 18$ cm, $b = 24$ cm and $c = 30$ cm.

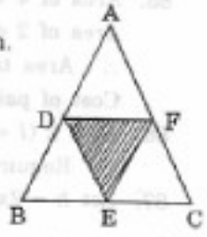
Perimeter $= (18 + 24 + 30)$ cm $= 72$ cm.

95. $a = 3$ cm, $b = 4$ cm and $c = 5$ cm.

It is a right-angled triangle with base $= 3$ cm and height $= 4$ cm.

$\therefore$ Its area $= \left(\frac{1}{2} \times 3 \times 4\right)$ cm^2 $= 6$ cm^2.

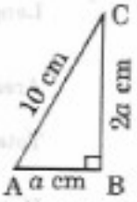
Area of required triangle $= \left(\frac{1}{4} \times 6\right)$ cm^2 $= \frac{3}{2}$ cm^2.



96. Let the sides be a cm and $2a$ cm.

Then, $a^2 + (2a)^2 = (10)^2 \Leftrightarrow 5a^2 = 100 \Leftrightarrow a^2 = 20.$

$\therefore$ Area $= \left(\frac{1}{2} \times a \times 2a\right) = a^2 = 20$ cm^2.



97. Let Base $= b$ cm and Height $= h$ cm.

$b + h + 26 = 60 \Leftrightarrow b + h = 34 \Leftrightarrow (b + h)^2 = (34)^2$ ...(i)

Also, $b^2 + h^2 = (26)^2$ ...(ii)

$\therefore \quad (b + h)^2 - (b^2 + h^2) = (34)^2 - (26)^2 \Leftrightarrow 2bh = (34 + 26)(34 - 26) = 480$

$\Leftrightarrow bh = 240 \Leftrightarrow \frac{1}{2}bh = 120.$

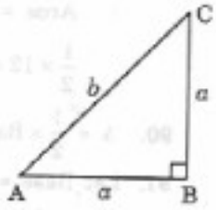
$\therefore$ Area $= 120$ cm^2.

98. Let the sides be a metres, a metres and b metres.

Then, $2a + b = 6 + 3\sqrt{2}$ and $b^2 = a^2 + a^2 = 2a^2 \Leftrightarrow b = \sqrt{2}\,a.$

$\therefore \quad 2a + \sqrt{2}\,a = 6 + 3\sqrt{2} \Leftrightarrow a = 3.$

$\therefore \quad$ Area $= \left(\frac{1}{2} \times 3 \times 3\right)$ m^2 $= 4.5$ m^2.



99. Let the smallest side be x cm.

Then, other sides are 13 cm and $(17 - x)$ cm.

Let $a = 13$, $b = x$ and $c = (17 - x)$. So, $s = 15$.

$\therefore \quad$ Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15 \times 2 \times (15-x)(x-2)}$

$\Leftrightarrow 30 \times (15 - x)(x - 2) = (30)^2 \Leftrightarrow (15 - x)(x - 2) = 30 \Leftrightarrow x^2 - 17x + 60 = 0$

$\Leftrightarrow (x - 12)(x - 5) = 0 \Leftrightarrow x = 12$ or $x = 5.$

$\therefore$ Smallest side $= 5$ cm.

100. Area of an equilateral triangle of side a cm $= \left(\frac{\sqrt{3}}{4}a^2\right)$ cm^2.

$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 24\sqrt{3} \Leftrightarrow a^2 = 96 \Leftrightarrow a = 4\sqrt{6}$ cm.

$\therefore$ Perimeter $= 3a = 12\sqrt{6}$ cm.

84. Area to be plastered $= [2\,(l + b) \times h] + (l \times b)$

$= \{[2\,(25 + 12) \times 6] + (25 \times 12)\}\ m^2$

$= (444 + 300)\ m^2 = 744\ m^2.$

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Total area of 4 walls $= (65 + 15)\ m^2 = 80\ m^2.$

$\therefore\ 2\,(l + b) \times h = 80 \Leftrightarrow 2 \times 5x \times 2x = 80 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2.$

Height of the room $= 4$ m.

88. $A_1 = \left(\frac{1}{2} \times 15 \times 12\right) cm^2 = 90\ cm^2.\ A_2 = 2A_1 = 180\ cm^2.$

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$\frac{1}{2} \times 12 \times \text{Height} = 30 \Rightarrow \text{Height} = 5$ cm.

90. $\Delta = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow 40 \times \text{Base} = \frac{1}{2} \times \text{Base} \times \text{Height} \Rightarrow \text{Height} = 80$ cm.

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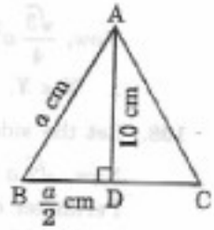
$\therefore$ Length of smallest side $= 12$ cm.

101. Let each side be a cm.

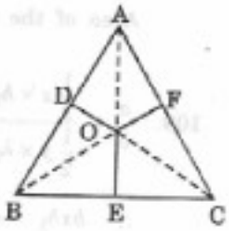


Then, $\left(\frac{a}{2}\right)^2 + (10)^2 = a^2 \Leftrightarrow \left(a^2 - \frac{a^2}{4}\right) = 100$

$\Leftrightarrow \frac{3a^2}{4} = 100 \Leftrightarrow a^2 = \frac{400}{3}$.

$\therefore$ Area $= \frac{\sqrt{3}}{4} \times a^2 = \left(\frac{\sqrt{3}}{4} \times \frac{400}{3}\right) \text{cm}^2 = \frac{100}{\sqrt{3}} \text{cm}^2$.

102. Let each side of the triangle be a cm.



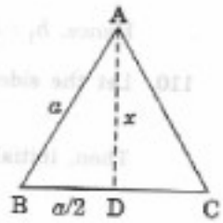
Then, ar (Δ AOB) + ar (Δ BOC) + ar (Δ AOC) = ar (Δ ABC)

$\Rightarrow \frac{1}{2} \times a \times \sqrt{3} + \frac{1}{2} \times a \times 2\sqrt{3} + \frac{1}{2} \times a \times 5\sqrt{3} = \frac{\sqrt{3}}{4} a^2$

$\Rightarrow \frac{a}{2}\sqrt{3}(1 + 2 + 5) = \frac{\sqrt{3}}{4} a^2 \Rightarrow a = 16$.

$\therefore$ Perimeter $= (3 \times 16) = 48$ cm.

103. Let the side of the triangle be a. Then,



$a^2 = \left(\frac{a}{2}\right)^2 + x^2 \Leftrightarrow \frac{3a^2}{4} = x^2 \Leftrightarrow a^2 = \frac{4x^2}{3}$.

$\therefore$ Area $= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times \frac{4}{3} x^2 = \frac{x^2}{\sqrt{3}} = \frac{x^2\sqrt{3}}{3}$.

104. Area of a square with side $a = a^2$ sq. units.

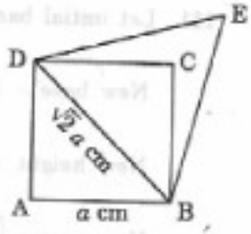
Area of a triangle with base $a = \left(\frac{1}{2} \times a \times h\right)$ sq. units.

$\therefore \quad a^2 = \frac{1}{2} \times a \times h \Leftrightarrow h = 2a$.

Hence, the altitude of the triangle is $2a$.

105. Let the side of the square be a cm.

Then, the length of its diagonal $= \sqrt{2}\,a$ cm.

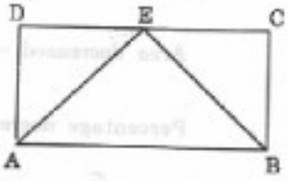


Area of equilateral triangle with side $\sqrt{2}a = \frac{\sqrt{3}}{4} \times (\sqrt{2}a)^2$

$= \frac{\sqrt{3}a^2}{2}$.

$\therefore$ Required ratio $= \frac{\sqrt{3}a^2}{2} : a^2 = \sqrt{3} : 2$.

106. Area of rectangle $= lb$ sq. units.



Area of the triangle $= \frac{1}{2} lb$ sq. units.

$\therefore$ Required ratio $= lb : \frac{1}{2} lb = 2 : 1$.

107. Let each side of the triangle be a cm and each side of the square be b cm.

Then, $X = \frac{\sqrt{3}}{4} a^2$ and $Y = b^2$, where $3a = 4b$, *i.e.*, $b = \frac{3a}{4}$.

$\therefore \quad X = \frac{\sqrt{3}}{4} a^2$ and $Y = \frac{9a^2}{16} \quad \left[\because b = \frac{3a}{4}\right]$

Now, $\frac{\sqrt{3}}{4}a^2 = \frac{1.732}{4}a^2 = 0.433\,a^2$ and $\frac{9a^2}{16} = 0.5625\,a^2$.

$\therefore$ X < Y.

108. Let the side of the square be a cm. Then, its diagonal = $\sqrt{2}\,a$ cm.

Now, $\sqrt{2}\,a = 12\sqrt{2} \Rightarrow a = 12$ cm.

Perimeter of the square = $4a$ = 48 cm. Perimeter of the equilateral triangle = 48 cm.

Each side of the triangle = 16 cm.

Area of the triangle = $\left(\frac{\sqrt{3}}{4} \times 16 \times 16\right)$ cm^2 = $(64\sqrt{3})$ cm^2.

109. $\frac{a}{b} = \frac{\frac{1}{2}x \times h_1}{\frac{1}{2}y \times h_2}$ $\left[\text{Ratio of areas} = \frac{a}{b},\ \text{Ratio of base} = x : y\right]$

$\therefore \quad bxh_1 = ayh_2 \Leftrightarrow \frac{h_1}{h_2} = \frac{ay}{bx}$.

Hence, $h_1 : h_2 = ay : bx$.

110. Let the sides be x cm and (80% of x) cm = $\frac{4x}{5}$ cm.

Then, initial area = $\frac{\sqrt{3}}{4}x^2$, final area = $\frac{\sqrt{3}}{4}\left(\frac{4x}{5}\right)^2 = \frac{16\sqrt{3}\,x^2}{100}$.

Decrease in area = $\left(\frac{\sqrt{3}}{4}x^2 - \frac{16\sqrt{3}}{100}x^2\right)$ cm^2 = $\frac{9\sqrt{3}\,x^2}{100}$ cm^2.

$\therefore$ Decrease% = $\left(\frac{9\sqrt{3}\,x^2}{100} \times \frac{4}{\sqrt{3}\,x^2} \times 100\right)\% = 36\%$.

111. Let initial base = b cm and initial height = h cm. Then, initial area = $\left(\frac{1}{2}bh\right)$ cm^2.

New base = (140% of b) cm = $\left(\frac{140b}{100}\right)$ cm = $\left(\frac{7b}{5}\right)$ cm.

New height = (60% of h) cm = $\left(\frac{60h}{100}\right)$ cm = $\left(\frac{3h}{5}\right)$ cm.

New area = $\left(\frac{1}{2} \times \frac{7b}{5} \times \frac{3h}{5}\right)$ cm^2 = $\left(\frac{21}{50}bh\right)$ cm^2.

Area decreased = $\left(\frac{1}{2}bh - \frac{21}{50}bh\right)$ cm^2 = $\left(\frac{4}{50}bh\right)$ cm^2.

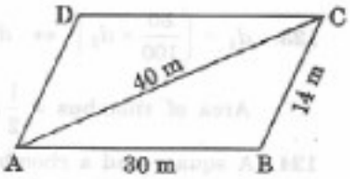
Percentage decrease = $\left(\frac{4bh}{50} \times \frac{2}{bh} \times 100\right)\% = 16\%$.

112. $A_1 = \frac{\sqrt{3}}{2}a^2$ and $A_2 = \frac{\sqrt{3}}{2}(2a)^2 = 4 \times \frac{\sqrt{3}}{2}a^2 = 4A_1$.

$\therefore$ K = 4.

113. Area of ||gm = (Base × Height) = (18 × 8) cm^2 = 144 cm^2.

114. Let ABCD be the given ||gm.



Area of ||gm ABCD = 2 × (area of Δ ABC).

Now, $a = 30$ m, $b = 14$ m, $c = 40$ m.

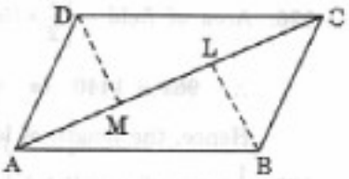
$\therefore \quad s = \frac{1}{2}(30 + 14 + 40)$ m = 42 m.

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 12 \times 28 \times 2}$ m^2 = 168 m^2.

Hence, area of ||gm ABCD = (2 × 168) m^2 = 336 m^2.

115. Let ABCD be the given ||gm.



Let AC = 70 cm.

Draw BL ⊥ AC and DM ⊥ AC.

Then, DM = BL = 27 cm.

Area of ||gm ABCD = ar (Δ ABC) + ar (Δ ACD)

$= \left[\left(\frac{1}{2} \times 70 \times 27\right) + \left(\frac{1}{2} \times 70 \times 27\right)\right]$ sq. cm = 1890 sq. cm.

116. Let the altitude of the triangle be h_1 and base of each be b.

Then, $\frac{1}{2} \times b \times h_1 = b \times h_2$, where $h_2 = 100$ m

$\Leftrightarrow \quad h_1 = 2h_2 = (2 \times 100)$ m = 200 m.

117. Let each have base = b and height = h. Then, $P = b \times h$, $R = b \times h$, $T = \frac{1}{2} \times b \times h$

So, P = R, P = 2T and $T = \frac{1}{2}R$ are all correct statements.

118. $\frac{1}{2} d_1 \times d_2 = 150 \Leftrightarrow \frac{1}{2} \times 10 \times d_2 = 150 \Leftrightarrow d_2 = 30$ cm.

119. $\frac{1}{2} d_1 \times 2d_1 = 25 \Leftrightarrow d_1^2 = 25 \Leftrightarrow d_1 = 5$.

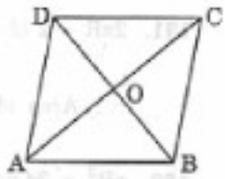
$\therefore$ Sum of lengths of diagonals = (5 + 10) cm = 15 cm.

120. Perimeter of the rhombus = 56 m. Each side of the rhombus = $\frac{56}{4}$ m = 14 m.

Height of the rhombus = 5 m.

$\therefore$ Area = (14 × 5) m^2 = 70 m^2.

121. Area = $\frac{1}{2} d_1 d_2 = \left(\frac{1}{2} \times 24 \times 10\right)$ cm^2 = 120 cm^2.



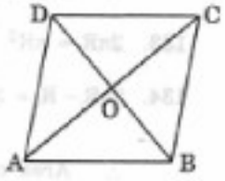
$OA = \frac{1}{2} d_1 = \left(\frac{1}{2} \times 24\right)$ cm = 12 cm.

$OB = \frac{1}{2} d_2 = \left(\frac{1}{2} \times 10\right)$ cm = 5 cm.

$AB^2 = OA^2 + OB^2 = (12)^2 + 5^2 = 169 \Leftrightarrow AB = 13$ cm.

$\therefore$ Perimeter = (13 × 4) cm = 52 cm.

122. AB = 26 cm and AC = 48 cm $\Rightarrow OA = \left(\frac{1}{2} \times 48\right)$ cm = 24 cm.



$OB^2 = AB^2 - OA^2 = (26)^2 - (24)^2 = (26 + 24)(26 - 24) = 100$

$\Rightarrow$ OB = 50 cm $\Rightarrow$ BD = 2 × OB = (2 × 50) cm = 100 cm.

$\therefore$ Area = $\frac{1}{2} \times AC \times BD = \left(\frac{1}{2} \times 48 \times 100\right)$ cm^2 = 2400 cm^2.

123. $d_1 = \left(\frac{80}{100} \times d_2\right) \Leftrightarrow d_1 = \frac{4d_2}{5}$.

Area of rhombus $= \frac{1}{2} d_1 d_2 = \left(\frac{1}{2} \times \frac{4d_2}{5} \times d_2\right) = \frac{2}{5}(d_2)^2$.

124. A square and a rhombus on the same base are equal in area.

125. Area of trapezium $= \left[\frac{1}{2} \times (1.5 + 2.5) \times 6.5\right] m^2 = 13\ m^2$.

126. Area of field $= \left[\frac{1}{2} \times (5x + 3x) \times 24\right] m^2 = (96x)\ m^2$.

$\therefore \quad 96x = 1440 \Leftrightarrow x = \frac{1440}{96} \Leftrightarrow x = 15$.

Hence, the length of longer parallel side $= (5x) = 75$ m.

127. $\frac{1}{2}$ (sum of parallel sides) $\times$ depth = Its area

$\Leftrightarrow \quad \frac{1}{2}(12 + 8) \times d = 840 \Leftrightarrow d = 84$ m.

128. Required $\% = \left[\frac{\pi \times (5)^2}{2\pi \times 5} \times 100\right]\% = 250\%$.

129. Speed $= 12$ km/hr $= \left(12 \times \frac{5}{18}\right)$ m/s $= \frac{10}{3}$ m/s.

Distance covered $= \left(20 \times 2 \times \frac{22}{7} \times 50\right)$ m $= \frac{44000}{7}$ m.

Time taken $= \frac{\text{Distance}}{\text{Speed}} = \left(\frac{44000}{7} \times \frac{3}{10}\right)$ s $= \left(\frac{4400 \times 3}{7} \times \frac{1}{60}\right)$ min

$= \frac{220}{7}$ min $= 31\frac{3}{7}$ min.

130. Area of the field grazed $= \left(\frac{22}{7} \times 14 \times 14\right)$ sq. ft $= 616$ sq. ft.

Number of days taken to graze the field $= \frac{616}{100}$ days $= 6$ days (approx.).

131. $2\pi R = 2(l + b) \Leftrightarrow 2\pi R = 2(26 + 18)$ cm $\Leftrightarrow R = \left(\frac{88}{2 \times 22} \times 7\right) = 14$ cm.

$\therefore$ Area of the circle $= \pi R^2 = \left(\frac{22}{7} \times 14 \times 14\right) cm^2 = 616\ cm^2$.

132. $\pi R^2 = 24.64 \Leftrightarrow R^2 = \left(\frac{24.64}{22} \times 7\right) = 7.84 \Leftrightarrow R = \sqrt{7.84} = 2.8$ cm.

$\therefore$ Circumference $= \left(2 \times \frac{22}{7} \times 2.8\right)$ cm $= 17.60$ m.

133. $2\pi R = \pi R^2 \Leftrightarrow R = 2 \Leftrightarrow 2R = 4$. Hence, diameter $= 4$.

134. $2\pi R - R = 37 \Leftrightarrow \left(\frac{44}{7} - 1\right) R = 37 \Leftrightarrow R = 7$.

$\therefore$ Area of the circle $= \left(\frac{22}{7} \times 7 \times 7\right) cm^2 = 154\ cm^2$.

135. $\pi R_1^2 + \pi R_2^2 = \pi R_3^2 \Leftrightarrow R_1^2 + R_2^2 = R_3^2 \Leftrightarrow (9)^2 + R_2^2 = (15)^2$

$\Leftrightarrow R_2^2 = (15)^2 - (9)^2 = 144 \Leftrightarrow R_2 = 12$ cm.

136. Side of the square $= \frac{44}{4}$ cm $= 11$ cm.

Area of the square $= (11 \times 11)$ cm^2 $= 121$ cm^2.

$2\pi R = 44 \Leftrightarrow 2 \times \frac{22}{7} \times R = 44 \Leftrightarrow R = 7$ cm.

Area of circle $= \pi R^2 = \left(\frac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

$\therefore$ Area of circle is larger by 33 cm^2.

137. Length of wire $= 2\pi \times R = \left(2 \times \frac{22}{7} \times 56\right)$ cm $= 352$ cm.

Side of the square $= \frac{352}{4}$ cm $= 88$ cm.

Area of the square $= (88 \times 88)$ cm^2 $= 7744$ cm^2.

138. Side of the square $= \sqrt{484}$ cm $= 22$ cm. Perimeter of the square $= (22 \times 4)$ cm $= 88$ cm.

$2\pi R = 88 \Leftrightarrow 2 \times \frac{22}{7} \times R = 88 \Leftrightarrow R = \left(88 \times \frac{7}{44}\right) = 14$ cm.

$\therefore$ Required area $= \pi R^2 = \left(\frac{22}{7} \times 14 \times 14\right)$ cm^2 $= 616$ cm^2.

139. Length of wire $= 2\pi R = \left(2 \times \frac{22}{7} \times 42\right)$ cm $= 264$ cm.

Perimeter of rectangle $= 2\,(6x + 5x)$ cm $= 22x$ cm.

$\therefore$ $22x = 264 \Leftrightarrow x = 12$.

Smaller side $= (5 \times 12)$ cm $= 60$ cm.

140. Total area of the field $= [(180 \times 120)] + 40000]$ m^2

$= (21600 + 40000)$ m^2 $= 61600$ m^2.

$\therefore$ $\pi R^2 = 61600 \Leftrightarrow R^2 = \left(61600 \times \frac{7}{22}\right) = (400 \times 7 \times 7)$ m

$\Leftrightarrow R = (20 \times 7)$ m $= 140$ m.

141. $\frac{\pi R_1^2}{\pi R_2^2} = \frac{16}{49} \Leftrightarrow \frac{R_1^2}{(14 \times 14)} = \frac{16}{49}$

$\Leftrightarrow R_1^2 = \frac{14 \times 14 \times 16}{49} \Leftrightarrow R_1 = \frac{14 \times 4}{7} = 8$ m.

142. $\frac{\pi R_1^2}{\pi R_2^2} = \frac{4}{9} \Leftrightarrow \frac{R_1^2}{R_2^2} = \frac{4}{9} \Leftrightarrow \frac{R_1}{R_2} = \frac{2}{3} \Leftrightarrow \frac{2\pi R_1}{2\pi R_2} = \frac{R_1}{R_2} = \frac{2}{3}$.

$\therefore$ Required ratio $= 2 : 3$.

143. Let the radius of the given circle be R cm and the side of the square be a cm.

Then, $2\pi R = 4a \Leftrightarrow \frac{R}{a} = \frac{2}{\pi}$.

$\therefore$ Ratio of their areas $= \frac{\pi R^2}{a^2} = \left(\pi \times \frac{4}{\pi^2}\right) = \left(\frac{4}{22} \times 7\right) = \frac{14}{11} = 14 : 11$.

144. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times 0.63\right)$ m $= \frac{99}{25}$ m.

Distance covered in 500 revolutions $= \left(\frac{99}{25} \times 500\right)$ m $= 1980$ m.

145. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times 20\right)$ cm $= \frac{880}{7}$ cm.

Required number of revolutions $= \left(17600 \times \frac{7}{880}\right) = 140.$

146. Distance covered in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times \frac{25}{100}\right)$ m $= \frac{11}{7}$ m.

$\therefore$ Required number of revolutions $= \left(11000 \times \frac{7}{11}\right) = 7000.$

147. Distance covered in 9 sec $= \left(2 \times \frac{22}{7} \times \frac{15}{2} \times 7\right)$ m $= 330$ m.

Distance covered in 1 sec $= \frac{330}{9}$ m $= \frac{110}{3}$ m.

$\therefore$ Required speed $= \left(\frac{110}{3} \times \frac{18}{5}\right)$ km/hr $= 132$ km/hr.

148. Distance covered in 10 sec $= \left(2 \times \frac{22}{7} \times \frac{35}{100} \times 40\right)$ m $= 88$ m.

Distance covered in 1 sec $= \frac{88}{10}$ m $= 8.8$ m.

$\therefore$ Speed $= 8.8$ m/s $= \left(8.8 \times \frac{18}{5}\right)$ km/hr $= 31.68$ km/hr.

149. Let each wheel make x revolutions per sec. Then,

$$\left[\left(2\pi \times \frac{7}{2} \times x\right) + (2\pi \times 7 \times x)\right] \times 10 = 1980$$

$$\Leftrightarrow \left(\frac{22}{7} \times 7 \times x\right) + \left(2 \times \frac{22}{7} \times 7 \times x\right) = 198 \Leftrightarrow 66x = 198 \Leftrightarrow x = 3.$$

Distance moved by smaller wheel in 3 revolutions $= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 3\right)$ cm $= 66$ cm.

$\therefore$ Speed of smaller wheel $= \frac{66}{3}$ m/s $= 22$ m/s.

150. Distance covered by smaller wheel in 1 revolution $= (2\pi \times 15)$ cm $= (30\pi)$ cm.
Distance covered by larger wheel in 1 revolution $= (2\pi \times 25)$ cm $= (50\pi)$ cm.

Let $k \times 30\pi = 15 \times 50\pi$. Then, $k = \left(\frac{15 \times 50\pi}{30\pi}\right) = 25.$

$\therefore$ Required number of revolutions $= 25.$

151. Let the diameter of the wheel be d metres.
Distance covered in 1 revolution $= (\pi d)$ m.
Distance covered in 113 revolutions $= (113\pi d)$ m.

$\therefore$ $113 \times \frac{22}{7} \times d = 226 \times 10 \Leftrightarrow d = \left(226 \times 10 \times \frac{7}{22} \times \frac{1}{113}\right)$ m $= 6\frac{4}{11}$ m.

152. Let the rear wheel make x revolutions. Then, the front wheel makes $(x + 10)$ revolutions.

$(x + 10) \times 3\pi = x \times 2\pi \Leftrightarrow 3x + 30 = 2x \Leftrightarrow x = 30.$

Distance travelled by the wagon $= (2\pi \times 30)$ ft $= (60\pi)$ ft.

153. Radius of the ground $= 17.5$ m. Radius of inner circle $= (17.5 - 1.4)$ m $= 16.1$ m.

Area of the garden $= \pi \times [(17.5)^2 - (16.1)^2]\ m^2 = \left[\frac{22}{7} \times (17.5 + 16.1)(17.5 - 16.1)\right] m^2$

$= \left(\frac{22}{7} \times 33.6 \times 1.4\right) m^2 = 147.84\ m^2.$

154. $2\pi R = 440 \Leftrightarrow 2 \times \frac{22}{7} \times R = 440 \Leftrightarrow R = \left(440 \times \frac{7}{44}\right) = 70$ m.

Inside radius $= (70 - 7)$ m $= 63$ m.

Area of the border $= \pi [(70)^2 - (63)^2]\ m^2$

$= \left[\frac{22}{7} \times (70 + 63) \times (70 - 63)\right] m^2 = 2926\ m^2.$

155. $\pi R_1^2 = 616 \Leftrightarrow R_1^2 = \left(616 \times \frac{7}{22}\right) = 196 \Leftrightarrow R_1 = 14$ cm.

$\pi R_2^2 = 154 \Leftrightarrow R_2^2 = \left(154 \times \frac{7}{22}\right) = 49 \Leftrightarrow R_2 = 7$ cm.

Breadth of the ring $= (R_1 - R_2)$ cm $= (14 - 7)$ cm $= 7$ cm.

156. $2\pi R_1 - 2\pi R_2 = 132 \Leftrightarrow 2\pi (R_1 - R_2) = 132 \Leftrightarrow (R_1 - R_2) = \left(\frac{132}{2 \times 22} \times 7\right) = 21$ m.

$\therefore$ Required width $= 21$ m.

157. Let the radius of the pool be R ft. Radius of the pool including the wall $= (R + 4)$ ft.

Area of the concrete wall $= \pi [(R + 4)^2 - R^2]$ sq. ft

$= [\pi (R + 4 + R)(R + 4 - R)]$ sq. ft $= 8\pi (R + 2)$ sq. ft.

$8\pi (R + 2) = \frac{11}{25} \pi R^2 \Leftrightarrow 11R^2 = 200 (R + 2) \Leftrightarrow 11R^2 - 200R - 400 = 0$

$\Leftrightarrow 11R^2 - 220R + 20R - 400 = 0$

$\Leftrightarrow 11R (R - 20) + 20 (R - 20) = 0$

$\Leftrightarrow (R - 20)(11R + 20) = 0 \Leftrightarrow R = 20.$

$\therefore$ Radius of the pool $= 20$ ft.

158. $\frac{2\pi R_1}{2\pi R_2} = \frac{23}{22} \Leftrightarrow \frac{R_1}{R_2} = \frac{23}{22} \Leftrightarrow R_1 = \frac{23}{22} R_2.$

Also, $R_1 - R_2 = 5$ m $\Leftrightarrow \frac{23R_2}{22} - R_2 = 5 \Leftrightarrow R_2 = 110.$

$\therefore$ Diameter of inner circle $= (2 \times 110)$ m $= 220$ m.

159. Area of the semi-circle $= \frac{1}{2} \pi R^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) m^2 = 77\ m^2.$

160. Perimeter of window $= \pi R + 2R = \left(\frac{22}{7} \times \frac{63}{2} + 63\right)$ cm $= (99 + 63)$ cm $= 162$ cm.

152. Let the rear wheel make x revolutions. Then, the front wheel makes $(x + 10)$ revolutions.

$(x + 10) \times 3\pi = x \times 2\pi \Leftrightarrow 3x + 30 = 2x \Leftrightarrow x = 30.$

Distance travelled by the wagon $= (2\pi \times 30)$ ft $= (60\pi)$ ft.

153. Radius of the ground $= 17.5$ m. Radius of inner circle $= (17.5 - 1.4)$ m $= 16.1$ m.

Area of the garden $= \pi \times [(17.5)^2 - (16.1)^2]\ m^2 = \left[\frac{22}{7} \times (17.5 + 16.1)(17.5 - 16.1)\right] m^2$

$= \left(\frac{22}{7} \times 33.6 \times 1.4\right) m^2 = 147.84\ m^2.$

154. $2\pi R = 440 \Leftrightarrow 2 \times \frac{22}{7} \times R = 440 \Leftrightarrow R = \left(440 \times \frac{7}{44}\right) = 70$ m.

Inside radius $= (70 - 7)$ m $= 63$ m.

Area of the border $= \pi [(70)^2 - (63)^2]\ m^2$

$= \left[\frac{22}{7} \times (70 + 63) \times (70 - 63)\right] m^2 = 2926\ m^2.$

155. $\pi R_1^{\,2} = 616 \Leftrightarrow R_1^{\,2} = \left(616 \times \frac{7}{22}\right) = 196 \Leftrightarrow R_1 = 14$ cm.

$\pi R_2^{\,2} = 154 \Leftrightarrow R_2^{\,2} = \left(154 \times \frac{7}{22}\right) = 49 \Leftrightarrow R_2 = 7$ cm.

Breadth of the ring $= (R_1 - R_2)$ cm $= (14 - 7)$ cm $= 7$ cm.

156. $2\pi R_1 - 2\pi R_2 = 132 \Leftrightarrow 2\pi (R_1 - R_2) = 132 \Leftrightarrow (R_1 - R_2) = \left(\frac{132}{2 \times 22} \times 7\right) = 21$ m.

$\therefore$ Required width $= 21$ m.

157. Let the radius of the pool be R ft. Radius of the pool including the wall $= (R + 4)$ ft.

Area of the concrete wall $= \pi [(R + 4)^2 - R^2]$ sq. ft

$= [\pi (R + 4 + R)(R + 4 - R)]$ sq. ft $= 8\pi (R + 2)$ sq. ft.

$$8\pi (R + 2) = \frac{11}{25}\pi R^2 \Leftrightarrow 11R^2 = 200 (R + 2) \Leftrightarrow 11R^2 - 200R - 400 = 0$$

$$\Leftrightarrow 11R^2 - 220R + 20R - 400 = 0$$

$$\Leftrightarrow 11R (R - 20) + 20 (R - 20) = 0$$

$$\Leftrightarrow (R - 20)(11R + 20) = 0 \Leftrightarrow R = 20.$$

$\therefore$ Radius of the pool $= 20$ ft.

158. $\frac{2\pi R_1}{2\pi R_2} = \frac{23}{22} \Leftrightarrow \frac{R_1}{R_2} = \frac{23}{22} \Leftrightarrow R_1 = \frac{23}{22} R_2.$

Also, $R_1 - R_2 = 5$ m $\Leftrightarrow \frac{23R_2}{22} - R_2 = 5 \Leftrightarrow R_2 = 110.$

$\therefore$ Diameter of inner circle $= (2 \times 110)$ m $= 220$ m.

159. Area of the semi-circle $= \frac{1}{2}\pi R^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) m^2 = 77\ m^2.$

160. Perimeter of window $= \pi R + 2R = \left(\frac{22}{7} \times \frac{63}{2} + 63\right)$ cm $= (99 + 63)$ cm $= 162$ cm.

161. Given : $\pi R + 2R = 36 \Leftrightarrow (\pi + 2)R = 36 \Leftrightarrow R = \dfrac{36}{\left(\dfrac{22}{7} + 2\right)}$ cm $= \left(\dfrac{36 \times 7}{36}\right)$ cm $= 7$ cm.

$\therefore$ Required area $= \pi R^2 = \left(\dfrac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

162. Length of each side of the square $= \sqrt{81}$ cm $= 9$ cm.

Length of wire $= (9 \times 4)$ cm $= 36$ cm.

$\pi R + 2R = 36 \Leftrightarrow (\pi + 2)R = 36 \Leftrightarrow R = \dfrac{36}{\left(\dfrac{22}{7} + 2\right)} = 7$ cm.

Area of the semi-circle $= \dfrac{1}{2}\pi R^2 = \left(\dfrac{1}{2} \times \dfrac{22}{7} \times 7 \times 7\right)$ cm^2 $= 77$ cm^2.

163. Area of the sector $= \left(\dfrac{1}{2} \times \text{arc} \times R\right) = \left(\dfrac{1}{2} \times 3.5 \times 5\right)$ cm^2 $= 8.75$ cm^2.

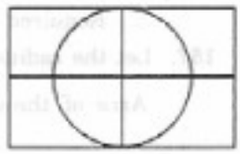
164. Area of the sector $= \dfrac{\pi R^2 0}{360} = \left(\dfrac{22}{7} \times 7 \times 7 \times \dfrac{108}{360}\right)$ cm^2 $= 46.2$ cm^2.

165. Side of the square $= \dfrac{120}{4}$ cm $= 30$ cm.

Radius of the required circle $= \left(\dfrac{1}{2} \times 30\right)$ cm $= 15$ cm.

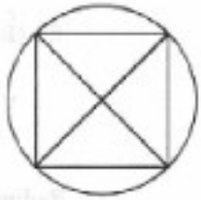
Area of the required circle $= [\pi \times (15)^2]$ cm^2 $= \left[\dfrac{22}{7} \times (15)^2\right]$ cm^2.

166. Radius of the required circle $= \left(\dfrac{1}{2} \times 14\right)$ cm $= 7$ cm.



Area of the circle $= \left(\dfrac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

167. $\pi R^2 = 220 \Leftrightarrow R^2 = \left(220 \times \dfrac{7}{22}\right) = 70$.



Now, $R = \dfrac{1}{2} \times$ (diagonal) $\Leftrightarrow$ diagonal $= 2R$.

$\therefore$ Area of the square $= \dfrac{1}{2} \times (\text{diagonal})^2$

$= \left(\dfrac{1}{2} \times 4R^2\right) = 2R^2 = (2 \times 70)$ cm^2 $= 140$ cm^2.

168. Given $R = 4$ cm. $R = \dfrac{1}{2} \times$ (diagonal of the square) $\Leftrightarrow$ diagonal $= 2R = 8$ cm.

Required area $= \pi R^2 - \dfrac{1}{2} \times (8)^2 = (\pi \times 16 - 32) = (16\pi - 32)$ cm^2.

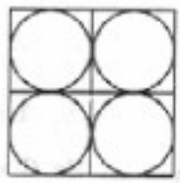
169. $2\pi R = 100 \Leftrightarrow R = \dfrac{100}{2\pi} = \dfrac{50}{\pi}$.

$R = \dfrac{1}{2} \times$ diagonal $\Leftrightarrow$ diagonal $= 2R = \dfrac{2 \times 50}{\pi} = \dfrac{100}{\pi}$.

$\therefore$ Area of the square $= \frac{1}{2} \times (\text{diagonal})^2 = \frac{1}{2} \times \left(\frac{100}{\pi}\right)^2$

$\Leftrightarrow \quad a^2 = \frac{1}{2} \times \left(\frac{100}{\pi}\right)^2 \Leftrightarrow \quad a = \frac{1}{\sqrt{2}} \times \frac{100}{\pi} = \frac{50\sqrt{2}}{\pi}$ cm.

170. Side of square paper $= \sqrt{784}$ cm $= 28$ cm.

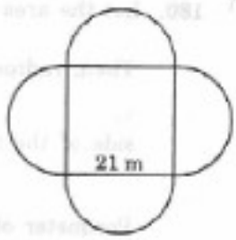


Radius of each circular plate $= \left(\frac{1}{4} \times 28\right)$ cm $= 7$ cm.

Circumference of each circular plate $= \left(2 \times \frac{22}{7} \times 7\right)$ cm $= 44$ cm.

171. Length of the fence $= 4\pi R$, where $R = \frac{21}{2}$ m

$= \left(4 \times \frac{22}{7} \times \frac{21}{2}\right)$ m $= 132$ m.



Cost of fencing $=$ Rs. $\left(132 \times \frac{25}{2}\right)$ $=$ Rs. 1650.

172. Radius of incircle of an equilateral triangle $= \frac{a}{2\sqrt{3}}$.

Radius of circumcircle of an equilateral triangle $= \frac{a}{\sqrt{3}}$.

$\therefore$ Required ratio $= \frac{\pi a^2}{12} : \frac{\pi a^2}{3} = \frac{1}{12} : \frac{1}{3} = 1 : 4$.

173. Radius of circumcircle $= \frac{a}{\sqrt{3}} = \frac{12}{\sqrt{3}}$ cm $= 4\sqrt{3}$ cm.

174. Radius of incircle $= \frac{a}{2\sqrt{3}} = \frac{42}{2\sqrt{3}}$ cm $= 7\sqrt{3}$ cm.

Area of incircle $= \left(\frac{22}{7} \times 49 \times 3\right)$ cm^2 $= 462$ cm^2.

175. Radius of incircle $= \frac{a}{2\sqrt{3}}$. Area of incircle $= \left(\frac{\pi \times a^2}{12}\right)$ cm^2.

$\therefore \quad \frac{\pi a^2}{12} = 154 \quad \Leftrightarrow \quad a^2 = \frac{154 \times 12 \times 7}{22} \quad \Leftrightarrow \quad a = 14\sqrt{3}$.

$\therefore$ Perimeter of the triangle $= (3 \times 14\sqrt{3})$ cm $= (42 \times 1.732)$ cm $= 72.7$ cm (approx.).

176. We have : $a = 6$, $b = 11$, $c = 15$. $s = \frac{1}{2}(6 + 11 + 15) = 16$.

Area of the triangle, $\Delta = \sqrt{16 \times 10 \times 5 \times 1} = 20\sqrt{2}$ cm^2.

Radius of incircle $= \frac{\Delta}{s} = \frac{20\sqrt{2}}{16} = \frac{5\sqrt{2}}{4}$ cm.

177. Let the radius of incircle be r cm. Then, $2\pi r = 88 \Leftrightarrow r = \left(88 \times \frac{7}{22} \times \frac{1}{2}\right) = 14$.

Semi-perimeter, $s = \left(\frac{30}{2}\right)$ cm $= 15$ cm.

$\therefore$ Area of the triangle $= r \times s = (14 \times 15)$ cm^2 $= 210$ cm^2.

178. Radius $= \dfrac{\text{Area}}{\text{Semi-perimeter}} = \left(\text{Area} \times \dfrac{2}{\text{Area}}\right) = 2.$

179. Let the perimeter of each be a.

Then, side of the equilateral triangle $= \dfrac{a}{3}$; side of the square $= \dfrac{a}{4}$;

radius of the circle $= \dfrac{a}{2\pi}$.

$\therefore \quad T = \dfrac{\sqrt{3}}{4} \times \left(\dfrac{a}{3}\right)^2 = \dfrac{\sqrt{3}\,a^2}{36}$; $S = \left(\dfrac{a}{4}\right)^2 = \dfrac{a^2}{16}$; $C = \pi \times \left(\dfrac{a}{2\pi}\right)^2 = \dfrac{a^2}{4\pi} = \dfrac{7a^2}{88}$.

So, $C > S > T$.

180. Let the area of each be a.

Then, radius of the circle $= \dfrac{\sqrt{a}}{\pi}$; side of the square $= \sqrt{a}$;

side of the triangle $= \sqrt{\dfrac{a \times 4}{\sqrt{3}}}$.

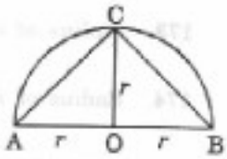
Perimeter of the circle $= 2\pi\sqrt{\dfrac{a}{\pi}} = 2\sqrt{\pi a} = 2\sqrt{3.14 \times a} = 2 \times 1.77\sqrt{a} = 3.54\sqrt{a}$.

Perimeter of the square $= 4\sqrt{a}$;

Perimeter of the triangle $= 3 \times \sqrt{\dfrac{4a}{1.732}} = 3 \times \sqrt{2.31a} = 3 \times 1.52\sqrt{a} = 4.56\sqrt{a}$.

Clearly, perimeter of the triangle is the greatest.

181. Required area $= \dfrac{1}{2} \times \text{base} \times \text{height} = \left(\dfrac{1}{2} \times 2r \times r\right) = r^2$.



182. Required area $= \dfrac{\pi}{2} \times \left(\dfrac{AC}{2}\right)^2 = \dfrac{\pi}{2} \times \dfrac{AC^2}{4} = \dfrac{\pi}{2} \times \dfrac{AB^2 + BC^2}{4}$

$= \dfrac{\pi}{2} \times \left(\dfrac{AB^2}{4} + \dfrac{BC^2}{4}\right) = \dfrac{\pi}{2} \times \left(\dfrac{AB}{2}\right)^2 + \dfrac{\pi}{2} \times \left(\dfrac{BC}{2}\right)^2 = 81 + 36 = 117 \text{ cm}^2$.

183. Let original radius be R cm. Then, original circumference $= (2\pi R)$ cm.

New radius $= (175\% \text{ of } R) \text{ cm} = \left(\dfrac{175}{100} \times R\right) \text{ cm} = \dfrac{7R}{4}$ cm.

New circumference $= \left(2\pi \times \dfrac{7R}{4}\right) \text{ cm} = \dfrac{7\pi R}{2}$ cm.

Increase in circumference $= \left(\dfrac{7\pi R}{2} - 2\pi R\right) \text{ cm} = \dfrac{3\pi R}{2}$ cm.

Increase% $= \left(\dfrac{3\pi R}{2} \times \dfrac{1}{2\pi R} \times 100\right)\% = 75\%$.

184. Let original diameter be d metres. Then, its circumference $= (\pi d)$ metres.

Time taken to cover $(8\pi d)$ m $= 40$ min.

New diameter $= (10d)$ m. Then, its circumference $= (\pi \times 10d)$ m.

$\therefore$ Time taken to go round it once $= \left(\dfrac{40}{8\pi d} \times 10\pi d\right)$ m $= 50$ min.

185. Let the original radius be R cm. New radius $= \left(\frac{106}{100}R\right)$ cm $= \left(\frac{53R}{50}\right)$ cm.

Original area $= \pi R^2$.

Increase in area $= \pi\left(\frac{53R}{50}\right)^2 - \pi R^2 = \pi R^2\left[\left(\frac{53}{50}\right)^2 - 1\right] = \frac{\pi R^2[(53)^2 - (50)^2]}{2500}$

$= \frac{\pi R^2 (103\times 3)}{2500}$ m^2.

Increase% $= \left(\frac{\pi R^2 \times 309}{2500}\times\frac{1}{\pi R^2}\times 100\right)\% = 12.36\%$.

186. Let the original radius be R cm.

New radius = (90% of R) cm $= \left(\frac{90}{100}\times R\right)$ cm $= \frac{9R}{10}$ cm.

Original area $= \pi R^2$.

Diminished area $= \left[\pi R^2 - \pi\left(\frac{9R}{10}\right)^2\right]$ cm^2 $= \left[\left(1-\frac{81}{100}\right)\pi R^2\right]$ cm^2 $= \left(\frac{19}{100}\pi R^2\right)$ cm^2.

Decrease% $= \left(\frac{19\pi R^2}{100}\times\frac{1}{\pi R^2}\times 100\right)\% = 19\%$.

187. Let the original radius be R cm. New radius = 2R.

Original area $= \pi R^2$, New area $= \pi(2R)^2 = 4\pi R^2$.

Increase in area $= (4\pi R^2 - \pi R^2) = 3\pi R^2$.

Increase% $= \left(\frac{3\pi R^2}{\pi R^2}\times 100\right)\% = 300\%$.

188. $2\pi R_1 = 4\pi$ and $2\pi R_2 = 8\pi \Rightarrow R_1 = 2$ and $R_2 = 4$

$\Rightarrow$ Original area $= (4\pi \times 2^2) = 16\pi$, Increased area $= (4\pi \times 4^2) = 64\pi$.

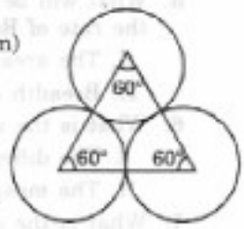
Thus, the area quadruples.

189. Required area = (Area of an equilateral Δ of side 7 cm)

$-$ (3 $\times$ area of sector with $\theta = 60°$ & $r = 3.5$ cm)

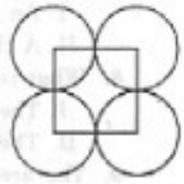
$= \left[\left(\frac{\sqrt{3}}{4}\times 7\times 7\right) - \left(3\times\frac{22}{7}\times 3.5\times 3.5\times\frac{60}{360}\right)\right]$ cm^2

$= \left(\frac{49\sqrt{3}}{4} - 11\times 0.5\times 3.5\right)$ cm^2 $= (21.217 - 19.25)$ cm^2 $= 1.967$ cm^2.



190. Required area $= \left(14\times 14 - 4\times\frac{1}{4}\times\frac{22}{7}\times 7\times 7\right)$ cm^2

$= (196 - 154)$ cm^2 $= 42$ cm^2.



191. Required area $= \left(63\times 63 - 4\times\frac{1}{4}\times\frac{22}{7}\times\frac{63}{2}\times\frac{63}{2}\right)$ m^2 $= 850.5$ m^2.

EXERCISE 24B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 11*) : *Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and*

Give answer (*a*) *if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;*

Give answer (*b*) *if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;*

Give answer (*c*) *if the data either in Statement I or in Statement II alone are sufficient to answer the question;*

Give answer (*d*) *if the data even in both Statements I and II together are not sufficient to answer the question;*

Give answer (*e*) *if the data in both Statements I and II together are necessary to answer the question.*

1. The area of a playground is 1600 m^2. What is its perimeter ? **(Bank P.O. 2003)**
   I. It is a perfect square playground.
   II. It costs Rs. 3200 to put a fence around the playground at the rate of Rs. 20 per metre.
2. What is the area of the rectangle ?
   I. The ratio of the length and the breadth is 3 : 2.
   II. The area of the rectangle is 3.6 times its perimeter.
3. Area of a square is equal to the area of a circle. What is the circumference of the circle ?
   I. The diagonal of the square is x inches.
   II. The side of the square is y inches. **(S.B.I.P.O. 2003)**
4. The area of a rectangle is equal to the area of a right-angled triangle. What is the length of the rectangle ? **(Bank P.O. 2003)**
   I. The base of the triangle is 40 cm.
   II. The height of the triangle is 50 cm.
5. What will be the cost of gardening a strip of land inside around a circular field, at the rate of Rs. 85 per sq. metre ?
   I. The area of the field is 1386 sq. metres.
   II. Breadth and length of the field are in the ratio of 3 : 5 respectively.
6. What is the area of the rectangle ? **(Bank P.O. 2003)**
   I. The difference between the sides is 5 cm.
   II. The measure of its diagonal is 10 cm.
7. What is the area of the circle ?
   I. An arc of length 4 cm subtends an angle of 60° at the centre.
   II. A chord of length 5 cm subtends an angle of 90° at the centre.
8. What is the area of the circle ? **(NABARD, 2002)**
   I. The circumference of the circle is 308 m.
   II. The radius of the circle is 28 m.
9. The area of a rectangle is equal to the area of a circle. What is the length of the rectangle ?
   I. The radius of the circle is equal to the breadth of the rectangle.
   II. The perimeter of the rectangle is 14 cm more than that of the circle.

10. What is the height of the triangle ? **(Bank P.O. 2002)**
I. The area of the triangle is 20 times its base.
II. The perimeter of the triangle is equal to the perimeter of a square of side 10 cm.

11. What will be the cost of painting the inner walls of a room if the rate of painting is Rs. 20 per square foot ? **(Bank P.O. 2000)**
I. Circumference of the floor is 44 feet.
II. The height of the wall of the room is 12 feet.

Directions (*Questions 12 to 15*) : *Each of the questions below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.*

12. What is the area of rectangular field ? **(Bank P.O. 2004)**
I. The perimeter of the field is 110 metres.
II. The length is 5 metres more than the width.
III. The ratio between length and width is 6 : 5 respectively.
(*a*) I and II only (*b*) Any two of the three (*c*) All I, II and III
(*d*) I, and either II or III only (*e*) None of these

13. What is the area of the hall ? **(Bank P.O. 2003)**
I. Material cost of flooring per square metre is Rs. 2.50.
II. Labour cost of flooring the hall is Rs. 3500.
III. Total cost of flooring the hall is Rs. 14,500.
(*a*) I and II only (*b*) II and III only (*c*) All I, II and III
(*d*) Any two of the three (*e*) None of these

14. What is the length of the diagonal of the given rectangle ?
I. The perimeter of the rectangle is 34 cm.
II. The difference between the length and breadth is 7 cm.
III. The length is 140% more than the breadth.
(*a*) Any two of the three (*b*) All I, II and III (*c*) I, and either II or III
(*d*) I and II only (*e*) II and III only

15. What is the cost of flooring the rectangular hall ? **(R.B.I. 2002)**
I. Length and breadth of the hall are in the respective ratio of 3 : 2.
II. Length of the hall is 48 m and cost of flooring is Rs. 85 per sq. m.
III. Perimeter of the hall is 160 m and cost of flooring is Rs. 85 per sq. m.
(*a*) I and II only (*b*) II and III only (*c*) III only
(*d*) I, and either II or III only (*e*) Any two of the three

16. What is the area of a right-angled triangle ? **(S.B.I.P.O. 2000)**
I. The perimeter of the triangle is 30 cm.
II. The ratio between the base and the height of the triangle is 5 : 12.
III. The area of the triangle is equal to the area of a rectangle of length 10 cm.
(*a*) I and II only (*b*) II and III only (*c*) I and III only
(*d*) III, and either I or II only (*e*) None of these

17. A path runs around a rectangular lawn. What is the width of the path ?
I. The length and breadth of the lawn are in the ratio of 3 : 1 respectively.
II. The width of the path is ten times the length of the lawn.
III. The cost of gravelling the path @ Rs. 50 per m^2 is Rs. 8832.
(*a*) All I, II and II (*b*) III, and either I or II (*c*) I and III only
(*d*) II and III only (*e*) None of these

18. What is the area of the isosceles triangle ?

I. Perimeter of the isosceles triangle is 18 metres.

II. Base of the triangle is 8 metres.

III. Height of the triangle is 3 metres.

(*a*) I and II only (*b*) II and III only (*c*) I and III only

(*d*) II, and either I or III only (*e*) Any two of the three

Directions (*Questions 19 to 22*) : *Each of the questions given below is followed by three statements. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is/are redundant and can be dispensed with while answering the given question.*

19. What is the cost of painting the two adjacent walls of a hall at Rs. 5 per m^2 which has no windows or doors ? **(S.B.I.P.O. 2001)**

I. The area of the hall is 24 sq. m.

II. The breadth, length and height of the hall are in the ratio of 4 : 6 : 5 respectively.

III. Area of one wall is 30 sq. m.

(*a*) I only (*b*) II only (*c*) III only

(*d*) Either I or III (*e*) All I, II and III are required

20. What is the area of the given rectangle ? **(Bank P.O. 1999)**

I. Perimeter of the rectangle is 60 cm.

II. Breadth of the rectangle is 12 cm.

III. Sum of two adjacent sides is 30 cm.

(*a*) I only (*b*) II only (*c*) III only

(*d*) I and II only (*e*) I or III only

21. What is the area of the given right-angled triangle ?

I. Length of the hypotenuse is 5 cm.

II. Perimeter of the triangle is four times its base.

III. One of the angles of the triangle is 60°.

(*a*) II only (*b*) III only (*c*) II or III only (*d*) II and III both

(*e*) Information given in all the three statements together is not sufficient to answer the question.

22. What will be the cost of painting the four walls of a room with length, width and height 5 m, 3 m and 8 m respectively ? The room has one door and one window.

I. Cost of painting per sq. m is Rs. 25.

II. Area of window is 2.25 sq. m which is half of the area of the door.

III. Area of the room is 15 sq. m. **(S.B.I.P.O. 1999)**

(*a*) I only (*b*) II only (*c*) III only

(*d*) II or III only (*e*) All I, II and III are required

ANSWERS

1. (*c*)	**2.** (*e*)	**3.** (*c*)	**4.** (*d*)	**5.** (*e*)	**6.** (*e*)	**7.** (*c*)	**8.** (*c*)
9. (*e*)	**10.** (*a*)	**11.** (*c*)	**12.** (*b*)	**13.** (*c*)	**14.** (*a*)	**15.** (*e*)	**16.** (*a*)
17. (*a*)	**18.** (*d*)	**19.** (*c*)	**20.** (*e*)	**21.** (*c*)	**22.** (*c*)		

SOLUTIONS

1. Area = 1600 m^2.

I. Side = $\sqrt{1600}$ m = 40 m. So, perimeter = (40 × 4) m = 160 m.

∴ I alone gives the answer.

II. Perimeter $= \dfrac{\text{Total cost}}{\text{Cost per metre}} = \dfrac{3200}{20}$ m $= 160$ m.

$\therefore$ II alone gives the answer.

$\therefore$ Correct answer is (*c*).

2. I. Let $l = 3x$ metres and $b = 2x$ metres. Then, area $= (6x^2)$ m^2.

II. Perimeter $= 2\,(3x + 2x)$ m $= (10x)$ m.

$\therefore \quad 6x^2 = 3.6 \times 10x \Leftrightarrow x = \dfrac{(3.6 \times 10)}{6} = 6.$

$\therefore$ $l = 18$ m and $b = 12$ m and so area can be obtained.

Thus, I and II together give the answer.

$\therefore$ Correct answer is (*e*).

3. I. Area of the circle = Area of the square $= \dfrac{1}{2}x^2$ sq. inches.

$$\Rightarrow \quad \pi r^2 = \frac{1}{2}x^2 \quad \Rightarrow \quad r = \sqrt{\frac{x^2}{2\pi}} = \frac{x}{\sqrt{2\pi}}.$$

$\therefore$ Circumference of the circle $= 2\pi r$, which can be obtained.

$\therefore$ I alone gives the answer.

II. Area of the circle = Area of the square $= y^2$ sq. inches.

$$\Rightarrow \quad \pi r^2 = y^2 \quad \Rightarrow \quad r = \frac{y}{\sqrt{\pi}}.$$

$\therefore$ Circumference of the circle $= 2\pi r$, which can be obtained.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

4. Given : Area of rectangle = Area of a right-angled triangle

$$\Rightarrow \quad l \times b = \frac{1}{2} \times \text{B} \times \text{H}$$

I gives, B = 40 cm.

II gives, H = 50 cm.

Thus, to find l, we need b also, which is not given.

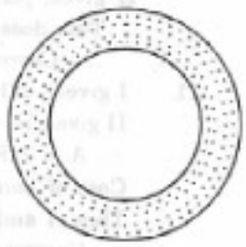
$\therefore$ Given data is not sufficient to give the answer.

$\therefore$ Correct answer is (*d*).

5. I. $\pi R_1^{\,2} = 1386 \Leftrightarrow R_1^{\,2} = \left(1386 \times \dfrac{7}{22}\right) \Leftrightarrow R_1 = 21$ m.

II. $R_2 = (21 - 1.4)$ m $= 19.6$ m.

$\therefore \quad$ Area $= \pi\,(R_1^{\,2} - R_2^{\,2}) = \dfrac{22}{7} \times [(21)^2 - (19.6)^2]$ m^2.



Thus, the required cost may be obtained.

$\therefore$ I and II together will give the answer.

$\therefore$ Correct answer is (*e*).

6. I. Let the sides be x cm and $(x + 5)$ cm.

II. $d = \sqrt{(x+5)^2 + x^2} \Leftrightarrow (x + 5)^2 + x^2 = (10)^2 \Leftrightarrow 2x^2 + 10x - 75 = 0$

$$\Leftrightarrow \quad x = \frac{-10 \pm \sqrt{100 + 600}}{4} = \frac{-10 + \sqrt{700}}{4} = \frac{-10 + 10\sqrt{7}}{4} = \frac{-10 + 10 \times 2.6}{4}$$

Thus, sides and therefore area may be known.

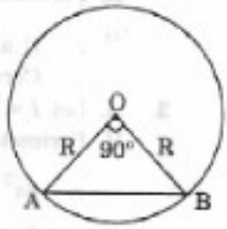
Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (*e*).

7. I. Length of arc $= \frac{2\pi R\theta}{360} \Leftrightarrow 4 = \left(\frac{2 \times \frac{22}{7} \times R \times 60}{360}\right)$.

This gives R and therefore, area of the circle $= \pi R^2$.

Thus, I only gives the answer.



II. $R^2 + R^2 = 5^2 \Leftrightarrow 2R^2 = 25 \Leftrightarrow R^2 = \frac{25}{2}$.

$\therefore$ Area of the circle $= \pi R^2 = \left(\frac{22}{7} \times \frac{25}{2}\right)$ sq. cm.

Thus, II only gives the answer.

$\therefore$ Correct answer is (*c*).

8. I. $2\pi R = 308 \Leftrightarrow 2 \times \frac{22}{7} \times R = 308 \Leftrightarrow R = \left(308 \times \frac{7}{44}\right) = 49$.

Thus, $A = \pi R^2$ can be obtained.

$\therefore$ I alone gives the answer.

II. R = 28 m gives $A = (\pi \times 28 \times 28)$ cm^2.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

9. Given : $l \times b = \pi R^2$. ...(*i*)

I gives, $R = b$. ...(*ii*)

From (*i*) and (*ii*), we get $l = \frac{\pi R^2}{b} = \frac{\pi R^2}{R} = \pi R$. ...(*iii*)

II gives, $2(l + b) = 2\pi R + 14 \Rightarrow l + b = \pi R + 7 \Rightarrow l + R = \pi R + 7$

$\Rightarrow l = \pi R - R + 7$

$\Rightarrow l = l - \frac{l}{\pi} + 7$ [Using (*iii*)]

$\Rightarrow l = 7\pi$.

Thus, I and II together give *l*.

$\therefore$ Correct answer is (*e*).

10. I. $A = 20 \times B \Rightarrow \frac{1}{2} \times B \times H = 20 \times B \Rightarrow H = 40$.

$\therefore$ I alone gives the answer.

II gives, perimeter of the triangle = 40 cm.

This does not give the height of the triangle.

$\therefore$ Correct answer is (*a*).

11. I gives, $2\pi R = 44$.

II gives, H = 12.

$\therefore$ $A = 2\pi RH = (44 \times 12)$.

Cost of painting = Rs. $(44 \times 12 \times 20)$.

Thus, I and II together give the answer.

$\therefore$ Correct answer is (*e*).

12. I. $2(l + b) = 110 \Rightarrow l + b = 55$.

II. $l = (b + 5) \Rightarrow l - b = 5$.

III. $\frac{l}{b} = \frac{6}{5} \Rightarrow 5l - 6b = 0$.

These are three equations in *l* and *b*. We may solve them pairwise.

$\therefore$ Any two of the three will give the answer.

$\therefore$ Correct answer is (*b*).

13. I. Material cost = Rs. 2.50 per m^2.

II. Labour cost = Rs. 3500.

III. Total cost = Rs. 14,500.

Let the area be A sq. metres.

$\therefore$ Material cost = Rs. (14500 − 3500) = Rs. 11,000.

$\therefore \quad \frac{5A}{2} = 11000 \Leftrightarrow A = \left(\frac{11000 \times 2}{5}\right) = 4400 \text{ m}^2.$

Thus, all I, II and III are needed to get the answer.

$\therefore$ Correct answer is (*c*).

14. I. $2(l + b) = 34 \Rightarrow l + b = 17$ ...(*i*)

II. $(l - b) = 7$ ...(*ii*)

III. $l = (100 + 140)\%$ of $b \Rightarrow l - \frac{240}{100} b = 0$

$\Rightarrow 100l - 240b = 0 \Rightarrow 5l - 12b = 0$ ...(*iii*)

These are 3 equations in l and b. We may solve them pairwise.

$\therefore$ Any two of the three will give the answer.

$\therefore$ Correct answer is (*a*).

15. I. Let $l = 3x$ metres and $b = 2x$ metres.

II. $l = 48$ m, Rate of flooring = Rs. 85 per m^2.

III. $2(l + b) = 160 \Leftrightarrow l + b = 80$, Rate of flooring = Rs. 85 per m^2.

From I and II, we get $3x = 48 \Leftrightarrow x = 16$

$\therefore$ $l = 48$ m, $b = 32$ m $\Rightarrow$ Area of floor = (48×32) m^2.

$\therefore$ Cost of flooring = Rs. $(48 \times 32 \times 85)$.

Thus, I and II give the answer.

From II and III, we get $l = 48$ m, $b = (80 - 48)$ m = 32 m.

$\therefore$ Area of floor and cost of flooring is obtained.

Thus, II and III give the answer.

From III and I, we get $3x + 2x = 80 \Leftrightarrow 5x = 80 \Leftrightarrow x = 16$.

$\therefore$ $l = (3 \times 16)$ m = 48 m and $b = (2 \times 16)$ m = 32 m.

$\therefore$ Area of floor and the cost of flooring is obtained.

Thus, III and I give the answer.

Hence, any two of the three will give the answer.

$\therefore$ Correct answer is (*e*).

16. **From II,** base : height = 5 : 12.

Let base = $5x$ and height = $12x$. Then, hypotenuse = $\sqrt{(5x)^2 + (12x)^2} = 13x$.

From I, perimeter of the triangle = 30 cm.

$\therefore \quad 5x + 12x + 13x = 30 \Leftrightarrow x = 1.$

So, base = $5x$ = 5 cm; height = $12x$ = 12 cm.

$\therefore \quad \text{Area} = \left(\frac{1}{2} \times 5 \times 12\right) \text{cm}^2 = 30 \text{ cm}^2.$

Thus, I and II together give the answer.

Clearly III is redundant, since the breadth of the rectangle is not given.

$\therefore$ Correct answer is (*a*).

17. III gives area of the path = $\frac{8832}{50}$ $m^2 = \frac{4416}{25}$ m^2.

II gives width of path = 10 × (Length of the lawn).

I gives length = $3x$ metres and breadth = x metres

Clearly, all the three will be required to find the width of the path.

$\therefore$ Correct answer is (*a*).

18. II gives base = 8 m.

I gives perimeter = 18 m.

III gives height = 3 m.

From II and I, we get :

$b = 8$ and $a + b + a = 18$ and so as = 5.

Thus, the three sides are 5 m, 5 m and 8 m.

From this, the area can be found out.

From II and III, we get : area = $\left(\frac{1}{2} \times 8 \times 3\right)$ m^2.

$\therefore$ Correct answer is (*d*).

19. **From II**, let $l = 4x$, $b = 6x$ and $h = 5x$

Then, area of the hall = $(24x^2)$ m^2.

From I. Area of the hall = 24 m^2.

From II and I, we get $24x^2 = 24 \Leftrightarrow x = 1$.

$\therefore$ $l = 4$ m, $b = 6$ m and $h = 5$ m.

Thus, area of two adjacent walls = $[(l \times h) + (b \times h)]$ m^2 can be found out and so the cost of painting two adjacent walls may be found out.

Thus, III is redundant.

$\therefore$ Correct answer is (*c*).

20. From I and II, we can find the length and breadth of the rectangle and therefore the area can be obtained.

So, III is redundant.

Also, from II and III, we can find the length and breadth and therefore the area can be obtained.

So, I is redundant.

$\therefore$ Correct answer is (*e*).

21. $\frac{BC}{AC} = \cos 60^\circ = \frac{1}{2} \Rightarrow BC = \frac{5}{2}$ cm $[\because AC = 5$ cm$]$

From I and III, we get :

$a = \frac{5}{2}$ cm, $b = 5$ cm and $\theta = 60^\circ$.

$\therefore$ $A = \frac{1}{2} ab \sin C$ gives the area.

Thus, I and III give the result.

$\therefore$ II is redundant.

Again, II gives $a + b + c = 4a \Rightarrow b + c = 3a \Rightarrow c = 3a - 5$ $[\because b = 5$ from I$]$

$a^2 + (3a - 5)^2 = 25$. This gives a and therefore c.

Now, area of $\Delta ABC = \frac{1}{2} \times a \times c$, which can be obtained.

Thus I and II give the area.

$\therefore$ III is redundant.

$\therefore$ Correct answer is (*c*).

22. From given length, breadth and height of the room, its area can be obtained.

So, III is redundant.

$\therefore$ Correct answer is (*c*).

25. VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = l, breadth = b and height = h units. Then,

1. ***Volume*** = $(l \times b \times h)$ cubic units.
2. ***Surface area*** = $2\,(lb + bh + lh)$ sq. units.
3. ***Diagonal*** = $\sqrt{l^2 + b^2 + h^2}$ units.

II. CUBE

Let each edge of a cube be of length a. Then,

1. ***Volume*** = a^3 cubic units.
2. ***Surface area*** = $6a^2$ sq. units.
3. ***Diagonal*** = $\sqrt{3}\,a$ units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. ***Volume*** = $(\pi r^2 h)$ cubic units.
2. ***Curved surface area*** = $(2\pi rh)$ sq. units.
3. ***Total surface area*** = $(2\pi rh + 2\pi r^2)$ sq. units
   $= 2\pi r\,(h + r)$ sq. units.

IV. CONE

Let radius of base = r and Height = h. Then,

1. ***Slant height***, $l = \sqrt{h^2 + r^2}$ units.
2. ***Volume*** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. ***Curved surface area*** = (πrl) sq. units.
4. ***Total surface area*** = $(\pi rl + \pi r^2)$ sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. ***Volume*** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. ***Surface area*** = $(4\pi r^2)$ sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. ***Volume*** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. ***Curved surface area*** = $(2\pi r^2)$ sq. units.
3. ***Total surface area*** = $(3\pi r^2)$ sq. units.

Remember : 1 litre = 1000 cm^3.

SOLVED EXAMPLES

Ex. 1. ***Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.***

Sol. Volume $= (16 \times 14 \times 7)\ m^3 = 1568\ m^3$.

Surface area $= [2\ (16 \times 14 + 14 \times 7 + 16 \times 7)]\ cm^2 = (2 \times 434)\ cm^2 = 868\ cm^2$.

Ex. 2. ***Find the length of the longest pole that can be placed in a room 12 m long, 8 m broad and 9 m high.***

Sol. Length of longest pole = Length of the diagonal of the room

$$= \sqrt{(12)^2 + 8^2 + 9^2} = \sqrt{289} = 17\ m.$$

Ex. 3. ***The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.***

Sol. Let the breadth of the wall be x metres.

Then, Height $= 5x$ metres and Length $= 40x$ metres.

$$\therefore \quad x \times 5x \times 40x = 12.8 \iff x^3 = \frac{12.8}{200} = \frac{128}{2000} = \frac{64}{1000}.$$

$$\text{So, } x = \frac{4}{10}\ m = \left(\frac{4}{10} \times 100\right)\ cm = 40\ cm.$$

Ex. 4. ***Find the number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar ?***

Sol. Volume of the wall $= (2400 \times 800 \times 60)$ cu. cm.

Volume of bricks = 90% of the volume of the wall

$$= \left(\frac{90}{100} \times 2400 \times 800 \times 60\right)\ \text{cu. cm.}$$

Volume of 1 brick $= (24 \times 12 \times 8)$ cu. cm.

$$\therefore \quad \text{Number of bricks} = \left(\frac{90}{100} \times \frac{2400 \times 800 \times 60}{24 \times 12 \times 8}\right) = 45000.$$

Ex. 5. ***Water flows into a tank 200 m × 150 m through a rectangular pipe 1.5 m × 1.25 m @ 20 kmph. In what time (in minutes) will the water rise by 2 metres ?***

Sol. Volume required in the tank $= (200 \times 150 \times 2)\ m^3 = 60000\ m^3$.

$$\text{Length of water column flown in 1 min.} = \left(\frac{20 \times 1000}{60}\right)\ m = \frac{1000}{3}\ m.$$

$$\text{Volume flown per minute} = \left(1.5 \times 1.25 \times \frac{1000}{3}\right)\ m^3 = 625\ m^3.$$

$$\therefore \quad \text{Required time} = \left(\frac{60000}{625}\right)\ \text{min.} = 96\ \text{min.}$$

Ex. 6. ***The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.***

Sol. Volume of the metal used in the box = External Volume − Internal Volume

$$= [(50 \times 40 \times 23) - (44 \times 34 \times 20)]\ cm^3$$

$$= 16080\ cm^3.$$

$$\therefore \quad \text{Weight of the metal} = \left(\frac{16080 \times 0.5}{1000}\right)\ kg = 8.04\ kg.$$

Ex. 7. ***The diagonal of a cube is $6\sqrt{3}$ cm. Find its volume and surface area.***

Sol. Let the edge of the cube be a.

$\therefore \quad \sqrt{3}\,a = 6\sqrt{3} \Rightarrow a = 6.$

So, Volume $= a^3 = (6 \times 6 \times 6)\ \text{cm}^3 = 216\ \text{cm}^3.$

Surface area $= 6a^2 = (6 \times 6 \times 6)\ \text{cm}^2 = 216\ \text{cm}^2.$

Ex. 8. ***The surface area of a cube is 1734 sq. cm. Find its volume.***

Sol. Let the edge of the cube be a. Then,

$6a^2 = 1734 \Rightarrow a^2 = 289 \Rightarrow a = 17$ cm.

$\therefore$ Volume $= a^3 = (17)^3\ \text{cm}^3 = 4913\ \text{cm}^3.$

Ex. 9. ***A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.***

Sol. Volume of the block $= (6 \times 12 \times 15)\ \text{cm}^3 = 1080\ \text{cm}^3.$

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm = 3 cm.

Volume of this cube $= (3 \times 3 \times 3)\ \text{cm}^3 = 27\ \text{cm}^3.$

Number of cubes $= \left(\frac{1080}{27}\right) = 40.$

Ex. 10. ***A cube of edge 15 cm is immersed completely in a rectangular vessel containing water. If the dimensions of the base of vessel are 20 cm × 15 cm, find the rise in water level.*** **(R.R.B. 2003)**

Sol. Increase in volume = Volume of the cube $= (15 \times 15 \times 15)\ \text{cm}^3.$

$\therefore$ Rise in water level $= \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{15 \times 15 \times 15}{20 \times 15}\right)\ \text{cm} = 11.25\ \text{cm}.$

Ex. 11. ***Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.***

Sol. Volume of new cube $= (1^3 + 6^3 + 8^3)\ \text{cm}^3 = 729\ \text{cm}^3.$

Edge of new cube $= \sqrt[3]{729}\ \text{cm} = 9\ \text{cm}.$

$\therefore$ Surface area of the new cube $= (6 \times 9 \times 9)\ \text{cm}^2 = 486\ \text{cm}^2.$

Ex. 12. ***If each edge of a cube is increased by 50%, find the percentage increase in its surface area.***

Sol. Let original length of each edge = a.

Then, original surface area $= 6a^2$.

New edge = (150% of a) $= \left(\frac{150}{100}\,a\right) = \frac{3a}{2}.$

New surface area $= 6 \times \left(\frac{3a}{2}\right)^2 = \frac{27}{2}a^2.$

Increase percent in surface area $= \left(\frac{15}{2}a^2 \times \frac{1}{6a^2} \times 100\right)\% = 125\%.$

Ex. 13. ***Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.***

Sol. Let their edges be a and b. Then,

$\frac{a^3}{b^3} = \frac{1}{27}$ or $\left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3$ or $\frac{a}{b} = \frac{1}{3}.$

$\therefore$ Ratio of their surface areas $= \frac{6a^2}{6b^2} = \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 = \frac{1}{9}$, *i.e.*, 1 : 9.

Ex. 14. ***Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.***

Sol. Volume $= \pi r^2 h = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 40\right)$ cm^3 $= 1540$ cm^3.

Curved surface area $= 2\pi rh = \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 40\right)$ cm^2 $= 880$ cm^2.

Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

$= \left[2 \times \frac{22}{7} \times \frac{7}{2} \times (40 + 3.5)\right]$ cm^2 $= 957$ cm^2.

Ex. 15. ***If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, then find the depth of the tank.***

Sol. Let the depth of the tank be h metres. Then,

$$\pi \times (7)^2 \times h = 1848 \iff h = \left(1848 \times \frac{7}{22} \times \frac{1}{7 \times 7}\right) = 12 \text{ m}.$$

Ex. 16. ***2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm in diameter. Find the length of the wire in metres.***

Sol. Let the length of the wire be h metres. Then,

$$\pi \times \left(\frac{0.50}{2 \times 100}\right)^2 \times h = \frac{2.2}{1000} \iff h = \left(\frac{2.2}{1000} \times \frac{100 \times 100}{0.25 \times 0.25} \times \frac{7}{22}\right) = 112 \text{ m}.$$

Ex. 17. ***How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron ?*** **(C.B.I. 1998)**

Sol. Volume of 1 rod $= \left(\frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 7\right)$ cu. m $= \frac{11}{5000}$ cu. m.

Volume of iron $= 0.88$ cu. m.

Number of rods $= \left(0.88 \times \frac{5000}{11}\right) = 400$.

Ex. 18. ***The radii of two cylinders are in the ratio 3 : 5 and their heights are in the ratio of 2 : 3. Find the ratio of their curved surface areas.***

Sol. Let the radii of the cylinders be $3x$, $5x$ and their heights be $2y$, $3y$ respectively. Then,

Ratio of their curved surface areas $= \frac{2\pi \times 3x \times 2y}{2\pi \times 5x \times 3y} = \frac{2}{5} = 2 : 5$.

Ex. 19. ***If 1 cubic cm of cast iron weighs 21 gms, then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.***

Sol. Inner radius $= \left(\frac{3}{2}\right)$ cm $= 1.5$ cm, Outer radius $= (1.5 + 1) = 2.5$ cm.

$\therefore$ Volume of iron $= [\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100]$ cm^3

$= \frac{22}{7} \times 100 \times [(2.5)^2 - (1.5)^2]$ cm^3 $= \left(\frac{8800}{7}\right)$ cm^3.

$\therefore$ Weight of the pipe $= \left(\frac{8800}{7} \times \frac{21}{1000}\right)$ kg $= 26.4$ kg.

Ex. 20. ***Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.***

Sol. Here, $r = 21$ cm and $h = 28$ cm.

$\therefore$ Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35$ cm.

$$\text{Volume} = \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\times\frac{22}{7}\times 21\times 21\times 28\right)\text{cm}^3 = 12936\ \text{cm}^3.$$

$$\text{Curved surface area} = \pi r l = \left(\frac{22}{7}\times 21\times 35\right)\text{cm}^2 = 2310\ \text{cm}^2.$$

$$\text{Total surface area} = (\pi r l + \pi r^2) = \left(2310 + \frac{22}{7}\times 21\times 21\right)\text{cm}^2 = 3696\ \text{cm}^2.$$

Ex. 21. ***Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.***

Sol. Here, $r = 7$m and $h = 24$ m.

So, $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$ m.

$$\text{Area of canvas} = \pi r l = \left(\frac{22}{7}\times 7\times 25\right)\text{m}^2 = 550\ \text{m}^2.$$

$$\therefore \quad \text{Length of canvas} = \left(\frac{\text{Area}}{\text{Width}}\right) = \left(\frac{550}{1.25}\right)\text{m} = 440\ \text{m}.$$

Ex. 22. ***The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.***

Sol. Let the radii of their bases be r and R and their heights be h and $2h$ respectively.

Then, $\dfrac{2\pi r}{2\pi \text{R}} = \dfrac{3}{4} \Rightarrow \dfrac{r}{\text{R}} = \dfrac{3}{4} \Rightarrow \text{R} = \dfrac{4}{3}r.$

$$\therefore \quad \text{Ratio of volumes} = \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{4}{3}r\right)^2 (2h)} = \frac{9}{32} = 9:32.$$

Ex. 23. ***The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and their heights are in the ratio 2 : 3. Find the ratio of their volumes.***

Sol. Let the radii of the cylinder and the cone be $3r$ and $4r$ and their heights be $2h$ and $3h$ respectively.

$$\therefore \quad \frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi\times(3r)^2\times 2h}{\frac{1}{3}\pi\times(4r^2)\times 3h} = \frac{9}{8} = 9:8.$$

Ex. 24. ***A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.***

Sol. Volume of the liquid in the cylindrical vessel

= Volume of the conical vessel

$$= \left(\frac{1}{3}\times\frac{22}{7}\times 12\times 12\times 50\right)\text{cm}^3 = \left(\frac{22\times 4\times 12\times 50}{7}\right)\text{cm}^3.$$

Let the height of the liquid in the vessel be h.

Then, $\dfrac{22}{7}\times 10\times 10\times h = \dfrac{22\times 4\times 12\times 50}{7}$ or $h = \left(\dfrac{4\times 12\times 50}{10\times 10}\right) = 24$ cm.

Ex. 25. ***Find the volume and surface area of a sphere of radius 10.5 cm.***

Sol. $$\text{Volume} = \frac{4}{3}\pi r^3 = \left(\frac{4}{3}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}\right)\text{cm}^3 = 4851\ \text{cm}^3.$$

$$\text{Surface area} = 4\pi r^2 = \left(4\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\right)\text{cm}^2 = 1386\ \text{cm}^2.$$

Ex. 26. ***If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area.***

Sol. Let original radius = R. Then, new radius $= \frac{150}{100}R = \frac{3R}{2}$.

Original volume $= \frac{4}{3}\pi R^3$, New volume $= \frac{4}{3}\pi\left(\frac{3R}{2}\right)^3 = \frac{9\pi R^3}{2}$.

Increase % in volume $= \left(\frac{19}{6}\pi R^3 \times \frac{3}{4\pi R^3} \times 100\right)\% = 237.5\%$.

Original surface area $= 4\pi R^2$. New surface area $= 4\pi\left(\frac{3R}{2}\right)^2 = 9\pi R^2$.

Increase % in surface area $= \left(\frac{5\pi R^2}{4\pi R^2} \times 100\right)\% = 125\%$.

Ex. 27. ***Find the number of lead balls, each 1 cm in diameter that can be made from a sphere of diameter 12 cm.***

Sol. Volume of larger sphere $= \left(\frac{4}{3}\pi \times 6 \times 6 \times 6\right)\text{ cm}^3 = 288\pi\text{ cm}^3$.

Volume of 1 small lead ball $= \left(\frac{4}{3}\pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)\text{ cm}^3 = \frac{\pi}{6}\text{ cm}^3$.

$\therefore$ Number of lead balls $= \left(288\pi \times \frac{6}{\pi}\right) = 1728$.

Ex. 28. ***How many spherical bullets can be made out of a lead cylinder 28 cm high and with base radius 6 cm, each bullet being 1.5 cm in diameter?*** **(R.R.B. 2003)**

Sol. Volume of cylinder $= (\pi \times 6 \times 6 \times 28)\text{ cm}^3 = (36 \times 28)\pi\text{ cm}^3$.

Volume of each bullet $= \left(\frac{4}{3}\pi \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right)\text{ cm}^3 = \frac{9\pi}{16}\text{ cm}^3$.

Number of bullets $= \frac{\text{Volume of cylinder}}{\text{Volume of each bullet}} = \left[(36 \times 28)\pi \times \frac{16}{9\pi}\right] = 1792$.

Ex. 29. ***A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire.***

Sol. Volume of sphere $= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right)\text{ cm}^3 = 972\pi\text{ cm}^3$.

Volume of wire $= (\pi \times 0.2 \times 0.2 \times h)\text{ cm}^3$.

$\therefore$ $972\pi = \pi \times \frac{2}{10} \times \frac{2}{10} \times h \Rightarrow h = (972 \times 5 \times 5)\text{ cm} = \left(\frac{972 \times 5 \times 5}{100}\right)\text{ m} = 243\text{ m}$.

Ex. 30. ***Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a sphere. Find the diameter of the sphere.***

Sol. Volume of sphere = Volume of 2 cones

$$= \left[\frac{1}{3}\pi \times (2.1)^2 \times 4.1 + \frac{1}{3}\pi \times (2.1)^2 \times 4.3\right]\text{cm}^3 = \frac{1}{3}\pi \times (2.1)^2 (8.4)\text{ cm}^3.$$

Let the radius of the sphere be R.

$\therefore$ $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi (2.1)^3 \times 4$ or $R = 2.1$ cm.

Hence, diameter of the sphere = 4.2 cm.

Ex. 31. ***A cone and a sphere have equal radii and equal volumes. Find the ratio of the diameter of the sphere to the height of the cone.***

Sol. Let radius of each be R and height of the cone be H.

Then, $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi R^2 H$ or $\frac{R}{H} = \frac{1}{4}$ or $\frac{2R}{H} = \frac{2}{4} = \frac{1}{2}$.

$\therefore$ Required ratio = 1 : 2.

Ex. 32. ***Find the volume, curved surface area and the total surface area of a hemisphere of radius 10.5 cm.***

Sol. Volume $= \frac{2}{3}\pi r^3 = \left(\frac{2}{3}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}\right)\text{cm}^3 = 2425.5\text{ cm}^3$.

Curved surface area $= 2\pi r^2 = \left(2\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\right)\text{cm}^2 = 693\text{ cm}^2$.

Total surface area $= 3\pi r^2 = \left(3\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}\right)\text{cm}^2 = 1039.5\text{ cm}^2$.

Ex. 33. ***A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?*** **(N.I.F.T. 2003)**

Sol. Volume of bowl $= \left(\frac{2}{3}\pi\times 9\times 9\times 9\right)\text{cm}^3 = 486\pi\text{ cm}^3$.

Volume of 1 bottle $= \left(\pi\times\frac{3}{2}\times\frac{3}{2}\times 4\right)\text{cm}^3 = 9\pi\text{ cm}^3$.

Number of bottles $= \left(\frac{486\pi}{9\pi}\right) = 54$.

Ex. 34. ***A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volumes.***

Sol. Let R be the radius of each.

Height of hemisphere = Its radius = R.

$\therefore$ Height of each = R.

Ratio of volumes $= \frac{1}{3}\pi R^2\times R : \frac{2}{3}\pi R^3 : \pi R^2\times R = 1 : 2 : 3$.

EXERCISE 25A

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The capacity of a tank of dimensions (8 m × 6 m × 2.5 m) is :
   (a) 120 litres (b) 1200 litres (c) 12000 litres (d) 120000 litres
2. Find the surface area of a 10 cm × 4 cm × 3 cm brick. **(R.R.B. 2001)**
   (a) 84 sq. cm (b) 124 sq. cm (c) 164 sq. cm (d) 180 sq. cm
3. A cistern 6 m long and 4 m wide contains water up to a depth of 1 m 25 cm. The total area of the wet surface is : **(S.S.C. 2004)**
   (a) 49 m^2 (b) 50 m^2 (c) 53.5 m^2 (d) 55 m^2
4. A boat having a length 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm when a man gets on it. The mass of man is : **(R.R.B. 2002)**
   (a) 12 kg (b) 60 kg (c) 72 kg (d) 96 kg

5. The area of the base of a rectangular tank is 6500 cm^2 and the volume of water contained in it is 2.6 cubic metres. The depth of water in the tank is :
(a) 3.5 m (b) 4 m (c) 5 m (d) 6 m

6. Given that 1 cu. cm of marble weighs 25 gms, the weight of a marble block 28 cm in width and 5 cm thick is 112 kg. The length of the block is :
(a) 26.5 cm (b) 32 cm (c) 36 cm (d) 37.5 cm

7. Half cubic metre of gold sheet is extended by hammering so as to cover an area of 1 hectare. The thickness of the sheet is :
(a) 0.0005 cm (b) 0.005 cm (c) 0.05 cm (d) 0.5 cm

8. In a shower, 5 cm of rain falls. The volume of water that falls on 1.5 hectares of ground is :
(a) 75 cu. m (b) 750 cu. m (c) 7500 cu. m (d) 75000 cu. m

9. The height of a wall is six times its width and the length of the wall is seven times its height. If volume of the wall be 16128 cu. m, its width is : **(C.B.I. 1998)**
(a) 4 m (b) 4.5 m (c) 5 m (d) 6 m

10. The volume of a rectangular block of stone is 10368 dm^3. Its dimensions are in the ratio of 3 : 2 : 1. If its entire surface is polished at 2 paise per dm^2, then the total cost will be :
(a) Rs. 31.50 (b) Rs. 31.68 (c) Rs. 63 (d) Rs. 63.36

11. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2. The volume of the cuboid is : **(S.S.C. 1999)**
(a) 24 cm^3 (b) 48 cm^3 (c) 64 cm^3 (d) 120 cm^3

12. The maximum length of a pencil that can be kept in a rectangular box of dimensions 8 cm × 6 cm × 2 cm, is :
(a) $2\sqrt{13}$ cm (b) $2\sqrt{14}$ cm (c) $2\sqrt{26}$ cm (d) $10\sqrt{2}$ cm

13. Find the length of the longest rod that can be placed in a room 16 m long, 12 m broad and $10\frac{2}{3}$ m high. **(S.S.C. 1999)**
(a) $22\frac{1}{3}$ m (b) $22\frac{2}{3}$ m (c) 23 m (d) 68 m

14. How many bricks, each measuring 25 cm × 11.25 cm × 6 cm, will be needed to build a wall 8 m × 6 m × 22.5 cm ? **(B.S.F. 2001)**
(a) 5600 (b) 6000 (c) 6400 (d) 7200

15. The number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the mortar occupies 5% of the volume of the wall, is : **(M.B.A. 2003)**
(a) 3040 (b) 5740 (c) 6080 (d) 8120

16. 50 men took a dip in a water tank 40 m long and 20 m broad on a religious day. If the average displacement of water by a man is 4 m^3, then the rise in the water level in the tank will be : **(N.I.F.T. 2000)**
(a) 20 cm (b) 25 cm (c) 35 cm (d) 50 cm

17. A tank 4 m long, 2.5 m wide and 1.5 m deep is dug in a field 31 m long and 10 m wide. If the earth dug out is evenly spread out over the field, the rise in level of the field is :
(a) 3.1 cm (b) 4.8 cm (c) 5 cm (d) 6.2 cm

18. A river 1.5 m deep and 36 m wide is flowing at the rate of 3.5 km per hour. The amount of water that runs into the sea per minute (in cubic metres) is :
(a) 3150 (b) 31500 (c) 6300 (d) 63000

19. A rectangular water tank is 80 m × 40 m. Water flows into it through a pipe 40 sq. cm at the opening at a speed of 10 km / hr. By how much, the water level will rise in the tank in half an hour ? **(M.B.A. 1997)**

(a) $\frac{3}{2}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{5}{8}$ cm (d) None of these

20. A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of areas of the four walls, the volume of the hall is :

(a) 720 (b) 900 (c) 1200 (d) 1800 **(L.I.C. A.A.O. 2003)**

21. The sum of the length, breadth and depth of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. It surface area is :

(a) 125 cm^2 (b) 236 cm^2 (c) 361 cm^2 (d) 486 cm^2

22. A swimming pool 9 m wide and 12 m long is 1 m deep on the shallow side and 4 m deep on the deeper side. Its volume is : **(M.A.T. 1998)**

(a) 208 m^3 (b) 270 m^3 (c) 360 m^3 (d) 408 m^3

23. A metallic sheet is of rectangular shape with dimensions 48 m × 36 m. From each of its corners, a square is cut off so as to make an open box. If the length of the square is 8 m, the volume of the box (in m^3) is : **(M.A.T. 2003)**

(a) 4830 (b) 5120 (c) 6420 (d) 8960

24. An open box is made of wood 3 cm thick. Its external dimensions are 1.46 m, 1.16 m and 8.3 dm. The cost of painting the inner surface of the box at 50 paise per 100 sq. cm is :

(a) Rs. 138.50 (b) Rs. 277 (c) Rs. 415. 50 (d) Rs. 554

25. A cistern of capacity 8000 litres measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is : **(S.S.C. 2003)**

(a) 90 cm (b) 1 dm (c) 1 m (d) 1.1 m

26. If a metallic cuboid weighs 16 kg, how much would a miniature cuboid of metal weigh, if all dimensions are reduced to one-fourth of the original ? **(D.M.R.C. 2003)**

(a) 0.25 kg (b) 0.50 kg (c) 0.75 kg (d) 1 kg

27. The areas of the three adjacent faces of a rectangular box which meet in a point are known. The product of these areas is equal to : **(Section Officers', 2003)**

(a) the volume of the box (b) twice the volume of the box

(c) the square of the volume of the box (d) the cube root of the volume of the box

28. If the areas of the three adjacent faces of a cuboidal box are 120 cm^2, 72 cm^2 and 60 cm^2 respectively, then find the volume of the box. **(S.S.C. 2002)**

(a) 720 cm^3 (b) 864 cm^3 (c) 7200 cm^3 (d) $(72)^2$ cm^3

29. If the areas of three adjacent faces of a rectangular block are in the ratio of 2 : 3 : 4 and its volume is 9000 cu. cm; then the length of the shortest side is :

(a) 10 cm (b) 15 cm (c) 20 cm (d) 30 cm

30. The perimeter of one face a of cube is 20 cm. Its volume must be : **(S.S.C. 1999)**

(a) 125 cm^3 (b) 400 cm^3 (c) 1000 cm^3 (d) 8000 cm^3

31. Total surface area of a cube whose side is 0.5 cm is : **(I.M.T. 2002)**

(a) $\frac{1}{4}$ cm^2 (b) $\frac{1}{8}$ cm^2 (c) $\frac{3}{4}$ cm^2 (d) $\frac{3}{2}$ cm^2

32. The cost of the paint is Rs. 36.50 per kg. If 1 kg of paint covers 16 square feet, how much will it cost to paint outside of a cube having 8 feet each side ?

(a) Rs. 692 (b) Rs. 768 (c) Rs. 876

(d) Rs. 972 (e) None of these **(Bank P.O. 2002)**

33. The dimensions of a piece of iron in the shape of a cuboid are 270 cm × 100 cm × 64 cm. If it is melted and recast into a cube, then the surface area of the cube will be :
(a) 14400 cm^2 (b) 44200 cm^2 (c) 57600 cm^2 (d) 86400 cm^2

34. The cost of painting the whole surface area of a cube at the rate of 13 paise per sq. cm is Rs. 343.98. Then the volume of the cube is : **(S.S.C. 2003)**
(a) 8500 cm^3 (b) 9000 cm^3 (c) 9250 cm^3 (d) 9261 cm^3

35. If the volume of a cube is 729 cm^3, then the surface area of the cube will be :
(a) 456 cm^2 (b) 466 cm^2 (c) 476 cm^2 (d) 486 cm^2

36. The length of an edge of a hollow cube open at one face is $\sqrt{3}$ metres. What is the length of the largest pole that it can accommodate ? **(M.A.T. 1997)**
(a) $\sqrt{3}$ metres (b) 3 metres (c) $3\sqrt{3}$ metres (d) $\frac{3}{\sqrt{3}}$ metres

37. What is the volume of a cube (in cubic cm) whose diagonal measures $4\sqrt{3}$ cm ?
(a) 8 (b) 16 (c) 27 (d) 64
(Hotel Management, 1999)

38. The surface area of a cube is 600 cm^2. The length of its diagonal is :
(a) $\frac{10}{\sqrt{3}}$ cm (b) $\frac{10}{\sqrt{2}}$ cm (c) $10\sqrt{2}$ cm (d) $10\sqrt{3}$ cm

39. If the numbers representing volume and surface area of a cube are equal, then the length of the edge of the cube in terms of the unit of measurement will be :
(a) 3 (b) 4 (c) 5 (d) 6

40. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge ?
(a) 10 (b) 100 (c) 1000 (d) 10000
(R.R.B. 2003)

41. A rectangular box measures internally 1.6 m long, 1 m broad and 60 cm deep. The number of cubical blocks each of edge 20 cm that can be packed inside the box is :
(a) 30 (b) 53 (c) 60 (d) 120

42. How many cubes of 3 cm edge can be cut out of a cube of 18 cm edge ?
(a) 36 (b) 216 (c) 218 (d) 432
(IGNOU, 2003)

43. A cuboidal block of 6 cm × 9 cm × 12 cm is cut up into an exact number of equal cubes. The least possible number of cubes will be : **(Section Officers', 2003)**
(a) 6 (b) 9 (c) 24 (d) 30

44. The size of a wooden block is 5 × 10 × 20 cm. How many such blocks will be required to construct a solid wooden cube of minimum size ?
(a) 6 (b) 8 (c) 12 (d) 16

45. An iron cube of side 10 cm is hammered into a rectangular sheet of thickness 0.5 cm. If the sides of the sheet are in the ratio 1 : 5, the sides are :
(a) 10 cm, 50 cm (b) 20 cm, 100 cm (c) 40 cm, 200 cm (d) None of these
(Hotel Management, 1997)

46. Three cubes of iron whose edges are 6 cm, 8 cm and 10 cm respectively are melted and formed into a single cube. The edge of the new cube formed is :
(a) 12 cm (b) 14 cm (c) 16 cm (d) 18 cm

47. Five equal cubes, each of side 5 cm, are placed adjacent to each other. The volume of the new solid formed will be :
(a) 125 cm^3 (b) 625 cm^3 (c) 15525 cm^3 (d) None of these

48. A cube of edge 5 cm is cut into cubes each of edge 1 cm. The ratio of the total surface area of one of the small cubes to that of the large cube is equal to : **(S.S.C. 2004)**
(a) 1 : 5 (b) 1 : 25 (c) 1 : 125 (d) 1 : 625

49. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cm side. What is the ratio of the total surface areas of the smaller cubes and the large cube ? **(M.A.T. 2004)**
(a) 2 : 1 (b) 3 : 2 (c) 25 : 18 (d) 27 : 20

50. Three cubes with sides in the ratio 3 : 4 : 5 are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. The sides of the cubes are : **(M.A.T. 2003)**
(a) 3 cm, 4 cm, 5 cm (b) 6 cm, 8 cm, 10 cm
(c) 9 cm, 12 cm, 15 cm (d) None of these

51. If the volumes of two cubes are in the ratio 27 : 1, the ratio of their edges is :
(a) 1 : 3 (b) 1 : 27 (c) 3 : 1 (d) 27 : 1
(S.S.C. 1999)

52. The volumes of two cubes are in the ratio 8 : 27. The ratio of their surface areas is :
(a) 2 : 3 (b) 4 : 9 (c) 12 : 9 (d) None of these
(Hotel Management, 2003)

53. Two cubes have volumes in the ratio 1 : 27. Then the ratio of the area of the face of one of the cubes to that of the other is :
(a) 1 : 3 (b) 1 : 6 (c) 1 : 9 (d) 1 : 12

54. If each edge of a cube is doubled, then its volume :
(a) is doubled (b) becomes 4 times
(c) becomes 6 times (d) becomes 8 times

55. If each edge of a cube is increased by 25%, then the percentage increase in its surface area is :
(a) 25% (b) 48.75% (c) 50% (d) 56.25%

56. A circular well with a diameter of 2 metres, is dug to a depth of 14 metres. What is the volume of the earth dug out ? **(S.S.C. 1999)**
(a) 32 m^3 (b) 36 m^3 (c) 40 m^3 (d) 44 m^3

57. The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base ? **(Bank P.O. 2003)**
(a) 1.4 m (b) 2.8 m (c) 14 m (d) 28 m (e) None of these

58. The volume of a right circular cylinder whose curved surface area is 2640 cm^2 and circumference of its base is 66 cm, is :
(a) 3465 cm^3 (b) 7720 cm^3 (c) 13860 cm^3 (d) 55440 cm^3

59. If the volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3, then the radius of the cylinder is equal to :
(a) π cm (b) 2 cm (c) 3 cm (d) 4 cm

60. The height of a right circular cylinder is 14 cm and its curved surface is 704 sq. cm. Then its volume is :
(a) 1408 cm^3 (b) 2816 cm^3 (c) 5632 cm^3 (d) 9856 cm^3

61. A closed metallic cylindrical box is 1.25 m high and its base radius is 35 cm. If the sheet metal costs Rs. 80 per m^2, the cost of the material used in the box is :
(a) Rs. 281.60 (b) Rs. 290 (c) Rs. 340.50 (d) Rs. 500

62. The curved surface area of a right circular cylinder of base radius r is obtained by multiplying its volume by :
(a) $2r$ (b) $\frac{2}{r}$ (c) $2r^2$ (d) $\frac{2}{r^2}$

63. The ratio of total surface area to lateral surface area of a cylinder whose radius is 20 cm and height 60 cm, is :
(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 5 : 3

64. A powder tin has a square base with side 8 cm and height 14 cm. Another tin has a circular base with diameter 8 cm and height 14 cm. The difference in their capacities is :
(a) 0 (b) 132 cm^3 (c) 137.1 cm^3 (d) 192 cm^3

65. The ratio between the radius of the base and the height of a cylinder is 2 : 3. If its volume is 12936 cu. cm, the total surface area of the cylinder is :
(a) 2587.2 cm^2 (b) 3080 cm^2 (c) 25872 cm^2 (d) 38808 cm^2

66. The radius of the cylinder is half its height and area of the inner part is 616 sq. cms. Approximately how many litres of milk can it contain ?
(a) 1.4 (b) 1.5 (c) 1.7 (d) 1.9 (e) 2.2
(S.B.I.P.O. 2000)

67. The sum of the radius of the base and the height of a solid cylinder is 37 metres. If the total surface area of the cylinder be 1628 sq. metres, its volume is :
(a) 3180 m^3 (b) 4620 m^3 (c) 5240 m^3 (d) None of these

68. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. Find the ratio of its diameter to its height. **(S.S.C. 2002)**
(a) 3 : 7 (b) 7 : 3 (c) 6 : 7 (d) 7 : 6

69. The height of a closed cylinder of given volume and the minimum surface area is :
(a) equal to its diameter (b) half of its diameter
(c) double of its diameter (d) None of these **(R.R.B. 2002)**

70. If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original one ?
(a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 8 : 1

71. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is :
(a) 4 : 9 (b) 9 : 4 (c) 20 : 27 (d) 27 : 20

72. Two right circular cylinders of equal volumes have their heights in the ratio 1 : 2. The ratio of their radii is : **(S.S.C. 1999)**
(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) $\sqrt{2} : 1$

73. X and Y are two cylinders of the same height. The base of X has diameter that is half the diameter of the base of Y. If the height of X is doubled, the volume of X becomes :
(a) equal to the volume of Y (b) double the volume of Y
(c) half the volume of Y (d) greater than the volume of Y
(C.B.I. 1997)

74. The radius of a wire is decreased to one-third and its volume remains the same. The new length is how many times the original length ?
(a) 1 time (b) 3 times (c) 6 times (d) 9 times

75. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by : **(S.S.C. 1999)**
(a) $10\frac{1}{2}$ cm (b) $11\frac{3}{7}$ cm (c) $12\frac{6}{7}$ cm (d) 14 cm

76. A well with 14 m inside diameter is dug 10 m deep. Earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. The height of the embankment is :
(a) $\frac{1}{2}$ m (b) $\frac{2}{3}$ m (c) $\frac{3}{4}$ m (d) $\frac{3}{5}$ m

77. Water flows through a cylindrical pipe of internal diameter 7 cm at 2 m per second. If the pipe is always half full, then what is the volume of water (in litres) discharged in 10 minutes ? **(S.S.C. 2003)**

(*a*) 2310 (*b*) 3850 (*c*) 4620 (*d*) 9240

78. The number of coins of radius 0.75 cm and thickness 0.2 cm to be melted to make a right circular cylinder of height 8 cm and base radius 3 cm is : **(S.S.C. 2003)**

(*a*) 460 (*b*) 500 (*c*) 600 (*d*) 640

79. Two cylindrical vessels with radii 15 cm and 10 cm and heights 35 cm and 15 cm respectively are filled with water. If this water is poured into a cylindrical vessel 15 cm in height, then the radius of the vessel is :

(*a*) 17.5 cm (*b*) 18 cm (*c*) 20 cm (*d*) 25 cm

80. 66 cubic centimetres of silver is drawn into a wire 1 mm in diameter. The length of the wire in metres will be : **(C.B.I. 1998)**

(*a*) 84 (*b*) 90 (*c*) 168 (*d*) 336

81. A hollow garden roller 63 cm wide with a girth of 440 cm is made of iron 4 cm thick. The volume of the iron used is :

(*a*) 54982 cm^3 (*b*) 56372 cm^3 (*c*) 57636 cm^3 (*d*) 58752 cm^3

82. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.4 cm thick. The volume of the metal is : **(S.S.C. 2003)**

(*a*) 280.52 cm^3 (*b*) 306.24 cm^3 (*c*) 310 cm^3 (*d*) 316 cm^3

83. What length of solid cylinder 2 cm in diameter mush be taken to cast into a hollow cylinder of external diameter 12 cm, 0.25 cm thick and 15 cm long ?

(*a*) 42.3215 cm (*b*) 44.0123 cm (*c*) 44.0625 cm (*d*) 44.6023 cm

84. A hollow iron pipe is 21 cm long and its external diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weighs 8 g/cm^3, then the weight of the pipe is :

(*a*) 3.6 kg (*b*) 3.696 kg (*c*) 36 kg (*d*) 36.9 kg **(S.S.C. 2004)**

85. A circular cylinder can hold 61.6 c.c. of water. If the height of the cylinder is 40 cm and the outer diameter is 16 mm, then the thickness of the material of the cylinder is :

(*a*) 0.2 mm (*b*) 0.3 mm (*c*) 1 mm (*d*) 2 mm

86. The radius of the base and height of a cone are 3 cm and 5 cm respectively whereas the radius of the base and height of a cylinder are 2 cm and 4 cm respectively. The ratio of the volume of cone to that of the cylinder is :

(*a*) 1 : 3 (*b*) 15 : 8 (*c*) 15 : 16 (*d*) 45 : 16

87. The curved surface of a right circular cone of height 15 cm and base diameter 16 cm is : **(S.S.C. 1999)**

(*a*) 60π cm^2 (*b*) 68π cm^2 (*c*) 120π cm^2 (*d*) 136π cm^2

88. What is the total surface area of a right circular cone of height 14 cm and base radius 7 cm ? **(Hotel Management, 2001)**

(*a*) 344.35 cm^2 (*b*) 462 cm^2 (*c*) 498.35 cm^2 (*d*) None of these

89. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is : **(S.S.C. 2000)**

(*a*) 12π cm^3 (*b*) 15π cm^3 (*c*) 16π cm^3 (*d*) 20π cm^3

90. The slant height of a right circular cone is 10 m and its height is 8 m. Find the area of its curved surface. **(R.R.B. 2003)**

(*a*) 30π m^2 (*b*) 40π m^2 (*c*) 60π m^2 (*d*) 80π m^2

91. If a right circular cone of height 24 cm has a volume of 1232 cm^3, then the area of its curved surface is : **(S.S.C. 2003)**

(*a*) 154 cm^2 (*b*) 550 cm^2 (*c*) 704 cm^2 (*d*) 1254 cm^2

4. When B runs 25 m, A runs $\frac{45}{2}$ m.

When B runs 1000 m, A runs $\left(\frac{45}{2} \times \frac{1}{25} \times 1000\right)$ m = 900 m.

$\therefore$ B beats A by 100 m.

5. To reach the winning post A will have to cover a distance of (500 – 140) m, *i.e.*, 360 m.
While A covers 3 m, B covers 4 m.

While A covers 360 m, B covers $\left(\frac{4}{3} \times 360\right)$ m = 480 m.

Thus, when A reaches the winning post, B covers 480 m and therefore remains 20 m behind.

$\therefore$ A wins by 20 m.

6. Ratio of the speeds of A and B = $\frac{5}{3} : 1 = 5 : 3$.

Thus, in a race of 5 m, A gains 2 m over B.
2 m are gained by A in a race of 5 m.

80 m will be gained by A in a race of $\left(\frac{5}{2} \times 80\right)$ m = 200 m.

$\therefore$ Winning post is 200 m away from the starting point.

7. A : B = 100 : 75 and B : C = 100 : 96.

$\therefore$ A : C = $\left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{100}{75} \times \frac{100}{96}\right) = \frac{100}{72} = 100 : 72$.

$\therefore$ A beats C by (100 – 72) m = 28 m.

8. A : B = 100 : 90 and A : C = 100 : 72.

B : C = $\frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{72} = \frac{90}{72}$.

When B runs 90 m, C runs 72 m.

When B runs 100 m, C runs $\left(\frac{72}{90} \times 100\right)$ m = 80 m.

$\therefore$ B can give C 20 m.

9. A : B = 100 : 90 and A : C = 100 : 87.

$\frac{B}{C} = \frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{87} = \frac{30}{29}$.

When B runs 30 m, C runs 29 m.

When B runs 180 m, C runs $\left(\frac{29}{30} \times 180\right)$ m = 174 m.

$\therefore$ B beats C by (180 – 174) m = 6 m.

10. A : B = 200 : 169 and A : C = 200 : 182.

$\frac{C}{B} = \left(\frac{C}{A} \times \frac{A}{B}\right) = \left(\frac{182}{200} \times \frac{200}{169}\right) = 182 : 169$.

When C covers 182 m, B covers 169 m.

When C covers 350 m, B covers $\left(\frac{169}{182} \times 350\right)$ m = 325 m.

11. A's speed = $\left(5 \times \frac{5}{18}\right)$ m/sec = $\frac{25}{18}$ m/sec.

92. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km^2. The height of the mountain is : (S.S.C. 2002)
(a) 2.2 km (b) 2.4 km (c) 3 km (d) 3.11 km

93. If the area of the base of a right circular cone is 3850 cm^2 and its height is 84 cm, then the curved surface area of the cone is :
(a) 10001 cm^2 (b) 10010 cm^2 (c) 10100 cm^2 (d) 11000 cm^2

94. Volume of a right circular cone having base radius 70 cm and curved surface area 40040 cm^2 is :
(a) 823400 cm^3 (b) 824000 cm^3 (c) 840000 cm^3 (d) 862400 cm^3

95. The radius and height of a right circular cone are in the ratio 3 : 4. If its volume is 96π cm^3, what is its slant height ? (C.B.I. 1997)
(a) 8 cm (b) 9 cm (c) 10 cm (d) 12 cm

96. The length of canvas 1.1 m wide required to build a conical tent of height 14 m and the floor area 346.5 sq. m is :
(a) 490 m (b) 525 m (c) 665 m (d) 860 m

97. If the radius of the base and the height of a right circular cone are doubled, then its volume becomes : (Asstt. Grade, 2003)
(a) 2 times (b) 3 times (c) 4 times (d) 8 times

98. If both the radius and height of a right circular cone are increased by 20%, its volume will be increased by : (S.S.C. 2004)
(a) 20% (b) 40% (c) 60% (d) 72.8%

99. If the height of a right circular cone is increased by 200% and the radius of the base is reduced by 50%, then the volume of the cone : (S.S.C. 2000)
(a) remains unaltered (b) decreases by 25%
(c) increases by 25% (d) increases by 50%

100. If the height of a cone be doubled and radius of base remains the same, then the ratio of the volume of the given cone to that of the second cone will be : (S.S.C. 2003)
(a) 1 : 2 (b) 2 : 1 (c) 1 : 8 (d) 8 : 1

101. Two cones have their heights in the ratio of 1 : 3 and radii 3 : 1. The ratio of their volumes is :
(a) 1 : 1 (b) 1 : 3 (c) 3 : 1 (d) 2 : 3

102. The radii of two cones are in the ratio 2 : 1, their volumes are equal. Find the ratio of their heights. (C.B.I. 1998)
(a) 1 : 8 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1

103. If the volumes of two cones are in the ratio of 1 : 4 and their diameters are in the ratio of 4 : 5, then the ratio of their heights is :
(a) 1 : 5 (b) 5 : 4 (c) 5 : 16 (d) 25 : 64

104. The volume of the largest right circular cone that can be cut out of a cube of edge 7 cm is : (M.A.T. 2002)
(a) 13.6 cm^3 (b) 89.8 cm^3 (c) 121 cm^3 (d) 147.68 cm^3

105. A cone of height 7 cm and base radius 3 cm is carved from a rectangular block of wood 10 cm × 5 cm × 2 cm. The percentage of wood wasted is :
(a) 34% (b) 46% (c) 54% (d) 66%

106. A right circular cone and a right circular cylinder have equal base and equal height. If the radius of the base and the height are in the ratio 5 : 12, then the ratio of the total surface area of the cylinder to that of the cone is :
(a) 3 : 1 (b) 13 : 9 (c) 17 : 9 (d) 34 : 9

107. A cylinder with base radius of 8 cm and height of 2 cm is melted to form a cone of height 6 cm. The radius of the cone will be : **(R.R.B. 2003)**
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

108. A right cylindrical vessel is full of water. How many right cones having the same radius and height as those of the right cylinder will be needed to store that water ?
(a) 2 (b) 3 (c) 4 (d) 8

109. A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to form cones, each of height 1 cm and base radius 1 mm. The number of cones is :
(a) 450 (b) 1350 (c) 4500 (d) 13500

110. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 24 cm ?
(a) 48 min. 15 sec. (b) 51 min. 12 sec. (c) 52 min. 1 sec. (d) 55 min.

111. A solid cylindrical block of radius 12 cm and height 18 cm is mounted with a conical block of radius 12 cm and height 5 cm. The total lateral surface of the solid thus formed is : **(Hotel Management, 1998)**
(a) 528 cm^2 (b) $1357\frac{5}{7}$ cm^2 (c) 1848 cm^2 (d) None of these

112. Consider the volumes of the following : **(Civil Services, 2002)**
1. A parallelopiped of length 5 cm, breadth 3 cm and height 4 cm
2. A cube of each side 4 cm
3. A cylinder of radius 3 cm and length 3 cm
4. A sphere of radius 3 cm

The volumes of these in the decreasing order is :
(a) 1, 2, 3, 4 (b) 1, 3, 2, 4 (c) 4, 2, 3, 1 (d) 4, 3, 2, 1

113. The volume of a sphere is 4851 cu. cm. Its curved surface area is :
(a) 1386 cm^2 (b) 1625 cm^2 (c) 1716 cm^2 (d) 3087 cm^2

114. The curved surface area of a sphere is 5544 sq. cm. Its volume is :
(a) 22176 cm^3 (b) 33951 cm^3 (c) 38808 cm^3 (d) 42304 cm^3

115. The volume of a sphere of radius r is obtained by multiplying its surface area by :
(a) $\frac{4}{3}$ (b) $\frac{r}{3}$ (c) $\frac{4r}{3}$ (d) $3r$

116. If the volume of a sphere is divided by its surface area, the result is 27 cm. The radius of the sphere is : **(R.R.B. 2003)**
(a) 9 cm (b) 36 cm (c) 54 cm (d) 81 cm

117. Spheres A and B have their radii 40 cm and 10 cm respectively. The ratio of the surface area of A to the surface area of B is : **(S.S.C. 2003)**
(a) 1 : 4 (b) 1 : 16 (c) 4 : 1 (d) 16 : 1

118. Surface area of a sphere is 2464 cm^2. If its radius be doubled, then the surface area of the new sphere will be :
(a) 4928 cm^2 (b) 9856 cm^2 (c) 19712 cm^2 (d) Data insufficient

119. If the radius of a sphere is doubled, how many times does its volume become ?
(a) 2 times (b) 4 times (c) 6 times (d) 8 times

120. If the radius of a sphere is increased by 2 cm, then its surface area increases by 352 cm^2. The radius of the sphere before the increase was : **(C.B.I. 2003)**
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

121. If the measured value of the radius is 1.5% larger, the percentage error (correct to one decimal place) made in calculating the volume of a sphere is : **(C.B.I. 1997)**
(a) 2.1 (b) 3.2 (c) 4.6 (d) 5.4

122. The volumes of two spheres are in the ratio of 64 : 27. The ratio of their surface areas is : **(R.R.B. 2002)**

(a) 1 : 2 (b) 2 : 3 (c) 9 : 16 (d) 16 : 9

123. If the surface areas of two spheres are in the ratio of 4 : 25, then the ratio of their volumes is :

(a) 4 : 25 (b) 25 : 4 (c) 125 : 8 (d) 8 : 125

124. If three metallic spheres of radii 6 cms, 8 cms and 10 cms are melted to form a single sphere, the diameter of the new sphere will be : **(D.M.R.C. 2003)**

(a) 12 cms (b) 24 cms (c) 30 cms (d) 36 cms

125. A solid metallic sphere of radius 8 cm is melted and recast into spherical balls each of radius 2 cm. The number of spherical balls, thus obtained, is :

(a) 16 (b) 48 (c) 64 (d) 82

126. A spherical ball of lead, 3 cm in diameter is melted and recast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm respectively. The diameter of the third ball is :

(a) 2.5 cm (b) 2.66 cm (c) 3 cm (d) 3.5 cm

127. If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the radius of each such ball is :

(a) 1.25 cm (b) 2.5 cm (c) 3.75 cm (d) 5 cm

128. A hollow spherical metallic ball has an external diameter 6 cm and is $\frac{1}{2}$ cm thick. The volume of metal used in the ball is : **(S.S.C. 2004)**

(a) $37\frac{2}{3}$ cm^3 (b) $40\frac{2}{3}$ cm^3 (c) $41\frac{2}{3}$ cm^3 (d) $47\frac{2}{3}$ cm^3

129. A solid piece of iron of dimensions $49 \times 33 \times 24$ cm is moulded into a sphere. The radius of the sphere is : **(Hotel Management, 1999)**

(a) 21 cm (b) 28 cm (c) 35 cm (d) None of these

130. How many bullets can be made out of a cube of lead whose edge measures 22 cm, each bullet being 2 cm in diameter ?

(a) 1347 (b) 2541 (c) 2662 (d) 5324

131. How many lead shots each 3 mm in diameter can be made from a cuboid of dimensions 9 cm × 11 cm × 12 cm ?

(a) 7200 (b) 8400 (c) 72000 (d) 84000

132. A sphere and a cube have equal surface areas. The ratio of the volume of the sphere to that of the cube is :

(a) $\sqrt{\pi} : \sqrt{6}$ (b) $\sqrt{2} : \sqrt{\pi}$ (c) $\sqrt{\pi} : \sqrt{3}$ (d) $\sqrt{6} : \sqrt{\pi}$

133. The ratio of the volume of a cube to that of a sphere which will fit inside the cube is :

(a) 4 : π (b) 4 : 3π (c) 6 : π (d) 2 : π

134. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. The radius of the sphere is :

(a) 3 cm (b) 4 cm (c) 6 cm (d) 12 cm **(S.S.C. 2002)**

135. The diameter of the iron ball used for the shot-put game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3}$ cm is made. What will be the diameter of the base of the cylinder ? **(S.S.C. 2004)**

(a) 14 cm (b) $\frac{14}{3}$ cm (c) 28 cm (d) $\frac{28}{3}$ cm

136. The volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is : **(C.B.I. 1997)**

(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$

137. How many spherical bullets can be made out of a lead cylinder 15 cm high and with base radius 3 cm, each bullet being 5 mm in diameter ?

(a) 6000 (b) 6480 (c) 7260 (d) 7800

138. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of spherical balls is :

(a) 12 (b) 16 (c) 24 (d) 48

139. The diameter of a sphere is 8 cm. It is melted and drawn into a wire of diameter 3 mm. The length of the wire is :

(a) 36.9 m (b) 37.9 m (c) 38.9 m (d) 39.9 m

140. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by : **(M.B.A. 2000)**

(a) $\frac{2}{9}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{9}{4}$ cm (d) $\frac{9}{2}$ cm

141. 12 spheres of the same size are made from melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is : **(S.S.C. 2000)**

(a) $\sqrt{3}$ cm (b) 2 cm (c) 3 cm (d) 4 cm

142. A cylindrical tub of radius 12 cm contains water upto a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. The radius of the ball is :

(a) 4.5 cm (b) 6 cm (c) 7.25 cm (d) 9 cm

143. A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is : **(C.B.I. 2003)**

(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm

144. A cone of height 9 cm with diameter of its base 18 cm is carved out from a wooden solid sphere of radius 9 cm. The percentage of the wood wasted is ? **(S.S.C. 2000)**

(a) 25% (b) 25π% (c) 50% (d) 75%

145. A metallic cone of radius 12 cm and height 24 cm is melted and made into spheres of radius 2 cm each. How many spheres are there ?

(a) 108 (b) 120 (c) 144 (d) 180

146. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. The height of the cone is : **(R.R.B. 2002)**

(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm

147. In what ratio are the volumes of a cylinder, a cone and a sphere, if each has the same diameter and the same height ?

(a) 1 : 3 : 2 (b) 2 : 3 : 1 (c) 3 : 1 : 2 (d) 3 : 2 : 1

148. The total surface area of a solid hemisphere of diameter 14 cm, is :

(a) 308 cm^2 (b) 462 cm^2 (c) 1232 cm^2 (d) 1848 cm^2

149. Volume of a hemisphere is 19404 cu. cm. Its radius is :

(a) 10.5 cm (b) 17.5 cm (c) 21 cm (d) 42 cm

150. The capacities of two hemispherical vessels are 6.4 litres and 21.6 litres. The areas of inner curved surfaces of the vessels will be in the ratio of :

(a) $\sqrt{2} : \sqrt{3}$ (b) 2 : 3 (c) 4 : 9 (d) 16 : 81

151. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, the volume of the beverage in the cylindrical vessel is : **(I.A.S. 1999)**

(a) $66\frac{2}{3}\%$ (b) $78\frac{1}{2}\%$ (c) 100%

(d) More than 100% (*i.e.*, some liquid will be left in the bowl).

152. A metallic hemisphere is melted and recast in the shape of a cone with the same base radius (R) as that of the hemisphere. If H is the height of the cone, then :

(a) $H = 2R$ (b) $H = 3R$ (c) $H = \sqrt{3}R$ (d) $H = \frac{2}{3}R$

(S.S.C. 1999)

153. A hemisphere of lead of radius 6 cm is cast into a right circular cone of height 75 cm. The radius of the base of the cone is :

(a) 1.4 cm (b) 2 cm (c) 2.4 cm (d) 4.2 cm

154. A hemisphere and a cone have equal bases. If their heights are also equal, then the ratio of their curved surfaces will be : **(S.S.C. 2002)**

(a) 1 : 2 (b) 2 : 1 (c) $1:\sqrt{2}$ (d) $\sqrt{2}:1$

155. A sphere of maximum volume is cut out from a solid hemisphere of radius *r*. The ratio of the volume of the hemisphere to that of the cut out sphere is :

(a) 3 : 2 (b) 4 : 1 (c) 4 : 3 (d) 7 : 4

ANSWERS

1. (d)	2. (c)	3. (a)	4. (b)	5. (b)	6. (b)	7. (b)	8. (b)
9. (a)	10. (d)	11. (b)	12. (c)	13. (b)	14. (c)	15. (c)	16. (b)
17. (c)	18. (a)	19. (c)	20. (c)	21. (b)	22. (b)	23. (b)	24. (b)
25. (a)	26. (a)	27. (c)	28. (c)	29. (b)	30. (a)	31. (d)	32. (c)
33. (d)	34. (d)	35. (d)	36. (b)	37. (d)	38. (d)	39. (d)	40. (c)
41. (d)	42. (b)	43. (c)	44. (b)	45. (b)	46. (a)	47. (b)	48. (b)
49. (c)	50. (b)	51. (c)	52. (b)	53. (c)	54. (d)	55. (d)	56. (d)
57. (e)	58. (c)	59. (b)	60. (b)	61. (a)	62. (b)	63. (c)	64. (d)
65. (b)	66. (b)	67. (b)	68. (b)	69. (a)	70. (b)	71. (c)	72. (d)
73. (c)	74. (d)	75. (b)	76. (b)	77. (c)	78. (d)	79. (d)	80. (a)
81. (d)	82. (b)	83. (c)	84. (b)	85. (c)	86. (c)	87. (d)	88. (c)
89. (a)	90. (c)	91. (b)	92. (b)	93. (b)	94. (d)	95. (c)	96. (b)
97. (d)	98. (d)	99. (b)	100. (a)	101. (c)	102. (b)	103. (d)	104. (b)
105. (a)	106. (c)	107. (d)	108. (b)	109. (d)	110. (b)	111. (d)	112. (d)
113. (a)	114. (c)	115. (b)	116. (d)	117. (d)	118. (b)	119. (d)	120. (d)
121. (c)	122. (d)	123. (d)	124. (b)	125. (c)	126. (a)	127. (d)	128. (d)
129. (a)	130. (b)	131. (d)	132. (d)	133. (c)	134. (c)	135. (c)	136. (a)
137. (b)	138. (d)	139. (b)	140. (c)	141. (d)	142. (d)	143. (b)	144. (d)
145. (a)	146. (b)	147. (c)	148. (b)	149. (c)	150. (c)	151. (c)	152. (a)
153. (c)	154. (d)	155. (b)					

SOLUTIONS

1. Capacity of the bank = Volume of the tank
$= \left(\frac{8 \times 100 \times 6 \times 100 \times 2.5 \times 100}{1000}\right)$ litres = 120000 litres.

2. Surface area = $[2\ (10 \times 4 + 4 \times 3 + 10 \times 3)]\ cm^2 = (2 \times 82)\ cm^2 = 164\ cm^2$.

3. Area of the wet surface = $[2\ (lb + bh + lh) - lb] = 2\ (bh + lh) + lb$
$= [2\ (4 \times 1.25 + 6 \times 1.25) + 6 \times 4]\ m^2 = 49\ m^2$.

4. Volume of water displaced = $(3 \times 2 \times 0.01)\ m^3 = 0.06\ m^3$.
∴ Mass of man = Volume of water displaced × Density of water
$= (0.06 \times 1000)$ kg = 60 kg.

5. Volume = $(2.6 \times 100 \times 100 \times 100)$ cu. cm.
Depth = $\frac{\text{Volume}}{\text{Area of the base}} = \left(\frac{2.6 \times 100 \times 100 \times 100}{6500}\right)$ cm = 400 cm = 4 m.

6. Let length = x cm. Then, $x \times 28 \times 5 \times \frac{25}{1000} = 112$
∴ $x = \left(112 \times \frac{1000}{25} \times \frac{1}{28} \times \frac{1}{5}\right)$ cm = 32 cm.

7. Volume of gold = $\left(\frac{1}{2} \times 100 \times 100 \times 100\right) cm^3$.
Area of sheet = $10000\ m^2 = (10000 \times 100 \times 100)\ cm^2$
∴ Thickness of the sheet = $\left(\frac{1 \times 100 \times 100 \times 100}{2 \times 10000 \times 100 \times 100}\right)$ cm = 0.005 cm.

8. Area = $(1.5 \times 10000)\ m^2 = 15000\ m^2$.
Depth = $\frac{5}{100}$ m = $\frac{1}{20}$ m.
∴ Volume = (Area × Depth) = $\left(15000 \times \frac{1}{20}\right) m^3 = 750\ m^3$.

9. Let the width of the wall be x metres.
Then, Height = $(6x)$ metres and Length = $(42x)$ metres.
∴ $42x \times x \times 6x = 16128 \Leftrightarrow x^3 = \left(\frac{16128}{42 \times 6}\right) = 64 \Leftrightarrow x = 4$.

10. Let the dimensions be $3x$, $2x$ and x respectively. Then,
$3x \times 2x \times x = 10368 \Leftrightarrow x^3 = \left(\frac{10368}{6}\right) = 1728 \Leftrightarrow x = 12$.
So, the dimensions of the block are 36 dm, 24 dm, and 12 dm.
Surface area = $[2\ (36 \times 24 + 24 \times 12 + 36 \times 12)]\ dm^2$
$= [2 \times 144\ (6 + 2 + 3)]\ dm^2 = 3168\ dm^2$.
∴ Cost of polishing = Rs. $\left(\frac{2 \times 3168}{100}\right)$ = Rs. 63.36.

11. Let the dimensions of the cuboid be x, $2x$ and $3x$.
Then, $2\ (x \times 2x + 2x \times 3x + x \times 3x) = 88$
$\Leftrightarrow \quad 2x^2 + 6x^2 + 3x^2 = 44 \Leftrightarrow 11x^2 = 44 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2$.
∴ Volume of the cuboid = $(2 \times 4 \times 6)\ cm^3 = 48\ cm^3$.

12. Required length $= \sqrt{8^2 + 6^2 + 2^2}$ cm $= \sqrt{104}$ cm $= 2\sqrt{26}$ cm.

13. Required length $= \sqrt{(16)^2 + (12)^2 + \left(\frac{32}{3}\right)^2}$ m $= \sqrt{256 + 144 + \frac{1024}{9}}$ m

$= \sqrt{\frac{4624}{9}}$ m $= \frac{68}{3}$ m $= 22\frac{2}{3}$ m.

14. Number of bricks $= \frac{\text{Volume of the wall}}{\text{Volume of 1 brick}} = \left(\frac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6}\right) = 6400.$

15. Volume of the bricks = 95% of volume of wall $= \left(\frac{95}{100} \times 600 \times 500 \times 50\right)$ cm^3.

Volume of 1 brick $= (25 \times 12.5 \times 7.5)$ cm^3.

$\therefore$ Number of bricks $= \left(\frac{95}{100} \times \frac{600 \times 500 \times 50}{25 \times 12.5 \times 7.5}\right) = 6080.$

16. Total volume of water displaced $= (4 \times 50)$ m^3 $= 200$ m^3.

$\therefore$ Rise in water level $= \left(\frac{200}{40 \times 20}\right)$ m $= 0.25$ m $= 25$ cm.

17. Volume of earth dug out $= \left(4 \times \frac{5}{2} \times \frac{3}{2}\right)$ m^3 $= 15$ m^3.

Area over which earth is spread $= \left(31 \times 10 - 4 \times \frac{5}{2}\right)$ m^2 $= 300$ m^2.

$\therefore$ Rise in level $= \left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{15}{300} \times 100\right)$ cm $= 5$ cm.

18. Length of water column flown in 1 min. $= \left(\frac{3.5 \times 1000}{60}\right)$ m $= \frac{175}{3}$ m.

$\therefore$ Volume flown per minute $= \left(\frac{175}{3} \times 36 \times \frac{3}{2}\right)$ m^3 $= 3150$ m^3.

19. Length of water column flown in 1 min. $= \left(\frac{10 \times 1000}{60}\right)$ m $= \frac{500}{3}$ m.

Volume flown per minute $= \left(\frac{500}{3} \times \frac{40}{100 \times 100}\right)$ m^3 $= \frac{2}{3}$ m^3.

Volume flown in half an hour $= \left(\frac{2}{3} \times 30\right)$ m^3 $= 20$ m^3.

$\therefore$ Rise in water level $= \left(\frac{20}{40 \times 80}\right)$ m $= \left(\frac{1}{160} \times 100\right)$ cm $= \frac{5}{8}$ cm.

20. $2(15 + 12) \times h = 2(15 \times 12)$ or $h = \frac{180}{27}$ m $= \frac{20}{3}$ m.

$\therefore$ Volume $= \left(15 \times 12 \times \frac{20}{3}\right)$ m^3 $= 1200$ m^3.

21. $(l + b + h) = 19$ and $\sqrt{l^2 + b^2 + h^2} = 5\sqrt{5}$ and so $(l^2 + b^2 + h^2) = 125$.

Now, $(l + b + h)^2 = 19^2 \Rightarrow (l^2 + b^2 + h^2) + 2(lb + bh + lh) = 361$

$\Rightarrow 2(lb + bh + lh) = (361 - 125) = 236.$

$\therefore$ Surface area $= 236$ cm^2.

22. Volume $= \left[12 \times 9 \times \left(\frac{1+4}{2}\right)\right] m^3 = (12 \times 9 \times 2.5)\, m^3 = 270\, m^3$.

23. Clearly, $l = (48 - 16)$ m $= 32$ m, $b = (36 - 16)$ m $= 20$ m, $h = 8$ m.

$\therefore$ Volume of the box $= (32 \times 20 \times 8)\, m^3 = 5120\, m^3$.

24. Internal length $= (146 - 6)$ cm $= 140$ cm.

Internal breadth $= (116 - 6)$ cm $= 110$ cm.

Internal depth $= (83 - 3)$ cm $= 80$ cm.

Area of inner surface $= [2\,(l + b) \times h] + lb$

$= [2\,(140 + 110) \times 80 + 140 \times 110]\, cm^2 = 55400\, cm^2$.

$\therefore$ Cost of painting $= \text{Rs.}\left(\frac{1}{2} \times \frac{1}{100} \times 55400\right) = \text{Rs. } 277$.

25. Let the thickness of the bottom be x cm.

Then, $[(330 - 10) \times (260 - 10) \times (110 - x)] = 8000 \times 1000$

$\Leftrightarrow \quad 320 \times 250 \times (110 - x) = 8000 \times 1000 \quad \Leftrightarrow \quad (110 - x) = \frac{8000 \times 1000}{320 \times 250} = 100$

$\Leftrightarrow \quad x = 10$ cm $= 1$ dm.

26. Let the dimensions of the bigger cuboid be x, y and z.

Then, Volume of the bigger cuboid $= xyz$.

Volume of the miniature cuboid $= \left(\frac{1}{4}x\right)\left(\frac{1}{4}y\right)\left(\frac{1}{4}z\right) = \frac{1}{64}xyz$.

$\therefore$ Weight of the miniature cuboid $= \left(\frac{1}{64} \times 16\right)$ kg $= 0.25$ kg.

27. Let length $= l$, breadth $= b$ and height $= h$. Then,

Product of areas of 3 adjacent faces $= (lb \times bh \times lh) = (lbh)^2 = (\text{Volume})^2$.

28. Let the length, breadth and height of the box be l, b and h respectively. Then,

Volume $= lbh = \sqrt{(lbh)^2} = \sqrt{lb \times bh \times lh} = \sqrt{120 \times 72 \times 60} = 720\, cm^3$.

29. Let $lb = 2x$, $bh = 3x$ and $lh = 4x$.

Then, $24x^3 = (lbh)^2 = 9000 \times 9000 \Rightarrow x^3 = 375 \times 9000 \Rightarrow x = 150$.

So, $lb = 300$, $bh = 450$, $lh = 600$ and $lbh = 9000$.

$\therefore \quad h = \frac{9000}{300} = 30$, $l = \frac{9000}{450} = 20$ and $b = \frac{9000}{600} = 15$.

Hence, shortest side $= 15$ cm.

30. Edge of the cube $= \left(\frac{20}{4}\right)$ cm $= 5$ cm.

$\therefore$ Volume $= (5 \times 5 \times 5)\, cm^3 = 125\, cm^3$.

31. Surface area $= \left[6 \times \left(\frac{1}{2}\right)^2\right] cm^2 = \frac{3}{2}\, cm^2$.

32. Surface area of the cube $= (6 \times 8^2)$ sq. ft. $= 384$ sq. ft.

Quantity of paint required $= \left(\frac{384}{16}\right)$ kg $= 24$ kg.

$\therefore$ Cost of painting $= \text{Rs. } (36.50 \times 24) = \text{Rs. } 876$.

33. Volume of the cube $= (270 \times 100 \times 64)\, cm^3$.

Edge of the cube $= \sqrt{270 \times 100 \times 64}$ cm $= (3 \times 10 \times 4)$ cm $= 120$ cm.

$\therefore$ Surface area $= (6 \times 120 \times 120)\, cm^2 = 86400\, cm^2$.

34. Surface area $= \left(\frac{34398}{13}\right) = 2646 \text{ cm}^2$.

$\therefore \quad 6a^2 = 2646 \Rightarrow a^2 = 441 \Rightarrow a = 21.$

So, Volume $= (21 \times 21 \times 21) \text{ cm}^3 = 9261 \text{ cm}^3$.

35. $a^3 = 729 \Rightarrow a = 9$.

$\therefore$ Surface area $= (6 \times 9 \times 9) \text{ cm}^2 = 486 \text{ cm}^2$.

36. Required length = Diagonal $= \sqrt{3}\,a = (\sqrt{3} \times \sqrt{3}) \text{ m} = 3 \text{ m}$.

37. $\sqrt{3}\,a = 4\sqrt{3} \Rightarrow a = 4$.

$\therefore$ Volume $= (4 \times 4 \times 4) \text{ cm}^3 = 64 \text{ cm}^3$.

38. $6a^2 = 600 \Rightarrow a^2 = 100 \Rightarrow a = 10$.

$\therefore$ Diagonal $= \sqrt{3}\,a = 10\sqrt{3} \text{ cm}$.

39. $a^3 = 6a^2 \Rightarrow a = 6$.

40. Number of cubes $= \left(\frac{100 \times 100 \times 100}{10 \times 10 \times 10}\right) = 1000$.

41. Number of blocks $= \left(\frac{160 \times 100 \times 60}{20 \times 20 \times 20}\right) = 120$.

42. Number of cubes $= \left(\frac{18 \times 18 \times 18}{3 \times 3 \times 3}\right) = 216$.

43. Volume of block $= (6 \times 9 \times 12) \text{ cm}^3 = 648 \text{ cm}^3$.

Side of largest cube = H.C.F. of 6 cm, 9 cm, 12 cm = 3 cm.

Volume of this cube $= (3 \times 3 \times 3) = 27 \text{ cm}^3$.

$\therefore$ Number of cubes $= \left(\frac{648}{27}\right) = 24$.

44. Side of smallest cube = L.C.M. of 5 cm, 10 cm, 20 cm = 20 cm.

Volume of the cube $= (20 \times 20 \times 20) \text{ cm}^3 = 8000 \text{ cm}^3$.

Volume of the block $= (5 \times 10 \times 20) \text{ cm}^3 = 1000 \text{ cm}^3$.

$\therefore$ Number of blocks $= \left(\frac{8000}{1000}\right) = 8$.

45. Let the sides of the sheet be x and $5x$. Then,

Volume of the sheet = Volume of the cube

$\Rightarrow \quad x \times 5x \times \frac{1}{2} = 10 \times 10 \times 10 \Rightarrow 5x^2 = 2000 \Rightarrow x^2 = 400 \Rightarrow x = 20.$

$\therefore$ The sides are 20 cm and 100 cm.

46. Volume of the new cube $= (6^3 + 8^3 + 10^3) \text{ cm}^3 = 1728 \text{ cm}^3$.

Let the edge of the new cube be a cm.

$\therefore \quad a^3 = 1728 \Rightarrow a = 12.$

47. The new solid formed is a cuboid of length 25 cm, breadth 5 cm and height 5 cm.

$\therefore$ Volume $= (25 \times 5 \times 5) \text{ cm}^3 = 625 \text{ cm}^3$.

48. Required ratio $= \frac{6 \times 1 \times 1}{6 \times 5 \times 5} = \frac{1}{25} = 1:25.$

49. Volume of the large cube $= (3^3 + 4^3 + 5^3) \text{ cm}^3 = 216 \text{ cm}^3$.

Let the edge of the large cube be a.

So, $a^3 = 216 \Rightarrow a = 6$ cm.

$\therefore$ Required ratio $= \frac{6 \times (3^2 + 4^2 + 5^2)}{6 \times 6^2} = \frac{50}{36} = 25:18.$

50. Let the sides of the three cubes be $3x$, $4x$ and $5x$.
Then, Volume of the new cube $= [(3x)^3 + (4x)^3 + (5x)^3] = 216x^3$.
Edge of the new cube $= (216x^3)^{1/3} = 6x$.
Diagonal of the new cube $= 6\sqrt{3}\,x$.
$\therefore \quad 6\sqrt{3}\,x = 12\sqrt{3} \Rightarrow x = 2$.
So, the sides of the cubes are 6 cm, 8 cm and 10 cm.

51. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{27}{1} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{3}{1}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{3}{1} \Leftrightarrow a : b = 3 : 1.$$

52. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{8}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{2}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{4}{9} \Leftrightarrow \frac{6a^2}{6b^2} = \frac{4}{9}.$$

53. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{1}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{1}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{1}{9}.$$

54. Let original edge $= a$. Then, volume $= a^3$.
New edge $= 2a$. So, new volume $= (2a)^3 = 8a^3$.
$\therefore$ Volume becomes 8 times.

55. Let original edge $= a$. Then, surface area $= 6a^2$.

$$\text{New edge} = \frac{125}{100}a = \frac{5a}{4}.$$

$$\text{New surface area} = 6 \times \left(\frac{5a}{4}\right)^2 = \frac{75a^2}{8}.$$

$$\text{Increase in surface area} = \left(\frac{75a^2}{8} - 6a^2\right) = \frac{27a^2}{8}.$$

$$\therefore \quad \text{Increase \%} = \left(\frac{27a^2}{8} \times \frac{1}{6a^2} \times 100\right)\% = 56.25\%.$$

56. Volume $= \pi r^2 h = \left(\frac{22}{7} \times 1 \times 1 \times 14\right) \text{m}^3 = 44 \text{ m}^3$.

57. Volume of the tank $= 246.4$ litres $= 246400 \text{ cm}^3$.
Let the radius of the base be r cm. Then,

$$\left(\frac{22}{7} \times r^2 \times 400\right) = 246400 \Leftrightarrow r^2 = \left(\frac{246400 \times 7}{22 \times 400}\right) = 196 \Leftrightarrow r = 14.$$

$\therefore$ Diameter of the base $= 2r = 28$ cm.

58. $2\pi r = 66 \Rightarrow r = \left(66 \times \frac{1}{2} \times \frac{7}{22}\right) = \frac{21}{2}$ cm.

$$\frac{2\pi rh}{2\pi r} = \left(\frac{2640}{66}\right) \Rightarrow h = 40 \text{ cm}.$$

$$\therefore \quad \text{Volume} = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40\right) \text{cm}^3 = 13860 \text{ cm}^3.$$

59. Let the radius and height be r cm each.

Then, $\frac{22}{7} \times r^2 \times r = \frac{176}{7} \Rightarrow r^3 = \left(\frac{176}{7} \times \frac{7}{22}\right) = 8 \Rightarrow r = 2$.

60. $\frac{2\pi rh}{h} = \frac{704}{14} \Rightarrow 2\pi r = \frac{704}{14}$.

$\therefore \quad r = \left(\frac{704}{14} \times \frac{1}{2} \times \frac{7}{22}\right) = 8$ cm.

$\therefore$ Volume $= \left(\frac{22}{7} \times 8 \times 8 \times 14\right)$ cm^3 $= 2816$ cm^3.

61. Total surface area $= 2\pi r(h + r) = \left[2 \times \frac{22}{7} \times \frac{35}{100} \times (1.25 + 0.35)\right]$ m^2

$= \left(2 \times \frac{22}{7} \times \frac{35}{100} \times \frac{16}{10}\right)$ m^2 $= 3.52$ m^2.

$\therefore$ Cost of the material $=$ Rs. $(3.52 \times 80) =$ Rs. 281.60.

62. Curved surface area $= 2\pi rh = (\pi r^2 h) \cdot \frac{2}{r} = \left(\text{Volume} \times \frac{2}{r}\right)$.

63. $\frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi rh + 2\pi r^2}{2\pi rh} = \frac{(h + r)}{h} = \frac{80}{60} = \frac{4}{3}$.

64. Difference in capacities $= \left(8 \times 8 \times 14 - \frac{22}{7} \times 4 \times 4 \times 14\right)$ cm^3 $= 192$ cm^3.

65. Let radius $= 2x$ and height $= 3x$. Then,

$\frac{22}{7} \times (2x)^2 \times 3x = 12936 \Leftrightarrow x^3 = \left(12936 \times \frac{7}{22} \times \frac{1}{12}\right) = 343 = 7^3$

$\therefore \quad x = 7$. So, radius $= 14$ cm and height $= 21$ cm.

$\therefore$ Total surface area $= 2 \times \frac{22}{7} \times 14 \times (21 + 14) = \left(2 \times \frac{22}{7} \times 14 \times 35\right)$ cm^2 $= 3080$ cm^2.

66. It is given that $r = \frac{1}{2}h$ and $2\pi rh + \pi r^2 = 616$ m^2

$\therefore \quad 2\pi \times \frac{1}{2}h \times h + \pi \times \frac{1}{4}h^2 = 616$

$\Rightarrow \quad \frac{5}{4} \times \frac{22}{7} \times h^2 = 616 \Rightarrow h^2 = \left(616 \times \frac{28}{110}\right) = \frac{28 \times 28}{5}$.

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{1}{4}h^2 \times h = \frac{22}{7} \times \frac{1}{4} \times \frac{28 \times 28}{5} \times \frac{28}{\sqrt{5}}$ cm^3

$= \left(\frac{22 \times 28 \times 28}{25} \times \sqrt{5}\right)$ cm^3 $= \left(\frac{22 \times 28 \times 28 \times 2.23}{25 \times 1000}\right)$ litres $= 1.53$ litre.

67. $(h + r) = 37$ and $2\pi r(h + r) = 1628$.

$\therefore \quad 2\pi r \times 37 = 1628$ or $r = \left(\frac{1628}{2 \times 37} \times \frac{7}{22}\right) = 7$.

So, $r = 7$ m and $h = 30$ m.

$\therefore$ Volume $= \left(\frac{22}{7} \times 7 \times 7 \times 30\right)$ m^3 $= 4620$ m^3.

68. $\frac{\pi r^2 h}{2\pi r h} = \frac{924}{264} \quad \Rightarrow \quad r = \left(\frac{924}{264} \times 2\right) = 7$ m.

And, $2\pi r h = 264 \quad \Rightarrow \quad h = \left(264 \times \frac{7}{22} \times \frac{1}{2} \times \frac{1}{7}\right) = 6$ m.

$\therefore$ Required ratio $= \frac{2r}{h} = \frac{14}{6} = 7:3.$

69. $V = \pi r^2 h$ and $S = 2\pi r h + 2\pi r^2$

$\Rightarrow S = 2\pi r (h + r)$, where $h = \frac{V}{\pi r^2}$

$\Rightarrow S = 2\pi r\left(\frac{V}{\pi r^2} + r\right) = \frac{2V}{r} + 2\pi r^2 \Rightarrow \frac{dS}{dr} = \frac{-2V}{r^2} + 4\pi r$ and $\frac{d^2S}{dr^2} = \left(\frac{4V}{r^3} + 4\pi\right) > 0$

$\therefore$ S is minimum when $\frac{dS}{dr} = 0$

$\Leftrightarrow \frac{-2V}{r^2} + 4\pi r = 0 \Leftrightarrow V = 2\pi r^3 \Leftrightarrow \pi r^2 h = 2\pi r^3 \Leftrightarrow h = 2r.$

70. Let original radius = R. Then, new radius $= \frac{R}{2}$.

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{R}{2}\right)^2 \times h}{\pi \times R^2 \times h} = \frac{1}{4}.$$

71. Let their radii be $2x$, $3x$ and heights be $5y$, $3y$.

Ratio of their volumes $= \frac{\pi \times (2x)^2 \times 5y}{\pi \times (3x)^2 \times 3y} = \frac{20}{27}.$

72. Let their heights be h and $2h$ and radii be r and R respectively. Then,

$\pi r^2 h = \pi R^2 (2h) \Rightarrow \frac{r^2}{R^2} = \frac{2h}{h} = \frac{2}{1} \Rightarrow \frac{r}{R} = \frac{\sqrt{2}}{1}$ *i.e.* $\sqrt{2}:1.$

73. Let the height of X and Y be h, and their radii be r and $2r$ respectively. Then,
Volume of $X = \pi r^2 h$ and Volume of $Y = \pi (2r)^2 h = 4\pi r^2 h$.
New height of $X = 2h$.
So, new volume of $X = \pi r^2 (2h) = 2\pi r^2 h = \frac{1}{2}(4\pi r^2 h) = \frac{1}{2} \times$ (Volume of Y).

74. Let original radius $= r$ and original length $= h$.

New radius $= \frac{r}{3}$ and let new length $= H$.

Then, $\pi r^2 h = \pi\left(\frac{r}{3}\right)^2 \times H$ or $H = 9h$.

75. Let the drop in the water level be h cm. Then,

$\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times h = 11000 \Leftrightarrow h = \left(\frac{11000 \times 7 \times 4}{22 \times 35 \times 35}\right)$ cm $= \frac{80}{7}$ cm $= 11\frac{3}{7}$ cm.

76. Volume of earth dug out = $\left(\frac{22}{7}\times 7\times 7\times 10\right)\text{m}^3 = 1540\text{ m}^3$.

Area of embankment = $\frac{22}{7}\times\left[(28)^2-(7)^2\right] = \left(\frac{22}{7}\times 35\times 21\right)\text{m}^2 = 2310\text{ m}^2$.

Height of embankment = $\left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{1540}{2310}\right)\text{m} = \frac{2}{3}\text{ m}$.

77. Volume of water flown in 1 sec. = $\left(\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 200\right)\text{cm}^3 = 7700\text{ cm}^3$.

Volume of water flown in 10 min. = $(7700\times 60\times 10)\text{ cm}^3$

$= \left(\frac{7700\times 60\times 10}{1000}\right)\text{litres} = 4620\text{ litres}$.

78. Volume of one coin = $\left(\frac{22}{7}\times\frac{75}{100}\times\frac{75}{100}\times\frac{2}{10}\right)\text{cm}^3 = \frac{99}{280}\text{ cm}^3$.

Volume of larger cylinder = $\left(\frac{22}{7}\times 3\times 3\times 8\right)\text{cm}^3$.

$\therefore$ Number of coins = $\left(\frac{22\times 9\times 8}{7}\times\frac{280}{99}\right) = 640$.

79. Let the radius of the vessel be R. Then,

$\pi R^2\times 15 = \pi\times(15)^2\times 35 + \pi\times(10)^2\times 15$

$\Leftrightarrow\ \pi R^2\times 15 = 9375\pi\ \Leftrightarrow\ R^2 = 625\ \Leftrightarrow\ R = 25$ cm.

80. Let the length of the wire be h.

Radius = $\frac{1}{2}$ mm = $\frac{1}{20}$ cm. Then,

$\frac{22}{7}\times\frac{1}{20}\times\frac{1}{20}\times h = 66\ \Leftrightarrow\ h = \left(\frac{66\times 20\times 20\times 7}{22}\right) = 8400\text{ cm} = 84\text{ m}$.

81. Circumference of the girth = 440 cm.

$\therefore\ 2\pi R = 440\ \Rightarrow\ R = \left(440\times\frac{1}{2}\times\frac{7}{22}\right) = 70$ cm.

So, Outer radius = 70 cm. Inner radius = (70 − 4) cm = 66 cm.

Volume of iron = $\pi\left[(70)^2-(66)^2\right]\times 63 = \left(\frac{22}{7}\times 136\times 4\times 63\right)\text{cm}^3 = 58752\text{ cm}^3$.

82. Internal radius = $\left(\frac{11.2}{2}\right)$ cm = 5.6 cm, External radius = (5.6 + 0.4) cm = 6 cm.

Volume of metal = $\left\{\frac{22}{7}\times\left[(6)^2-(5.6)^2\right]\times 21\right\}\text{cm}^3 = (66\times 11.6\times 0.4)\text{ cm}^3 = 306.24\text{ cm}^3$.

83. External radius = 6 cm, Internal radius = (6 − 0.25) cm = 5.75 cm.

Volume of material in hollow cylinder

$= \left\{\frac{22}{7}\times\left[(6)^2-(5.75)^2\right]\times 15\right\}\text{cm}^3 = \left(\frac{22}{7}\times 11.75\times 0.25\times 15\right)\text{cm}^3$

$= \left(\frac{22}{7}\times\frac{1175}{100}\times\frac{25}{100}\times 15\right)\text{cm}^3 = \left(\frac{11\times 705}{56}\right)\text{cm}^3$.

Let the length of solid cylinder be h. Then,

$\frac{22}{7}\times 1\times 1\times h = \left(\frac{11\times 705}{56}\right)\ \Leftrightarrow\ h = \left(\frac{11\times 705}{56}\times\frac{7}{22}\right)\text{cm} = 44.0625\text{ cm}$.

84. External radius = 4 cm, Internal radius = 3 cm.

Volume of iron $= \left\{\frac{22}{7} \times [(4)^2 - (3)^2] \times 21\right\} \text{cm}^3 = \left(\frac{22}{7} \times 7 \times 1 \times 21\right) \text{cm}^3 = 462 \text{ cm}^3$.

$\therefore$ Weight of iron = (462 × 8) gm = 3696 gm = 3.696 kg.

85. Let the internal radius of the cylinder be x. Then,

$$\frac{22}{7} \times r^2 \times 40 = \frac{616}{10} \Leftrightarrow r^2 = \left(\frac{616 \times 7}{10 \times 22 \times 40}\right) = 0.49 \Leftrightarrow r = 0.7.$$

So, internal radius = 0.7 cm = 7 mm.

$\therefore$ Thickness = (8 − 7) mm = 1 mm.

86. $\frac{\text{Volume of cone}}{\text{Volume of cylinder}} = \frac{\frac{1}{3} \times \pi \times (3)^2 \times 5}{\pi \times (2)^2 \times 4} = \frac{45}{48} = \frac{15}{16}$.

87. $h = 15$ cm, $r = 8$ cm. So, $l = \sqrt{r^2 + h^2} = \sqrt{8^2 + (15)^2} = 17$ cm.

$\therefore$ Curved surface area $= \pi rl = (\pi \times 8 \times 17) \text{ cm}^2 = 136\pi \text{ cm}^2$.

88. $h = 14$ cm, $r = 7$ cm. So, $l = \sqrt{(7)^2 + (14)^2} = \sqrt{245} = 7\sqrt{5}$ cm.

$\therefore$ Total surface area $= \pi rl + \pi r^2 = \left(\frac{22}{7} \times 7 \times 7\sqrt{5} + \frac{22}{7} \times 7 \times 7\right) \text{cm}^2$

$= [154(\sqrt{5} + 1)] \text{ cm}^2 = (154 \times 3.236) \text{ cm}^2 = 498.35 \text{ cm}^2$.

89. Clearly, we have $r = 3$ cm and $h = 4$ cm.

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times 3^2 \times 4\right) \text{cm}^3 = 12\pi \text{ cm}^3$.

90. $l = 10$ m, $h = 8$ m. So, $r = \sqrt{l^2 - h^2} = \sqrt{(10)^2 - 8^2} = 6$ m.

$\therefore$ Curved surface area $= \pi rl = (\pi \times 6 \times 10) \text{ m}^2 = 60\pi \text{ m}^2$.

91. $\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232 \Leftrightarrow r^2 = \left(\frac{1232 \times 7 \times 3}{22 \times 24}\right) = 49 \Leftrightarrow r = 7.$

Now, $r = 7$ cm, $h = 24$ cm. So, $l = \sqrt{(7)^2 + (24)^2} = 25$ cm.

$\therefore$ Curved surface area $= \left(\frac{22}{7} \times 7 \times 25\right) \text{cm}^2 = 550 \text{ cm}^2$.

92. Let the radius of the base be r km. Then,

$$\pi r^2 = 1.54 \Rightarrow r^2 = \left(\frac{1.54 \times 7}{22}\right) = 0.49 \Rightarrow r = 0.7 \text{ km}.$$

Now, $l = 2.5$ km, $r = 0.7$ km.

$\therefore$ $h = \sqrt{(2.5)^2 - (0.7)^2} \text{ km} = \sqrt{6.25 - 0.49} \text{ km} = \sqrt{5.76} \text{ km} = 2.4 \text{ km}$.

So, height of the mountain = 2.4 km.

93. $\pi r^2 = 3850 \Rightarrow r^2 = \left(\frac{3850 \times 7}{22}\right) = 1225 \Rightarrow r = 35.$

Now, $r = 35$ cm, $h = 84$ cm.

So, $l = \sqrt{(35)^2 + (84)^2} = \sqrt{1225 + 7056} = \sqrt{8281} = 91$ cm.

$\therefore$ Curved surface area $= \left(\frac{22}{7} \times 35 \times 91\right) \text{ cm}^2 = 10010 \text{ cm}^2$.

94. $\frac{22}{7} \times 70 \times l = 40040 \Rightarrow l = \left(\frac{40040 \times 7}{22 \times 70}\right) = 182.$

Now, $l = 182$ cm, $r = 70$ cm.

So, $h = \sqrt{(182)^2 - (70)^2} = \sqrt{252 \times 112} = 168$ cm.

$\therefore$ Volume $= \left(\frac{1}{3} \times \frac{22}{7} \times 70 \times 70 \times 168\right) \text{cm}^3 = 862400 \text{ cm}^3.$

95. Let the radius and the height of the cone be $3x$ and $4x$ respectively. Then,

$\frac{1}{3} \times \pi \times (3x)^2 \times 4x = 96\pi \Leftrightarrow 36x^3 = (96 \times 3) \Leftrightarrow x^3 = \left(\frac{96 \times 3}{36}\right) = 8 \Leftrightarrow x = 2.$

$\therefore$ Radius $= 6$ cm, Height $= 8$ cm.

Slant height $= \sqrt{6^2 + 8^2}$ cm $= \sqrt{100}$ cm $= 10$ cm.

96. $\pi r^2 = 346.5 \Rightarrow r^2 = \left(346.5 \times \frac{7}{22}\right) = \frac{441}{4} \Rightarrow r = \frac{21}{2}.$

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{\frac{441}{4} + (14)^2} = \sqrt{\frac{1225}{4}} = \frac{35}{2}.$

So, area of canvas needed $= \pi r l = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2}\right) \text{m}^2 = \left(\frac{33 \times 35}{2}\right) \text{m}^2.$

$\therefore$ Length of canvas $= \left(\frac{33 \times 35}{2 \times 1.1}\right) \text{m} = 525 \text{ m}.$

97. Let the original radius and height of the cone be r and h respectively.

Then, new radius $= 2r$. New height $= 2h$.

$\therefore \frac{\text{New Volume}}{\text{Original Volume}} = \frac{\frac{1}{3} \times \pi \times (2r)^2 \times 2h}{\frac{1}{3} \times \pi \times r^2 \times h} = \frac{8}{1}.$

98. Let the original radius and height of the cone be r and h respectively.

Then, Original volume $= \frac{1}{3} \pi r^2 h.$

New radius $= \frac{120}{100} r = \frac{6}{5} r$, New height $= \frac{6}{5} h.$

New volume $= \frac{1}{3} \pi \times \left(\frac{6}{5} r\right)^2 \times \left(\frac{6}{5} h\right) = \frac{216}{125} \times \frac{1}{3} \pi r^2 h.$

Increase in volume $= \frac{91}{125} \times \frac{1}{3} \pi r^2 h.$

$\therefore$ Increase % $= \left(\frac{\frac{91}{125} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100\right) \% = 72.8\%.$

99. Let the original radius and height of the cone be r and h respectively.

Then, original volume $= \frac{1}{3} \pi r^2 h.$

New radius $= \frac{r}{2}$ and new height $= 3h.$

New volume $= \frac{1}{3} \times \pi \times \left(\frac{r}{2}\right)^2 \times 3h = \frac{3}{4} \times \frac{1}{3} \pi r^2 h.$

$\therefore$ Decrease % $= \left(\frac{\frac{1}{4} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100 \right) \% = 25\%.$

100. Required ratio $= \frac{\frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 \times (2h)} = \frac{1}{2}.$

101. Let their heights be x, $3x$ and their radii be $3y$, y.

Then, Ratio of volumes $= \frac{\frac{1}{3} \times \pi \times (3y)^2 \times x}{\frac{1}{3} \times \pi \times y^2 \times (3x)} = \frac{9}{3} = 3 : 1.$

102. Let their radii be $2x$, x and their heights be h and H respectively. Then,

$\frac{1}{3} \times \pi \times (2x)^2 \times h = \frac{1}{3} \times \pi \times x^2 \times H$ or $\frac{h}{H} = \frac{1}{4}.$

103. Let their radii be $4x$ and $5x$, and their heights be h and H respectively. Then,

$\frac{\frac{1}{3} \times \pi \times (4x)^2 \times h}{\frac{1}{3} \times \pi \times (5x)^2 \times H} = \frac{1}{4}$ or $\frac{h}{H} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}.$

104. Volume of the largest cone

= Volume of the cone with diameter of base 7 cm and height 7 cm

$= \left(\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 7 \right) \text{cm}^3 = \left(\frac{269.5}{3} \right) \text{cm}^3 = 89.8 \text{ cm}^3.$

105. Volume of the block $= (10 \times 5 \times 2) \text{ cm}^3 = 100 \text{ cm}^3.$

Volume of the cone carved out $= \left(\frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7 \right) \text{cm}^3 = 66 \text{ cm}^3.$

$\therefore$ Wood wasted $= (100 - 66)\% = 34\%.$

106. Let their radius and height be $5x$ and $12x$ respectively.

Slant height of the cone, $l = \sqrt{(5x)^2 + (12x)^2} = 13x.$

$\frac{\text{Total surface area of cylinder}}{\text{Total surface area of cone}} = \frac{2\pi r (h + r)}{\pi r (l + r)} = \frac{2(h + r)}{(l + r)} = \frac{2 \times (12x + 5x)}{(13x + 5x)} = \frac{34x}{18x} = \frac{17}{9}.$

107. Let the radius of the cone be r cm.

Then, $\frac{1}{3} \pi \times r^2 \times 6 = \pi \times 8 \times 8 \times 2 \Leftrightarrow r^2 = \left(\frac{8 \times 8 \times 2 \times 3}{6} \right) = 64 \Leftrightarrow r = 8 \text{ cm}.$

108. Let radius of each be r and height of each be h.

Then, number of cones needed $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 cone}} = \frac{\pi r^2 h}{\frac{1}{3} \pi r^2 h} = 3.$

109. Volume of cylinder $= (\pi \times 3 \times 3 \times 5) \text{ cm}^3 = 45\pi \text{ cm}^3.$

Volume of 1 cone $= \left(\frac{1}{3} \pi \times \frac{1}{10} \times \frac{1}{10} \times 1 \right) \text{cm}^3 = \frac{\pi}{300} \text{ cm}^3.$

$\therefore$ Number of cones $= \left(45\pi \times \frac{300}{\pi} \right) = 13500.$

110. Volume flown in conical vessel $= \frac{1}{3}\pi \times (20)^2 \times 24 = 3200\pi$.

Volume flown in 1 min. $= \left(\pi \times \frac{2.5}{10} \times \frac{2.5}{10} \times 1000\right) = 62.5\pi$.

$\therefore$ Time taken $= \left(\frac{3200\pi}{62.5\pi}\right) = 51$ min. 12 sec.

111. Slant height of the cone, $l = \sqrt{(12)^2 + (5)^2} = 13$ cm.

Lateral surface of the solid = Curved surface of cone + Curved surface of cylinder + Surface area of bottom

$= \pi rl + 2\pi rh + \pi r^2$, where h is the height of the cylinder

$= \pi r\,(l + h + r) = \left[\frac{22}{7} \times 12 \times (13 + 18 + 12)\right] \text{cm}^2$

$= \left(\frac{22}{7} \times 12 \times 43\right) \text{cm}^2 = \left(\frac{11352}{7}\right) \text{cm}^2 = 1621\frac{5}{7} \text{cm}^2$.

112. Volume of parallelopiped $= (5 \times 3 \times 4)\ \text{cm}^3 = 60\ \text{cm}^3$.

Volume of cube $= (4)^3\ \text{cm}^3 = 64\ \text{cm}^3$.

Volume of cylinder $= \left(\frac{22}{7} \times 3 \times 3 \times 3\right) \text{cm}^3 = 84.86\ \text{cm}^3$.

Volume of sphere $= \left(\frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3\right) = 113.14\ \text{cm}^3$.

113. $\frac{4}{3} \times \frac{22}{7} \times R^3 = 4851 \Rightarrow R^3 = \left(4851 \times \frac{3}{4} \times \frac{7}{22}\right) = \left(\frac{21}{2}\right)^3 \Rightarrow R = \frac{21}{2}$.

$\therefore$ Curved surface area $= \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2 = 1386\ \text{cm}^2$.

114. $4\pi R^2 = 5544 \Rightarrow R^2 = \left(5544 \times \frac{1}{4} \times \frac{7}{22}\right) = 441 \Rightarrow R = 21$.

$\therefore$ Volume $= \left(\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\right) \text{cm}^3 = 38808\ \text{cm}^3$.

115. Volume $= \frac{4}{3}\pi r^3 = \frac{r}{3}(4\pi r^2) = \frac{r}{3} \times$ Surface area.

116. $\frac{\frac{4}{3}\pi R^3}{4\pi R^2} = 27 \Rightarrow R = 81$ cm.

117. Let the radii of A and B be r and R respectively.

$\therefore$ Required ratio $= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{40}{10}\right)^2 = 16:1$.

118. Let the original radius be r.

Then, original surface area $= 4\pi r^2 = 2464\ \text{cm}^2$ (given).

New radius $= 2r$.

$\therefore$ New surface area $= 4\pi\,(2r)^2 = 4 \times 4\pi r^2 = (4 \times 2464)\ \text{cm}^2 = 9856\ \text{cm}^2$.

119. Let the original radius be r. Then, original volume $= \frac{4}{3}\pi r^3$.

New radius $= 2r$.

$\therefore$ New volume $= \frac{4}{3}\pi\,(2r)^3 = 8 \times \frac{4}{3}\pi r^3 = 8 \times$ original volume.

120. $4\pi (r+2)^2 - 4\pi r^2 = 352 \Leftrightarrow (r+2)^2 - r^2 = \left(352 \times \frac{7}{22} \times \frac{1}{4}\right) = 28.$

$\Leftrightarrow (r+2+r)(r+2-r) = 28 \Leftrightarrow 2r+2 = 14 \Rightarrow r = \left(\frac{14}{2} - 1\right) = 6$ cm.

121. Let the correct radius be 100 cm. Then, measured radius = 101.5 cm.

$\therefore$ Error in volume $= \frac{4}{3}\pi[(101.5)^3 - (100)^3]$ cm^3

$= \frac{4}{3}\pi(1045678.375 - 1000000)$ cm^3 $= \left(\frac{4}{3} \times \pi \times 45678.375\right)$ cm^3.

$\therefore$ Error % $= \left\{\frac{\frac{4}{3}\pi(45678.375)}{\frac{4}{3}\pi(100 \times 100 \times 100)} \times 100\right\}\% = 4.56\% = 4.6\%$ (app.).

122. Let their radii be R and r. Then,

$$\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \frac{64}{27} = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{4}{3}.$$

Ratio of surface areas $= \frac{4\pi R^2}{4\pi r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$.

123. Let their radii be R and r. Then,

$$\frac{4\pi R^2}{4\pi r^2} = \frac{4}{25} \Rightarrow \left(\frac{R}{r}\right)^2 = \left(\frac{2}{5}\right)^2 \Rightarrow \frac{R}{r} = \frac{2}{5}.$$

$\therefore$ Ratio of volumes $= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \left(\frac{R}{r}\right)^3 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$.

124. Volume of new sphere $= \left[\frac{4}{3}\pi \times (6)^3 + \frac{4}{3}\pi \times (8)^3 + \frac{4}{3}\pi \times (10)^3\right]$ cm^3

$= \left\{\frac{4}{3}\pi[(6)^3 + (8)^3 + (10)^3]\right\}$ cm^3

$= \left(\frac{4}{3}\pi \times 1728\right)$ cm^3 $= \left[\frac{4}{3}\pi \times (12)^3\right]$ cm^3.

Let the radius of the new sphere be R. Then,

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (12)^3 \Rightarrow R = 12$ cm.

$\therefore$ Diameter $= 2R = 24$ cm.

125. Volume of bigger sphere $= \left[\frac{4}{3}\pi \times (8)^3\right]$ cm^3 $= \left(\frac{4}{3}\pi \times 512\right)$ cm^3.

Volume of 1 ball $= \left[\frac{4}{3}\pi \times (2)^3\right]$ cm^3 $= \left(\frac{4}{3}\pi \times 8\right)$ cm^3.

$\therefore$ Number of balls $= \left(\frac{\frac{4}{3}\pi \times 512}{\frac{4}{3}\pi \times 8}\right) = \frac{512}{8} = 64.$

126. Let the radius of the third ball be R cm. Then,

$$\frac{4}{3}\pi\times\left(\frac{3}{4}\right)^3+\frac{4}{3}\pi\times(1)^3+\frac{4}{3}\pi\times R^3=\frac{4}{3}\pi\times\left(\frac{3}{2}\right)^3$$

$$\Rightarrow\quad \frac{27}{64}+1+R^3=\frac{27}{8}\ \Rightarrow\ R^3=\frac{125}{64}=\left(\frac{5}{4}\right)^3\ \Rightarrow\ R=\frac{5}{4}.$$

$\therefore$ Diameter of the third ball $= 2R = \frac{5}{2}$ cm $= 2.5$ cm.

127. Volume of each ball $= \frac{1}{8}\times\left(\frac{4}{3}\pi\times10\times10\times10\right)\text{cm}^3$.

$$\therefore\quad \frac{4}{3}\pi R^3=\frac{1}{8}\times\frac{4}{3}\pi\times10\times10\times10\ \Rightarrow\ R^3=\left(\frac{10}{2}\right)^3=5^3\ \Rightarrow\ R=5.$$

128. External radius $= 3$ cm, Internal radius $= (3 - 0.5)$ cm $= 2.5$ cm.

$$\text{Volume of the metal} = \left[\frac{4}{3}\times\frac{22}{7}\times\{(3)^3-(2.5)^3\}\right]\text{cm}^3$$

$$=\left(\frac{4}{3}\times\frac{22}{7}\times\frac{91}{8}\right)\text{cm}^3=\left(\frac{143}{3}\right)\text{cm}^3=47\frac{2}{3}\text{cm}^3.$$

129. Volume of the solid $= (49\times33\times24)\ \text{cm}^3$.

Let the radius of the sphere be r.

Then, $\frac{4}{3}\pi r^3=(49\times33\times24)\ \Leftrightarrow\ r^3=\left(\frac{49\times33\times24\times3\times7}{4\times22}\right)=(21)^3\ \Leftrightarrow\ r=21.$

130. Number of bullets $= \dfrac{\text{Volume of the cube}}{\text{Volume of 1 bullet}}=\left(\dfrac{22\times22\times22}{\frac{4}{3}\times\frac{22}{7}\times1\times1\times1}\right)=2541.$

131. Volume of each lead shot $=\left[\frac{4}{3}\pi\times\left(\frac{0.3}{2}\right)^3\right]\text{cm}^3=\left(\frac{4}{3}\times\frac{22}{7}\times\frac{27}{8000}\right)\text{cm}^3=\frac{99}{7000}\text{cm}^3.$

$\therefore$ Number of lead shots $=\left(9\times11\times12\times\frac{7000}{99}\right)=84000.$

132. $4\pi R^2=6a^2\ \Rightarrow\ \frac{R^2}{a^2}=\frac{3}{2\pi}\ \Rightarrow\ \frac{R}{a}=\frac{\sqrt{3}}{\sqrt{2\pi}}.$

$$\frac{\text{Volume of sphere}}{\text{Volume of cube}}=\frac{\frac{4}{3}\pi R^3}{a^3}=\frac{4}{3}\pi\cdot\left(\frac{R}{a}\right)^3=\frac{4}{3}\pi\cdot\frac{3\sqrt{3}}{2\pi\sqrt{2\pi}}=\frac{2\sqrt{3}}{\sqrt{2\pi}}=\frac{\sqrt{12}}{\sqrt{2\pi}}=\frac{\sqrt{6}}{\sqrt{\pi}}.$$

133. Let the edge of the cube be a. Then, volume of the cube $= a^3$.

Radius of the sphere $= (a/2)$.

$$\text{Volume of the sphere}=\frac{4}{3}\pi\left(\frac{a}{2}\right)^3=\frac{\pi a^3}{6}.$$

$\therefore$ Required ratio $= a^3:\frac{\pi a^3}{6}=6:\pi.$

134. $4\pi R^2=2\pi\times6\times12\ \Rightarrow\ R^2=\left(\frac{6\times12}{2}\right)=36\ \Rightarrow\ R=6$ cm.

135. Let the radius of the cylinder be R.

Then, $\pi \times R^2 \times \frac{7}{3} = \frac{4}{3}\pi \times 7 \times 7 \times 7$

$\Rightarrow \quad R^2 = \left(\frac{4 \times 7 \times 7 \times 7}{3} \times \frac{3}{7}\right) = 196 = (14)^2 \Rightarrow R = 14$ cm.

$\therefore$ Diameter $= 2R = 28$ cm.

136. Required volume = Volume of a sphere of radius 1 cm

$$= \left(\frac{4}{3}\pi \times 1 \times 1 \times 1\right) \text{cm}^3 = \frac{4}{3}\pi \text{ cm}^3.$$

137. Volume of cylinder $= \pi \times (3)^2 \times 15 = 135\pi$ cm^3.

Radius of 1 bullet $= \frac{5}{2}$ mm $= \frac{5}{20}$ cm $= \frac{1}{4}$ cm.

Volume of 1 bullet $= \left(\frac{4}{3}\pi \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$ cm$^3 = \frac{\pi}{48}$ cm^3.

$\therefore$ Number of bullets $= \left(135\pi \times \frac{48}{\pi}\right) = 6480$.

138. Let the radius of the cylindrical rod be r.

Then, height of the rod $= 8r$ and radius of one ball $= \frac{r}{2}$.

$\therefore$ Number of balls $= \dfrac{\pi \times r^2 \times 8r}{\frac{4}{3}\pi \times \left(\frac{r}{2}\right)^3} = \left(\frac{8 \times 8 \times 3}{4}\right) = 48.$

139. Let the length of the wire be h.

Then, $\pi \times \frac{3}{20} \times \frac{3}{20} \times h = \frac{4}{3}\pi \times 4 \times 4 \times 4$

$\Leftrightarrow \quad h = \left(\frac{4 \times 4 \times 4 \times 4 \times 20 \times 20}{3 \times 3 \times 3}\right)$ cm $= \left(\frac{102400}{27}\right)$ cm $= 3792.5$ cm $= 37.9$ m.

140. Let the rise in the water level be h cm.

Then, $\pi \times 4 \times 4 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{3 \times 3}{4}\right) = \frac{9}{4}$ cm.

141. Let the radius of each sphere be r cm.

Then, Volume of 12 spheres = Volume of cylinder

$\Rightarrow \quad 12 \times \frac{4}{3}\pi \times r^3 = \pi \times 8 \times 8 \times 2 \Rightarrow r^3 = \left(\frac{8 \times 8 \times 2 \times 3}{12 \times 4}\right) = 8 \Rightarrow r = 2$ cm.

$\therefore$ Diameter of each sphere $= 2r = 4$ cm.

142. Let the radius of the ball be r cm.

Volume of ball = Volume of water displaced by it

$\therefore \quad \frac{4}{3}\pi r^3 = \pi \times 12 \times 12 \times 6.75 \Rightarrow r^3 = 9 \times 9 \times 9 \Rightarrow r = 9$ cm.

143. Let the height of the cone be h cm. Then,

$\frac{1}{3}\pi \times 6 \times 6 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{36 \times 3}{36}\right) = 3$ cm.

144. Volume of sphere $= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) cm^3$.

Volume of cone $= \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right) cm^3$.

Volume of wood wasted $= \left[\left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) - \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right)\right] cm^3$

$= (\pi \times 9 \times 9 \times 9)\ cm^3$.

$\therefore$ Required percentage $= \left(\frac{\pi \times 9 \times 9 \times 9}{\frac{4}{3}\pi \times 9 \times 9 \times 9} \times 100\right)\% = \left(\frac{3}{4} \times 100\right)\% = 75\%$.

145. Number of spheres $= \frac{\text{Volume of cone}}{\text{Volume of 1 sphere}} = \frac{\frac{1}{3}\pi \times 12 \times 12 \times 24}{\frac{4}{3}\pi \times 2 \times 2 \times 2} = 108$.

146. Volume of material in the sphere $= \left[\frac{4}{3}\pi \times \{(4)^3 - (2)^3\}\right] cm^3 = \left(\frac{4}{3}\pi \times 56\right) cm^3$.

Let the height of the cone be h cm.

Then, $\frac{1}{3}\pi \times 4 \times 4 \times h = \left(\frac{4}{3}\pi \times 56\right) \Leftrightarrow h = \left(\frac{4 \times 56}{4 \times 4}\right) = 14$ cm.

147. Let radius = R and height = H. Then,

Ratio of their volumes $= \pi R^2 H : \frac{1}{3}\pi R^2 H : \frac{4}{3}\pi R^3 = H : \frac{1}{3}H : \frac{4}{3}R$

$= H : \frac{1}{3}H : \frac{4}{3} \times \frac{H}{2}$ $\left[\text{In sphere, } H = 2R \text{ or } R = \frac{H}{2}\right]$

$= 3 : 1 : 2$.

148. Total surface area $= 3\pi R^2 = \left(3 \times \frac{22}{7} \times 7 \times 7\right) cm^2 = 462\ cm^2$.

149. Let the radius be R cm. Then,

$\frac{2}{3} \times \frac{22}{7} \times R^3 = 19404 \Leftrightarrow R^3 = \left(19404 \times \frac{21}{44}\right) = (21)^3 \Leftrightarrow R = 21$ cm.

150. Let their radii be R and r. Then,

$\frac{\frac{2}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{6.4}{21.6} \Leftrightarrow \left(\frac{R}{r}\right)^3 = \frac{8}{27} = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{R}{r} = \frac{2}{3}$.

$\therefore$ Ratio of curved surface areas $= \frac{2\pi R^2}{2\pi r^2} = \left(\frac{R}{r}\right)^2 = \frac{4}{9}$.

151. Let the height of the vessel be x. Then, radius of the bowl = radius of the vessel $= \frac{x}{2}$.

Volume of the bowl, $V_1 = \frac{2}{3}\pi\left(\frac{x}{2}\right)^3 = \frac{1}{12}\pi x^3$.

Volume of the vessel, $V_2 = \pi\left(\frac{x}{2}\right)^2 x = \frac{1}{4}\pi x^3$.

Since $V_2 > V_1$, so the vessel can contain 100% of the beverage filled in the bowl.

152. $\frac{2}{3}\pi R^3 = \frac{1}{3}\pi R^2 H \Rightarrow H = 2R.$

153. Let the radius of the cone be R cm. Then,

$$\frac{1}{3}\pi \times R^2 \times 75 = \frac{2}{3}\pi \times 6 \times 6 \times 6$$

$$\Leftrightarrow R^2 = \left(\frac{2\times 6\times 6\times 6}{75}\right) = \left(\frac{144}{25}\right) = \left(\frac{12}{5}\right)^2 \Leftrightarrow R = \frac{12}{5}\text{cm} = 2.4\text{ cm}.$$

154. Let the radius of each be R. Height of hemisphere, H = R.

So, height of cone = height of hemisphere = R.

Slant height of cone = $\sqrt{R^2 + R^2} = \sqrt{2}\,R$.

$$\frac{\text{Curved surface area of hemisphere}}{\text{Curved surface area of cone}} = \frac{2\pi R^2}{\pi R \times \sqrt{2}\,R} = \sqrt{2} : 1.$$

155. Volume of hemisphere = $\frac{2}{3}\pi r^3$.

Volume of biggest sphere = Volume of sphere with diameter $r = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{1}{6}\pi r^3$

$$\therefore \text{ Required ratio} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{6}\pi r^3} = \frac{4}{1} \text{ i.e. } 4:1.$$

EXERCISE 25B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 10*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the weight of the iron beam ?
   I. The beam is 9 m long, 40 cm wide and 20 cm high.
   II. Iron weighs 50 kg per cubic metre.
2. What is the volume of 32 metre high cylindrical tank ? **(Bank P.O. 2003)**
   I. The area of its base is 154 m^2.
   II. The diameter of the base is 14 m.
3. What is the volume of a cube ? **(Bank P.O. 2003)**
   I. The area of each face of the cube is 64 square metres.
   II. The length of one side of the cube is 8 metres.

4. What is the total cost of painting the inner surface of an open box at the rate of 50 paise per 100 sq. cm ?
   I. The box is made of wood 3 cm thick.
   II. The external dimensions of the box are 50 cm, 40 cm and 23 cm.
5. What is the capacity of a cylindrical tank ? **(I.B.P.S. 2002)**
   I. Radius of the base is half of its height which is 28 metres.
   II. Area of the base is 616 sq. metres and its height is 28 metres.
6. What is the volume of the cylinder ? **(Bank P.O. 2003)**
   I. Height is equal to the diameter.
   II. Perimeter of the base is 352 cm.
7. What will be the total cost of whitewashing the conical tomb at the rate of 80 paise per square metre ?
   I. The diameter and the slant height of the tomb are 28 m and 50 m.
   II. The height of the tomb is 48 m and the area of its base is 616 sq. m.
8. What is the height of a circular cone ? **(Bank P.O. 1999)**
   I. The area of that cone is equal to the area of a rectangle whose length is 33 cm.
   II. The area of the base of that cone is 154 sq. cm.
9. Is a given rectangular block, a cube ? **(M.A.T. 1999)**
   I. At least 2 faces of the rectangular block are squares.
   II. The volume of the block is 64.
10. A spherical ball of given radius x cm is melted and made into a right circular cylinder. What is the height of the cylinder ? **(S.B.I.P.O. 2003)**
    I. The volume of the cylinder is equal to the volume of the ball.
    II. The area of the base of the cylinder is given.

Directions (*Questions 11-13*) : *Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.*

11. What is the capacity of the cylindrical tank ? **(R.B.I. 2003)**
    I. The area of the base is 61,600 sq. cm.
    II. The height of the tank is 1.5 times the radius.
    III. The circumference of base is 880 cm.
    (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
    (*d*) Any two of the three (*e*) Only II and either I or III
12. A solid metallic cone is melted and recast into a sphere. What is the radius of the sphere ?
    I. The radius of the base of the cone is 2.1 cm.
    II. The height of the cone is four times the radius of its base.
    III. The height of the cone is 8.4 cm.
    (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
    (*d*) Any two of the three (*e*) All I, II and III
13. What is the total surface area of the cone ?
    I. The area of the base of the cone is 154 cm^2.
    II. The curved surface area of the cone is 550 cm^2.
    III. The volume of the cone is 1232 cm^3.
    (*a*) I, and either II or III (*b*) II, and either I or III
    (*c*) III, and either I or II (*d*) Any two of the three
    (*e*) None of these

ANSWERS

1. (e) 2. (c) 3. (c) 4. (e) 5. (c) 6. (e) 7. (c) 8. (d)
9. (d) 10. (b) 11. (e) 12. (d) 13. (a)

SOLUTIONS

1. I gives, $l = 9$ m, $b = \frac{40}{100}$ m $= \frac{2}{5}$ m and $h = \frac{20}{100}$ m $= \frac{1}{5}$ m.

This gives, volume $= (l \times b \times h) = \left(9 \times \frac{2}{5} \times \frac{1}{5}\right) m^3 = \frac{18}{25} m^3$.

II gives, weight of iron is 50 kg / m^3.

$\therefore$ Weight $= \left(\frac{18}{25} \times 50\right)$ kg $= 36$ kg.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (e).

2. Given, height = 32 m.

I gives, area of the base = 154 m^2.

$\therefore$ Volume = (area of the base × height) = (154 × 32) m^3 = 4928 m^3.

Thus, I alone gives the answer.

II gives, radius of the base = 7 m.

$\therefore$ Volume $= \pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 32\right) m^3 = 4928\ m^3$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

3. Let each edge be a metres. Then,

I. $a^2 = 64 \Rightarrow a = 8$ m $\Rightarrow$ Volume = (8 × 8 × 8) m^3 = 512 m^3.

Thus, I alone gives the answer.

II. $a = 8$ m $\Rightarrow$ Volume = (8 × 8 × 8) m^3 = 512 m^3.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (c).

4. I gives, thickness of the wall of the box = 3 cm.

II gives, Internal length = (50 − 6) cm = 44 cm, Internal breadth = (40 − 6) = 34 cm,
Internal height = (23 − 3) cm = 20 cm.

Area to be painted = (area of 4 walls + area of floor) = $[2(l + b) \times h + (l \times b)]$
$= [2(44 + 34) \times 20 + (44 \times 34)]\ cm^2 = 4616\ cm^2$.

Cost of painting = Rs. $\left(\frac{1}{2 \times 100} \times 4616\right)$ = Rs. 23.08.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (e).

5. I gives, $h = 28$ m and $r = 14$ cm.

$\therefore$ Capacity $= \pi r^2 h$, which can be obtained.

Thus, I alone gives the answer.

II gives, $\pi r^2 = 616$ m^2 and $h = 28$ m.

$\therefore$ Capacity $= (\pi r^2 \times h) = (616 \times 28)$ m^3.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

6. I gives, $h = 2r$.

II gives, $2\pi r = 352 \Rightarrow r = \left(\frac{352}{2} \times \frac{7}{22}\right)$ cm $= 56$ cm.

From I and II, we have $r = 56$ cm, $h = (2 \times 56)$ cm $= 112$ cm.

Thus, we can find the volume.

$\therefore$ Correct answer is (*e*).

7. I gives, $r = 14$ m, $l = 50$ m.

$\therefore$ Curved surface $= \pi rl = \left(\frac{22}{7} \times 14 \times 50\right)$ m$^2 = 2200$ m^2.

Cost of whitewashing $=$ Rs. $\left(2200 \times \frac{80}{100}\right)$ $=$ Rs. 1760.

Thus, I alone gives the answer.

II gives, $h = 48$ m, $\pi r^2 = 616$ m^2.

These results give r and h and so l can be found out.

$\therefore$ Curved surface $= \pi rl$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

8. II gives the value of r.

But, in I, the breadth of rectangle is not given.

So, we cannot find the surface area of the cone.

Hence, the height of the cone cannot be determined.

$\therefore$ Correct answer is (*d*).

9. I gives, any two of l, b, h are equal.

II gives, $lbh = 64$.

From I and II, the values of l, b, h may be (1, 1, 64), (2, 2, 16), (4, 4, 4).

Thus, the block may be a cube or cuboid.

$\therefore$ Correct answer is (*d*).

10. Clearly, I is not needed, since it is evident from the given question.

From II, we get radius of the base of the cylinder.

Now, $\frac{4}{3}\pi x^3 = \pi r^2 h$ in which x and r are known.

$\therefore$ h can be determined.

$\therefore$ Correct answer is (*b*).

11. Capacity $= \pi r^2 h$.

I gives, $\pi r^2 = 61600$. This gives r.

II gives, $h = 1.5\ r$.

Thus, I and II give the answer.

Again, III gives $2\pi r = 880$. This gives r.

So, II and III also give the answer.

$\therefore$ Correct answer is (*e*).

12. $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$.

Now r and h can be determined from any two of I, II and III.

Thus, R can be calculated.

$\therefore$ Correct answer is (*d*).

13 Total surface area of the cone = $(\pi rl + \pi r^2)$ cm^2.

I gives, $\pi r^2 = 154$. Thus, we can find r.

II gives, $\pi rl = 550$.

From I and II we get the answer.

III gives, $\frac{1}{3}\pi r^2 h = 1232$.

From I and III, we can find h and therefore, l.

Hence the surface area can be determined.

$\therefore$ Correct answer is (a).

26. RACES AND GAMES OF SKILL

IMPORTANT FACTS

Races : *A contest of speed in running, riding, driving, sailing or rowing is called a race.*

Race Course : *The ground or path on which contests are made is called a race course.*

Starting Point : *The point from which a race begins is known as a starting point.*

Winning Point or Goal : *The point set to bound a race is called a winning point or a goal.*

Winner : *The person who first reaches the winning point is called a winner.*

Dead Heat Race : *If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.*

Start : Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that '*A gives B, a start of 12 metres*'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only (100 – 12) = 88 metres.

In a 100 m race, '*A can give B 12 m*' or '*A can give B a start of 12 m*' or '*A beats B by 12 m*' means that while A runs 100 m, B runs (100 – 12) = 88 m.

Games : '*A game of 100, means that the person among the contestants who scores 100 points first is the winner*'.

If A scores 100 points while B scores only 80 points, then we say that '*A can give B 20 points*'.

SOLVED EXAMPLES

Ex. 1. ***In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.***

Sol. Clearly, B covers 28 m in 7 seconds.

$\therefore$ B's time over the course $= \left(\frac{7}{28} \times 1000\right)$ sec $= 250$ seconds.

$\therefore$ A's time over the course $= (250 - 7)$ sec $= 243$ sec $= 4$ min. 3 sec.

Ex. 2. ***A runs $1\frac{3}{4}$ times as fast as B. If A gives B a start of 84 m, how far must the winning post be so that A and B might reach it at the same time?***

Sol. Ratio of the rates of A and B $= \frac{7}{4} : 1 = 7 : 4$.

So, in a race of 7 m, A gains 3 m over B.

$\therefore$ 3 m are gained by A in a race of 7 m.

$\therefore$ 84 m are gained by A in a race of $\left(\frac{7}{3} \times 84\right)$ m $= 196$ m.

$\therefore$ Winning post must be 196 m away from the starting point.

Ex. 3. ***A can run 1 km in 3 min. 10 sec. and B can cover the same distance in 3 min. 20 sec. By what distance can A beat B?***

Sol. Clearly, A beats B by 10 sec.

Distance covered by B in 10 sec. $= \left(\frac{1000}{200} \times 10\right)$ m $= 50$ m.

$\therefore$ A beats B by 50 metres.

Ex. 4. ***In a 100 m race, A runs at 8 km per hour. If A gives B a start of 4 m and still beats him by 15 seconds, what is the speed of B ?***

Sol. Time taken by A to cover 100 m $= \left(\frac{60 \times 60}{8000} \times 100\right)$ sec $= 45$ sec.

$\therefore$ B covers $(100 - 4)$ m $= 96$ m in $(45 + 15)$ sec $= 60$ sec.

$\therefore$ B's speed $= \left(\frac{96 \times 60 \times 60}{60 \times 1000}\right)$ km/hr $= 5.76$ km/hr.

Ex. 5. ***A, B and C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many metre's start can B give C ?***

Sol. While A covers 1000 m, B covers $(1000 - 40)$ m $= 960$ m and C covers $(1000 - 64)$ m or 936 m.

When B covers 960 m, C covers 936 m.

When B covers 1000 m, C covers $\left(\frac{936}{960} \times 1000\right)$ m $= 975$ m.

$\therefore$ B can give C a start of $(1000 - 975)$ or 25 m.

Ex. 6. ***In a game of 80 points, A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60 ?***

Sol. A : B = 80 : 75, A : C = 80 : 65.

$$\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{75}{80} \times \frac{80}{65}\right) = \frac{15}{13} = \frac{60}{52} = 60 : 52.$$

$\therefore$ In a game of 60, B can give C 8 points.

EXERCISE 26

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. In a 100 m race, A covers the distance in 36 seconds and B in 45 seconds. In this race A beats B by :
   (a) 20 m (b) 25 m (c) 22.5 m (d) 9 m
2. In a 200 metres race A beats B by 35 m or 7 seconds. A's time over the course is :
   (a) 40 sec (b) 47 sec (c) 33 sec (d) None of these
3. In a 300 m race A beats B by 22.5 m or 6 seconds. B's time over the course is :
   (a) 86 sec (b) 80 sec (c) 76 sec (d) None of these
4. A can run 22.5 m while B runs 25 m. In a kilometre race B beats A by :
   (a) 100 m (b) $111\frac{1}{9}$ m (c) 25 m (d) 50 m
5. In a 500 m race, the ratio of the speeds of two contestants A and B is 3 : 4. A has a start of 140 m. Then, A wins by :
   (a) 60 m (b) 40 m (c) 20 m (d) 10 m

6. A runs $1\frac{2}{3}$ times as fast as B. If A gives B a start of 80 m, how far must the winning post be so that A and B might reach it at the same time ?

(a) 200 m (b) 300 m (c) 270 m (d) 160 m

7. In a 100 m race, A can beat B by 25 m and B can beat C by 4 m. In the same race, A can beat C by :

(a) 21 m (b) 26 m (c) 28 m (d) 29 m

8. In a 100 m race, A can give B 10 m and C 28 m. In the same race B can give C :

(a) 18 m (b) 20 m (c) 27 m (d) 9 m

9. In a 100 m race, A beats B by 10 m and C by 13 m. In a race of 180 m, B will beat C by :

(a) 5.4 m (b) 4.5 m (c) 5 m (d) 6 m

10. In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 350 m, C will beat B by :

(a) 22.75 m (b) 25 m (c) 19.5 m (d) $7\frac{4}{7}$ m

11. A and B take part in a 100 m race. A runs at 5 km per hour. A gives B a start of 8 m and still beats him by 8 seconds. The speed of B is :

(a) 5.15 kmph (b) 4.14 kmph (c) 4.25 kmph (d) 4.4 kmph

12. In a game of 100 points, A can give B 20 points and C 28 points. Then, B can give C :

(a) 8 points (b) 10 points (c) 14 points (d) 40 points

13. At a game of billiards, A can give B 15 points in 60 and A can give C 20 points in 60. How many points can B give C in a game of 90 ?

(a) 30 points (b) 20 points (c) 10 points (d) 12 points

ANSWERS

1. (a)	2. (c)	3. (b)	4. (a)	5. (c)	6. (a)	7. (c)	8. (b)
9. (d)	10. (b)	11. (b)	12. (b)	13. (c)			

SOLUTIONS

1. Distance covered by B in 9 sec. $= \left(\frac{100}{45} \times 9\right)$ m = 20 m.

∴ A beats B by 20 metres.

2. B runs 35 m in 7 sec.

∴ B covers 200 m in $\left(\frac{7}{35} \times 200\right)$ = 40 sec.

B's time over the course = 40 sec.

∴ A's time over the course = (40 − 7) sec = 33 sec.

3. B runs $\frac{45}{2}$ m in 6 sec.

∴ B covers 300 m in $\left(6 \times \frac{2}{45} \times 300\right)$ sec = 80 sec.

Time taken by A to cover 100 m = $\left(100 \times \frac{18}{25}\right)$ sec = 72 sec.

$\therefore$ Time taken by B to cover 92 m = (72 + 8) sec = 80 sec.

$\therefore$ B's speed = $\left(\frac{92}{80} \times \frac{18}{5}\right)$ kmph = 4.14 kmph.

12. A : B = 100 : 80 and A : C = 100 : 72.

$\therefore$ $\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{80}{100} \times \frac{100}{72}\right) = \frac{10}{9} = \frac{100}{90}$ = 100 : 90.

$\therefore$ B can give C 10 points.

13. A : B = 60 : 45 and A : C = 60 : 40.

$\therefore$ $\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{45}{60} \times \frac{60}{40}\right) = \frac{45}{40} = \frac{90}{80}$ = 90 : 80.

$\therefore$ B can give C 10 points in a game of 90.

27. CALENDAR

IMPORTANT FACTS AND FORMULAE

Under this heading we mainly deal with finding the day of the week on a particular given date. The process of finding it lies on obtaining the number of odd days.

I. **Odd Days** : Number of days more than the complete number of weeks in a given period is the number of odd days during that period.

II. **Leap Year** : Every year which is divisible by 4 is called a leap year.

Thus, each one of the years 1992, 1996, 2004, 2008, 2012, etc. is a leap year.

Every 4th century is a leap year but no other century is a leap year.

Thus, each one of 400, 800, 1200, 1600, 2000, etc. is a leap year.

None of 1900, 2010, 2020, 2100, etc. is a leap year.

An year which is not a leap year is called an ordinary year.

III. (*i*) *An ordinary year has 365 days.* (*ii*) *A leap year has 366 days.*

IV. **Counting of Odd Days :**

(*i*) 1 ordinary year = 365 days = (52 weeks + 1 day).

∴ *An ordinary year has 1 odd day.*

(*ii*) 1 leap year = 366 days = (52 weeks + 2 days).

∴ *A leap year has 2 odd days.*

(*iii*) 100 years = 76 ordinary years + 24 leap years

= [(76 × 52) weeks + 76 days] + [(24 × 52) weeks + 48 days]

= 5200 weeks + 124 days = (5217 weeks + 5 days).

∴ *100 years contain 5 odd days.*

200 years contain 10 and therefore 3 odd days.

300 years contain 15 and therefore 1 odd day.

400 years contain (20 + 1) *and therefore 0 odd day.*

Similarly, each one of 800, 1200, 1600, 2000, etc. contains 0 odd days.

Remark : (7*n* + *m*) odd days, where *m* < 7 is equivalent to *m* odd days.

Thus, 8 odd days ≡ 1 odd day etc.

V.

No. of odd days	0	1	2	3	4	5	6
Day	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.

SOLVED EXAMPLES

Ex. 1. ***What was the day of the week on 16th July, 1776 ?***

Sol. 16th July, 1776 = (1775 years + Period from 1st Jan., 1776 to 16th July, 1776)

Counting of odd days :

1600 years have 0 odd day. 100 years have 5 odd days.

75 years = (18 leap years + 57 ordinary years)

= [(18 × 2) + (57 × 1)] odd days = 93 odd days

= (13 weeks + 2 days) = 2 odd days.

∴ 1775 years have (0 + 5 + 2) odd days = 7 odd days = 0 odd day.

Jan. Feb. March April May June July

31 + 29 + 31 + 30 + 31 + 30 + 16 = 198 days

= (28 weeks + 2 days) = 2 odd days.

∴ Total number of odd days = (0 + 2) = 2. Required day was **'Tuesday'**.

Ex. 2. ***What was the day of the week on 15th August, 1947?***

Sol. 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th Aug., 1947)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

47 years = (11 leap years + 36 ordinary years)

= [(11 × 2) + (36 × 1)] odd days = 58 odd days = 2 odd days.

Jan. Feb. March April May June July Aug.

31 + 28 + 31 + 30 + 31 + 30 + 31 + 15

= 227 days = (32 weeks + 3 days) = 3 odd days.

Total number of odd days = (0 + 1 + 2 + 3) odd days = 6 odd days.

Hence, the required day was **'Saturday'**.

Ex. 3. ***What was the day of the week on 16th April, 2000?***

Sol. 16th April, 2000 = (1999 years + Period from 1st Jan., 2000 to 16th April, 2000)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

99 years = (24 leap years + 75 ordinary years)

= [(24 × 2) + (75 × 1)] odd days = 123 odd days

= (17 weeks + 4 days) = 4 odd days.

Jan. Feb. March April

31 + 29 + 31 + 16 = 107 days = (15 weeks + 2 days) = 2 odd days.

Total number of odd days = (0 + 1 + 4 + 2) odd days = 7 odd days = 0 odd day.

Hence, the required day was **'Sunday'**.

Ex. 4. ***On what dates of July 2004 did Monday fall?***

Sol. Let us find the day on 1st July, 2004.

2000 years have 0 odd day. 3 ordinary years have 3 odd days.

Jan. Feb. March April May June July

31 + 29 + 31 + 30 + 31 + 30 + 1

= 183 days = (26 weeks + 1 day) = 1 odd day.

Total number of odd days = (0 + 3 + 1) odd days = 4 odd days.

∴ 1st July 2004 was 'Thursday'.

Thus, 1st Monday in July 2004 was on 5th July.

Hence, during July 2004, Monday fell on 5th, 12th, 19th and 26th.

Ex. 5. ***Prove that the calendar for the year 2003 will serve for the year 2014.***

Sol. In order that the calendar for the year 2003 and 2014 be the same, 1st January of both the years must be on the same day of the week.

For this, the number of odd days between 31st Dec., 2002 and 31st Dec., 2013 must be the same.

We know that an ordinary year has 1 odd day and a leap year has 2 odd days.

During this period, there are 3 leap years, namely 2004, 2008 and 2012 and 8 ordinary years.

Total number of odd days = (6 + 8) days = 0 odd day.

Hence, the calendar for 2003 will serve for the year 2014.

Ex. 6. ***Prove that any date in March of a year is the same day of the week as the corresponding date in November that year.***

Sol. We will show that the number of odd days between last day of February and last day of October is zero.

March	April	May	June	July	Aug.	Sept.	Oct.
31	+ 30	+ 31	+ 30	+ 31	+ 31	+ 30	+ 31

= 241 days = 35 weeks = 0 odd day.

∴ Number of odd days during this period = 0.

Thus, 1st March of an year will be the same day as 1st November of that year.

Hence, the result follows.

EXERCISE 27

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer :*

1. January 1, 2004 was a Thursday. What day of the week lies on Jan. 1, 2005 ?
   (*a*) Thursday (*b*) Friday (*c*) Saturday (*e*) Sunday
2. On 8th March, 2005, Wednesday falls. What day of the week was it on 8th March, 2004 ?
   (*a*) Sunday (*b*) Monday (*c*) Tuesday (*d*) Wednesday
3. The calendar for the year 2005 is the same as for the year :
   (*a*) 2010 (*b*) 2011 (*c*) 2012 (*d*) 2013
4. On what dates of April 2001 did Sunday fall ?
   (*a*) 1st, 8th, 15th, 22nd, 29th (*b*) 2nd, 9th, 16th, 23rd, 30th
   (*c*) 4th, 11th, 18th, 25th (*d*) 6th, 13th, 20th, 27th
5. What will be the day of the week on 1st January, 2010 ?
   (*a*) Friday (*b*) Saturday (*c*) Sunday (*d*) Monday
6. What was the day of the week on 17th June, 1998 ?
   (*a*) Monday (*b*) Tuesday (*c*) Wednesday (*d*) Thursday
7. What was the day of the week on 28th May, 2003 ?
   (*a*) Friday (*b*) Saturday (*c*) Sunday (*d*) Monday
8. Today is Friday. After 62 days, it will be :
   (*a*) Saturday (*b*) Monday (*c*) Tuesday (*d*) Thursday
9. The last day of a century cannot be :
   (*a*) Monday (*b*) Wednesday (*c*) Friday (*d*) Tuesday
10. The first Republic Day of India was celebrated on 26th January, 1950. It was :
    (*a*) Tuesday (*b*) Wednesday (*c*) Thursday (*d*) Friday

SOLUTIONS

1. The year 2004 being a leap year, it has 2 odd days. So, first day of 2005 will be 2 days beyond Thursday and so it will be Saturday.
2. The year 2004 being a leap year, it has 2 odd days.
   So, the day on 8th March, 2005 will be two days beyond the day on 8th March, 2004.
   But, 8th March, 2005 is Wednesday. So, 8th March, 2004 is Monday.
3. Count the number of days from 2005 onwards to get 0 odd day.

Year	2005	2006	2007	2008	2010	2011
Odd days	1	1	1	2	1	1 = 7 or 0 odd day.

∴ Calendar for the year 2005 is the same as that for the year 2012.

4. Find the day on 1st April, 2001. 2000 years contain 2 odd days.

Jan. Feb. March April

31 + 28 + 31 + 1 = 91 days = 13 weeks 0 day = 0 odd day.

Sunday fell on 1st, 8th, 15th, 22nd and 29th of April 2001.

5. 2000 years have 2 odd days.

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Odd days	1	1	1	2	1	1	1	2	1

= 11 odd days = 4 odd days.

1st January, 2010 has 1 odd day. Total number of odd days = (2 + 4 + 1) = 7 = 0.

∴ 1st January, 2010 will be a Sunday.

6. 1600 years have 0 odd day. 300 years have 1 odd day.

97 years = 24 leap years + 73 ordinary years

= [(24 × 2) + (73 × 1)] odd days = 121 odd days

= (17 weeks + 2 days) odd days = 2 odd days.

Jan. Feb. March April May June

31 + 28 + 31 + 30 + 31 + 17 = 168 days = 0 odd day.

Total number of odd days = (0 + 1 + 2 + 0) = 3 odd days.

Hence, the required day was **'Wednesday'.**

7. 2000 years have 2 odd days.

The years 2001 and 2002 have (1 + 1) = 2 odd days.

Jan. Feb. March April May

31 + 28 + 31 + 30 + 28

= 148 days = 21 weeks + 1 day = 1 odd day.

Total number of odd days = (2 + 2 + 1) = 5.

∴ The required day was **'Friday'.**

8. Each day of the week is repeated after 7 days. So, after 63 days, it will be Friday.

Hence, after 62 days, it will be Thursday.

9. 100 years contain 5 odd days. So, last day of 1st century is 'Friday'.

200 years contain (5 × 2) = 10 odd days = 3 odd days.

So, last day of 2nd century is 'Wednesday'.

300 years contain (5 × 3) = 15 odd days = 1 odd day.

∴ Last day of 3rd century is 'Monday'.

400 years contain 0 odd day.

∴ Last day of 4th century is 'Sunday'.

Since the order is continually kept in successive cycles, we see that the last day of a century cannot be Tuesday, Thursday or Saturday.

10. 26th Jan., 1950 = (1949 years + Period from 1st Jan., 1950 to 26th Jan., 1950)

1600 years have 0 odd day. 300 years have 1 odd day.

49 years = (12 leap years + 37 ordinary years)

= [(12 × 2) + (37 × 1)] odd days = 61 odd days = 5 odd days.

Number of days from 1st Jan. to 26th Jan. = 26 = 5 odd days.

Total number of odd days = (0 + 1 + 5 + 5) = 11 = 4 odd days.

∴ The required day was **'Thursday'.**

28. CLOCKS

IMPORTANT FACTS

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

A clock has two hands, the smaller one is called the *hour hand* or *short hand* while the larger one is called the *minute hand* or *long hand*.

(*i*) In 60 minutes, the minute hand gains 55 minutes on the hour hand.

(*ii*) In every hour, both the hands coincide once.

(*iii*) The hands are in the same straight line when they are coincident or opposite to each other.

(*iv*) When the two hands are at right angles, they are 15 minute spaces apart.

(*v*) When the hands are in opposite directions, they are 30 minute spaces apart.

(*vi*) Angle traced by hour hand in 12 hrs = 360°.

(*vii*) Angle traced by minute hand in 60 min. = 360°.

Too Fast and Too Slow : *If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.*

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

SOLVED EXAMPLES

Ex. 1. ***Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.***

Sol. Angle traced by the hour hand in 12 hours = 360°.

Angle traced by it in 3 hrs 25 min. *i.e.* $\frac{41}{12}$ hrs = $\left(\frac{360}{12}\times\frac{41}{12}\right)^\circ = 102\frac{1}{2}^\circ$.

Angle traced by minute hand in 60 min. = 360°.

Angle traced by it in 25 min. = $\left(\frac{360}{60}\times 25\right)^\circ = 150^\circ$.

$\therefore$ Required angle = $\left(150^\circ - 102\frac{1}{2}^\circ\right) = 47\frac{1}{2}^\circ$.

Ex. 2. ***At what time between 2 and 3 o'clock will the hands of a clock be together?***

Sol. At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, *i.e.* they are 10 min. spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

$\therefore$ 10 minutes will be gained in $\left(\frac{60}{55}\times 10\right)$ min. = $10\frac{10}{11}$ min.

$\therefore$ The hands will coincide at $10\frac{10}{11}$ min. past 2.

Ex. 3. ***At what time between 4 and 5 o'clock will the hands of a clock be at right angle ?***

Sol. At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand.

Now, when the two hands are at right angles, they are 15 min. spaces apart.

So, they are at right angles in following two cases.

Case I. *When minute hand is 15 min. spaces behind the hour hand :*

In this case min. hand will have to gain (20 – 15) = 5 minute spaces.

55 min. spaces are gained by it in 60 min.

5 min. spaces will be gained by it in $\left(\frac{60}{55}\times 5\right)$ min. $= 5\frac{5}{11}$ min.

$\therefore$ They are at right angles at $5\frac{5}{11}$ min. past 4.

Case II. *When the minute hand is 15 min. spaces ahead of the hour hand :*

To be in this position, the minute hand will have to gain (20 + 15) = 35 minute spaces.

55 min. spaces are gained in 60 min.

35 min. spaces are gained in $\left(\frac{60}{55}\times 35\right)$ min. $= 38\frac{2}{11}$ min.

$\therefore$ They are at right angles at $38\frac{2}{11}$ min. past 4.

Ex. 4. ***Find at what time between 8 and 9 o'clock will the hands of a clock be in the same straight line but not together.***

Sol. At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, *i.e.* the two hands are 20 min. spaces apart.

To be in the same straight line but not together they will be 30 minute spaces apart.

So, the minute hand will have to gain (30 – 20) = 10 minute spaces over the hour hand.

55 minute spaces are gained in 60 min.

10 minute spaces will be gained in $\left(\frac{60}{55}\times 10\right)$ min. $= 10\frac{10}{11}$ min.

$\therefore$ The hands will be in the same straight line but not together at $10\frac{10}{11}$ min. past 8.

Ex. 5. ***At what time between 5 and 6 o'clock are the hands of a clock 3 minutes apart ?***

Sol. At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.

Case I. *Minute hand is 3 min. spaces behind the hour hand.*

In this case, the minute hand has to gain (25 – 3) = 22 minute spaces.

55 min. are gained in 60 min.

22 min. are gained in $\left(\frac{60}{55}\times 22\right)$ min. = 24 min.

$\therefore$ The hands will be 3 min. apart at 24 min. past 5.

Case II. *Minute hand is 3 min. spaces ahead of the hour hand.*

In this case, the minute hand has to gain (25 + 3) = 28 minute spaces.

55 min. are gained in 60 min.

28 min. are gained in $\left(\frac{60}{55}\times 28\right) = 31\frac{5}{11}$ min.

$\therefore$ The hands will be 3 min. apart at $31\frac{5}{11}$ min. past 5.

Ex. 6. ***The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose ?***

Sol. In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.

55 min. are gained in 60 min.

60 min. are gained in $\left(\frac{60}{55} \times 60\right)$ min. $= 65\frac{5}{11}$ min.

But, they are together after 65 min.

$\therefore$ Gain in 65 min. $= \left(65\frac{5}{11} - 65\right) = \frac{5}{11}$ min.

Gain in 24 hours $= \left(\frac{5}{11} \times \frac{60 \times 24}{65}\right)$ min. $= 10\frac{10}{43}$ min.

$\therefore$ The clock gains $10\frac{10}{43}$ minutes in 24 hours.

Ex. 7. ***A watch which gains uniformly, is 5 min. slow at 8 o'clock in the morning on Sunday and it is 5 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct ?***

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours
= 180 hours

$\therefore$ The watch gains $\left(5 + 5\frac{4}{5}\right)$ min. or $\frac{54}{5}$ min. in 180 hrs.

Now $\frac{54}{5}$ min. are gained in 180 hrs.

$\therefore$ 5 min. are gained in $\left(180 \times \frac{5}{54} \times 5\right)$ hrs. = 83 hrs 20 min. = 3 days 11 hrs 20 min.

$\therefore$ Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.

$\therefore$ It will be correct at 20 min. past 7 p.m. on Wednesday.

Ex. 8. ***A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day ?***

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.

Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$\therefore$ $\frac{356}{15}$ hrs of this clock = 24 hours of correct clock.

89 hrs of this clock $= \left(24 \times \frac{15}{356} \times 89\right)$ hrs of correct clock.

= 90 hrs of correct clock.

So, the correct time is 11 p.m.

Ex. 9. ***A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 1 p.m. on the following day ?***

Sol. Time from 8 a.m. on a day to 1 p.m. on the following day = 29 hours.

24 hours 10 min. of this clock = 24 hours of the correct clock.

$\frac{145}{6}$ hrs of this clock = 24 hrs of the correct clock

29 hrs of this clock = $\left(24 \times \frac{6}{145} \times 29\right)$ hrs of the correct clock

= 28 hrs 48 min. of correct clock

∴ The correct time is 28 hrs 48 min. after 8 a.m.

This is 48 min. past 12.

EXERCISE 28

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer* :

1. A clock is started at noon. By 10 minutes past 5, the hour hand has turned through :
   (a) 145° (b) 150° (c) 155° (d) 160°
2. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon ? **(I.A.S. 2000)**
   (a) 144° (b) 150° (c) 168° (d) 180°
3. At 3.40, the hour hand and the minute hand of a clock form an angle of :
   (a) 120° (b) 125° (c) 130° (d) 135°
4. The angle between the minute hand and the hour hand of a clock when the time is 8.30, is :
   (a) 80° (b) 75° (c) 60° (d) 105°
5. The angle between the minute hand and the hour hand of a clock when the time is 4.20, is :
   (a) 0° (b) 10° (c) 5° (d) 20°
6. At what angle the hands of a clock are inclined at 15 minutes past 5 ?
   (a) $58\frac{1}{2}°$ (b) 64° (c) $67\frac{1}{2}°$ (d) $72\frac{1}{2}°$
   (L.I.C.A.A.O. 2003)
7. The reflex angle between the hands of a clock at 10.25 is : **(S.C.R.A. 1996)**
   (a) 180° (b) $192\frac{1}{2}°$ (c) 195° (d) $197\frac{1}{2}°$
8. How many times do the hands of a clock coincide in a day ?
   (a) 20 (b) 21 (c) 22 (d) 24
9. How many times in a day, the hands of a clock are straight ?
   (a) 22 (b) 24 (c) 44 (d) 48
10. How many times are the hands of a clock at right angle in a day ? **(I.A.S. 1997)**
    (a) 22 (b) 24 (c) 44 (d) 48
11. How many times in a day, are the hands of a clock in straight line but opposite in direction ? **(R.R.B. 2003)**
    (a) 20 (b) 22 (c) 24 (d) 48
12. How much does a watch lose per day, if its hands coincide every 64 minutes ?
    (a) $32\frac{8}{11}$ min. (b) $36\frac{5}{11}$ min. (c) 90 min. (d) 96 min.
13. At what time, in minutes, between 3 o'clock and 4 o'clock, both the needles will coincide each other ? **(R.R.B. 2002)**
    (a) $5\frac{1}{11}''$ (b) $12\frac{4}{11}''$ (c) $13\frac{4}{11}''$ (d) $16\frac{4}{11}''$

14. At what time between 9 and 10 o'clock will the hands of a watch be together ?
(a) 45 min. past 9 (b) 50 min. past 9
(c) $49\frac{1}{11}$ min. past 9 (d) $48\frac{2}{11}$ min. past 9

15. At what time between 7 and 8 o'clock will the hands of a clock be in the same straight line but, not together ? **(A.A.O. Exam. 2003)**
(a) 5 min. past 7 (b) $5\frac{2}{11}$ min. past 7
(c) $5\frac{3}{11}$ min. past 7 (d) $5\frac{5}{11}$ min. past 7

16. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions ?
(a) 45 min. past 4 (b) 40 min. past 4
(c) $50\frac{4}{11}$ min. past 4 (d) $54\frac{6}{11}$ min. past 4

17. At what time between 5.30 and 6 will the hands of a clock be at right angles ?
(a) $43\frac{5}{11}$ min. past 5 (b) $43\frac{7}{11}$ min. past 5
(c) 40 min. past 5 (d) 45 min. past 5

18. A watch which gains uniformly is 2 minutes low at noon on Monday and is 4 min. 48 sec fast at 2 p.m. on the following Monday. When was it correct ? **(R.R.B. 2001)**
(a) 2 p.m. on Tuesday (b) 2 p.m. on Wednesday
(c) 3 p.m. on Thursday (d) 1 p.m. on Friday

19. A watch which gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 o'clock, the true time is :
(a) $59\frac{7}{12}$ min. past 3 (b) 4 p.m.
(c) $58\frac{7}{11}$ min. past 3 (d) $2\frac{3}{11}$ min. past 4

ANSWERS

1. (c)	2. (d)	3. (c)	4. (b)	5. (b)	6. (c)	7. (d)	8. (c)	9. (c)	10. (c)
11. (b)	12. (a)	13. (d)	14. (c)	15. (d)	16. (d)	17. (b)	18. (b)	19. (b)	

SOLUTIONS

1. Angle traced by hour hand in 12 hrs = 360°.
Angle traced by hour hand in 5 hrs 10 min. *i.e.* $\frac{31}{6}$ hrs. $= \left(\frac{360}{12} \times \frac{31}{6}\right)^\circ = 155^\circ$.

2. Angle traced by the hour hand in 6 hours $= \left(\frac{360}{12} \times 6\right)^\circ = 180^\circ$.

3. Angle traced by hour hand in 12 hrs. = 360°.
Angle traced by it in $\frac{11}{3}$ hrs $= \left(\frac{360}{12} \times \frac{11}{3}\right)^\circ = 110^\circ$.
Angle traced by minute hand in 60 min. = 360°.

Angle traced by it in 40 min. $= \left(\frac{360}{60} \times 40\right)^\circ = 240^\circ$.

$\therefore$ Required angle $(240 - 110)^\circ = 130^\circ$.

4. Angle traced by hour hand in $\frac{17}{2}$ hrs $= \left(\frac{360}{12} \times \frac{17}{2}\right)^\circ = 255$.

Angle traced by min. hand in 30 min. $= \left(\frac{360}{60} \times 30\right)^\circ = 180$.

$\therefore$ Required angle $= (255 - 180)^\circ = 75^\circ$.

5. Angle traced by hour hand in $\frac{13}{3}$ hrs $= \left(\frac{360}{12} \times \frac{13}{3}\right)^\circ = 130^\circ$.

Angle traced by min. hand in 20 min. $= \left(\frac{360}{60} \times 20\right)^\circ = 120^\circ$.

$\therefore$ Required angle $= (130 - 120)^\circ = 10^\circ$.

6. Angle traced by hour hand in $\frac{21}{4}$ hrs $= \left(\frac{360}{12} \times \frac{21}{4}\right)^\circ = 157\frac{1}{2}^\circ$.

Angle traced by min. hand in 15 min. $= \left(\frac{360}{12} \times 15\right)^\circ = 90^\circ$.

$\therefore$ Required angle $= \left(157\frac{1}{2}\right)^\circ - 90^\circ = 67\frac{1}{2}^\circ$.

7. Angle traced by hour hand in $\frac{125}{12}$ hrs $= \left(\frac{360}{12} \times \frac{125}{12}\right)^\circ = 312\frac{1}{2}^\circ$.

Angle traced by minute hand in 25 min $= \left(\frac{360}{60} \times 25^\circ\right) = 150^\circ$.

$\therefore$ Reflex angle $= 360^\circ - \left(312\frac{1}{2} - 150\right)^\circ = 360^\circ - 162\frac{1}{2}^\circ = 197\frac{1}{2}^\circ$.

8. The hands of a clock coincide 11 times in every 12 hours (Since between 11 and 1, they coincide only once, *i.e.* at 12 o'clock).

$\therefore$ The hands coincide 22 times in a day.

9. In 12 hours, the hands coincide or are in opposite direction 22 times.

$\therefore$ In 24 hours, the hands coincide or are in opposite direction 44 times a day.

10. In 12 hours, they are at right angles 22 times.

$\therefore$ In 24 hours, they are at right angles 44 times.

11. The hands of a clock point in opposite directions (in the same straight line) 11 times in every 12 hours (Because between 5 and 7 they point in opposite directions at 6 o'clock only). So, in a day, the hands point in the opposite directions 22 times.

12. 55 min. spaces are covered in 60 min.

60 min. spaces are covered in $\left(\frac{60}{55} \times 60\right)$ min. $= 65\frac{5}{11}$ min.

Loss in 64 min. $= \left(65\frac{5}{11} - 64\right) = \frac{16}{11}$ min.

Loss in 24 hrs $= \left(\frac{16}{11} \times \frac{1}{64} \times 24 \times 60\right)$ min. $= 32\frac{8}{11}$ min.

13. At 3 o'clock, the minute hand is 15 min. spaces apart from the hour hand.
To be coincident, it must gain 15 min. spaces.
55 min. are gained in 60 min.

15 min. are gained in $\left(\frac{60}{55}\times 15\right)$ min. $= 16\frac{4}{11}$ min.

$\therefore$ The hands are coincident at $16\frac{4}{11}$ min. past 3.

14. To be together between 9 and 10 o'clock, the minute hand has to gain 45 min. spaces.
55 min. spaces gained in 60 min.

45 min. spaces are gained in $\left(\frac{60}{55}\times 45\right)$ min. or $49\frac{1}{11}$ min.

$\therefore$ The hands are together at $49\frac{1}{11}$ min. past 9.

15. When the hands of the clock are in the same straight line but not together, they are 30 minute spaces apart.
At 7 o'clock, they are 25 min. spaces apart.
$\therefore$ Minute hand will have to gain only 5 min. spaces.
55 min. spaces are gained in 60 min.

5 min. spaces are gained in $\left(\frac{60}{55}\times 5\right)$ min $= 5\frac{5}{11}$ min.

$\therefore$ Required time $= 5\frac{5}{11}$ min. past 7.

16. At 4 o'clock, the hands of the watch are 20 min. spaces apart.
To be in opposite directions, they must be 30 min. spaces apart.
$\therefore$ Minute hand will have to gain 50 min. spaces.
55 min. spaces are gained in 60 min.

50 min. spaces are gained in $\left(\frac{60}{55}\times 50\right)$ min. or $54\frac{6}{11}$ min.

$\therefore$ Required time $= 54\frac{6}{11}$ min. past 4.

17. At 5 o'clock, the hands are 25 min. spaces apart.
To be at right angles and that too between 5.30 and 6, the minute hand has to gain $(25 + 15) = 40$ min. spaces.
55 min. spaces are gained in 60 min.

40 min. spaces are gained in $\left(\frac{60}{55}\times 40\right)$ min. $= 43\frac{7}{11}$ min.

$\therefore$ Required time $= 43\frac{7}{11}$ min. past 5.

18. Time from 12 p.m. on Monday to 2 p.m. on the following Monday = 7 days 2 hours
= 170 hours.

$\therefore$ The watch gains $\left(2+4\frac{4}{5}\right)$ min. or $\frac{34}{5}$ min. in 170 hrs.

Now, $\frac{34}{5}$ min. are gained in 170 hrs.

$\therefore$ 2 min. are gained in $\left(170 \times \frac{5}{34} \times 2\right)$ hrs = 50 hrs.

$\therefore$ Watch is correct 2 days 2 hrs. after 12 p.m. on Monday *i.e.* it will be correct at 2 p.m. on Wednesday.

19. Time from 7 a.m. to 4.15 p.m. = 9 hrs 15 min. = $\frac{37}{4}$ hrs.

3 min. 5 sec. of this clock = 3 min. of the correct clock.

$\Rightarrow$ $\frac{37}{720}$ hrs of this clock = $\frac{1}{20}$ hrs of the correct clock

$\Rightarrow$ $\frac{37}{4}$ hrs of this clock = $\left(\frac{1}{20} \times \frac{720}{37} \times \frac{37}{4}\right)$ hrs of the correct clock

= 9 hrs of the correct clock

$\therefore$ The correct time is 9 hrs after 7 a.m. *i.e.* 4 p.m.

29. STOCK AND SHARES

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus (in the name of the company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them ***shares*** of the company.

IMPORTANT FACTS AND FORMULAE

1. **Stock-capital** : *The total amount of money needed to run the company is called the* ***stock-capital***.
2. **Shares or Stock** : *The whole capital is divided into small units, called* ***shares*** *or* ***stock***.

   For each investment, the company issues a *share-certificate*, showing the value of each share and the number of shares held by a person.

   The person who subscribes in shares or stock is called a ***share holder*** *or* ***stock holder***.
3. **Dividend** : *The annual profit distributed among share holders is called* ***dividend***.

   Dividend is paid annually as per share or as a percentage.
4. **Face Value** : *The value of a share or stock printed on the share-certificate is called its* ***Face Value*** *or* ***Nominal Value*** *or* ***Par Value***.
5. **Market Value** : The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be :

   (*i*) ***At premium*** or ***Above par***, if its market value is more than its face value.

   (*ii*) ***At par***, if its market value is the same as its face value.

   (*iii*) ***At discount*** or ***Below par***, if its market value is less than its face value.

   Thus, if a Rs. 100 stock is quoted at a premium of 16, then market value of the stock = Rs. (100 + 16) = Rs. 116.

   Likewise, if a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. (100 − 7) = Rs. 93.
6. **Brokerage** : *The broker's charge is called* ***brokerage***.

   (*i*) When stock is purchased, brokerage is added to the cost price.

   (*ii*) When stock is sold, brokerage is subtracted from the selling price.

   Remember :

   (*i*) The face value of a share always remains the same.

   (*ii*) The market value of a share changes from time to time.

   (*iii*) Dividend is always paid on the face value of a share.

   (*iv*) Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{Face value of 1 share}}.$$

Thus, by a Rs. 100, 9% stock at 120, we mean that :

(*i*) Face Value (N.V.) of stock = Rs. 100.

(*ii*) Market Value (M.V.) of stock = Rs. 120.

(*iii*) Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.

(*iv*) An investment of Rs. 120 gives an annual income of Rs. 9.

(*v*) Rate of interest p.a. = Annual income from an investment of Rs. 100

$$= \left(\frac{9}{120} \times 100\right)\% = 7\frac{1}{2}\%.$$

SOLVED EXAMPLES

Ex. 1. ***Find the cost of :***

(*i*) ***Rs. 7200, 8% stock at 90;***

(*ii*) ***Rs. 4500, 8.5% stock at 4 premium;***

(*iii*) ***Rs. 6400, 10% stock at 15 discount.***

Sol. (*i*) Cost of Rs. 100 stock = Rs. 90.

Cost of Rs. 7200 stock = Rs. $\left(\frac{90}{100} \times 7200\right)$ = Rs. 6480.

(*ii*) Cost of Rs. 100 stock = Rs. (100 + 4) = Rs. 104.

Cost of Rs. 4500 stock = Rs. $\left(\frac{104}{100} \times 4500\right)$ = Rs. 4680.

(*iii*) Cost of Rs. 100 stock = Rs. (100 − 15) = Rs. 85.

Cost of Rs. 6400 stock = Rs. $\left(\frac{85}{100} \times 6400\right)$ = Rs. 5440.

Ex. 2. ***Find the cash required to purchase Rs. 3200, $7\frac{1}{2}$% stock at 107 $\left(\text{brokerage } \frac{1}{2}\%\right)$.***

Sol. Cash required to purchase Rs. 100 stock = Rs. $\left(107 + \frac{1}{2}\right)$ = Rs. $\frac{215}{2}$.

Cash required to purchase Rs. 3200 stock = Rs. $\left(\frac{215}{2} \times \frac{1}{100} \times 3200\right)$ = Rs. 3440.

Ex. 3. ***Find the cash realised by selling Rs. 2440, 9.5% stock at 4 discount $\left(\text{brokerage } \frac{1}{4}\%\right)$.***

Sol. By selling Rs. 100 stock, cash realised = Rs. $\left[(100 - 4) - \frac{1}{4}\right]$ = Rs. $\frac{383}{4}$.

By selling Rs. 2400 stock, cash realised = Rs. $\left(\frac{383}{4} \times \frac{1}{100} \times 2400\right)$ = Rs. 2296.

Ex. 4. ***Find the annual income derived from Rs. 2500, 8% stock at 106.***

Sol. Income from Rs. 100 stock = Rs. 8.

Income from Rs. 2500 stock = Rs. $\left(\frac{8}{100} \times 2500\right)$ = Rs. 200.

Ex. 5. ***Find the annual income derived by investing Rs. 6800 in 10% stock at 136.***

Sol. By investing Rs. 136, income obtained = Rs. 10.

By investing Rs. 6800, income obtained = Rs. $\left(\frac{10}{136} \times 6800\right)$ = Rs. 500.

Ex. 6. ***Which is better investment ? $7\frac{1}{2}\%$ stock at 105 or $6\frac{1}{2}\%$ stock at 94.***

Sol. Let the investment in each case be Rs. (105×94).

Case I : $7\frac{1}{2}\%$ stock at 105 :

On investing Rs. 105, income = Rs. $\frac{15}{2}$.

On investing Rs. (105×94), income = Rs. $\left(\frac{15}{2} \times \frac{1}{105} \times 105 \times 94\right)$ = Rs. 705.

Case II : $6\frac{1}{2}\%$ stock at 94 :

On investing Rs. 94, income = Rs. $\frac{13}{2}$.

On investing Rs. (105×94), income = Rs. $\left(\frac{13}{2} \times \frac{1}{94} \times 105 \times 94\right)$ = Rs. 682.50.

Clearly, the income from $7\frac{1}{2}\%$ stock at 105 is more.

Hence, the investment in $7\frac{1}{2}\%$ stock at 105 is better.

Ex. 7. ***Find the cost of 96 shares of Rs. 10 each at $\frac{3}{4}$ discount, brokerage being $\frac{1}{4}$ per share.*** **(L.I.C. 2003)**

Sol. Cost of 1 share = Rs. $\left[\left(10 - \frac{3}{4}\right) + \frac{1}{4}\right]$ = Rs. $\frac{19}{2}$.

Cost of 96 shares = Rs. $\left(\frac{19}{2} \times 96\right)$ = Rs. 912.

Ex. 8. ***Find the income derived from 88 shares of Rs. 25 each at 5 premium, brokerage being $\frac{1}{4}$ per share and the rate of dividend being $7\frac{1}{2}\%$ per annum. Also, find the rate of interest on the investment.***

Sol. Cost of 1 share = Rs. $\left(25 + 5 + \frac{1}{4}\right)$ = Rs. $\frac{121}{4}$.

Cost of 88 shares = Rs. $\left(\frac{121}{4} \times 88\right)$ = Rs. 2662.

$\therefore$ Investment made = Rs. 2662.

Face value of 88 shares = Rs. (88×25) = Rs. 2200.

Dividend on Rs. 100 = $\frac{15}{2}$.

Dividend on Rs. 2200 = Rs. $\left(\frac{15}{2} \times \frac{1}{100} \times 2200\right)$ = Rs. 165.

$\therefore$ Income derived = Rs. 165.

Rate of interest on investment = $\left(\frac{165}{2662} \times 100\right)$ = 6.2%.

Ex. 9. ***A man buys Rs. 25 shares in a company which pays 9% dividend. The money invested is such that it gives 10% on investment. At what price did he buy the shares?***

Sol. Suppose he buys each share for Rs. x.

Then, $\left(25 \times \frac{9}{100}\right) = \left(x \times \frac{10}{100}\right)$ or $x = 22.50$.

$\therefore$ Cost of each share = Rs. 22.50.

Ex. 10. ***A man sells Rs. 5000, 12% stock at 156 and invests the proceeds partly in 8% stock at 90 and 9% stock at 108. He thereby increases his income by Rs. 70. How much of the proceeds were invested in each stock?***

Sol. S.P. of Rs. 5000 stock = Rs. $\left(\frac{156}{100} \times 5000\right)$ = Rs. 7800.

Income from this stock = Rs. $\left(\frac{12}{100} \times 5000\right)$ = Rs. 600.

Let investment in 8% stock be x and that in 9% stock = $(7800 - x)$.

$\therefore \quad \left(x \times \frac{8}{90}\right) + (7800 - x) \times \frac{9}{108} = (600 + 70)$

$\Leftrightarrow \quad \frac{4x}{45} + \frac{7800 - x}{12} = 670 \quad \Leftrightarrow \quad 16x + 117000 - 15x = (670 \times 180) \quad \Leftrightarrow \quad x = 3600.$

$\therefore$ Money invested in 8% stock at 90 = Rs. 3600.

Money invested in 9% at 108 = Rs. (7800 – 3600) = Rs. 4200.

EXERCISE 29

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The cost price of a Rs. 100 stock at 4 discount, when brokerage is $\frac{1}{4}\%$ is :
   (a) Rs. 95.75 (b) Rs. 96 (c) Rs. 96.25 (d) Rs. 104.25
2. The cash realised on selling a 14% stock at Rs. 106.25, brokerage being $\frac{1}{4}\%$, is :
   (a) Rs. 105.50 (b) Rs. 106 (c) Rs. 106.50 (d) Rs. 113.75
3. How many shares of market value Rs. 25 each can be purchased for Rs. 12750, brokerage being 2% ? **(M.A.T. 2002)**
   (a) 450 (b) 500 (c) 550 (d) 600
4. A man invests in a 16% stock at 128. The interest obtained by him is :
   (a) 8% (b) 12% (c) 12.5% (d) 16%
5. The income derived from a Rs. 100, 13% stock at Rs. 105, is :
   (a) Rs. 5 (b) Rs. 8 (c) Rs. 13 (d) Rs. 18
6. A man invested Rs. 4455 in Rs. 10 shares quoted at Rs. 8.25. If the rate of dividend be 12%, his annual income is :
   (a) Rs. 207.40 (b) Rs. 534.60 (c) Rs. 648 (d) Rs. 655.60
7. A man invested Rs. 14,400 in Rs. 100 shares of a company at 20% premium. If the company declares 5% dividend at the end of the year, then how much does he get ?
   (a) Rs. 500 (b) Rs. 600 (c) Rs. 650 (d) Rs. 720
   (Hotel Management, 2003)
8. A 6% stock yields 8%. The market value of the stock is :
   (a) Rs. 48 (b) Rs. 75 (c) Rs. 96 (d) Rs. 133.33

9. A 9% stock yields 8%. The market value of the stock is :
(a) Rs. 72 (b) Rs. 92 (c) Rs. 112.50 (d) Rs. 116.50

10. A 12% stock yielding 10% is quoted at :
(a) Rs. 83.33 (b) Rs. 110 (c) Rs. 112 (d) Rs. 120

11. By investing Rs. 1620 in 8% stock, Michael earns Rs. 135. The stock is then quoted at :
(a) Rs. 80 (b) Rs. 96 (c) Rs. 106 (d) Rs. 108

12. To produce an annual income of Rs. 1200 from a 12% stock at 90, the amount of stock needed is :
(a) Rs. 10,000 (b) Rs. 10,800 (c) Rs. 14,400 (d) Rs. 16,000

13. In order to obtain an income of Rs. 650 from 10% stock at Rs. 96, one must make an investment of :
(a) Rs. 3100 (b) Rs. 6240 (c) Rs. 6500 (d) Rs. 9600

14. By investing in $16\frac{2}{3}\%$ stock at 64, one earns Rs. 1500. The investment made is :
(a) Rs. 5640 (b) Rs. 5760 (c) Rs. 7500 (d) Rs. 9600

15. A man invested Rs. 1552 in a stock at 97 to obtain an income of Rs. 128. The dividend from the stock is :
(a) 7.5% (b) 8% (c) 9.7% (d) None of these.

16. A man bought 20 shares of Rs. 50 at 5 discount, the rate of dividend being $13\frac{1}{2}\%$. The rate of interest obtained is :
(a) $12\frac{1}{2}\%$ (b) $13\frac{1}{2}\%$ (c) 15% (d) $16\frac{2}{3}\%$

17. A man buys Rs. 20 shares paying 9% dividend. The man wants to have an interest of 12% on his money. The market value of each share is :
(a) Rs. 12 (b) Rs. 15 (c) Rs. 18 (d) Rs. 21

18. A man buys Rs. 50 shares in a company which pays 10% dividend. If the man gets 12.5% on his investment, at what price did he buy the shares ? **(L.I.C.A.A.O. 2003)**
(a) Rs. 37.50 (b) Rs. 40 (c) Rs. 48 (d) Rs. 52

19. The market value of a 10.5% stock, in which an income of Rs. 756 is derived by investing Rs. 9000, brokerage being $\frac{1}{4}\%$, is :
(a) Rs. 108.25 (b) Rs. 112.20 (c) Rs. 124.75 (d) Rs. 125.25

20. Sakshi invests a part of Rs. 12,000 in 12% stock at Rs. 120 and the remainder in 15% stock at Rs. 125. If his total dividend per annum is Rs. 1360, how much does he invest in 12% stock at Rs. 120 ?
(a) Rs. 4000 (b) Rs. 4500 (c) Rs. 5500 (d) Rs. 6000

21. Rs. 9800 are invested partly in 9% stock at 75 and 10% stock at 80 to have equal amount of incomes. The investment in 9% stock is :
(a) Rs. 4800 (b) Rs. 5000 (c) Rs. 5400 (d) Rs. 5600

22. A man invests some money partly in 9% stock at 96 and partly in 12% stock at 120. To obtain equal dividends from both, he must invest the money in the ratio :
(a) 3 : 4 (b) 3 : 5 (c) 4 : 5 (d) 16 : 15

23. Which is better investment — 11% stock at 143 or $9\frac{3}{4}\%$ stock at 117 ?
(a) 11% stock at 143 (b) $9\frac{3}{4}\%$ stock at 117 (c) Both are equally good
(d) Cannot be compared, as the total amount of investment is not given

24. Which is better investment, 12% stock at par with an income tax at the rate of 5 paise per rupee or $14\frac{2}{7}\%$ stock at 120 free from income tax ?

(*a*) 12% stock (*b*) $14\frac{2}{7}\%$ stock (*c*) Both are equally good

(*d*) Cannot be compared

25. A invested some money in 10% stock at 96. If B wants to invest in an equally good 12% stock, he must purchase a stock worth of :

(*a*) Rs. 80 (*b*) Rs. 115.20 (*c*) Rs. 120 (*d*) Rs. 125.40

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*b*)	**4.** (*c*)	**5.** (*c*)	**6.** (*c*)	**7.** (*b*)	**8.** (*b*)	**9.** (*c*)	**10.** (*d*)
11. (*b*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*b*)	**16.** (*c*)	**17.** (*b*)	**18.** (*b*)	**19.** (*c*)	**20.** (*a*)
21. (*b*)	**22.** (*d*)	**23.** (*b*)	**24.** (*b*)	**25.** (*b*)					

SOLUTIONS

1. C.P. = Rs. $\left(100 - 4 + \frac{1}{4}\right)$ = Rs. 96.25.

2. Cash realised = Rs. (106.25 – 0.25) = Rs. 106.

3. C.P. of each share = Rs. (25 + 2% of 25) = Rs. 25.50.

∴ Number of shares = $\left(\frac{12750}{25.50}\right)$ = 500.

4. By investing Rs. 128, income derived = Rs. 16.

By investing Rs. 100, income derived = Rs. $\left(\frac{16}{128} \times 100\right)$ = Rs. 12.5.

∴ Interest obtained = 12.5%.

5. Income on Rs. 100 stock = Rs. 13.

6. Number of shares = $\left(\frac{4455}{8.25}\right)$ = 540.

Face value = Rs. (540 × 10) = Rs. 5400.

Annual income = Rs. $\left(\frac{12}{100} \times 5400\right)$ = Rs. 648.

7. Number of shares = $\left(\frac{14400}{120}\right)$ = 120.

Face value = Rs. (100 × 120) = Rs. 12000.

Annual income = Rs. $\left(\frac{5}{100} \times 12000\right)$ = Rs. 600.

8. For an income of Rs. 8, investment = Rs. 100.

For an income of Rs. 6, investment = Rs. $\left(\frac{100}{8} \times 6\right)$ = Rs. 75.

∴ Market value of Rs. 100 stock = Rs. 75.

9. To obtain Rs. 8, investment = Rs. 100.

To obtain Rs. 9, investment = Rs. $\left(\frac{100}{8} \times 9\right)$ = Rs. 112.50.

∴ Market value of Rs. 100 stock = Rs. 112.50.

10. To earn Rs. 10, money invested = Rs. 100.

To earn Rs. 12, money invested = Rs. $\left(\frac{100}{10} \times 12\right)$ = Rs. 120.

∴ Market value of Rs. 100 stock = Rs. 120.

11. To earn Rs. 135, investment = Rs. 1620.

To earn Rs. 8, investment = Rs. $\left(\frac{1620}{135} \times 8\right)$ = Rs. 96.

∴ Market value of Rs. 100 stock = Rs. 96.

12. For an income of Rs. 12, stock needed = Rs. 100.

For an income of Rs. 1200, stock needed = Rs. $\left(\frac{100}{12} \times 1200\right)$ = Rs. 10,000.

13. To obtain Rs. 10, investment = Rs. 96.

To obtain Rs. 650, investment = Rs. $\left(\frac{96}{10} \times 650\right)$ = Rs. 6240.

14. To earn Rs. $\frac{50}{3}$, investment = Rs. 64.

To earn Rs. 1500, investment = Rs. $\left(64 \times \frac{3}{50} \times 1500\right)$ = Rs. 5760.

15. By investing Rs. 1552, income = Rs. 128.

By investing Rs. 97, income = Rs. $\left(\frac{128}{1552} \times 97\right)$ = Rs. 8.

∴ Dividend = 8%.

16. Investment = Rs. $[20 \times (50 - 5)]$ = Rs. 900.

Face value = Rs. (50×20) = Rs. 1000.

Dividend = Rs. $\left(\frac{27}{2} \times \frac{1000}{100}\right)$ = Rs. 135.

Interest obtained = $\left(\frac{135}{900} \times 100\right)\%$ = 15%.

17. Dividend on Rs. 20 = Rs. $\left(\frac{9}{100} \times 20\right)$ = Rs. $\frac{9}{5}$.

Rs. 12 is an income on Rs. 100.

∴ Rs. $\frac{9}{5}$ is an income on Rs. $\left(\frac{100}{12} \times \frac{9}{5}\right)$ = Rs. 15.

18. Dividend on 1 share = Rs. $\left(\frac{10}{100} \times 50\right)$ = Rs. 5.

Rs. 12.50 is an income on an investment of Rs. 100.

Rs. 5 is an income on an investment of Rs. $\left(100 \times \frac{2}{25} \times 5\right)$ = Rs. 40.

∴ Cost of 1 share = Rs. 40.

19. For an income of Rs. 756, investment = Rs. 9000.

For an income of Rs. $\frac{21}{2}$, investment = Rs. $\left(\frac{9000}{756}\times\frac{21}{2}\right)$ = Rs. 125.

$\therefore$ For a Rs. 100 stock, investment = Rs. 125.

Market value of Rs. 100 stock = Rs. $\left(125-\frac{1}{4}\right)$ = Rs. 124.75.

20. Let investment in 12% stock be Rs. x.

Then, investment in 15% stock = Rs. $(12000 - x)$.

$$\frac{12}{120}\times x+\frac{15}{125}\times(12000-x)=1360$$

$$\Leftrightarrow \frac{x}{10}+\frac{3}{25}(12000-x)=1360$$

$$\Leftrightarrow 5x+72000-6x=1360\times 50 \Leftrightarrow x=4000.$$

21. Let the investment in 9% stock be Rs. x.

Then, investment in 10% stock = Rs. $(9800 - x)$.

$$\frac{9}{75}\times x=\frac{10}{80}\times(9800-x) \Leftrightarrow \frac{3x}{25}=\frac{9800-x}{8}$$

$$\Leftrightarrow 24x=9800\times 25-25x \Leftrightarrow 49x=9800\times 25 \Leftrightarrow x=5000.$$

22. For an income of Re. 1 in 9% stock at 96, investment = Rs. $\left(\frac{96}{9}\right)$ = Rs. $\frac{32}{3}$.

For an income of Re. 1 in 12% stock at 120, investment = Rs. $\left(\frac{120}{12}\right)$ = Rs. 10.

$\therefore$ Ratio of investments = $\frac{32}{3}:10=32:30=16:15$.

23. Let investment in each case be Rs. (143×117).

Income in 1st case = Rs. $\left(\frac{11}{143}\times 143\times 117\right)$ = Rs. 1287.

Income in 2nd case = Rs. $\left(\frac{39}{4\times 117}\times 143\times 117\right)$ = Rs. 1394.25.

Clearly, $9\frac{3}{4}\%$ stock at 117 is better.

24. Let investment in each case = Rs. (100×120).

Income from 12% stock = Rs. $\left(\frac{12}{100}\times 100\times 120\right)$ = Rs. 1440.

Net income = Rs. $\left(1440-\frac{5}{100}\times 1440\right)$ = Rs. 1368.

Income from $14\frac{2}{7}\%$ stock = Rs. $\left(\frac{100}{7\times 20}\times 100\times 120\right)$ = Rs. 1428.57.

Clearly, $14\frac{2}{7}\%$ stock is better.

25. For an income of Rs. 10, investment = Rs. 96.

For an income of Rs. 12, investment = Rs. $\left(\frac{96}{10}\times 12\right)$ = Rs. 115.20.

30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation : Let n be a positive integer. Then, factorial n, denoted by $\lfloor n$ or n ! is defined as :

$$n ! = n (n - 1) (n - 2) \ldots\ldots 3.2.1.$$

Examples : (*i*) $5 ! = (5 \times 4 \times 3 \times 2 \times 1) = 120$; (*ii*) $4 ! = (4 \times 3 \times 2 \times 1) = 24$ etc.

We define, $0 ! = 1$.

Permutations : *The different arrangements of a given number of things by taking some or all at a time, are called permutations.*

Ex. 1. All permutations (or arrangements) made with the letters *a, b, c* by taking two at a time are (***ab, ba, ac, ca, bc, cb***).

Ex. 2. All permutations made with the letters *a, b, c,* taking all at a time are : (***abc, acb, bac, bca, cab, cba***).

Number of Permutations : Number of all permutations of n things, taken r at a time, is given by :

$$^nP_r = n (n - 1) (n - 2) \ldots\ldots (n - r + 1) = \frac{n !}{(n - r) !}$$

Examples : (*i*) $^6P_2 = (6 \times 5) = 30$. (*ii*) $^7P_3 = (7 \times 6 \times 5) = 210$.

Cor. ***Number of all permutations of n things, taken all at a time = n !***

An Important Result : *If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1 + p_2 + \ldots\ldots + p_r) = n$.*

Then, ***number of permutations of these n objects is*** :

$$\frac{n !}{(p_1 !) \cdot (p_2 !) \ldots\ldots (p_r !)}.$$

Combinations : *Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.*

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by *a, b, c,* taking two at a time are ***ab, bc, ca.***

Ex. 3. The only combination that can be formed of three letters *a, b, c* taken all at a time is ***abc.***

Ex. 4. Various groups of 2 out of four persons A, B, C, D are :

AB, AC, AD, BC, BD, CD.

Ex. 5. Note that *ab* and *ba* are two different permutations but they represent the same combination.

Number of Combinations : The number of all combinations of n things, taken r at a time is :

$$^nC_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2)\ldots\text{to } r \text{ factors}}{r!}.$$

Note that : $^nC_n = 1$ *and* $^nC_0 = 1$.

An Important Result : $^nC_r = {}^nC_{(n-r)}$.

Example : (*i*) $^{11}C_4 = \frac{(11\times10\times9\times8)}{(4\times3\times2\times1)} = 330.$

(*ii*) $^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16\times15\times14}{3!} = \frac{16\times15\times14}{3\times2\times1} = 560.$

SOLVED EXAMPLES

Ex. 1. *Evaluate* : $\frac{30!}{28!}$

Sol. We have, $\frac{30!}{28!} = \frac{30\times29\times(28!)}{28!} = (30\times29) = 870.$

Ex. 2. *Find the value of* (*i*) $^{60}P_3$ (*ii*) $^4P_4$

Sol. (*i*) $^{60}P_3 = \frac{60!}{(60-3)!} = \frac{60!}{57!} = \frac{60\times59\times58\times(57!)}{57!} = (60\times59\times58) = 205320.$

(*ii*) $^4P_4 = 4! = (4\times3\times2\times1) = 24.$

Ex. 3. *Find the value of* (*i*) $^{10}C_3$ (*ii*) $^{100}C_{98}$ (*iii*) $^{50}C_{50}$

Sol. (*i*) $^{10}C_3 = \frac{10\times9\times8}{3!} = \frac{10\times9\times8}{3\times2\times1} = 120.$

(*ii*) $^{100}C_{98} = {}^{100}C_{(100-98)} = {}^{100}C_2 = \left(\frac{100\times99}{2\times1}\right) = 4950.$

(*iii*) $^{50}C_{50} = 1.$ [$\because {}^nC_n = 1$]

Ex. 4. ***How many words can be formed by using all letters of the word 'BIHAR' ?***

Sol. The word BIHAR contains 5 different letters.

$\therefore$ Required number of words = $^5P_5 = 5! = (5\times4\times3\times2\times1) = 120.$

Ex. 5. ***How many words can be formed by using all the letters of the word 'DAUGHTER' so that the vowels always come together ?***

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.

Then, the letters to be arranged are DGHTR (AUE).

These 6 letters can be arranged in $^6P_6 = 6! = 720$ ways.

The vowels in the group (AUE) may be arranged in $3! = 6$ ways.

$\therefore$ Required number of words = $(720 \times 6) = 4320.$

Ex. 6. ***How many words can be formed from the letters of the word 'EXTRA', so that the vowels are never together ?***

Sol. The given word contains 5 different letters.

Taking the vowels EA together, we treat them as one letter.

Then, the letters to be arranged are XTR (EA).

These letters can be arranged in 4 ! = 24 ways.

The vowels EA may be arranged amongst themselves in 2 ! = 2 ways.

Number of words, each having vowels together = (24 × 2) = 48.

Total number of words formed by using all the letters of the given words
= 5 ! = (5 × 4 × 3 × 2 × 1) = 120.

Number of words, each having vowels never together = (120 − 48) = 72.

Ex. 7. ***How many words can be formed from the letters of the word 'DIRECTOR' so that the vowels are always together ?***

Sol. In the given word, we treat the vowels IEO as one letter.

Thus, we have DRCTR (IEO).

This group has 6 letters of which R occurs 2 times and others are different.

Number of ways of arranging these letters = $\frac{6!}{2!}$ = 360.

Now 3 vowels can be arranged among themselves in 3 ! = 6 ways.

∴ Required number of ways = (360 × 6) = 2160.

Ex. 8. ***In how many ways can a cricket eleven be chosen out of a batch of 15 players ?***

Sol. Required number of ways = $^{15}C_{11} = {}^{15}C_{(15-11)} = {}^{15}C_4$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365.$$

Ex. 9. ***In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies ?***

Sol. (3 men out 6) and (2 ladies out of 5) are to be chosen.

∴ Required number of ways = $({}^6C_3 \times {}^5C_2) = \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1}\right) = 200.$

EXERCISE 30

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The value of $^{75}P_2$ is :
   (*a*) 2775 (*b*) 150 (*c*) 5550 (*d*) None of these
2. How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed ?
   (*a*) 40 (*b*) 400 (*c*) 5040 (*d*) 2520
3. How many words with or without meaning, can be formed by using all the letters of the word, 'DELHI', using each letter exactly once ?
   (*a*) 10 (*b*) 25 (*c*) 60 (*d*) 120
4. In how many ways can the letters of the word 'APPLE' be arranged ?
   (*a*) 720 (*b*) 120 (*c*) 60 (*d*) 180 (*e*) None of these
5. In how many ways can the letters of the word 'LEADER' be arranged ?
   (*a*) 72 (*b*) 144 (*c*) 360 (*d*) 720 (*e*) None of these
   (Bank P.O. 2003)

6. In how many different ways can the letters of the word 'RUMOUR' be arranged ?
(a) 180 (b) 90 (c) 30 (d) 720 (e) None of these
(Bank P.O. 2003)

7. How many words can be formed by using all the letters of the word, 'ALLAHABAD' ?
(a) 3780 (b) 1890 (c) 7560 (d) 2520 (e) None of these

8. How many arrangements can be made out of the letters of the word 'ENGINEERING' ?
(a) 277200 (b) 92400 (c) 69300 (d) 23100 (e) None of these

9. How many words can be formed from the letters of the word 'SIGNATURE' so that the vowels always come together ?
(Bank P.O. 2003)
(a) 720 (b) 1440 (c) 2880 (d) 3600 (e) 17280

10. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together ?
(M.B.A. 2002)
(a) 120 (b) 720 (c) 4320 (d) 2160 (e) None of these

11. In how many different ways can the letters of the word 'SOFTWARE' be arranged in such a way that the vowels always come together ?
(Bank P.O. 2003)
(a) 120 (b) 360 (c) 1440 (d) 13440 (e) 720

12. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together ?
(Bank P.O. 2002)
(a) 360 (b) 480 (c) 720 (d) 5040 (e) None of these

13. In how many different ways can the letters of the word 'JUDGE' be arranged in such a way that the vowels always come together ?
(S.B.I.P.O. 2001)
(a) 48 (b) 120 (c) 124 (d) 160 (e) None of these

14. In how many different ways can the letters of the word 'AUCTION' be arranged in such a way that the vowels always come together ?
(S.B.I.P.O. 2000)
(a) 30 (b) 48 (c) 144 (d) 576 (e) None of these

15. In how many different ways can the letters of the word 'BANKING' be arranged so that the vowels always come together ?
(Bank P.O. 2003)
(a) 120 (b) 240 (c) 360 (d) 540 (e) 720

16. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together ?
(S.B.I.P.O. 2003)
(a) 810 (b) 1440 (c) 2880 (d) 50400 (e) 5760

17. In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together ?
(a) 10080 (b) 4989600 (c) 120960 (d) None of these

18. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions ?
(Bank P.O. 2002)
(a) 32 (b) 48 (c) 36 (d) 60 (e) 120

19. In how many different ways can the letters of the word 'MACHINE' be arranged so that the vowels may occupy only the odd positions ?
(a) 210 (b) 576 (c) 144 (d) 1728 (e) 3456

20. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women ?
(Bank P.O. 2003)
(a) 63 (b) 90 (c) 126 (d) 45 (e) 135

21. In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women ?
(Bank P.O. 2003)
(a) 266 (b) 5040 (c) 11760 (d) 86400 (e) None of these

22. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done ?
(M.B.A. 2002)
(a) 564 (b) 645 (c) 735 (d) 756 (e) None of these

23. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there ?
(a) 159 (b) 194 (c) 205 (d) 209 (e) None of these
(S.B.I.P.O. 2000)

24. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw ?
(a) 32 (b) 48 (c) 64 (d) 96 (e) None of these
(Bank P.O. 1998)

25. How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated ? **(S.S.C. 2000)**
(a) 5 (b) 10 (c) 15 (d) 20

26. In how many ways can 21 books on English and 19 books on Hindi be placed in a row on a shelf so that two books on Hindi may not be together ?
(a) 3990 (b) 1540 (c) 1995 (d) 3672 (e) None of these

27. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed ?
(a) 210 (b) 1050 (c) 25200 (d) 21400 (e) None of these

ANSWERS

1. (c)	2. (c)	3. (d)	4. (c)	5. (c)	6. (a)
7. (c)	8. (a)	9. (e)	10. (c)	11. (e)	12. (c)
13. (a)	14. (d)	15. (e)	16. (d)	17. (c)	18. (c)
19. (b)	20. (a)	21. (c)	22. (d)	23. (d)	24. (c)
25. (d)	26. (b)	27. (e)			

SOLUTIONS

1. $^{75}P_2 = \dfrac{75!}{(75-2)!} = \dfrac{75!}{73!} = \dfrac{75 \times 74 \times (73!)}{73!} = (75 \times 74) = 5550.$

2. 'LOGARITHM' contains 10 different letters.
Required number of words = Number of arrangements of 10 letters, taking 4 at a time
$= {}^{10}P_4 = (10 \times 9 \times 8 \times 7) = 5040.$

3. The word 'DELHI' contains 5 different letters.
Required number of words = Number of arrangements of 5 letters, taken all at a time
$= {}^5P_5 = 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

4. The word 'APPLE' contains 5 letters, 1A, 2P, 1L and 1E.
$\therefore$ Required number of ways $= \dfrac{5!}{(1!)(2!)(1!)(1!)} = 60.$

5. The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.
$\therefore$ Required number of ways $= \dfrac{6!}{(1!)(2!)(1!)(1!)(2!)} = 360.$

6. The word 'RUMOUR' contains 6 letters, namely 2R, 2U, 1M and 1U.
$\therefore$ Required number of ways $= \dfrac{6!}{(2!)(2!)(1!)(1!)} = 180.$

7. The word 'ALLAHABAD' contains 9 letters, namely 4A, 2L, 1H, 1B and 1D.

$\therefore$ Requisite number of words $= \dfrac{9\,!}{(4\,!)(2\,!)(2\,!)(1\,!)(1\,!)} = 7560.$

8. The word 'ENGINEERING' contains 11 letters, namely 3E, 3N, 2G, 2I and 1R.

$\therefore$ Required number of arrangements $= \dfrac{11\,!}{(3\,!)(3\,!)(2\,!)(2\,!)(1\,!)} = 277200.$

9. The word 'SIGNATURE' contains 9 different letters.

When the vowels IAUE are taken together, they can be supposed to form an entity, treated as one letter.

Then, the letters to be arranged are SGNTR (IAUE).

These 6 letters can be arranged in ${}^{6}P_{6} = 6\,! = 720$ ways.

The vowels in the group (IAUE) can be arranged amongst themselves in ${}^{4}P_{4} = 4\,! = 24$ ways.

$\therefore$ Required number of words $= (720 \times 24) = 17280.$

10. The word 'OPTICAL' contains 7 different letters.

When the vowels OIA are always together, they can be supposed to form one letter.

Then, we have to arrange the letters PTCL (OIA).

Now, 5 letters can be arranged in $5\,! = 120$ ways.

The vowels (OIA) can be arranged among themselves in $3\,! = 6$ ways.

$\therefore$ Required number of ways $= (120 \times 6) = 720.$

11. The word 'SOFTWARE' contains 8 different letters.

When the vowels OAE are always together, they can be supposed to form one letter.

Thus, we have to arrange the letters SFTWR (OAE).

Now, 5 letters can be arranged in $6\,! = 720$ ways.

The vowels (OAE) can be arranged among themselves in $3\,! = 6$ ways.

$\therefore$ Required number of ways $= (720 \times 6) = 4320.$

12. The word 'LEADING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter.

Then, we have to arrange the letters LDNG (EAI).

Now, 5 letters can be arranged in $5\,! = 120$ ways.

The vowels (EAI) can be arranged among themselves in $3\,! = 6$ ways.

$\therefore$ Required number of ways $= (120 \times 6) = 720.$

13. The word 'JUDGE' has 5 different letters.

When the vowels UE are always together, they can be supposed to form one letter.

Then, we have to arrange the letters JDG (UE).

Now, 4 letters can be arranged in $4\,! = 24$ ways.

The vowels (UE) can be arranged among themselves in $2\,! = 2$ ways.

$\therefore$ Required number of ways $= (24 \times 2) = 48.$

14. The word 'AUCTION' has 7 different letters.

When the vowels AUIO are always together, they can be supposed to form one letter.

Then, we have to arrange the letters CTN (AUIO).

Now, 4 letters can be arranged in $4\,! = 24$ ways.

The vowels (AUIO) can be arranged among themselves in $4\,! = 24$ ways.

$\therefore$ Required number of ways $= (24 \times 24) = 576.$

15. In the word 'BANKING', we treat the two vowels AI as one letter. Thus, we have BNKNG (AI).

This has 6 letters of which N occurs 2 times and the rest are different.

Number of ways of arranging these letters $= \frac{6!}{(2!)(1!)(1!)(1!)(1!)} = 360.$

Now, 2 vowels AI can be arranged in $2! = 2$ ways.

$\therefore$ Required number of ways $= (360 \times 2) = 720.$

16. In the word 'CORPORATION', we treat the vowels OOAIO as one letter. Thus, we have CRPRTN (OOAIO).

This has 7 letters of which R occurs 2 times and the rest are different.

Number of ways of arranging these letters $= \frac{7!}{2!} = 2520.$

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged in $\frac{5!}{3!} = 20$ ways.

$\therefore$ Required number of ways $= (2520 \times 20) = 50400.$

17. In the word 'MATHEMATICS' we treat the vowels AEAI as one letter. Thus, we have MTHMTCS (AEAI).

Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different.

$\therefore$ Number of ways of arranging these letters $= \frac{8!}{(2!)(2!)} = 10080.$

Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.

Number of ways of arranging these letters $= \frac{4!}{2!} = 12.$

$\therefore$ Required number of words $= (10080 \times 12) = 120960.$

18. There are 6 letters in the given word, out of which there are 3 vowels and 3 consonants. Let us mark these positions as under :

$$\left(1\right)\left(2\right)\left(3\right)\left(4\right)\left(5\right)\left(6\right)$$

Now, 3 vowels can be placed at any of the three places out of 4, marked 1, 3, 5.

Number of ways of arranging the vowels $= {}^3P_3 = 3! = 6.$

Also, the 3 consonants can be arranged at the remaining 3 positions.

Number of ways of these arrangements $= {}^3P_3 = 3! = 6.$

Total number of ways $= (6 \times 6) = 36.$

19. There are 7 letters in the given word, out of which there are 3 vowels and 4 consonants. Let us mark the positions to be filled up as follows :

$$\left(1\right)\left(2\right)\left(3\right)\left(4\right)\left(5\right)\left(6\right)\left(7\right)$$

Now, 3 vowels can be placed at any of the three places, out of the four marked 1, 3, 5, 7.

$\therefore$ Number of ways of arranging the vowels $= {}^4P_3 = (4 \times 3 \times 2) = 24.$

Also, the 4 consonants at the remaining 4 positions may be arranged in

$= {}^4P_4 = 4! = 24$ ways.

$\therefore$ Required number of ways $= (24 \times 24) = 576.$

20. Required number of ways $= ({}^7C_5 \times {}^3C_2) = ({}^7C_2 \times {}^3C_1) = \left(\frac{7 \times 6}{2 \times 1} \times 3\right) = 63.$

21. Required number of ways = $({}^{8}C_5 \times {}^{10}C_6)$

$= ({}^{8}C_3 \times {}^{10}C_4) = \left(\frac{8\times7\times6}{3\times2\times1} \times \frac{10\times9\times8\times7}{4\times3\times2\times1}\right) = 11760.$

22. We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only)

∴ Required number of ways $= ({}^{7}C_3 \times {}^{6}C_2) + ({}^{7}C_4 \times {}^{6}C_1) + ({}^{7}C_5)$

$= \left(\frac{7\times6\times5}{3\times2\times1} \times \frac{6\times5}{2\times1}\right) + ({}^{7}C_3 \times {}^{6}C_1) + ({}^{7}C_2)$

$= 525 + \left(\frac{7\times6\times5}{3\times2\times1} \times 6\right) + \left(\frac{7\times6}{2\times1}\right)$

$= (525 + 210 + 21) = 756.$

23. We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

∴ Required number of ways $= ({}^{6}C_1 \times {}^{4}C_3) + ({}^{6}C_2 \times {}^{4}C_2) + ({}^{6}C_3 \times {}^{4}C_1) + ({}^{6}C_4)$

$= ({}^{6}C_1 \times {}^{4}C_1) + ({}^{6}C_2 \times {}^{4}C_2) + ({}^{6}C_3 \times {}^{4}C_1) + ({}^{6}C_2)$

$= (6\times4) + \left(\frac{6\times5}{2\times1} \times \frac{4\times3}{2\times1}\right) + \left(\frac{6\times5\times4}{3\times2\times1} \times 4\right) + \left(\frac{6\times5}{2\times1}\right)$

$= (24 + 90 + 80 + 15) = 209.$

24. We may have (1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

∴ Required number of ways $= ({}^{3}C_1 \times {}^{6}C_2) + ({}^{3}C_2 \times {}^{6}C_1) + ({}^{3}C_3)$

$= \left(3 \times \frac{6\times5}{2\times1}\right) + \left(\frac{3\times2}{2\times1} \times 6\right) + 1 = (45 + 18 + 1) = 64.$

25. Since each desired number is divisible by 5, so we must have 5 at the unit place. So, there is 1 way of doing it.

Tens place can be filled by any of the remaining 5 numbers.

So, there are 5 ways of filling the tens place.

The hundreds place can now be filled by any of the remaining 4 digits. So, there are 4 ways of filling it.

∴ Required number of numbers $= (1 \times 5 \times 4) = 20.$

26. In order that two books on Hindi are never together, we must place all these books as under :

X E X E X E X ..... X E X

where E denotes the position of an English book and X that of a Hindi book.

Since there are 21 books on English, the number of places marked X are therefore, 22.

Now, 19 places out of 22 can be chosen in ${}^{22}C_{19} = {}^{22}C_3 = \frac{22\times21\times20}{3\times2\times1} = 1540$ ways.

Hence, the required number of ways = 1540.

27. Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$= ({}^{7}C_3 \times {}^{4}C_2) = \left(\frac{7\times6\times5}{3\times2\times1} \times \frac{4\times3}{2\times1}\right) = 210.$

Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging 5 letters among themselves

$= 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

∴ Required number of words $= (210 \times 120) = 25200.$

31. PROBABILITY

IMPORTANT FACTS AND FORMULAE

1. **Experiment** : An operation which can produce some well-defined outcomes is called an experiment.
2. **Random Experiment** : An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

   Examples of Performing a Random Experiment :

   (*i*) Rolling an unbiased dice.

   (*ii*) Tossing a fair coin.

   (*iii*) Drawing a card from a pack of well-shuffled cards.

   (*iv*) Picking up a ball of certain colour from a bag containing balls of different colours.

   Details :

   (*i*) When we throw a coin. Then either a Head (H) or a Tail (T) appears.

   (*ii*) A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

   (*iii*) A pack of cards has 52 cards.

   It has 13 cards of each suit, namely **Spades, Clubs, Hearts** and **Diamonds.**

   Cards of spades and clubs are **black cards.**

   Cards of hearts and diamonds are **red cards.**

   There are 4 honours of each suit.

   These are **Aces, Kings, Queens** and **Jacks.**

   These are called **face cards.**
3. **Sample Space** : When we perform an experiment, then the set S of all possible outcomes is called the **Sample Space.**

   Examples of Sample Spaces :

   (*i*) In tossing a coin, S = {H, T}.

   (*ii*) If two coins are tossed, then S = {HH, HT, TH, T T}.

   (*iii*) In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.
4. **Event** : Any subset of a sample space is called an event.
5. **Probability of Occurrence of an Event :**

   Let S be the sample space and let E be an event.

   Then, $E \subset S$.

   $\therefore \quad P(E) = \dfrac{n(E)}{n(S)}$.
6. **Results on Probability :**

   (*i*) $P(S) = 1$ (*ii*) $0 \le P(E) \le 1$ (*iii*) $P(\phi) = 0$

   (*iv*) For any events A and B, we have :

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   (*v*) If $\overline{A}$ denotes (not-A), then $P(\overline{A}) = 1 - P(A)$.

SOLVED EXAMPLES

Ex. 1. ***In a throw of a coin, find the probability of getting a head.***

Sol. Here S = {H, T} and E = {H}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}.$$

Ex. 2. ***Two unbiased coins are tossed. What is the probability of getting at most one head ?***

Sol. Here S = {HH, HT, TH, TT}.

Let E = event of getting at most one head.

$\therefore$ E = {TT, HT, TH}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}.$$

Ex. 3. ***An unbiased die is tossed. Find the probability of getting a multiple of 3.***

Sol. Here S = {1, 2, 3, 4, 5, 6}.

Let E be the event of getting a multiple of 3.

Then, E = {3, 6}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$$

Ex. 4. ***In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.***

Sol. Here, $n(S) = (6 \times 6) = 36$.

Let E = Event of getting a total more than 7

= {(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}.$$

Ex. 5. ***A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.***

Sol. Let S be the sample space. Then,

$$n(S) = \text{Number of ways of drawing 2 balls out of } (6+4) = {}^{10}C_2 = \frac{(10 \times 9)}{(2 \times 1)} = 45.$$

Let E = Event of getting both balls of the same colour. Then,

$n(E)$ = Number of ways of drawing (2 balls out of 6) or (2 balls out of 4)

$$= ({}^{6}C_2 + {}^{4}C_2) = \frac{(6 \times 5)}{(2 \times 1)} + \frac{(4 \times 3)}{(2 \times 1)} = (15 + 6) = 21.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}.$$

Ex. 6. ***Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6 ?***

Sol. Clearly, $n(S) = 6 \times 6 = 36$.

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then

E = {(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)}

$\therefore$ $n(E) = 14$.

$$\text{Hence, } P(E) = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}.$$

Ex. 7. *Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queens ?*

Sol. We have $n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$

Let A = event of getting both black cards;
B = event of getting both queens.

$\therefore$ $A \cap B$ = event of getting queens of black cards.

$\therefore$ $n(A) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325$, $n(B) = {}^{4}C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6$ and $n(A \cap B) = {}^{2}C_2 = 1.$

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{325}{1326}$; $P(B) = \frac{n(B)}{n(S)} = \frac{6}{1326}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{1326}.$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}\right) = \frac{330}{1326} = \frac{55}{221}.$

EXERCISE 31

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer :*

1. In a simultaneous throw of two coins, the probability of getting at least one head is :

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

2. Three unbiased coins are tossed. What is the probability of getting at least 2 heads ?

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

3. Three unbiased coins are tossed. What is the probability of getting at most two heads ?

   (a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{7}{8}$

4. In a single throw of a die, what is the probability of getting a number greater than 4 ?

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

5. In a simultaneous throw of two dice, what is the probability of getting a total of 7 ?

   (a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

6. What is the probability of getting a sum 9 from two throws of a dice ?

   (a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$ **(M.B.A. 2002)**

7. In a simultaneous throw of two dice, what is the probability of getting a doublet ?

   (a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{7}$

8. In a simultaneous throw of two dice, what is the probability of getting a total of 10 or 11 ?

   (a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{7}{12}$ (d) $\frac{5}{36}$

9. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even ? **(Asstt. PF Commissioner's Exam, 2002)**

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{5}{16}$

10. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3 ?

(a) $\frac{3}{10}$ (b) $\frac{3}{20}$ (c) $\frac{2}{5}$ (d) $\frac{1}{2}$

11. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5 ?

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{8}{15}$ (d) $\frac{9}{20}$

12. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize ?

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{2}{7}$ (d) $\frac{5}{7}$

13. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card ?

(a) $\frac{1}{13}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{9}{52}$

14. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is :

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{1}{26}$ (d) $\frac{1}{52}$

15. One card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card or a king ?

(a) $\frac{1}{2}$ (b) $\frac{6}{13}$ (c) $\frac{7}{13}$ (d) $\frac{27}{52}$

16. From a pack of 52 cards, one card is drawn at random. What is the probability that the card drawn is a ten or a spade ?

(a) $\frac{4}{13}$ (b) $\frac{1}{4}$ (c) $\frac{1}{13}$ (d) $\frac{1}{26}$

17. The probability that a card drawn from a pack of 52 cards will be a diamond or a king, is :

(a) $\frac{2}{13}$ (b) $\frac{4}{13}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$

18. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings ? **(M.B.A. 2002; Railways, 2002)**

(a) $\frac{1}{15}$ (b) $\frac{25}{57}$ (c) $\frac{35}{256}$ (d) $\frac{1}{221}$

19. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is : **(M.B.A. 2000)**

(a) $\frac{3}{20}$ (b) $\frac{29}{34}$ (c) $\frac{47}{100}$ (d) $\frac{13}{102}$

20. Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings, is :

(a) $\frac{7}{13}$ (b) $\frac{3}{26}$ (c) $\frac{63}{221}$ (d) $\frac{55}{221}$

21. A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white ?

(a) $\frac{3}{4}$ (b) $\frac{4}{7}$ (c) $\frac{1}{8}$ (d) $\frac{3}{7}$

22. A box contains 5 green, 4 yellow and 3 white marbles. Three marbles are drawn at random. What is the probability that they are not of the same colour ?

(a) $\frac{3}{44}$ (b) $\frac{3}{55}$ (c) $\frac{52}{55}$ (d) $\frac{41}{44}$

(Bank P.O. 2000)

23. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is : **(M.B.A. 2002)**

(a) $\frac{1}{22}$ (b) $\frac{3}{22}$ (c) $\frac{2}{91}$ (d) $\frac{2}{77}$

24. A bag contains 6 white and 4 red balls. Three balls are drawn at random. What is the probability that one ball is red and the other two are white ?

(a) $\frac{1}{2}$ (b) $\frac{1}{12}$ (c) $\frac{3}{10}$ (d) $\frac{7}{12}$

25. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue ? **(Bank P.O. 2003)**

(a) $\frac{10}{21}$ (b) $\frac{11}{21}$ (c) $\frac{2}{7}$ (d) $\frac{5}{7}$

26. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green ? **(Bank P.O. 2002)**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{7}{19}$ (d) $\frac{8}{21}$ (e) $\frac{9}{21}$

27. A box contains 10 black and 10 white balls. The probability of drawing two balls of the same colour, is :

(a) $\frac{9}{19}$ (b) $\frac{9}{38}$ (c) $\frac{10}{19}$ (d) $\frac{5}{19}$

28. A box contains 4 red balls, 5 green balls and 6 white balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green ?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{7}{15}$

29. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is :

(a) $\frac{21}{46}$ (b) $\frac{25}{117}$ (c) $\frac{1}{50}$ (d) $\frac{3}{25}$

30. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The chance that exactly 2 of them are children, is :

(a) $\frac{1}{9}$ (b) $\frac{1}{5}$ (c) $\frac{1}{12}$ (d) $\frac{10}{21}$

31. A box contains 20 electric bulbs, out of which 4 are defective. Two bulbs are chosen at random from this box. The probability that at least one of these is defective, is :

(a) $\frac{4}{19}$ (b) $\frac{7}{19}$ (c) $\frac{12}{19}$ (d) $\frac{21}{95}$

32. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi ?

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{3}{10}$

33. Two dice are tossed. The probability that the total score is a prime number is :

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{7}{9}$

34. A speaks truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other, narrating the same incident ?

(a) 5% (b) 15% (c) 35% (d) 45%

(Bank P.O. 2000)

35. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is (1 / 7) and the probability of wife's selection is (1 / 5). What is the probability that only one of them is selected ?

(a) $\frac{4}{5}$ (b) $\frac{2}{7}$ (c) $\frac{8}{15}$ (d) $\frac{4}{7}$

ANSWERS

1. (d)	2. (b)	3. (d)	4. (b)	5. (a)	6. (c)	7. (a)	8. (d)	9. (b)
10. (a)	11. (d)	12. (c)	13. (b)	14. (c)	15. (c)	16. (a)	17. (b)	18. (d)
19. (d)	20. (d)	21. (b)	22. (d)	23. (c)	24. (a)	25. (a)	26. (d)	27. (a)
28. (b)	29. (a)	30. (d)	31. (b)	32. (a)	33. (b)	34. (c)	35. (b)	

SOLUTIONS

1. Here S = {HH, HT, TH, T T}.

Let E = event of getting at least one head = {HT, TH, HH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$.

2. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at least two heads = {THH, HTH, HHT, HHH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$.

3. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at most two heads.

Then, E = {TTT, TTH, THT, HTT, THH, HTH, HHT}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$.

4. When a die is thrown, we have S = {1, 2, 3, 4, 5, 6}.

Let E = event of getting a number greater than 4 = {5, 6}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.

5. We know that in a simultaneous throw of two dice, $n(S) = 6 \times 6 = 36$.

Let E = event of getting a total of 7 = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.

6. In two throws of a die, $n(S) = (6 \times 6) = 36$.

Let E = event of getting a sum 9 = {(3, 6), (4, 5), (5, 4), (6, 3)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$.

7. In a simultaneous throw of two dice, $n(S) = (6 \times 6) = 36$.
Let E = event of getting a doublet = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

8. In a simultaneous throw of two dice, we have $n(S) = (6 \times 6) = 36$.
Let E = event of getting a total of 10 or 11 = {(4, 6), (5, 5), (6, 4), (5, 6), (6, 5)}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}.$$

9. In a simultaneous throw of two dice, we have $n(S) = (6 \times 6) = 36$.
Let E = event of getting two numbers whose product is even.
Then, E = {(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)}.

$\therefore \quad n(E) = 27.$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}.$$

10. Here, S = {1, 2, 3, 4, ..., 19, 20}.
Let E = event of getting a multiple of 3 = {3, 6, 9, 12, 15, 18}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}.$$

11. Here, S = {1, 2, 3, 4, ..., 19, 20}.
Let E = event of getting a multiple of 3 or 5 = {3, 6, 9, 12, 15, 18, 5, 10, 20}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}.$$

12. $P(\text{getting a prize}) = \dfrac{10}{(10+25)} = \dfrac{10}{35} = \dfrac{2}{7}.$

13. Clearly, there are 52 cards, out of which there are 16 face cards.

$$\therefore \quad P(\text{getting a face card}) = \frac{16}{52} = \frac{4}{13}.$$

14. Here, $n(S) = 52$.
Let E = event of getting a queen of club or a king of heart.
Then, $n(E) = 2$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}.$$

15. Here, $n(S) = 52$.
There are 26 red cards (including 2 kings) and there are 2 more kings.
Let E = event of getting a red card or a king.
Then, $n(E) = 28$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}.$$

16. Here, $n(S) = 52$.
There are 13 spades (including one ten) and there are 3 more tens.
Let E = event of getting a ten or a spade.
Then, $n(E) = (13 + 3) = 16$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}.$$

17. Here, $n(S) = 52$.

There are 13 cards of diamond (including one king) and there are 3 more kings.

Let E = event of getting a diamond or a king.

Then, $n(E) = (13 + 3) = 16$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}.$$

18. Let S be the sample space. Then,

$$n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 2 kings out of 4.

$$\therefore \quad n(E) = {}^{4}C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221}.$$

19. Let S be the sample space. Then,

$$n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 1 spade and 1 heart.

$\therefore$ $n(E)$ = number of ways of choosing 1 spade out of 13 and 1 heart out of 13

$$= ({}^{13}C_1 \times {}^{13}C_1) = (13 \times 13) = 169.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{169}{1326} = \frac{13}{102}.$$

20. Clearly, $n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{2} = 1326.$

Let E_1 = event of getting both red cards,

E_2 = event of getting both kings.

Then, $E_1 \cap E_2$ = event of getting 2 kings of red cards.

$$\therefore \quad n(E_1) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325; \; n(E_2) = {}^{4}C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6;$$

$$n(E_1 \cap E_2) = {}^{2}C_2 = 1.$$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{325}{1326}; \; P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{1326}; \; P(E_1 \cap E_2) = \frac{1}{1326}.$$

$$\therefore \quad P(\text{both red or both kings}) = P(E_1 \cup E_2)$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}\right) = \frac{330}{1326} = \frac{55}{221}.$$

21. Total number of balls = (6 + 8) = 14.

Number of white balls = 8.

$$P(\text{drawing a white ball}) = \frac{8}{14} = \frac{4}{7}.$$

22. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 3 marbles out of 12

$$= {}^{12}C_3 = \frac{(12 \times 11 \times 10)}{(3 \times 2 \times 1)} = 220.$$

Let E be the event of drawing 3 balls of the same colour.

Then, E = event of drawing (3 balls out of 5) or (3 balls out of 4) or (3 balls out of 3)

$\Rightarrow \quad n(E) = ({}^5C_3 + {}^4C_3 + {}^3C_3) = ({}^5C_2 + {}^4C_1 + 1) = \dfrac{(5 \times 4)}{(2 \times 1)} + 4 + 1 = 15.$

$\Rightarrow \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{15}{220} = \dfrac{3}{44}.$

$\therefore \quad$ Required probability $= \left(1 - \dfrac{3}{44}\right) = \dfrac{41}{44}.$

23. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 3 balls out of 15 $= {}^{15}C_3 = \dfrac{(15 \times 14 \times 13)}{(3 \times 2 \times 1)} = 455.$

Let E = event of getting all the 3 red balls.

$\therefore \quad n(E) = {}^5C_3 = {}^5C_2 = \dfrac{(5 \times 4)}{(2 \times 1)} = 10.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{10}{455} = \dfrac{2}{91}.$

24. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 3 balls out of 10

$= {}^{10}C_3 = \dfrac{(10 \times 9 \times 8)}{(3 \times 2 \times 1)} = 120.$

Let E = event of drawing 1 red and 2 white balls

$\therefore \quad n(E)$ = Number of ways of drawing 1 red ball out of 4 and 2 white balls out of 6

$= ({}^4C_1 \times {}^6C_2) = \left(4 \times \dfrac{6 \times 5}{2 \times 1}\right) = 60.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{60}{120} = \dfrac{1}{2}.$

25. Total number of balls = (2 + 3 + 2) = 7.

Let S be the sample space. Then,

$n(S)$ = Number of ways of drawing 2 balls out of 7 $= {}^7C_2 = \dfrac{(7 \times 6)}{(2 \times 1)} = 21.$

Let E = Event of drawing 2 balls, none of which is blue.

$\therefore \quad n(E)$ = Number of ways of drawing 2 balls out of (2 + 3) balls

$= {}^5C_2 = \dfrac{(5 \times 4)}{(2 \times 1)} = 10.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{10}{21}.$

26. Total number of balls = (8 + 7 + 6) = 21.

Let E = event that the ball drawn is neither red nor green

= event that the ball drawn is red.

$\therefore \quad n(E) = 8$

$\therefore \quad P(E) = \dfrac{8}{21}.$

27. Total number of balls = 20.

Let S be the sample space. Then,

$n(S)$ = Number of ways of drawing 2 balls out of 20 $= {}^{20}C_2 = \dfrac{(20 \times 19)}{(2 \times 1)} = 190.$

Let E = event of drawing 2 balls of the same colour

$n(E) = ({}^{10}C_2 + {}^{10}C_2) = 2 \times \left(\frac{10 \times 9}{2 \times 1}\right) = 90.$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{90}{190} = \frac{9}{19}.$

28. Total number of balls = (4 + 5 + 6) = 15.

$\therefore \quad n(S) = 15.$

Let E_1 = event of drawing a red ball

and E_2 = event of drawing a green ball.

Then, $E_1 \cap E_2 = \phi$.

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \left(\frac{4}{15} + \frac{5}{15}\right) = \frac{9}{15} = \frac{3}{5}.$

29. Let S be the sample space and E be the event of selecting 1 girl and 2 boys. Then,

$n(S)$ = Number of ways of selecting 3 students out of 25

$= {}^{25}C_3 = \frac{(25 \times 24 \times 23)}{(3 \times 2 \times 1)} = 2300.$

$n(E) = ({}^{10}C_1 \times {}^{15}C_2) = \left\{10 \times \frac{(15 \times 14)}{(2 \times 1)}\right\} = 1050.$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1050}{2300} = \frac{21}{46}.$

30. Let S be the sample space and E be the event of choosing four persons such that 2 of them are children. Then,

$n(S)$ = Number of ways of choosing 4 persons out of 9

$= {}^{9}C_4 = \frac{(9 \times 8 \times 7 \times 6)}{(4 \times 3 \times 2 \times 1)} = 126.$

$n(E)$ = Number of ways of choosing 2 children out of 4 and 2 persons out of (3 + 2) persons

$= ({}^{4}C_2 \times {}^{5}C_2) = \frac{(4 \times 3)}{(2 \times 1)} \times \frac{(5 \times 4)}{(2 \times 1)} = 60.$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{60}{126} = \frac{10}{21}.$

31. $P(\text{None is defective}) = \frac{{}^{16}C_2}{{}^{20}C_2} = \left(\frac{16 \times 15}{2 \times 1} \times \frac{2 \times 1}{20 \times 19}\right) = \frac{12}{19}.$

$P(\text{at least one is defective}) = \left(1 - \frac{12}{19}\right) = \frac{7}{19}.$

32. $P(E) = \frac{30}{100} = \frac{3}{10}$, $P(H) = \frac{20}{100} = \frac{1}{5}$ and $P(E \cap H) = \frac{10}{100} = \frac{1}{10}.$

$P(E \text{ or } H) = P(E \cup H)$

$= P(E) + P(H) - P(E \cap H)$

$= \left(\frac{3}{10} + \frac{1}{5} - \frac{1}{10}\right) = \frac{4}{10} = \frac{2}{5}.$

33. Clearly, $n(S) = (6 \times 6) = 36$.

Let E = Event that the sum is a prime number.

Then, E = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}

$\therefore \quad n(E) = 15$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}.$

34. Let A = Event that A speaks the truth

and B = Event that B speaks the truth.

Then, $P(A) = \dfrac{75}{100} = \dfrac{3}{4}$, $P(B) = \dfrac{80}{100} = \dfrac{4}{5}$.

$\therefore \quad P(\overline{A}) = \left(1 - \dfrac{3}{4}\right) = \dfrac{1}{4}$ and $P(\overline{B}) = \left(1 - \dfrac{4}{5}\right) = \dfrac{1}{5}$.

P (A and B contradict each other)

= P [(A speaks the truth and B tells a lie) or (A tells a lie and B speaks the truth)]

$= P[(A \text{ and } \overline{B}) \text{ or } (\overline{A} \text{ and } B)]$

$= P(A \text{ and } \overline{B}) + P(\overline{A} \text{ and } B)$

$= P(A) \cdot P(\overline{B}) + P(\overline{A}) \cdot P(B)$

$= \left(\dfrac{3}{4} \times \dfrac{1}{5}\right) + \left(\dfrac{1}{4} \times \dfrac{4}{5}\right) = \left(\dfrac{3}{20} + \dfrac{1}{5}\right) = \dfrac{7}{20} = \left(\dfrac{7}{20} \times 100\right)\% = 35\%.$

$\therefore$ A and B contradict each other in 35% of the cases.

35. Let A = Event that the husband is selected

and B = Event that the wife is selected.

Then, $P(A) = \dfrac{1}{7}$ and $P(B) = \dfrac{1}{5}$.

$\therefore \quad P(\overline{A}) = \left(1 - \dfrac{1}{7}\right) = \dfrac{6}{7}$ and $P(\overline{B}) = \left(1 - \dfrac{1}{5}\right) = \dfrac{4}{5}$.

$\therefore$ Required probability = P [(A and not B) or (B and not A)]

$= P[(A \text{ and } \overline{B}) \text{ or } (B \text{ and } \overline{A})]$

$= P(A \text{ and } \overline{B}) + P(B \text{ and } \overline{A})$

$= P(A) \cdot P(\overline{B}) + P(B) \cdot P(\overline{A}) = \left(\dfrac{1}{7} \times \dfrac{4}{5}\right) + \left(\dfrac{1}{5} \times \dfrac{6}{7}\right) = \dfrac{10}{35} = \dfrac{2}{7}.$

32. TRUE DISCOUNT

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that :

Sum due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. (156 – 100) = Rs. 56 = (Sum due) – (P.W.).

We define : ***T.D. = Interest on P.W.***

Amount = (P.W.) + (T.D.).

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

1. $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (\text{R} \times \text{T})} = \dfrac{100 \times \text{T.D.}}{\text{R} \times \text{T}}$.

2. $\text{T.D.} = \dfrac{(\text{P.W.}) \times \text{R} \times \text{T}}{100} = \dfrac{\text{Amount} \times \text{R} \times \text{T}}{100 + (\text{R} \times \text{T})}$.

3. $\text{Sum} = \dfrac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$.

4. (S.I.) – (T.D.) = S.I. on T.D.

5. When the sum is put at compound interest, then $\text{P.W.} = \dfrac{\text{Amount}}{\left(1 + \dfrac{\text{R}}{100}\right)^{\text{T}}}$.

SOLVED EXAMPLES

Ex. 1. ***Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.***

Sol. $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (\text{R} \times \text{T})} = \text{Rs.}\left[\dfrac{100 \times 930}{100 + (8 \times 3)}\right] = \text{Rs.}\left(\dfrac{100 \times 930}{124}\right) = \text{Rs. } 750.$

T.D. = (Amount) – (P.W.) = Rs. (930 – 750) = Rs. 180.

Ex. 2. ***The true discount on a bill due 9 months hence at 12% per annum is Rs. 540. Find the amount of the bill and its present worth.***

Sol. Let amount be Rs. x. Then,

$$\frac{x \times \text{R} \times \text{T}}{100 + (\text{R} \times \text{T})} = \text{T.D.} \Rightarrow \frac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)} = 540 \Rightarrow x = \left(\frac{540 \times 109}{9}\right) = \text{Rs. } 6540.$$

$\therefore$ Amount = Rs. 6540.

P.W. = Rs. (6540 – 540) = Rs. 6000.

Ex. 3. ***The true discount on a certain sum of money due 3 years hence is Rs. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.***

Sol. T.D. = Rs. 250 and S.I. = Rs. 375.

$$\therefore \quad \text{Sum due} = \frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \text{Rs.}\left(\frac{375 \times 250}{375 - 250}\right) = \text{Rs. } 750.$$

$$\text{Rate} = \left(\frac{100 \times 375}{750 \times 3}\right)\% = 16\frac{2}{3}\%.$$

Ex. 4. ***The difference between the simple interest and true discount on a certain sum of money for 6 months at $12\frac{1}{2}\%$ per annum is Rs. 25. Find the sum.***

Sol. Let the sum be Rs. x. Then,

$$\text{T.D.} = \frac{x \times \frac{25}{2} \times \frac{1}{2}}{100 + \left(\frac{25}{2} \times \frac{1}{2}\right)} = \left(x \times \frac{25}{4} \times \frac{4}{425}\right) = \frac{x}{17}.$$

$$\text{S.I.} = \left(x \times \frac{25}{2} \times \frac{1}{2} \times \frac{1}{100}\right) = \frac{x}{16}.$$

$$\therefore \quad \frac{x}{16} - \frac{x}{17} = 25 \Rightarrow 17x - 16x = 25 \times 16 \times 17 \Rightarrow x = 6800.$$

Hence, sum due = Rs. 6800.

Ex. 5. ***A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement he gains Rs. 40. What is the amount of the bill, if the money be worth $12\frac{1}{2}\%$?***

Sol. Let the sum be Rs. x. Then,

$$\left[\frac{x}{2} + \frac{\frac{x}{2} \times 100}{100 + \left(\frac{25}{2} \times 2\right)}\right] - \frac{x \times 100}{100 + \left(\frac{25}{2} \times 1\right)} = 40 \Rightarrow \frac{x}{2} + \frac{2x}{5} - \frac{8x}{9} = 40 \Rightarrow x = 3600.$$

$\therefore$ Amount of the bill = Rs. 3600.

EXERCISE 32

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The present worth of Rs. 2310 due $2\frac{1}{2}$ years hence, the rate of interest being 15% per annum, is :
   (a) Rs. 1750 (b) Rs. 1680 (c) Rs. 1840 (d) Rs. 1443.75
2. If the true discount on a sum due 2 years hence at 14% per annum be Rs. 168, the sum due is :
   (a) Rs. 768 (b) Rs. 968 (c) Rs. 1960 (d) Rs. 2400
3. The true discount on Rs. 2562 due 4 months hence is Rs. 122. The rate percent is :
   (a) 12% (b) $13\frac{1}{3}\%$ (c) 15% (d) 14%

4. The true discount on Rs. 1760 due after a certain time at 12% per annum is Rs. 160. The time after which it is due is :
(a) 6 months (b) 8 months (c) 9 months (d) 10 months

5. The true discount on a bill due 9 months hence at 16% per annum is Rs. 189. The amount of the bill is :
(a) Rs. 1386 (b) Rs. 1764 (c) Rs. 1575 (d) Rs. 2268

6. The interest on Rs. 750 for 2 years is the same as the true discount on Rs. 960 due 2 years hence. If the rate of interest is the same in both cases, it is :
(a) 12% (b) 14% (c) 15% (d) $16\frac{2}{3}\%$

7. The simple interest and the true discount on a certain sum for a given time and at a given rate are Rs. 85 and Rs. 80 respectively. The sum is :
(a) Rs. 1800 (b) Rs. 1450 (c) Rs. 1360 (d) Rs. 6800

8. If Rs. 10 be allowed as true discount on a bill of Rs. 110 due at the end of a certain time, then the discount allowed on the same sum due at the end of double the time is :
(a) Rs. 20 (b) Rs. 21.81 (c) Rs. 22 (d) Rs. 18.33

9. A man wants to sell his scooter. There are two offers, one at Rs. 12,000 cash and the other at a credit of Rs. 12,880 to be paid after 8 months, money being at 18% per annum. Which is the better offer ?
(a) Rs. 12,000 in cash (b) Rs. 12,880 at credit (c) Both are equally good

10. Goods were bought for Rs. 600 and sold the same day for Rs. 688.50 at a credit of 9 months and thus gaining 2%. The rate of interest per annum is :
(a) $16\frac{2}{3}\%$ (b) $14\frac{1}{2}\%$ (c) $13\frac{1}{3}\%$ (d) 15%

11. The present worth of Rs. 1404 due in two equal half-yearly instalments at 8% per annum simple interest is :
(a) Rs. 1325 (b) Rs. 1300 (c) Rs. 1350 (d) Rs. 1500

12. A trader owes a merchant Rs. 10,028 due 1 year hence. The trader wants to settle the account after 3 months. If the rate of interest is 12% per annum, how much cash should he pay ?
(a) Rs. 9025.20 (b) Rs. 9200 (c) Rs. 9600 (d) Rs. 9560

13. A man buys a watch for Rs. 1950 in cash and sells it for Rs. 2200 at a credit of 1 year. If the rate of interest is 10% per annum, the man :
(a) gains Rs. 55 (b) gains Rs. 50 (c) loses Rs. 30 (d) gains Rs. 30

14. A man purchased a cow for Rs. 3000 and sold it the same day for Rs. 3600, allowing the buyer a credit of 2 years. If the rate of interest be 10% per annum, then the man has a gain of :
(a) 0% (b) 5% (c) 7.5% (d) 10%

15. A owes B, Rs. 1573 payable $1\frac{1}{2}$ years hence. Also B owes A, Rs. 1444.50 payable 6 months hence. If they want to settle the account forthwith, keeping 14% as the rate of interest, then who should pay and how much ?
(a) A, Rs. 28.50 (b) B, Rs. 37.50 (c) A, Rs. 50 (d) B, Rs. 50

16. A has to pay Rs. 220 to B after 1 year. B asks A to pay Rs. 110 in cash and defer the payment of Rs. 110 for 2 years. A agrees to it. If the rate of interest be 10% per annum, in this mode of payment :
(a) There is no gain or loss to any one (b) A gains Rs. 7.34
(c) A loses Rs. 7.34 (d) A gains Rs. 11

17. Rs. 20 is the true discount on Rs. 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same ?
(a) Rs. 10 (b) Rs. 10.40 (c) Rs. 15.20 (d) Rs. 13

ANSWERS

1. (b) 2. (a) 3. (c) 4. (d) 5. (b) 6. (b) 7. (c) 8. (d) 9. (a)
10. (a) 11. (a) 12. (b) 13. (b) 14. (a) 15. (d) 16. (b) 17. (b)

SOLUTIONS

1. P.W. = Rs. $\left[\frac{100 \times 2310}{100 + \left(15 \times \frac{5}{2}\right)}\right]$ = Rs. 1680.

2. P.W. = $\frac{100 \times \text{T.D.}}{\text{R} \times \text{T}} = \frac{100 \times 168}{14 \times 2} = 600$.

   ∴ Sum = (P.W. + T.D.) = Rs. (600 + 168) = Rs. 768.

3. P.W. = Rs. (2562 − 122) = Rs. 2440.

   ∴ S.I. on Rs. 2440 for 4 months is Rs. 122.

   ∴ Rate = $\left(\frac{100 \times 122}{2440 \times \frac{1}{3}}\right)\%$ = 15%.

4. P.W. = Rs. (1760 − 160) = Rs. 1600.

   ∴ S.I. on Rs. 1600 at 12% is Rs. 160.

   ∴ Time = $\left(\frac{100 \times 160}{1600 \times 12}\right) = \frac{5}{6}$ years = $\left(\frac{5}{6} \times 12\right)$ months = 10 months.

5. Let P.W. be Rs. x. Then, S.I. on Rs. x at 16% for 9 months = Rs. 189.

   ∴ $x \times 16 \times \frac{9}{12} \times \frac{1}{100} = 189$ or $x = 1575$.

   ∴ P.W. = Rs. 1575.

   ∴ Sum due = P.W. + T.D. = Rs. (1575 + 189) = Rs. 1764.

6. S.I. on Rs. 750 = T.D. on Rs. 960.

   This means P.W. of Rs. 960 due 2 years hence is Rs. 750.

   ∴ T.D. = Rs. (960 − 750) = Rs. 210.

   Thus, S.I. on Rs. 750 for 2 years is Rs. 210.

   ∴ Rate = $\left(\frac{100 \times 210}{750 \times 2}\right)\%$ = 14%.

7. Sum = $\frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \frac{85 \times 80}{(85 - 80)}$ = Rs. 1360.

8. S.I. on Rs. (110 − 10) for a certain time = Rs. 10.

   S.I. on Rs. 100 for double the time = Rs. 20.

   T.D. on Rs. 120 = Rs. (120 − 100) = Rs. 20.

   T.D. on Rs. 110 = Rs. $\left(\frac{20}{120} \times 110\right)$ = Rs 18.33.

9. P.W. of Rs. 12,880 due 8 months hence

   = Rs. $\left[\frac{12880 \times 100}{100 + \left(18 \times \frac{8}{12}\right)}\right]$ = Rs. $\left(\frac{12880 \times 100}{112}\right)$ = Rs. 11500.

   Clearly, Rs. 12,000 in cash is a better offer.

10. S.P. = 102% of Rs. 600 = Rs. $\left(\frac{102}{100} \times 600\right)$ = Rs. 612.

Now, P.W. = Rs. 612 and sum = Rs. 688.50.

∴ T.D. = Rs. (688.50 − 612) = Rs. 76.50.

Thus, S.I. on Rs. 612 for 9 months is Rs. 76.50.

∴ Rate = $\left(\frac{100 \times 76.50}{612 \times \frac{3}{4}}\right)\% = 16\frac{2}{3}\%$.

11. Required sum = P.W. of Rs. 702 due 6 months hence + P.W. of Rs. 702 due 1 year hence

$$= \text{Rs.}\left[\left(\frac{100 \times 702}{100 + 8 \times \frac{1}{2}}\right) + \left(\frac{100 \times 702}{100 + (8 \times 1)}\right)\right] = \text{Rs. } (675 + 650) = \text{Rs. } 1325.$$

12. Required money = P.W. of Rs. 10028 due 9 months hence

$$= \text{Rs.}\left[\frac{10028 \times 100}{100 + \left(12 \times \frac{9}{12}\right)}\right] = \text{Rs. } 9200.$$

13. S.P. = P.W. of Rs. 2200 due 1 year hence = Rs. $\left[\frac{2200 \times 100}{100 + (10 \times 1)}\right]$ = Rs. 2000.

∴ Gain = Rs. (2000 − 1950) = Rs. 50.

14. C.P. = Rs. 3000. S.P. = Rs. $\left[\frac{3600 \times 100}{100 + (10 \times 2)}\right]$ = Rs. 3000.

Gain = 0%.

15. A owes = P.W. of Rs. 1573 due $\frac{3}{2}$ years hence

$$= \text{Rs.}\left[\frac{1573 \times 100}{100 + \left(14 \times \frac{3}{2}\right)}\right] = \text{Rs.}\left(\frac{1573 \times 100}{121}\right) = \text{Rs. } 1300.$$

B owes = P.W. of Rs. 1444.50 due 6 months hence

$$= \text{Rs.}\left[\frac{1444.50 \times 100}{100 + \left(14 \times \frac{1}{2}\right)}\right] = \text{Rs.}\left(\frac{1444.50 \times 100}{107}\right) = \text{Rs. } 1350.$$

∴ B must pay Rs. 50 to A.

16. A has to pay = P.W. of Rs. 220 due 1 year hence = Rs. $\left[\frac{220 \times 100}{100 + (10 \times 1)}\right]$ = Rs. 200.

A actually pays = Rs. 110 + P.W. of Rs. 110 due 2 years hence

$$= \left[110 + \frac{110 \times 100}{100 + (10 \times 2)}\right] = \text{Rs. } 192.66.$$

∴ A gains = Rs. (200 − 192.66) = Rs. 7.34.

17. S.I. on Rs. (260 − 20) for a given time = Rs. 20.

S.I. on Rs. 240 for half the time = Rs. 10.

T.D. on Rs. 250 = Rs. 10.

∴ T.D. on Rs. 260 = Rs. $\left(\frac{10}{250} \times 260\right)$ = Rs. 10.40.

33. BANKER'S DISCOUNT

IMPORTANT CONCEPTS

Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.

The date exactly after 5 months is called *nominally due date*. Three days (known as *grace days*) are added to it to get a date, known as *legally due date*.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (*i.e.*, Rs. 10,000 in this case) for the period from the date on which the bill was discounted (*i.e.*, paid by the banker) and the legally due date. This amount is known as ***Banker's Discount*** **(B.D.)**

Thus, **B.D.** *is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.*

Banker's Gain (B.G.) = (B.D.) – (T.D.) *for the unexpired time.*

Note : When the date of the bill is not given, grace days are not to be added.

IMPORTANT FORMULAE

1. B.D. = S.I. on bill for unexpired time.
2. B.G. = (B.D.) – (T.D.) = S.I. on T.D. $= \frac{(\text{T.D.})^2}{\text{P.W.}}$.
3. T.D. $= \sqrt{\text{P.W.} \times \text{B.G.}}$
4. B.D. $= \left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}\right)$.
5. T.D. $= \left[\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})}\right]$.
6. Amount $= \left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$.
7. T.D. $= \left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}\right)$

SOLVED EXAMPLES

Ex. 1. ***A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.***

Sol. Face value of the bill = Rs. 6000.

Date on which the bill was drawn = July 14 at 5 months.

Nominally due date = December 14. Legally due date = December 17.

Date on which the bill was discounted = October 5.

Unexpired time : Oct. Nov. Dec.

$$26 + 30 + 17 = 73 \text{ days} = \frac{1}{5} \text{ year}.$$

$\therefore$ B.D. = S.I. on Rs. 6000 for $\frac{1}{5}$ year = Rs. $\left(6000 \times 10 \times \frac{1}{5} \times \frac{1}{100}\right)$ = Rs. 120.

T.D. = Rs. $\left[\frac{6000 \times 10 \times \frac{1}{5}}{100 + \left(10 \times \frac{1}{5}\right)}\right]$ = Rs. $\left(\frac{12000}{102}\right)$ = Rs. 117.64.

$\therefore$ B.G. = (B.D.) – (T.D.) = Rs. (120 – 117.64) = Rs. 2.36.

Money received by the holder of the bill = Rs. (6000 – 120) = Rs. 5880.

Ex. 2. ***If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?***

Sol. B.G. = S.I. on T.D. = Rs. $\left(120 \times 15 \times \frac{1}{2} \times \frac{1}{100}\right)$ = Rs. 9.

$\therefore$ (B.D.) – (T.D.) = Rs. 9.

$\therefore$ B.D. = Rs. (120 + 9) = Rs. 129.

Ex. 3. ***The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.***

Sol. S.I. on Rs. 1800 = T.D. on Rs. 1872.

$\therefore$ P.W. of Rs. 1872 is Rs. 1800.

$\therefore$ Rs. 72 is S.I. on Rs. 1800 at 12%.

$\therefore$ Time = $\left(\frac{100 \times 72}{12 \times 1800}\right)$ year = $\frac{1}{3}$ year = 4 months.

Ex. 4. ***The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.***

Sol. Sum = $\left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$ = Rs. $\left(\frac{120 \times 110}{120 - 110}\right)$ = Rs. 1320.

Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.

$\therefore$ Rate = $\left(\frac{100 \times 120}{1320 \times \frac{2}{3}}\right)\%$ = $13\frac{7}{11}\%$.

Ex. 5. ***The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.***

Sol. T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$

$\therefore$ B.G. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$ = Rs. $\left(\frac{110 \times 110}{1100}\right)$ = Rs. 11.

$\therefore$ B.D. = (T.D. + B.G.) = Rs. (110 + 11) = Rs. 121.

Ex. 6. ***The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.***

Sol. Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}}$.

$\therefore$ $\frac{\text{T.D.}}{\text{B.G.}} = \frac{\text{Sum}}{\text{B.D.}} = \frac{1650}{165} = \frac{10}{1}$.

Thus, if B.G. is Re 1, T.D. = Rs. 10.

If B.D. is Rs. 11, T.D. = Rs. 10. If B.D. is Rs. 165, T.D. = Rs. $\left(\frac{10}{11} \times 165\right)$ = Rs. 150.

And, B.G. = Rs. (165 – 150) = Rs. 15.

Ex. 7. *What rate percent does a man get for his money when in discounting a bill due 10 months hence, he deducts 10% of the amount of the bill?*

Sol. Let, amount of the bill = Rs. 100. Money deducted = Rs. 10.

Money received by the holder of the bill = Rs. (100 − 10) = Rs. 90.

∴ S.I. on Rs. 90 for 10 months = Rs. 10.

∴ $\text{Rate} = \left(\frac{100 \times 10}{90 \times \frac{10}{12}}\right)\% = 13\frac{1}{3}\%$.

EXERCISE 33

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer :*

1. The true discount on a bill of Rs. 540 is Rs. 90. The banker's discount is :
   (*a*) Rs. 60 (*b*) Rs. 108 (*c*) Rs. 110 (*d*) Rs. 112
2. The present worth of a certain bill due sometime hence is Rs. 800 and the true discount is Rs. 36. The banker's discount is :
   (*a*) Rs. 37 (*b*) Rs. 37.62 (*c*) Rs. 34.38 (*d*) Rs. 38.98
3. The present worth of a certain sum due sometime hence is Rs. 1600 and the true discount is Rs. 160. The banker's gain is :
   (*a*) Rs. 20 (*b*) Rs. 24 (*c*) Rs. 16 (*d*) Rs. 12
4. The banker's gain of a certain sum due 2 years hence at 10% per annum is Rs. 24. The present worth is :
   (*a*) Rs. 480 (*b*) Rs. 520 (*c*) Rs. 600 (*d*) Rs. 960
5. The banker's gain on a bill due 1 year hence at 12% per annum is Rs. 6. The true discount is :
   (*a*) Rs. 72 (*b*) Rs. 36 (*c*) Rs. 54 (*d*) Rs. 50
6. The banker's discount on a bill due 4 months hence at 15% is Rs. 420. The true discount is :
   (*a*) Rs. 400 (*b*) Rs. 360 (*c*) Rs. 480 (*d*) Rs. 320
7. The banker's gain on a sum due 3 years hence at 12% per annum is Rs. 270. The banker's discount is :
   (*a*) Rs. 960 (*b*) Rs. 840 (*c*) Rs. 1020 (*d*) Rs. 760
8. The present worth of a sum due sometime hence is Rs. 576 and the banker's gain is Rs. 16. The true discount is :
   (*a*) Rs. 36 (*b*) Rs. 72 (*c*) Rs. 48 (*d*) Rs. 96
9. The banker's discount on Rs. 1600 at 15% per annum is the same as true discount on Rs. 1680 for the same time and at the same rate. The time is :
   (*a*) 3 months (*b*) 4 months (*c*) 6 months (*d*) 8 months
10. The banker's discount on a sum of money for $1\frac{1}{2}$ years is Rs. 558 and the true discount on the same sum for 2 years is Rs. 600. The rate percent is :
    (*a*) 10% (*b*) 13% (*c*) 12% (*d*) 15%
11. The banker's discount of a certain sum of money is Rs. 72 and the true discount on the same sum for the same time is Rs. 60. The sum due is :
    (*a*) Rs. 360 (*b*) Rs. 432 (*c*) Rs. 540 (*d*) Rs. 1080

12. The banker's discount on a certain sum due 2 years hence is $\frac{11}{10}$ of the true discount. The rate percent is :

(a) 11% (b) 10% (c) 5% (d) 5.5%

13. The banker's gain on a certain sum due $1\frac{1}{2}$ years hence is $\frac{3}{25}$ of the banker's discount. The rate percent is :

(a) $5\frac{1}{5}\%$ (b) $9\frac{1}{9}\%$ (c) $8\frac{1}{8}\%$ (d) $6\frac{1}{6}\%$

ANSWERS

1. (b)	2. (b)	3. (c)	4. (c)	5. (d)	6. (a)	7. (c)
8. (d)	9. (b)	10. (c)	11. (a)	12. (c)	13. (b)	

SOLUTIONS

1. P.W. = Rs. (540 − 90) = Rs. 450.

∴ S.I. on Rs. 450 = Rs. 90

S.I. on Rs. 540 = Rs. $\left(\frac{90}{450} \times 540\right)$ = Rs. 108.

∴ B.D. = Rs. 108.

2. B.G. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$ = Rs. $\left(\frac{36 \times 36}{800}\right)$ = Rs. 1.62.

∴ B.D. = (T.D. + B.G.) = Rs. (36 + 1.62) = Rs. 37.62.

3. B.G. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$ = Rs. $\left(\frac{160 \times 160}{1600}\right)$ = Rs. 16.

4. T.D. = $\left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}\right)$ = Rs. $\left(\frac{24 \times 100}{10 \times 2}\right)$ = Rs. 120.

∴ P.W. = $\frac{100 \times \text{T.D.}}{\text{Rate} \times \text{Time}}$ = Rs. $\left(\frac{100 \times 120}{10 \times 2}\right)$ = Rs. 600.

5. T.D. = $\frac{\text{B.G.} \times 100}{\text{R} \times \text{T}}$ = Rs. $\left(\frac{6 \times 100}{12 \times 1}\right)$ = Rs. 50.

6. T.D. = $\frac{\text{B.D.} \times 100}{100 + (\text{R} \times \text{T})}$ = Rs. $\left[\frac{420 \times 100}{100 + \left(15 \times \frac{1}{3}\right)}\right]$ = Rs. $\left(\frac{420 \times 100}{105}\right)$ = Rs. 400.

7. T.D. = $\left(\frac{\text{B.G.} \times 100}{\text{R} \times \text{T}}\right)$ = Rs. $\left(\frac{270 \times 100}{12 \times 3}\right)$ = Rs. 750.

∴ B.D. = Rs. (750 + 270) = Rs. 1020.

8. T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$ = $\sqrt{576 \times 16}$ = 96.

9. S.I. on Rs. 1600 = T.D. on Rs. 1680.

∴ Rs. 1600 is the P.W. of Rs. 1680, *i.e.*, Rs. 80 is S.I. on Rs. 1600 at 15%.

∴ Time = $\left(\frac{100 \times 80}{1600 \times 15}\right)$ year = $\frac{1}{3}$ year = 4 months.

10. B.D. for $\frac{3}{2}$ years = Rs. 558. B.D. for 2 years = Rs. $\left(558 \times \frac{2}{3} \times 2\right)$ = Rs. 744.

T.D. for 2 years = Rs. 600.

$\therefore$ Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}$ = Rs. $\left(\frac{744 \times 600}{144}\right)$ = Rs. 3100.

Thus, Rs. 744 is S.I. on Rs. 3100 for 2 years.

$\therefore$ Rate = $\left(\frac{100 \times 744}{3100 \times 2}\right)\%$ = 12%.

11. Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}$ = Rs. $\left(\frac{72 \times 60}{72 - 60}\right)$ = Rs. $\left(\frac{72 \times 60}{12}\right)$ = Rs. 360.

12. Let T.D. be Re 1. Then, B.D. = Rs. $\frac{11}{10}$ = Rs. 1.10.

$\therefore$ Sum = Rs. $\left(\frac{1.10 \times 1}{1.10 - 1}\right)$ = Rs. $\left(\frac{110}{10}\right)$ = Rs. 11.

$\therefore$ S.I. on Rs. 11 for 2 years is Rs. 1.10.

$\therefore$ Rate = $\left(\frac{100 \times 1.10}{11 \times 2}\right)\%$ = 5%.

13. Let, B.D. = Re 1. Then, B.G. = Re $\frac{3}{25}$.

$\therefore$ T.D. = (B.D. − B.G.) = Re $\left(1 - \frac{3}{25}\right)$ = Re $\frac{22}{25}$.

Sum = $\left(\frac{1 \times \frac{22}{25}}{1 - \frac{22}{25}}\right)$ = Rs. $\frac{22}{3}$.

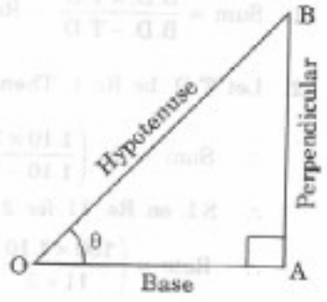
S.I. on Rs. $\frac{22}{3}$ for $1\frac{1}{2}$ years is Re 1.

$\therefore$ Rate = $\left(\frac{100 \times 1}{\frac{22}{3} \times \frac{3}{2}}\right)\%$ = $9\frac{1}{9}\%$.

34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1. We already know that :
   In a rt. angled ΔOAB, where $\angle BOA = \theta$,



   (*i*) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{OB}$;

   (*ii*) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OA}{OB}$;

   (*iii*) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{OA}$;

   (*iv*) $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{OB}{AB}$;

   (*v*) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA}$;

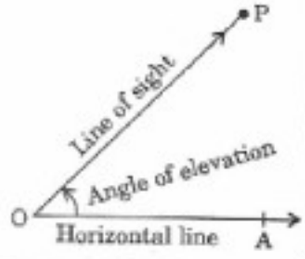
   (*vi*) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB}$.

2. **Trigonometrical Identities :**
   (*i*) $\sin^2\theta + \cos^2\theta = 1$. (*ii*) $1 + \tan^2\theta = \sec^2\theta$. (*iii*) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$.

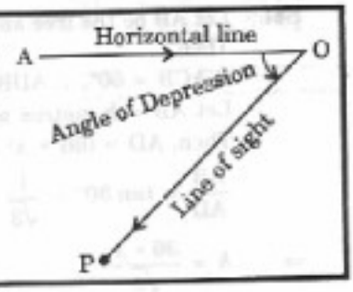
3. **Values of T-ratios :**

θ	$0°$	$(\pi/6)$ $30°$	$(\pi/4)$ $45°$	$(\pi/3)$ $60°$	$(\pi/2)$ $90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

4. **Angle of Elevation** : Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.
   $\therefore$ Angle of elevation of P from O = $\angle AOP$.



5. Angle of Depression : Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



SOLVED EXAMPLES

Ex. 1. ***If the height of a pole is $2\sqrt{3}$ metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.***

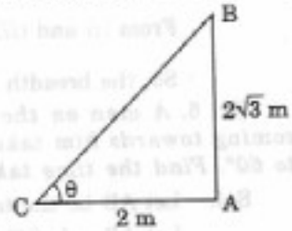
Sol. Let AB be the pole and AC be its shadow.

Let angle of elevation, $\angle ACB = \theta$.

Then, AB = $2\sqrt{3}$ m, AC = 2 m.

$$\tan\theta = \frac{AB}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = 60°.$$

So, the angle of elevation is 60°.



Ex. 2. ***A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.***

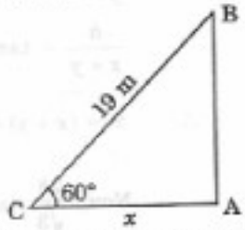
Sol. Let AB be the wall and BC be the ladder.

Then, $\angle ACB = 60°$ and BC = 19 m.

Let AC = x metres

$$\frac{AC}{BC} = \cos 60° \Rightarrow \frac{x}{19} = \frac{1}{2} \Rightarrow x = \frac{19}{2} = 9.5.$$

$\therefore$ Distance of the foot of the ladder from the wall = 9.5 m.



Ex. 3. ***The angle of elevation of the top of a tower at a point on the ground is 30°. On walking 24 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.***

Sol. Let AB be the tower and C and D be the points of observation. Then,

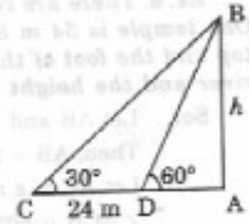
$$\frac{AB}{AD} = \tan 60° = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}.$$

$$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}.$$

$$CD = (AC - AD) = \left(h\sqrt{3} - \frac{h}{\sqrt{3}}\right).$$

$\therefore$ $$h\sqrt{3} - \frac{h}{\sqrt{3}} = 24 \Rightarrow h = 12\sqrt{3} = (12 \times 1.73) = 20.76.$$

Hence, the height of the tower is 20.76 m.



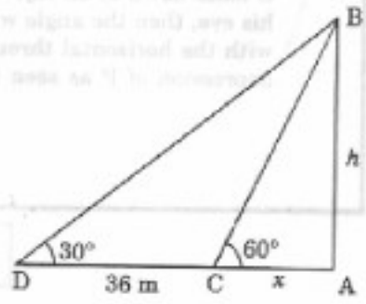
Ex. 4. ***A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 36 m from the bank, he finds the angle to be 30°. Find the breadth of the river.***

Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$\angle ACB = 60°$, $\angle ADB = 30°$ and CD = 36 m.

Let AB = h metres and AC = x metres.

Then, AD = $(36 + x)$ metres.



$$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{36+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{36+x}{\sqrt{3}} \qquad ...(i)$$

$$\frac{AB}{AC} = \tan 60° = \sqrt{3} \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow \quad h = \sqrt{3}\,x \qquad ...(ii)$$

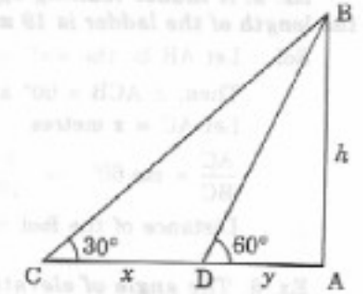
From (i) and (ii), we get : $\frac{36+x}{\sqrt{3}} = \sqrt{3}\,x \Rightarrow x = 18$ m.

So, the breadth of the river = 18 m.

Ex. 5. ***A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.***

Sol. Let AB be the tower and C and D be the two positions of the boat.

Let AB = h, CD = x and AD = y.



$$\frac{h}{y} = \tan 60° = \sqrt{3} \Rightarrow y = \frac{h}{\sqrt{3}}.$$

$$\frac{h}{x+y} = \tan 30° = \frac{1}{\sqrt{3}} \rightarrow x + y = \sqrt{3}\,h.$$

$$\therefore \quad x = (x+y) - y = \left(\sqrt{3}\,h - \frac{h}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}.$$

Now, $\frac{2h}{\sqrt{3}}$ is covered in 10 min.

$\therefore$ $\frac{h}{\sqrt{3}}$ will be covered in $\left(10 \times \frac{\sqrt{3}}{2h} \times \frac{h}{\sqrt{3}}\right) = 5$ min.

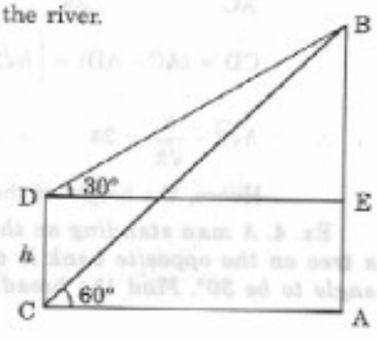
Hence, required time = 5 minutes.

Ex. 6. ***There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.***

Sol. Let AB and CD be the two temples and AC be the river.

Then, AB = 54 m.

Let AC = x metres and CD = h metres.



$\angle ACB = 60°$, $\angle EDB = 30°$.

$$\frac{AB}{AC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad AC = \frac{AB}{\sqrt{3}} = \frac{54}{\sqrt{3}} = \left(\frac{54}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = 18\sqrt{3} \text{ m.}$$

DE = AC = $18\sqrt{3}$ m.

$$\frac{BE}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BE = \left(18\sqrt{3} \times \frac{1}{\sqrt{3}}\right) = 18 \text{ m.}$$

$$\therefore \quad CD = AE = AB - BE = (54 - 18) \text{ m} = 36 \text{ m.}$$

So, Width of the river = AC = $18\sqrt{3}$ m = (18 × 1.73) m = 31.14 m.

Height of the other temple = CD = 18 m.

EXERCISE 34

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is : **(R.R.B. 2003)**
   (a) 30° (b) 45° (c) 60° (d) 90°
2. From a point P on a level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, the distance of point P from the foot of the tower is :
   (a) 149 m (b) 156 m (c) 173 m (d) 200 m **(R.R.B. 2002)**
3. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is :
   (a) 2.3 m (b) 4.6 m (c) 7.8 m (d) 9.2 m
4. An observer 1.6 m tall is $20\sqrt{3}$ m away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The height of the tower is :
   (a) 21.6 m (b) 23.2 m (c) 24.72 m (d) None of these
5. Two ships are sailing in the sea on the two sides of a lighthouse. The angles of elevation of the top of the lighthouse as observed from the two ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is :
   (a) 173 m (b) 200 m (c) 273 m (d) 300 m
6. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of elevation becomes 60°. What is the distance between the base of the tower and the point P ? **(Bank P.O. 1999)**
   (a) $4\sqrt{3}$ units (b) 8 units (c) 12 units
   (d) Data inadequate (e) None of these
7. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top of the tower increases by 15°. The height of the tower is :
   (a) 17.3 m (b) 21.9 m (c) 27.3 m (d) 30 m
8. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water ?
   (a) 32 kmph (b) 36 kmph (c) 38 kmph
   (d) 40 kmph (e) 42 kmph **(S.B.I.P.O. 1999)**
9. On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, the distance between the objects is :
   (a) 63.5 m (b) 76.9 m (c) 86.7 m (d) 90 m

10. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower ?

(*a*) 14 min. 35 sec. (*b*) 15 min. 49 sec. (*c*) 16 min. 23 sec. (*d*) 18 min. 5 sec.

(R.R.B. 2002)

11. The top of a 15 metre high tower makes an angle of elevation of 60° with the bottom of an electric pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole ?

(*a*) 5 metres (*b*) 8 metres (*c*) 10 metres

(*d*) 12 metres (*e*) None of these

ANSWERS

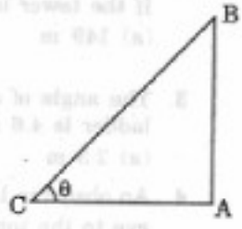
1. (*a*)	**2.** (*c*)	**3.** (*d*)	**4.** (*a*)	**5.** (*c*)	**6.** (*d*)
7. (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*c*)	**11.** (*e*)	

SOLUTIONS

1. Let AB be the tree and AC be its shadow.

Let $\angle ACB = \theta$.

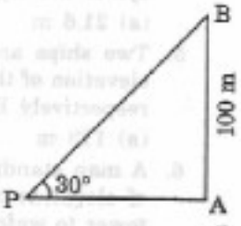
Then, $\frac{AC}{AB} = \sqrt{3} \Rightarrow \cot\theta = \sqrt{3} \Rightarrow \theta = 30°$.



2. Let AB be the tower. Then, $\angle APB = 30°$ and AB = 100 m.

$\frac{AB}{AP} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AP = (AB \times \sqrt{3}) = 100\sqrt{3}$ m.

$= (100 \times 1.73)$ m $= 173$ m.

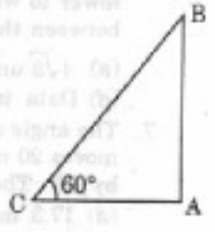


3. Let AB be the wall and BC be the ladder.

Then, $\angle ACB = 60°$ and AC = 4.6 m.

$\frac{AC}{BC} = \cos 60° = \frac{1}{2}$

$\Rightarrow BC = 2 \times AC = (2 \times 4.6)$ m $= 9.2$ m.



4. Let AB be the observer and CD be the tower.

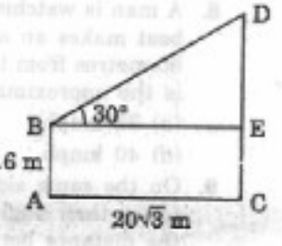
Draw BE ⊥ CD.

Then, CE = AB = 1.6 m, BE = AC = $20\sqrt{3}$ m.

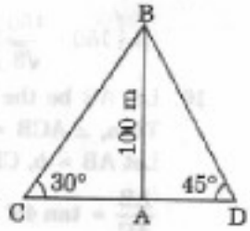
$\frac{DE}{BE} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow DE = \frac{20\sqrt{3}}{\sqrt{3}}$ m $= 20$ m.

$\therefore CD = CE + DE = (1.6 + 20)$ m $= 21.6$ m.



5. Let AB be the lighthouse and C and D be the positions of the ships. Then,
AB = 100 m, $\angle ACB = 30°$ and $\angle ADB = 45°$.



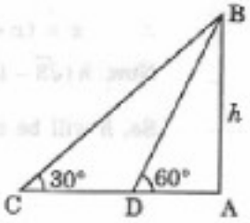
$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = 100\sqrt{3}$ m.

$\frac{AB}{AD} = \tan 45° = 1 \Rightarrow AD = AB = 100$ m.

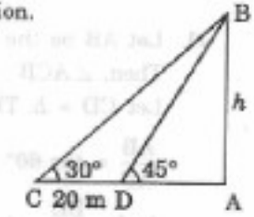
$\therefore$ CD = (AC + AD) = $(100\sqrt{3} + 100)$ m

$= 100(\sqrt{3} + 1)$ m $= (100 \times 2.73)$ m $= 273$ m.

6. One of AB, AD and CD must have been given.
So, the data is inadequate.



7. Let AB be the tower and C and D be the points of observation.
Then, $\angle ACB = 30°$, $\angle ADB = 45°$ and CD = 20 m.
Let AB = h.



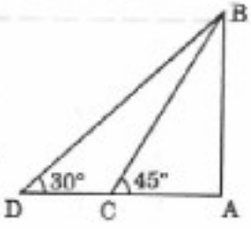
Then, $\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}$.

And, $\frac{AB}{AD} = \tan 45° = 1 \Rightarrow AD = AB = h$.

$CD = 20 \Rightarrow (AC - AD) = 20 \Rightarrow h\sqrt{3} - h = 20$.

$\therefore h = \frac{20}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 10(\sqrt{3}+1)$ m $= (10 \times 2.73)$ m $= 27.3$ m.

8. Let AB be the tower and C and D be the two positions of the boats.
Then, $\angle ACB = 45°$, $\angle ADB = 30°$ and AC = 60 m.
Let AB = h.



Then, $\frac{AB}{AC} = \tan 45° = 1 \Rightarrow AB = AC \Rightarrow h = 60$ m.

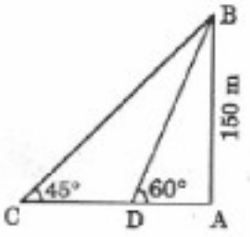
And, $\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AD = (AB \times \sqrt{3}) = 60\sqrt{3}$ m.

$\therefore$ CD = (AD − AC) = $60(\sqrt{3} - 1)$ m.

Hence, required speed = $\left[\frac{60(\sqrt{3}-1)}{5}\right]$ m/s = (12×0.73) m/s

$= \left(12 \times 0.73 \times \frac{18}{5}\right)$ km/hr = 31.5 km/hr ≈ 32 km/hr.

9. Let AB be the tower and C and D be the objects.
Then, AB = 150 m, $\angle ACB = 45°$ and $\angle ADB = 60°$.



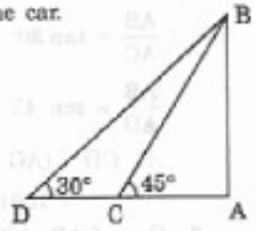
$\frac{AB}{AD} = \tan 60° = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{150}{\sqrt{3}}$ m.

$\frac{AB}{AC} = \tan 45° = 1 \Rightarrow AC = AB = 150$ m.

$\therefore$ CD = (AC − AD)

$= \left(150 - \frac{150}{\sqrt{3}}\right)$ m $= \left[\frac{150(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right]$ m $= 50(3-\sqrt{3})$ m $= (50 \times 1.27)$ m $= 63.5$ m.

10. Let AB be the tower and C and D be the two positions of the car.



Then, $\angle ACB = 45°$, $\angle ADB = 30°$.

Let AB $= h$, CD $= x$ and AC $= y$.

$\frac{AB}{AC} = \tan 45° = 1 \Rightarrow \frac{h}{y} = 1 \Rightarrow y = h.$

$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow x + y = \sqrt{3}\,h.$

$\therefore \quad x = (x+y) - y = \sqrt{3}\,h - h = h(\sqrt{3}-1).$

Now, $h(\sqrt{3}-1)$ is covered in 12 min.

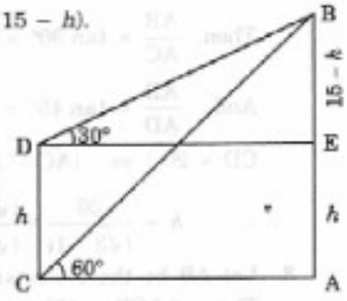
So, h will be covered in $\left[\frac{12}{h(\sqrt{3}-1)} \times h\right] = \frac{12}{(\sqrt{3}-1)}$ min.

$= \left(\frac{1200}{73}\right)$ min. $\approx$ 16 min. 23 sec.

11. Let AB be the tower and CD be the electric pole.

Then, $\angle ACB = 60°$, $\angle EDB = 30°$ and AB $= 15$ m.

Let CD $= h$. Then, BE $=$ (AB $-$ AE) $=$ (AB $-$ CD) $= (15 - h)$.



$\frac{AB}{AC} = \tan 60° = \sqrt{3} \Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{15}{\sqrt{3}}.$

And, $\frac{BE}{DE} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow DE = (BE \times \sqrt{3}) = \sqrt{3}(15 - h).$

$AC = DE \Rightarrow \frac{15}{\sqrt{3}} = \sqrt{3}(15 - h)$

$\Rightarrow 3h = (45 - 15) \Rightarrow h = 10$ m.

35. ODD MAN OUT AND SERIES

EXERCISE 35

Directions : ***Find the odd man out :***

1. 3, 5, 7, 12, 17, 19
   (*a*) 19 (*b*) 17 (*c*) 13 (*d*) 12
2. 10, 14, 16, 18, 21, 24, 26
   (*a*) 26 (*b*) 24 (*c*) 21 (*d*) 18
3. 3, 5, 9, 11, 14, 17, 21
   (*a*) 21 (*b*) 17 (*c*) 14 (*d*) 9
4. 1, 4, 9, 16, 23, 25, 36
   (*a*) 9 (*b*) 23 (*c*) 25 (*d*) 36
5. 6, 9, 15, 21, 24, 28, 30
   (*a*) 28 (*b*) 21 (*c*) 24 (*d*) 30
6. 41, 43, 47, 53, 61, 71, 73, 81
   (*a*) 61 (*b*) 71 (*c*) 73 (*d*) 81
7. 16, 25, 36, 72, 144, 196, 225
   (*a*) 36 (*b*) 72 (*c*) 196 (*d*) 225
8. 10, 25, 45, 54, 60, 75, 80
   (*a*) 10 (*b*) 45 (*c*) 54 (*d*) 75
9. 1, 4, 9, 16, 20, 36, 49
   (*a*) 1 (*b*) 9 (*c*) 20 (*d*) 49
10. 8, 27, 64, 100, 125, 216, 343
    (*a*) 27 (*b*) 100 (*c*) 125 (*d*) 343
11. 1, 5, 14, 30, 50, 55, 91
    (*a*) 5 (*b*) 50 (*c*) 55 (*d*) 91
12. 385, 462, 572, 396, 427, 671, 264
    (*a*) 385 (*b*) 427 (*c*) 671 (*d*) 264
13. 835, 734, 642, 751, 853, 981, 532
    (*a*) 751 (*b*) 853 (*c*) 981 (*d*) 532
14. 331, 482, 551, 263, 383, 242, 111
    (*a*) 263 (*b*) 383 (*c*) 242 (*d*) 111
15. 2, 5, 10, 17, 26, 37, 50, 64
    (*a*) 50 (*b*) 26 (*c*) 37 (*d*) 64
16. 19, 28, 39, 52, 67, 84, 102
    (*a*) 52 (*b*) 102 (*c*) 84 (*d*) 67
17. 253, 136, 352, 460, 324, 631, 244
    (*a*) 136 (*b*) 324 (*c*) 352 (*d*) 631
18. 2, 5, 10, 50, 500, 5000
    (*a*) 0 (*b*) 5 (*c*) 10 (*d*) 5000
19. 4, 5, 7, 10, 14, 18, 25, 32
    (*a*) 7 (*b*) 14 (*c*) 18 (*d*) 33

Directions : ***Find out the wrong number in each sequence :***

20. 22, 33, 66, 99, 121, 279, 594
(*a*) 33 (*b*) 121 (*c*) 279 (*d*) 594

21. 36, 54, 18, 27, 9,18.5, 4.5
(*a*) 4.5 (*b*) 18.5 (*c*) 54 (*d*) 18

22. 582, 605, 588, 611, 634, 617, 600
(*a*) 634 (*b*) 611 (*c*) 605 (*d*) 600

23. 46080, 3840, 384, 48, 24, 2, 1
(*a*) 1 (*b*) 2 (*c*) 24 (*d*) 384

24. 1, 8, 27, 64, 124, 216, 343
(*a*) 8 (*b*) 27 (*c*) 64 (*d*) 124

25. 5, 16, 6, 16, 7, 16, 9
(*a*) 9 (*b*) 7 (*c*) 6 (*d*) None of these

26. 6, 13, 18, 25, 30, 37, 40
(*a*) 25 (*b*) 30 (*c*) 37 (*d*) 40

27. 56, 72, 90, 110, 132, 150
(*a*) 72 (*b*) 110 (*c*) 132 (*d*) 150

28. 8, 13, 21, 32, 47, 63, 83
(*a*) 47 (*b*) 63 (*c*) 32 (*d*) 83

29. 25, 36, 49, 81, 121, 169, 225
(*a*) 36 (*b*) 49 (*c*) 121 (*d*) 169

30. 1, 2, 6, 15, 31, 56, 91
(*a*) 31 (*b*) 91 (*c*) 56 (*d*) 15

31. 52, 51, 48, 43, 34, 27, 16
(*a*) 27 (*b*) 34 (*c*) 43 (*d*) 48

32. 105, 85, 60, 30, 0, – 45, – 90
(*a*) 0 (*b*) 85 (*c*) – 45 (*d*) 60

33. 4, 6, 8, 9, 10, 11, 12
(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 9

34. 125, 127, 130, 135, 142, 153, 165
(*a*) 130 (*b*) 142 (*c*) 153 (*d*) 165

35. 16, 36, 64, 81, 100, 144, 190
(*a*) 81 (*b*) 100 (*c*) 190 (*d*) 36

36. 125, 123, 120, 115, 108, 100, 84
(*a*) 123 (*b*) 115 (*c*) 100 (*d*) 84

37. 3, 10, 21, 36, 55, 70, 105
(*a*) 105 (*b*) 70 (*c*) 36 (*d*) 55

38. 4, 9, 19, 39, 79, 160, 319
(*a*) 319 (*b*) 160 (*c*) 79 (*d*) 39

39. 10, 14, 28, 32, 64, 68, 132
(*a*) 32 (*b*) 68 (*c*) 132 (*d*) 28

40. 8, 27, 125, 343, 1331
(*a*) 1331 (*b*) 343 (*c*) 125 (*d*) None of these

Directions : ***Insert the missing number*** **:**

41. 4, – 8, 16, – 32, 64, (.....)
(a) 128 (b) – 128 (c) 192 (d) – 192

42. 5, 10, 13, 26, 29, 58, 61, (.....)
(a) 122 (b) 64 (c) 125 (d) 128

43. 1, 4, 9, 16, 25, 36, 49, (.....)
(a) 54 (b) 56 (c) 64 (d) 81

44. 1, 8, 27, 64, 125, 216, (.....)
(a) 354 (b) 343 (c) 392 (d) 245

45. 11, 13, 17, 19, 23, 29, 31, 37, 41, (.....)
(a) 43 (b) 47 (c) 53 (d) 51

46. 16, 33, 65, 131, 261, (.....)
(a) 523 (b) 521 (c) 613 (d) 721

47. 3, 7, 6, 5, 9, 3, 12, 1, 15, (.....)
(a) 18 (b) 13 (c) – 1 (d) 3

48. 15, 31, 63, 127, 255, (.....)
(a) 513 (b) 511 (c) 517 (d) 523

49. 2, 6, 12, 20, 30, 42, 56, (.....)
(a) 60 (b) 64 (c) 72 (d) 70

50. 8, 24, 12, 36, 18, 54, (.....)
(a) 27 (b) 108 (c) 68 (d) 72

51. 165, 195, 255,285, 345, (.....)
(a) 375 (b) 420 (c) 435 (d) 390

52. 7, 26, 63, 124, 215, 342, (.....)
(a) 481 (b) 511 (c) 391 (d) 421

53. 2, 4, 12, 48, 240, (.....)
(a) 960 (b) 1440 (c) 1080 (d) 1920

54. 8, 7, 11, 12, 14, 17, 17, 22, (.....)
(a) 27 (b) 20 (c) 22 (d) 24

55. 10, 5, 13, 10, 16, 20, 19, (.....)
(a) 22 (b) 40 (c) 38 (d) 23

56. 1, 2, 4, 8, 16, 32, 64, (.....), 256
(a) 148 (b) 128 (c) 154 (d) 164

57. 71, 76, 69, 74, 67, 72, (.....)
(a) 77 (b) 65 (c) 80 (d) 76

58. 9, 12, 11, 14, 13, (.....), 15
(a) 12 (b) 16 (c) 10 (d) 17

59. Complete the series : 2, 5, 9, 19, 37, ...
(a) 76 (b) 74 (c) 75 (d) None of these

60. Find the wrong number in the series : 3, 8, 15, 24, 34, 48, 63
(a) 15 (b) 24 (c) 34 (d) 48 (e) 63

61. Find the wrong number in the series : 2, 9, 28, 65, 126, 216, 344
(a) 2 (b) 28 (c) 65 (d) 126 (e) 216

62. Find out the wrong number in the series : 5, 15, 30, 135, 405, 1215, 3645
(a) 3645 (b) 1215 (c) 405 (d) 30 (e) 15

63. Find out the wrong number in the series : 125, 106, 88, 76, 65, 58, 53
(a) 125 (b) 106 (c) 88 (d) 76 (e) 65

Directions : ***Find out the wrong number in the series :***

64. 190, 166, 145, 128, 112, 100, 91
(a) 100 (b) 166 (c) 145 (d) 128 (e) 112

65. 1, 1, 2, 6, 24, 96, 720
(a) 720 (b) 96 (c) 24 (d) 6 (e) 2

66. 40960, 10240, 2560, 640, 200, 40, 10
(a) 640 (b) 40 (c) 200 (d) 2560 (e) 10240

67. 64, 71, 80, 91, 104, 119, 135, 155
(a) 71 (b) 80 (c) 104 (d) 119 (e) 135

68. 7, 8, 18, 57, 228, 1165, 6996
(a) 8 (b) 18 (c) 57 (d) 228 (e) 1165

69. 3, 7, 15, 27, 63, 127, 255
(a) 7 (b) 15 (c) 27 (d) 63 (e) 127

70. 19, 26, 33, 46, 59, 74, 91
(a) 26 (b) 33 (c) 46 (d) 59 (e) 74

71. 2880, 480, 92, 24, 8, 4, 4
(a) 480 (b) 92 (c) 24 (d) 8 (e) 4

72. 445, 221, 109, 46, 25, 11, 4
(a) 221 (b) 109 (c) 46 (d) 25 (e) 11

73. 3, 7, 15, 39, 63, 127, 255, 511
(a) 7 (b) 15 (c) 39 (d) 63 (e) 127

74. 1, 3, 10, 21, 64, 129, 356, 777
(a) 10 (b) 21 (c) 64 (d) 129 (e) 356

75. 196, 169, 144, 121, 100, 80, 64
(a) 169 (b) 144 (c) 121 (d) 100 (e) 80

76. 6, 12, 48, 100, 384, 768, 3072
(a) 768 (b) 384 (c) 100 (d) 48 (e) 12

77. 10, 26, 74, 218, 654, 1946, 5834
(a) 26 (b) 74 (c) 218 (d) 654 (e) 1946

78. 15, 16, 34, 105, 424, 2124, 12576
(a) 16 (b) 34 (c) 105 (d) 424 (e) 2124

79. 2807, 1400, 697, 347, 171, 84, 41, 20
(a) 697 (b) 347 (c) 171 (d) 84 (e) 41

80. 32, 36, 41, 61, 86, 122, 171, 235
(a) 41 (b) 61 (c) 86 (d) 122 (e) 171

81. 3, 4, 9, 22.5, 67.5, 202.5, 810
(a) 4 (b) 9 (c) 22.5 (d) 67.5 (e) 202.5

82. 1, 2, 8, 33, 148, 760, 4626
(a) 2 (b) 8 (c) 33 (d) 148 (e) 760

83. 3, 8, 18, 46, 100, 210, 432
(a) 8 (b) 18 (c) 46 (d) 100 (e) 210

84. 789, 645, 545, 481, 440, 429, 425
(a) 645 (b) 545 (c) 481 (d) 440 (e) 429

85. 1060, 510, 242, 106, 46, 16, 3
(a) 510 (b) 242 (c) 106 (d) 46 (e) 16

86. 5, 8, 20, 42, 124, 246, 736
(a) 8 (b) 20 (c) 42 (d) 124 (e) 246

87. 2, 3, 6, 15, 52.5, 157.5, 630
(a) 3 (b) 6 (c) 15 (d) 52.5 (e) 157.5

88. 888, 440, 216, 104, 48, 22, 6
(a) 440 (b) 216 (c) 104 (d) 48 (e) 22

89. 4, 5, 15, 49, 201, 1011, 6073
(a) 5 (b) 15 (c) 49 (d) 201 (e) 1011

ANSWERS

1. (d)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (c)
10. (b)	**11.** (b)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (b)	**18.** (d)
19. (c)	**20.** (c)	**21.** (b)	**22.** (a)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (d)	**27.** (d)
28. (a)	**29.** (a)	**30.** (b)	**31.** (b)	**32.** (a)	**33.** (b)	**34.** (d)	**35.** (c)	**36.** (c)
37. (b)	**38.** (b)	**39.** (c)	**40.** (d)	**41.** (b)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (a)
46. (a)	**47.** (c)	**48.** (b)	**49.** (c)	**50.** (a)	**51.** (c)	**52.** (b)	**53.** (b)	**54.** (b)
55. (b)	**56.** (b)	**57.** (b)	**58.** (b)	**59.** (c)	**60.** (c)	**61.** (e)	**62.** (d)	**63.** (c)
64. (d)	**65.** (b)	**66.** (c)	**67.** (e)	**68.** (d)	**69.** (c)	**70.** (b)	**71.** (b)	**72.** (c)
73. (c)	**74.** (e)	**75.** (e)	**76.** (c)	**77.** (d)	**78.** (e)	**79.** (b)	**80.** (a)	**81.** (a)
82. (e)	**83.** (b)	**84.** (d)	**85.** (c)	**86.** (b)	**87.** (d)	**88.** (e)	**89.** (a)	

SOLUTIONS

1. Each of the numbers except 12, is a prime number.
2. Each of the numbers except 21, is an even number.
3. Each of the numbers except 14, is an odd number.
4. Each of the given numbers except 23, is a perfect square.
5. Each of the numbers except 28, is a multiple of 3.
6. Each of the numbers except 81, is a prime number.
7. Each of the numbers except 72, is a perfect square.
8. Each of the numbers except 54, is a multiple of 5.
9. The pattern is $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$. But, instead of 5^2, it is 20, which is to be turned out.
10. The pattern is $2^3, 3^3, 4^3, 5^3, 6^3, 7^3$. But, 100 is not a perfect cube.
11. The pattern is 1^2, $1^2 + 2^2$, $1^2 + 2^2 + 3^2$, $1^2 + 2^2 + 3^2 + 4^2$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$. But, 50 is not of this pattern.
12. In each number except 427, the middle digit is the sum of the other two.
13. In each number except 751, the difference of third and first digit is the middle one.
14. In each number except 383, the product of first and third digits is the middle one.
15. The pattern is $x^2 + 1$, where $x = 1, 2, 3, 4, 5, 6, 7, 8$ etc. But, 64 is out of pattern.
16. The pattern is $x^2 + 3$, where $x = 4, 5, 6, 7, 8, 9$ etc. But, 102 is out of pattern.
17. Sum of the digits in each number, except 324 is 10.
18. Pattern is 1st × 2nd = 3rd; 2nd × 3rd = 4th; 3rd × 4th = 5th.
But, 4th × 5th = 50 × 500 = 25000 ≠ 5000 = 6th.

19. 2nd = (1st + 1); 3rd = (2nd + 2); 4th = (3rd + 3); 5th = (4th + 4).
But, 18 = 6th ≠ 5th + 5 = 14 + 5 = 19.
20. Each number except 279 is a multiple of 11.
21. The terms are alternately multiplied by 1.5 and divided by 3. However, 18.5 does not satisfy it.
22. Alternately 23 is added and 17 is subtracted from the terms. So, 634 is wrong.
23. The terms are successively divided by 12, 10, 8, 6, ... etc. So, 24 is wrong.
24. The numbers are 1^3, 2^3, 3^3, 4^3 etc. So, 124 is wrong; it must have been 5^3 *i.e.*, 125.
25. Terms at odd places are 5, 6, 7, 8 etc. and each term at even place is 16.
So, 9 is wrong.
26. The difference between two successive terms from the beginning are 7, 5, 7, 5, 7, 5.
So, 40 is wrong.
27. The numbers are 7×8, 8×9, 9×10, 10×11, 11×12, 12×13. So, 150 is wrong.
28. Go on adding 5, 8, 11, 14, 17, 20.
So, the number 47 is wrong and must be replaced by 46.
29. The numbers are squares of odd natural numbers, starting from 5 upto 15.
So, 36 is wrong.
30. Add 1^2, 2^2, 3^2, 4^2, 5^2, 6^2. So, 91 is wrong.
31. Subtract 1, 3, 5, 7, 9, 11 from successive numbers. So, 34 is wrong.
32. Subtract 20, 25, 30, 35, 40, 45 from successive numbers. So, 0 is wrong.
33. Each number is a composite number except 11.
34. Prime numbers 2, 3, 5, 7, 11, 13 are to be added successively. So, 165 is wrong.
35. Each number is the square of a composite number except 190.
36. Prime numbers 2, 3, 5, 7, 11, 13 have successively been subtracted.
So, 100 is wrong. It must be (108 – 11) *i.e.*, 97.
37. The pattern is 1×3, 2×5, 3×7, 4×9, 5×11, 6×13, 7×15 etc.
38. Double the number and add 1 to it, to get the next number. So, 160 is wrong.
39. Alternately, we add 4 and double the next.
So, 132 is wrong. It must be (68×2) *i.e.*, 136.
40. The numbers are cubes of primes *i.e.*, 2^3, 3^3, 5^3, 7^3, 11^3. Clearly, none is wrong.
41. Each number is the preceding number multiplied by – 2.
So, the required number is – 128.
42. Numbers are alternately multiplied by 2 and increased by 3.
So, the missing number = $61 \times 2 = 122$.
43. Numbers are 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2. So, the next number is $8^2 = 64$.
44. Numbers are 1^3, 2^3, 3^3, 4^3, 5^3, 6^3. So, the missing number is $7^3 = 343$.
45. Numbers are all primes. The next prime is 43.
46. Each number is twice the preceding one with 1 added or subtracted alternately.
So, the next number is $(2 \times 261 + 1) = 523$.
47. There are two series, beginning respectively with 3 and 7. In one 3 is added and in another 2 is subtracted. The next number is $1 - 2 = -1$.
48. Each number is double the preceding one plus 1.
So, the next number is $(255 \times 2) + 1 = 511$.
49. The pattern is 1×2, 2×3, 3×4, 4×5, 5×6, 6×7, 7×8.
So, the next number is $8 \times 9 = 72$.
50. Numbers are alternately multiplied by 3 and divided by 2.
So, the next number = $54 \div 2 = 27$.

51. Each number is 15 multiplied by a prime number *i.e.*, 15×11, 15×13, 15×17, 15×19, 15×23. So, the next number is $15 \times 29 = 435$.
52. Numbers are $(2^3 - 1)$, $(3^3 - 1)$, $(4^3 - 1)$, $(5^3 - 1)$, $(6^3 - 1)$, $(7^3 - 1)$ etc.
So, the next number is $(8^3 - 1) = (512 - 1) = 511$.
53. Go on multiplying the given numbers by 2, 3, 4, 5, 6. So, the correct next number is 1440.
54. There are two series (8, 11, 14, 17, 20) and (7, 12, 17, 22) increasing by 3 and 5 respectively.
55. There are two series (10, 13, 16, 19) and (5, 10, 20, 40), one increasing by 3 and the other multiplied by 2.
56. Each previous number is multiplied by 2.
57. Alternately, we add 5 and subtract 7.
58. Alternately, we add 3 and subtract 1.
59. Second number is one more than twice the first; third number is one less than twice the second; fourth number is one more than twice the third; fifth number is one less than the fourth. Therefore, the sixth number is one more than twice the fifth.
So, the missing number is 75.
60. The difference between consecutive terms are respectively 5, 7, 9, 11 and 13.
So, 34 is a wrong number.
61. $2 = (1^3 + 1)$; $9 = (2^3 + 1)$; $28 = (3^3 + 1)$; $65 = (4^3 + 1)$; $125 = (5^3 + 1)$; $216 \neq (6^3 + 1)$ and $344 = (7^3 + 1)$. So, 216 is a wrong number.
62. Multiply each term by 3 to obtain the next term. Hence, 30 is a wrong number.
63. Go on subtracting prime numbers, 19, 17, 13, 11, 7, 5 from the numbers to get the next number. So, 88 is wrong.
64. Go on subtracting 24, 21, 18, 15, 12, 9 from the numbers to get the next number.
Clearly, 128 is wrong.
65. Go on multiplying with 1, 2, 3, 4, 5, 6 to get the next number. So, 96 is wrong.
66. Go on dividing by 4 to get the next number. So, 200 is wrong.
67. Go on adding 7, 9, 11, 13, 15, 17, 19 respectively to obtain the next number.
So, 135 is wrong.
68. Let the given numbers be A, B, C, D, E, F, G. Then,
A, $A \times 1$, $B \times 2 + 2$, $C \times 3 + 3$, $D \times 4 + 4$, $E \times 5 + 5$, $F \times 6 + 6$ are the required numbers.
Clearly, 228 is wrong.
69. Go on multiplying the number by 2 and adding 1 to it to get the next number.
So, 27 is wrong.
70. Go on adding 7, 9, 11, 13, 15, 17 respectively to obtain the next number.
So, 33 is wrong.
71. Go on dividing by 6, 5, 4, 3, 2, 1 respectively to obtain the next number.
Clearly, 92 is wrong.
72. Go on subtracting 3 and dividing the result by 2 to obtain the next number.
Clearly, 46 is wrong.
73. Go on multiplying 2 and adding 1 to get the next number. So, 39 is wrong.
74. $A \times 2 + 1$, $B \times 3 + 1$, $C \times 2 + 1$, $D \times 3 + 1$ and so on. So, 356 is wrong.
75. Numbers must be $(14)^2$, $(13)^2$, $(11)^2$, $(10)^2$, $(9)^2$, $(8)^2$. So, 80 is wrong.
76. Each even term of the series is obtained by multiplying the previous term by 2.
2nd term = (1st term) $\times$ 2 = $6 \times 2 = 12$; 4th term = (3rd term) $\times$ 2 = $48 \times 2 = 96$;
6th term = (5th term) $\times$ 2 = $384 \times 2 = 768$.
$\therefore$ 4th term should be 96 instead of 100.

77. 2nd term = (1st term) × 3 − 4 = 10 × 3 − 4 = 26;
3rd term = (2nd term) × 3 − 4 = 26 × 3 − 4 = 74;
4th term = (3rd term) × 3 − 4 = 74 × 3 − 4 = 218;
5th term = (4th term) × 3 − 4 = 218 × 3 − 4 = 650.
∴ 5th term must be 650 instead of 654.

78. 2nd term = (1st term) × 1 + 1 = 15 × 1 + 1 = 16;
3rd term = (2nd term) × 2 + 2 = 16 × 2 + 2 = 34;
4th term = (3rd term) × 3 + 3 = 34 × 3 + 3 = 105;
5th term = (4th term) × 4 + 4 = 105 × 4 + 4 = 424;
6th term = (5th term) × 5 + 5 = 425 × 5 + 5 = 2125.
∴ 6th term should be 2125 instead of 2124.

79. 7th term = (8th term) × 2 + 1 = 20 × 2 + 1 = 41;
6th term = (7th term) × 2 + 2 = 41 × 2 + 2 = 84;
5th term = (6th term) × 2 + 3 = 84 × 2 + 3 = 171;
4th term = (5th term) × 2 + 4 = 171 × 2 + 4 = 346.
∴ 4th term should be 346 instead of 347.

80. 2nd term = (1st term) + 2^2 = 32 + 4 = 36; 3rd term = (2nd term) + 3^2 = 36 + 9 = 45;
4th term = (3rd term) + 4^2 = 45 + 16 = 61; 5th term = (4th term) + 5^2 = 61 + 25 = 86.
∴ 3rd term should be 45 instead of 41.

81. There are two sequences (3, 9, 67.5, 810) and (4, 22.5, 202.5).
Pattern is : (1st term × 3), (2nd term × 7.5), (3rd term × 12) for the first sequence and (1st term × 5), (2nd term × 9) and so on for the second sequence.

82. 2nd term = (1st term × $1 + 1^2$) = $1 \times 1 + 1^2 = 2$;
3rd term = (2nd term × $2 + 2^2$) = $2 \times 2 + 2^2 = 8$;
4th term = (3rd term × $3 + 3^2$) = $8 \times 3 + 3^2 = 33$;
5th term = (4th term × $4 + 4^2$) = $33 \times 4 + 4^2 = 148$;
6th term = (5th term × $5 + 5^2$) = $148 \times 5 + 5^2 = 765$.
∴ 760 is wrong.

83. 2nd term = (1st term × 2 + 2) = 3 × 2 + 2 = 8;
3rd term = (2nd term × 2 + 4) = 8 × 2 + 4 = 20;
4th term = (3rd term × 2 + 6) = 20 × 2 + 6 = 46;
5th term = (4th term × 2 + 8) = 46 × 2 + 8 = 100 and so on.
∴ 18 is wrong.

84. 2nd term = 1st term − $(12)^2$ = 789 − 144 = 645;
3rd term = (2nd term) − $(10)^2$ = 645 − 100 = 545;
4th term = (3rd term) − $(8)^2$ = 545 − 64 = 481;
5th term = (4th term) − $(6)^2$ = 481 − 36 = 445.
∴ 440 is wrong.

85. 2nd term = (1st term − 30) ÷ 2 = $\left(\frac{1050-30}{2}\right)$ = 510;

3rd term = (2nd term − 26) ÷ 2 = $\left(\frac{510-26}{2}\right)$ = 242;

4th term = (3rd term − 22) ÷ 2 = $\left(\frac{242-22}{2}\right)$ = 110.

∴ 106 is wrong.

86. 2nd term = (1st term $\times$ 2 $-$ 2) = $(5 \times 2 - 2) = 8$;
3rd term = (2nd term $\times$ 3 $-$ 2) = $(8 \times 3 - 2) = 22$;
4th term = (3rd term $\times$ 2 $-$ 2) = $(22 \times 2 - 2) = 42$;
5th term = (4th term $\times$ 3 $-$ 2) = $(42 \times 3 - 2) = 124$ and so on.
$\therefore$ 20 is wrong.

87. 2nd term = (1st term $\times$ 1.5) = $2 \times 1.5 = 3$; 3rd term = (2nd term $\times$ 2) = $3 \times 2 = 6$;
4th term = (3rd term $\times$ 2.5) = $6 \times 2.5 = 15$; 5th term = (4th term $\times$ 3) = $15 \times 3 = 45$.
$\therefore$ 52.5 is wrong.

88. 2nd term = $\left(\frac{\text{1st term} - 8}{2}\right) = \left(\frac{888 - 8}{2}\right) = 440$;

3rd term = $\left(\frac{\text{2nd term} - 8}{2}\right) = \left(\frac{440 - 8}{2}\right) = 216$;

4th term = $\left(\frac{\text{3rd term} - 8}{2}\right) = \left(\frac{216 - 8}{2}\right) = 104$;

5th term = $\left(\frac{\text{4th term} - 8}{2}\right) = \left(\frac{104 - 8}{2}\right) = 48$;

6th term = $\left(\frac{\text{5th term} - 8}{2}\right) = \left(\frac{48 - 8}{2}\right) = 20$.

$\therefore$ 22 is wrong.

89. 2nd term = (1st term $\times$ 1 + 2) = $(4 \times 1 + 2) = 6$;
3rd term = (2nd term $\times$ 2 + 3) = $(6 \times 2 + 3) = 15$;
4th term = (3rd term $\times$ 3 + 4) = $(15 \times 3 + 4) = 49$;
5th term = (4th term $\times$ 4 + 5) = $(49 \times 4 + 5) = 210$ and so on.
$\therefore$ 5 is wrong.

36. TABULATION

This section comprises of questions in which certain data regarding common disciplines as production over a period of a few years : imports, exports, incomes of employees in a factory, students applying for and qualifying a certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given questions on the basis of comparative analysis of the data.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the *tabular form*. In order to avoid some heads again and again, tables are made consisting of horizontal lines called *rows* and vertical lines called *columns* with distinctive heads, known as *captions*. Units of measurements are given with the captions.

SOLVED EXAMPLES

Ex. 1. ***The following table gives the sales of batteries manufactured by a company over the years. Study the table and answer the questions that follow :***

(S.B.I.P.O. 1998)

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEARS (NUMBERS IN THOUSANDS)

Year	TYPES OF BATTERIES					
	4AH	7AH	32AH	35AH	55AH	Total
1992	75	144	114	102	108	543
1993	90	126	102	84	126	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery ?
   (*a*) 4AH (*b*) 7AH (*c*) 32AH (*d*) 35AH (*e*) 55AH
2. What is the difference in the number of 35AH batteries sold in 1993 and 1997 ?
   (*a*) 24000 (*b*) 28000 (*c*) 35000 (*d*) 39000 (*e*) 42000
3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year :
   (*a*) 1994 (*b*) 1995 (*c*) 1996 (*d*) 1997 (*e*) 1998
4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997 ?
   (*a*) 4AH (*b*) 7AH (*c*) 32AH (*d*) 35AH (*e*) 55AH
5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992 ?
   (*a*) 28% (*b*) 31% (*c*) 33% (*d*) 34% (*e*) 37%

Sol. 1. (*c*) : The total sales (in thousands) of all the seven years for various batteries are :

For 4AH = 75 + 90 + 96 + 105 + 90 + 105 + 115 = 676

For 7AH = 144 + 126 + 114 + 90 + 75 + 60 + 85 = 694

For 32AH = 114 + 102 + 75 + 150 + 135 + 165 + 160 = 901

For 35 AH = 102 + 84 + 105 + 90 + 75 + 45 + 100 = 601

For 55 AH = 108 + 126 + 135 + 75 + 90 + 120 + 145 = 799.

Clearly, sales are maximum in case of 32AH batteries.

2. (*d*) : Required difference = [(84 − 45) × 1000] = 39000.

3. (*d*) : The percentages of sales of 4AH batteries to the total sales in different years are :

For 1992 $= \left(\frac{75}{543} \times 100\right)\% = 13.81\%$; **For 1993** $= \left(\frac{90}{528} \times 100\right)\% = 17.05\%$;

For 1994 $= \left(\frac{96}{525} \times 100\right)\% = 18.29\%$; **For 1995** $= \left(\frac{105}{510} \times 100\right)\% = 20.59\%$;

For 1996 $= \left(\frac{96}{465} \times 100\right)\% = 19.35\%$; **For 1997** $= \left(\frac{105}{495} \times 100\right)\% = 21.21\%$;

For 1998 $= \left(\frac{115}{605} \times 100\right)\% = 19.01\%$.

Clearly, the percentage is maximum in 1997.

4. (*b*) : From the table it is clear that the sales of 7AH batteries have been decreasing continuously from 1992 to 1997.

5. (*d*) : Required Percentage $= \left[\frac{(145 - 108)}{108} \times 100\right]\% = 34.26\% = 34\%$.

Ex. 2. ***Study the following table carefully and answer these questions :*** (S.B.I.P.O. 2002)

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEARS

Year / State	1997		1998		1999		2000		2001	
	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

1. Combining the states P and Q together in 1998, what is the percentage of the candidates qualified to that of the candidates appeared ?

(*a*) 10.87% (*b*) 11.49% (*c*) 12.35% (*d*) 12.54% (*e*) 13.05%

2. The percentage of the total number of qualified candidates to the total number of appeared candidates among all the five states in 1999 is :

(*a*) 11.49% (*b*) 11.84% (*c*) 12.21% (*d*) 12.57% (*e*) 12.73%

3. What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together ?

(*a*) 12.36% (*b*) 12.16% (*c*) 11.47% (*d*) 11.15% (*e*) None of these

4. What is the average of candidates who appeared from State Q during the given yeas ?
(a) 8700 (b) 8760 (c) 8810 (d) 8920 (e) 8990

5. In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates ?
(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

6. Total number of candidates qualified from all the states together in 1997 is approximately what percentage of the total number of candidates qualified from all the states together in 1998 ?
(a) 72% (b) 77% (c) 80% (d) 83% (e) 86%

Sol. 1. (c) : Required Percentage $= \left[\frac{(1020+1240)}{(8800+9500)} \times 100\right]\% = \left(\frac{2260}{18300} \times 100\right)\%$

$= 12.35\%$.

2. (b) : Required Percentage $= \left[\frac{(850+920+890+980+1350)}{(7400+8450+7800+8700+9800)} \times 100\right]\%$

$= \left(\frac{4990}{42150} \times 100\right)\% = 11.84\%$.

3. (d) : Required Percentage $= \left[\frac{(840+1050+920+980+1020)}{(7500+9200+8450+9200+8800)} \times 100\right]\%$

$= \left(\frac{4810}{43150} \times 100\right)\% = 11.15\%$.

4. (e) : Required average $= \frac{8100+9500+8700+9700+8950}{5} = \frac{44950}{5} = 8990$.

5. (e) : The percentages of candidates qualified to candidates appeared from State P during different years are :

For 1997 $= \left(\frac{780}{6400} \times 100\right)\% = 12.19\%$; **For 1998** $= \left(\frac{1020}{8800} \times 100\right)\% = 11.59\%$;

For 1999 $= \left(\frac{890}{7800} \times 100\right)\% = 11.41\%$; **For 2000** $= \left(\frac{1010}{8750} \times 100\right)\% = 11.54\%$;

For 2001 $= \left(\frac{1250}{9750} \times 100\right)\% = 12.82\%$.

$\therefore$ Maximum percentage is for the year 2001.

6. (c) : Required Percentage $= \left[\frac{(720+840+780+950+870)}{(980+1050+1020+1240+940)} \times 100\right]\%$

$= \left(\frac{4160}{5230} \times 100\right)\% = 79.54\% = 80\%$.

Ex. 3. ***The following table gives the percentage of marks obtained by seven students in six different subjects in an examination. Study the table and answer the questions based on it. The numbers in the brackets give the maximum marks in each subject.***
(Bank P.O. 2003)

Subjects (Max. Marks) / Student	Maths (150)	Chemistry (130)	Physics (120)	Geography (100)	History (60)	Computer Science (40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tarun	65	35	50	77	80	80

1. What was the aggregate of marks obtained by Sajal in all the six subjects ?
(a) 409 (b) 419 (c) 429 (d) 439 (e) 449
2. What is the overall percentage of Tarun ?
(a) 52.5% (b) 55% (c) 60% (d) 63% (e) 64.5%
3. What are the average marks obtained by all the seven students in Physics ? (rounded off to two digits after decimal)
(a) 77.26 (b) 89.14 (c) 91.37 (d) 96.11 (e) 103.21
4. The number of students who obtained 60% and above marks in all the subjects is :
(a) 1 (b) 2 (c) 3 (d) None (e) None of these
5. In which subject is the overall percentage the best ?
(a) History (b) Maths (c) Physics (d) Chemistry (e) Geography

Sol. **1.** (e) : Aggregate marks obtained by Sajal
= [(90% of 150) + (60% of 130) + (70% of 120) + (70% of 100) + (90% of 60) + (70% of 40)] = 135 + 78 + 84 + 70 + 54 + 28 = 449.

2. (c) : Aggregate marks obtained by Tarun
= [(65% of 150) + (35% of 130) + (50% of 120) + (77% of 100) + (80% of 60) + (80% of 40)] = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.
Total maximum marks (of all the six subjects)
= (150 + 130 + 120 + 100 + 60 + 40) = 600.

$$\text{Overall percentage of Tarun} = \left(\frac{360}{600} \times 100\right)\% = 60\%.$$

3. (b) : Average marks obtained in Physics by all the seven students

$$= \frac{1}{7} \times [(90\% \text{ of } 120) + (80\% \text{ of } 120) + (70\% \text{ of } 120) + (80\% \text{ of } 120) + (85\% \text{ of } 120) + (65\% \text{ of } 120) + (50\% \text{ of } 120)]$$

$$= \frac{1}{7} \times [(90 + 80 + 70 + 80 + 85 + 65 + 50)\% \text{ of } 120]$$

$$= \frac{1}{7} \times [520\% \text{ of } 120] = \frac{624}{7} = 89.14.$$

4. (b) : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.

5. (b) : We shall find the overall percentage (for all the seven students) with respect to each subject.
The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.

Therefore, overall percentage for :

(*i*) Maths $= \left[\frac{1}{7} \times (90 + 100 + 90 + 80 + 80 + 70 + 65)\right]\%$

$= \left[\frac{1}{7} \times (575)\right]\% = 82.14\%.$

(*ii*) Chemistry $= \left[\frac{1}{7} \times (50 + 80 + 60 + 65 + 65 + 75 + 35)\right]\%$

$= \left[\frac{1}{7} \times (430)\right]\% = 61.43\%.$

(*iii*) Physics $= \left[\frac{1}{7} \times (90 + 80 + 70 + 80 + 85 + 65 + 50)\right]\%$

$= \left[\frac{1}{7} \times (520)\right]\% = 74.29\%.$

(*iv*) Geography $= \left[\frac{1}{7} \times (60 + 40 + 70 + 80 + 95 + 85 + 77)\right]\%$

$= \left[\frac{1}{7} \times (507)\right]\% = 72.43\%.$

(*v*) History $= \left[\frac{1}{7} \times (70 + 80 + 90 + 60 + 50 + 40 + 80)\right]\%$

$= \left[\frac{1}{7} \times (470)\right]\% = 67.14\%.$

(*vi*) Computer Science $= \left[\frac{1}{7} \times (80 + 70 + 70 + 60 + 90 + 60 + 80)\right]\%$

$= \left[\frac{1}{7} \times (510)\right]\% = 72.86\%.$

Clearly, this percentage is highest for Maths.

Ex. 4. ***Study the following table carefully and answer the questions given below :*** **(Bank P.O. 2001)**

CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION

Marks out of 50 / Subject	40 and above	30 and above	20 and above	10 and above	0 and above
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
(Aggregate) Average	7	27	73	87	100

1. The number of students scoring less than 40% marks in aggregate is :
   (*a*) 13 (*b*) 19 (*c*) 20 (*d*) 27 (*e*) 34
2. If at least 60% marks in Physics are required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics ?
   (*a*) 27 (*b*) 32 (*c*) 34 (*d*) 41 (*e*) 68
3. What is the difference between the number of students passed with 30 as cut-off marks in Chemistry and those passed with 30 as cut-off marks in aggregate ?
   (*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6 (*e*) 7

4. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately :
(a) 21% (b) 27% (c) 29% (d) 31% (e) 34%

5. If it is known that at least 23 students were eligible for a Symposium on Chemistry, the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range :
(a) 40-50 (b) 30-40 (c) 20-30 (d) Below 20 (e) Cannot be determined

Sol. 1. (d) : We have 40% of 50 = $\left(\frac{40}{100} \times 50\right)$ = 20.

∴ Required number = Number of students scoring less than 20 marks in aggregate
= 100 – number of students scoring 20 and above marks in aggregate = 100 – 73 = 27.

2. (b) : We have 60% of 50 = $\left(\frac{60}{100} \times 50\right)$ = 30.

∴ Required number = Number of students scoring 30 and above marks in Physics = 32.

3. (d) : Required difference = (Number of students scoring 30 and above marks in Chemistry) – (Number of students scoring 30 and above marks in aggregate) = 27 – 21 = 6.

4. (c) : Number of students getting at least 60% marks in Chemistry
= Number of students getting 30 and above marks in Chemistry = 21.
Number of students getting at least 40% marks in aggregate
= Number of students getting 20 and above marks in aggregate = 73.

∴ Required Percentage = $\left(\frac{21}{73} \times 100\right)$% = 28.77% = 29%.

5. (c) : Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

EXERCISE 36

Directions (Questions 1 to 6) : ***Study the following table and answer the questions based on it.*** **(Bank P.O. 2003)**

NUMBER OF CANDIDATES APPEARED, QUALIFIED AND SELECTED IN A COMPETITIVE EXAMINATION FROM FIVE STATES DELHI, H.P., U.P., PUNJAB AND HARYANA OVER THE YEARS 1994 TO 1998

	Delhi			H.P.			U.P.			Punjab			Haryana		
Year	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.
1997	8000	850	94	7800	810	82	7500	720	78	8200	680	85	6400	700	75
1998	4800	500	48	7500	800	65	5600	620	85	6800	600	70	7100	650	75
1999	7500	640	82	7400	560	70	4800	400	48	6500	525	65	5200	350	55
2000	9500	850	90	8800	920	86	7000	650	70	7800	720	84	6400	540	60
2001	9000	800	70	7200	850	75	8500	950	80	5700	485	60	4500	600	75

1. In the year 1997, which state had the lowest percentage of candidates selected over the candidates appeared ?
(a) Delhi (b) H.P. (c) U.P. (d) Punjab (e) Haryana

2. The percentage of candidates qualified from Punjab over those appeared from Punjab is highest in the year :
(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

3. The percentage of candidates selected from U.P. over those qualified from U.P. is highest in the year :
(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

4. The number of candidates selected from Haryana during the period under review is approximately what percent of the number selected from Delhi during this period ?
(a) 79.5% (b) 81% (c) 84.5% (d) 88.5% (e) 92.5%

5. For which state the average number of candidates selected over the years is the maximum ?
(a) Delhi (b) H.P. (c) U.P. (d) Punjab (e) Haryana

6. What is the approximate percentage of total number of candidates selected to the total number of candidates qualified for all the five states together during the year 1999 ?
(a) 10% (b) 11% (c) 12% (d) 13% (e) 14%

Directions (Questions 7 to 11) : ***Study the following table to answer the questions that are given below it.*** **(R.B.I. 2003)**

EXPENDITURES OF A COMPANY (IN LAKH RUPEES) PER ANNUM OVER THE GIVEN YEARS

Item of Expenditure / Year	Salary	Fuel and Transport	Bonus	Interest on Loans	Taxes
1998	288	98	3.00	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

7. The ratio between the total expenditure on Taxes for all the years and the total expenditure on Fuel and Transport for all the years respectively is approximately :
(a) 4 : 7 (b) 10 : 13 (c) 15 : 18 (d) 5 : 8 (e) 2 : 3

8. The total expenditure of the Company over these items during the year 2000 is :
(a) Rs. 544.44 lakhs (b) Rs. 501.11 lakhs (c) Rs. 446.46 lakhs
(d) Rs. 478.87 lakhs (e) Rs. 612.13 lakhs

9. What is the average amount of interest per year which the Company had to pay during this period ?
(a) Rs. 32.43 lakhs (b) Rs. 33.72 lakhs (c) Rs. 34.18 lakhs
(d) Rs. 35.69 lakhs (e) Rs. 36.66 lakhs

10. Total expenditure on all these items in 1998 was approximately what percent of the total expenditure in 2002 ?
(a) 62% (b) 66% (c) 69% (d) 71% (e) 73%

11. The total amount of bonus paid by the Company during the given period is approximately what percent of the total amount of salary paid during this period ?
(a) 0.1% (b) 0.5% (c) 1% (d) 1.25% (e) 1.11%

Directions (Questions 12 to 16) : ***A school has four sections A, B, C, D of Class IX students. The results of half-yearly and annual examinations are shown in the table given below. Answer the questions based on this table.*** **(Bank P.O. 2000)**

Result	Number of Students			
	Section A	Section B	Section C	Section D
Students failed in both Exams	28	23	17	27
Students failed in half-yearly but passed in Annual Exams	14	12	8	13
Students passed in half-yearly but failed in Annual Exams	6	17	9	15
Students passed in both Exams	64	55	46	76

12. How many students are there in Class IX in the school ?
(*a*) 336 (*b*) 189 (*c*) 335 (*d*) 286 (*e*) 430

13. Which section has the minimum failure rate in half-yearly examination ?
(*a*) A (*b*) B (*c*) C
(*d*) D (*e*) Cannot be determined

14. Which section has the maximum success rate in annual examination ?
(*a*) A (*b*) B (*c*) C
(*d*) D (*e*) Cannot be determined

15. Which section has the maximum pass percentage in at least one of the two examinations ?
(*a*) A (*b*) B (*c*) C
(*d*) D (*e*) Cannot be determined

16. If the number of students passing an examination be considered a criteria for comparison of difficulty level of two examinations, which of the following statements is true in this context ?
(*a*) Half-yearly examinations were more difficult.
(*b*) Annual examinations were more difficult.
(*c*) Both the examinations had almost the same difficulty level.
(*d*) The two examinations cannot be compared for difficulty level.
(*e*) For students of Sections A and B, the annual examinations seem to be more difficult as compared to the half-yearly examinations.

Directions (Questions 17 to 21) : ***The following table shows the number of new employees added to different categories of employees in a Company and also the number of employees from these categories who left the company every year since the foundation of the Company in 1995.*** **(Bank P.O. 2001)**

Year	Managers		Technicians		Operators		Accountants		Peons	
	New	Left	New	Left	New	Left	New	Left	New	Left
1995	760	—	1200	—	880	—	1160	—	820	—
1996	280	120	272	120	256	104	200	100	184	96
1997	179	92	240	128	240	120	224	104	152	88
1998	148	88	236	96	208	100	248	96	196	80
1999	160	72	256	100	192	112	272	88	224	120
2000	193	96	288	112	248	144	260	92	200	104

17. During the period between 1995 and 2000, the total number of Operators who left the Company is what percent of the total number of Operators who joined the Company ?
(a) 19% (b) 21% (c) 27% (d) 29% (e) 32%

18. For which of the following categories the percentage increase in the number of employees working in the Company from 1995 to 2000 was the maximum ?
(a) Managers (b) Technicians (c) Operators (d) Accountants (e) Peons

19. What is the difference between the total number of Technicians added to the Company and the total number of Accountants added to the Company during the years 1996 to 2000 ?
(a) 128 (b) 112 (c) 96 (d) 88 (e) 72

20. What was the total number of Peons working in the Company in the year 1999 ?
(a) 1312 (b) 1192 (c) 1088 (d) 968 (e) 908

21. What is the pooled average of the total number of employees of all categories in the year 1997 ?
(a) 1325 (b) 1285 (c) 1265 (d) 1235 (e) 1195

Directions (Questions 22 to 25) : ***The following table gives the percentage distribution of population of five states, P, Q, R, S and T on the basis of poverty line and also on the basis of sex. Study the table and answer the questions based on it.***

(Bank P.O. 2000)

State	Percentage of Population below Poverty Line	Proportion of Males and Females	
		Below Poverty Line	Above Poverty Line
		M : F	M : F
P	35	5 : 6	6 : 7
Q	25	3 : 5	4 : 5
R	24	1 : 2	2 : 3
S	19	3 : 2	4 : 3
T	15	5 : 3	3 : 2

22. What will be the number of females above poverty line in the State S if it is known that the population of State S is 7 million ?
(a) 3 million (b) 2.43 million (c) 1.33 million
(d) 5.7 million (e) 1.61 million

23. If the male population above poverty line for State R is 1.9 million, then the total population of State R is :
(a) 4.5 million (b) 4.85 million (c) 5.35 million
(d) 6.25 million (e) 7.6 million

24. What will be the male population above poverty line for State P if the female population below poverty line for State P is 2.1 million ?
(a) 2.1 million (b) 2.3 million (c) 2.7 million
(d) 3.3 million (e) 3.4 million

25. If the population of males below poverty line for State Q is 2.4 million and that for State T is 6 million, then the total populations of states Q and T are in the ratio :
(a) 1 : 3 (b) 2 : 5 (c) 3 : 7 (d) 4 : 9 (e) 5 : 12

ANSWERS

1. (d)	2. (d)	3. (b)	4. (d)	5. (a)	6. (d)	7. (b)	8. (a)	9. (e)
10. (c)	11. (c)	12. (e)	13. (d)	14. (a)	15. (d)	16. (c)	17. (d)	18. (a)
19. (d)	20. (b)	21. (e)	22. (b)	23. (d)	24. (d)	25. (b)		

SOLUTIONS

1. The percentages of candidates selected over the candidates appeared in 1997, for various states are :

(*i*) For Delhi $= \left(\frac{94}{8000} \times 100\right)\% = 1.175\%$; (*ii*) For H.P. $= \left(\frac{82}{7800} \times 100\right)\% = 1.051\%$;

(*iii*) For U.P. $= \left(\frac{78}{7500} \times 100\right)\% = 1.040\%$; (*iv*) For Punjab $= \left(\frac{85}{8200} \times 100\right)\% = 1.037\%$;

(*v*) For Haryana $= \left(\frac{75}{6400} \times 100\right)\% = 1.172\%$.

Clearly, this percentage is lowest for Punjab.

2. The percentages of candidates qualified from Punjab over those appeared from Punjab during different years are :

For 1997 $= \left(\frac{680}{8200} \times 100\right)\% = 8.29\%$; **For 1998** $= \left(\frac{600}{6800} \times 100\right)\% = 8.82\%$;

For 1999 $= \left(\frac{525}{6500} \times 100\right)\% = 8.08\%$; **For 2000** $= \left(\frac{720}{7800} \times 100\right)\% = 9.23\%$;

For 2001 $= \left(\frac{485}{5700} \times 100\right)\% = 8.51\%$.

Clearly, this percentage is highest for the year 2000.

3. The percentages of candidates selected from U.P. over those qualified from U.P. during different years are :

For 1997 $= \left(\frac{78}{720} \times 100\right)\% = 10.83\%$; **For 1998** $= \left(\frac{85}{620} \times 100\right)\% = 13.71\%$;

For 1999 $= \left(\frac{48}{400} \times 100\right)\% = 12\%$; **For 2000** $= \left(\frac{70}{650} \times 100\right)\% = 10.77\%$;

For 2001 $= \left(\frac{80}{950} \times 100\right)\% = 8.42\%$.

Clearly, this percentage is highest for the year 1998.

4. Required Percentage $= \left[\frac{(75 + 75 + 55 + 60 + 75)}{(94 + 48 + 82 + 90 + 70)} \times 100\right]\%$

$= \left(\frac{340}{384} \times 100\right)\% = 88.54\% = 88.5\%$.

5. The average number of candidates selected over the given period for various states are :

For Delhi $= \frac{94 + 48 + 82 + 90 + 70}{5} = \frac{384}{5} = 76.8$

For H.P. $= \frac{82 + 65 + 70 + 86 + 75}{5} = \frac{378}{5} = 75.6$

For U.P. $= \frac{78+85+48+70+80}{5} = \frac{361}{5} = 72.2$

For Punjab $= \frac{85+70+65+84+60}{5} = \frac{364}{4} = 72.8$

For Haryana $= \frac{75+75+55+60+75}{5} = \frac{340}{5} = 68.$

Clearly, this average is maximum for Delhi.

6. Required Percentage $= \left[\frac{(82+70+48+65+55)}{(640+560+400+525+350)} \times 100\right]\%$

$= \left(\frac{320}{2475} \times 100\right)\% = 12.93\% \approx 13\%.$

7. Required Ratio $= \frac{(83+108+74+88+98)}{(98+112+101+133+142)} = \frac{451}{586} = \frac{1}{1.3} = \frac{10}{13}.$

8. Total expenditure of the Company during 2000
   = Rs. (324 + 101 + 3.84 + 41.6 + 74) lakhs = Rs. 544.44 lakhs.

9. Average amount of interest paid by the Company during the given period
   = Rs. $\left(\frac{23.4+32.5+41.6+36.4+49.4}{5}\right)$ lakhs = Rs. $\left(\frac{183.3}{5}\right)$ lakhs
   = Rs. 36.66 lakhs.

10. Required Percentage $= \left[\frac{(288+98+3.00+23.4+83)}{(420+142+3.96+49.4+98)} \times 100\right]\%$

$= \left(\frac{495.4}{713.36} \times 100\right)\% = 69.45\%.$

11. Required Percentage $= \left[\frac{(3.00+2.52+3.84+3.68+3.96)}{(288+342+324+336+420)} \times 100\right]\%$

$= \left(\frac{17}{1710} \times 100\right)\% = 1\%.$

12. Since the classification of the students on the basis of their results and sections form independent groups, so the total number of students in the class :
   = (28 + 23 + 17 + 27 + 14 + 12 + 8 + 13 + 6 + 17 + 9 + 15 + 64 + 55 + 46 + 76) = 430.

13. Total number of failures in half-yearly exams in a section
   = [(Number of students failed in both exams) + (Number of students failed in half-yearly but passed in Annual exams)] in that section

   ∴ Failure rate in half-yearly exams in **Section A**

$= \left[\frac{\text{Number of students of Section A failed in half-yearly}}{\text{Total number of students in Section A}} \times 100\right]\%$

$= \left[\frac{(28+14)}{(28+14+6+64)} \times 100\right]\% = \left(\frac{42}{112} \times 100\right)\% = 37.5\%$

Similarly, failure rate in half-yearly exams in :

Section B $= \left[\frac{(23+12)}{(23+12+17+55)} \times 100\right]\% = \left(\frac{35}{107} \times 100\right)\% = 32.71\%$

Section C $= \left[\frac{(17+8)}{(17+8+9+46)} \times 100\right]\% = \left(\frac{25}{80} \times 100\right)\% = 31.25\%$

Section D $= \left[\frac{(27+13)}{(27+13+15+76)} \times 100\right]\% = \left(\frac{40}{131} \times 100\right)\% = 30.53\%$

Clearly, the failure rate is minimum for Section D.

14. Total number of students passed in annual exams in a section
= [(Number of students failed in half-yearly but passed in annual exams) + (Number of students passed in both exams)] in that section

∴ Success rate in annual examination in **Section A**

$$= \left[\frac{\text{Number of students of Section A passed in annual exams}}{\text{Total number of students in Section A}} \times 100\right]\%$$

$$= \left[\frac{(14+64)}{(28+14+6+64)} \times 100\right]\% = \left(\frac{78}{112} \times 100\right)\% = 69.64\%$$

Similarly, success rate in annual examinations in :

Section B $= \left[\frac{(12+55)}{(23+12+17+55)} \times 100\right]\% = \left(\frac{67}{107} \times 100\right)\% = 62.62\%$

Section C $= \left[\frac{(8+46)}{(17+8+9+46)} \times 100\right]\% = \left(\frac{54}{80} \times 100\right)\% = 67.5\%$

Section D $= \left[\frac{(13+76)}{(27+13+15+76)} \times 100\right]\% = \left(\frac{89}{131} \times 100\right)\% = 67.94\%$

Clearly, the success rate in annual examination is maximum for Section A.

15. Pass percentages in at least one of the two examinations for different sections are :

For **Section A** $= \left[\frac{(14+6+64)}{(28+14+6+64)} \times 100\right]\% = \left(\frac{84}{112} \times 100\right)\% = 75\%$

For **Section B** $= \left[\frac{(12+17+55)}{(23+12+17+55)} \times 100\right]\% = \left(\frac{84}{107} \times 100\right)\% = 78.5\%$

For **Section C** $= \left[\frac{(8+9+46)}{(17+8+9+46)} \times 100\right]\% = \left(\frac{63}{80} \times 100\right)\% = 78.75\%$

For **Section D** $= \left[\frac{(13+15+76)}{(27+13+15+76)} \times 100\right]\% = \left(\frac{104}{131} \times 100\right)\% = 79.39\%$

Clearly, the pass percentage is maximum for Section D.

16. Number of students who passed half-yearly exams in the school
= (Number of students passed in half-yearly but failed in annual exams) + (Number of students passed in both exams) = (6 + 17 + 9 + 15) + (64 + 55 + 46 + 76) = 288

Also, Number of students who passed annual exams in the school
= (Number of students failed in half-yearly but passed in annual exams) + (Number of students passed in both exams) = (14 + 12 + 8 + 13) + (64 + 55 + 46 + 76) = 288

Since, the number of students passed in half-yearly = the number of students passed in annual exams, therefore, it can be inferred that both the examinations had almost the same difficulty level.

Thus, Statements (*a*), (*b*) and (*d*) are false and Statement (*c*) is true.

Also, number of students from Sections A and B who passed the annual exams
= (14 + 12) + (64 + 55) = 145

And, number of students from Sections A and B who passed the half-yearly exams
= (6 + 17) + (64 + 55) = 142.

Since the number of students of Sections A and B who passed the annual exams is greater than those who passed the half-yearly exams it implies that for students of Sections A and B, the half-yearly exams were more difficult as compared to annual exams.

Hence, Statement (*e*) is false.

17. Total number of Operators who left the Company during 1996-2000
$= (104 + 120 + 100 + 112 + 144) = 580.$

Total number of Operators who joined the Company during 1996-2000
$= (880 + 256 + 240 + 208 + 192 + 248) = 2024.$

$\therefore$ Required Percentage $= \left(\frac{580}{2024} \times 100\right)\% = 28.66\% = 29\%.$

18. Number of Managers working in the Company :

In **1995** = 760.

In **2000** = $(760 + 280 + 179 + 148 + 160 + 193) - (120 + 92 + 88 + 72 + 96) = 1252.$

$\therefore$ Percentage increase in the number of Managers
$= \left[\frac{(1252 - 760)}{760} \times 100\right]\% = 64.74\%.$

Number of Technicians working in the Company :

In **1995** = 1200.

In **2000** = $(1200 + 272 + 240 + 236 + 256 + 288) - (120 + 128 + 96 + 100 + 112)$
$= 1936.$

$\therefore$ Percentage increase in the number of Technicians
$= \left[\frac{(1936 - 1200)}{1200} \times 100\right]\% = 61.33\%.$

Number of Operators working in the Company :

In **1995** = 880.

In **2000** = $(880 + 256 + 240 + 208 + 192 + 248) - (104 + 120 + 100 + 112 + 144)$
$= 1444.$

$\therefore$ Percentage increase in the number of Operators
$= \left[\frac{(1444 - 880)}{880} \times 100\right]\% = 64.09\%$

Number of Accountants working in the Company :

In **1995** = 1160.

In **2000** = $(1160 + 200 + 224 + 248 + 272 + 260) - (100 + 104 + 96 + 88 + 92)$
$= 1884.$

$\therefore$ Percentage increase in the number of Accountants
$= \left[\frac{(1884 - 1160)}{1160} \times 100\right]\% = 62.41\%$

Number of Peons working in the Company :

In **1995** = 820.

In **2000** = $(820 + 184 + 152 + 196 + 224 + 200) - (96 + 88 + 80 + 120 + 104)$
$= 1288.$

$\therefore$ Percentage increase in the number of Peons
$= \left[\frac{(1288 - 820)}{820} \times 100\right]\% = 57.07\%$

Clearly, the percentage increase is maximum in case of Managers.

19. Required difference = (272 + 240 + 236 + 256 + 288) − (200 + 224 + 248 + 272 + 260) = 88.

20. Total number of Peons working in the Company in 1999
= (820 + 184 + 152 + 196 + 224) − (96 + 88 + 80 + 120) = 1192.

21. Total number of employees of various categories working in the Company in 1997 are :

Managers = (760 + 280 + 179) − (120 + 92) = 1007
Technicians = (1200 + 272 + 240) − (120 + 128) = 1464
Operators = (880 + 256 + 240) − (104 + 120) = 1152
Accountants = (1160 + 200 + 224) − (100 + 104) = 1380
Peons = (820 + 184 + 152) − (96 + 88) = 972

∴ Pooled average of all the five categories of employees working in the Company in 1997 = $\frac{1}{5} \times (1007 + 1464 + 1152 + 1380 + 972) = \frac{1}{5} \times 5975 = 1195$.

22. Total population of State S = 7 million.

∴ Population above poverty line = [(100 − 19)% of 7] million
= (81% of 7) million = 5.67 million.

And so, the number of females above poverty line in State S = $\left(\frac{3}{7} \times 5.67\right)$ million
= 2.43 million.

23. Let the total population of State R be x million.

Then, population of State R above poverty line = [(100 − 24)% of x] million
= $\left(\frac{76}{100} \times x\right)$ million.

And so, male population of State R above poverty line = $\left[\frac{2}{5} \times \left(\frac{76}{100} \times x\right)\right]$ million

But, it is given that male population of State R above poverty line = 1.9 million

∴ $\frac{2}{5} \times \left(\frac{76}{100} \times x\right) = 1.9 \Rightarrow x = \frac{5 \times 100 \times 1.9}{76 \times 2} = 6.25$.

∴ Total population of State R = 6.25 million.

24. Female population below poverty line for State P = 2.1 million.

Let the male population below poverty line for State P be x million.

Then, $5 : 6 = x : 2.1 \Rightarrow x = \frac{2.1 \times 5}{6} = 1.75$

∴ Population below poverty line for State P = (2.1 + 1.75) million = 3.85 million.

Let the population above poverty line for State P be y million.

Since, 35% of the total population of State P is below poverty line, therefore, 65% of the total population of State P is above poverty line *i.e.*, the ratio of population below poverty line to that above poverty line for State P is 35 : 65.

∴ $35 : 65 = 3.85 : y \Rightarrow y = \frac{65 \times 3.85}{35} = 7.15$

i.e., population above poverty line for State P = 7.15 million and so, male population above poverty line for State P = $\left(\frac{6}{13} \times 7.15\right)$ million = 3.3 million.

25. **For State Q :**

Male population below poverty line = 2.4 million.

Let the female population below poverty line be x million.

Then, $3 : 5 = 2.4 : x \Rightarrow x = \frac{5 \times 2.4}{3} = 4$

$\therefore$ Total population below poverty line = (2.4 + 4) = 6.4 million.

If N_q be the total population of State Q, then,

25% of N_q = 6.4 million $\Rightarrow N_q = \left(\frac{6.4 \times 100}{25}\right)$ million = 25.6 million.

For State T :

Male population below poverty line = 6 million.

Let the female population below poverty line be y million.

Then, $5 : 3 = 6 : y \Rightarrow y = \frac{3 \times 6}{5} = 3.6$

$\therefore$ Total population below poverty line = (6 + 3.6) = 9.6 million.

If N_t be the total population of State T, then

15% of N_t = 9.6 million $\Rightarrow N_t = \left(\frac{9.6 \times 100}{15}\right)$ million = 64 million.

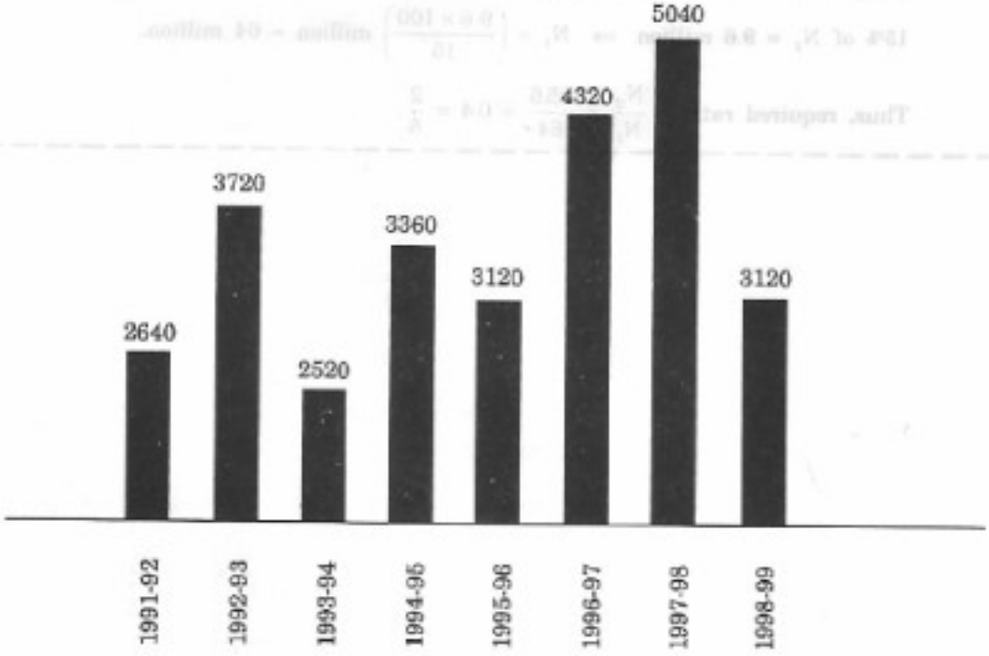
Thus, required ratio = $\frac{N_q}{N_t} = \frac{25.6}{64} = 0.4 = \frac{2}{5}$.

37. BAR GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale. One of the parameters is plotted on the horizontal axis and the other on the vertical axis. The candidate is required to understand the given information and thereafter answer the given questions on the basis of data analysis.

Ex. 1. ***The bar graph given below shows the foreign exchange reserves of a country (in million US $) from 1991-92 to 1998-99. Answer the questions based on this graph.*** **(Bank P.O. 2001)**

FOREIGN EXCHANGE RESERVES OF A COUNTRY
(in million US $)



1. The foreign exchange reserves in 1997-98 was how many times that in 1994-95 ?
   (a) 0.7 (b) 1.2 (c) 1.4 (d) 1.5 (e) 1.8
2. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94 ?
   (a) 100 (b) 150 (c) 200 (d) 620 (e) 2520
3. For which year, the percent increase of foreign exchange reserves over the previous year, is the highest ?
   (a) 1992-93 (b) 1993-94 (c) 1994-95 (d) 1996-97 (e) 1997-98

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review ?

(a) 95% (b) 110% (c) 115% (d) 125% (e) 140%

5. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves, is :

(a) 2 : 6 (b) 3 : 4 (c) 3 : 5 (d) 4 : 4 (e) 5 : 3

Sol. 1. (d) : Required ratio $= \frac{5040}{3360} = 1.5.$

2. (a) : Foreign exchange reserves in 1997-98 = 5040 million US $

Foreign exchange reserves in 1993-94 = 2520 million US $.

$\therefore$ Increase = (5040 − 2520) = 2520 million US $.

$\therefore$ Percentage increase $= \left(\frac{2520}{2520} \times 100\right)\% = 100\%.$

3. (a) : There is an increase in foreign exchange reserves during the years 1992-93, 1994-95, 1996-97 and 1997-98 as compared to previous year (as shown by bar-graph).

The percentage increase in reserves during these years compared to previous year are :

(i) For **1992-93** $= \left[\frac{(3720 - 2640)}{2640} \times 100\right]\% = 40.91\%$

(ii) For **1994-95** $= \left[\frac{(3360 - 2520)}{2520} \times 100\right]\% = 33.33\%$

(iii) For **1996-97** $= \left[\frac{(4320 - 3120)}{3120} \times 100\right]\% = 38.46\%$

(iv) For **1997-98** $= \left[\frac{(5040 - 4320)}{4320} \times 100\right]\% = 16.67\%$

Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period

$= \left[\frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)\right]$ million US $

= 3480 million US $.

Foreign exchange reserves in 1996-97 = 4320 million US $.

$\therefore$ Required Percentage $= \left(\frac{4320}{3480} \times 100\right)\% = 124.14\% \approx 125\%.$

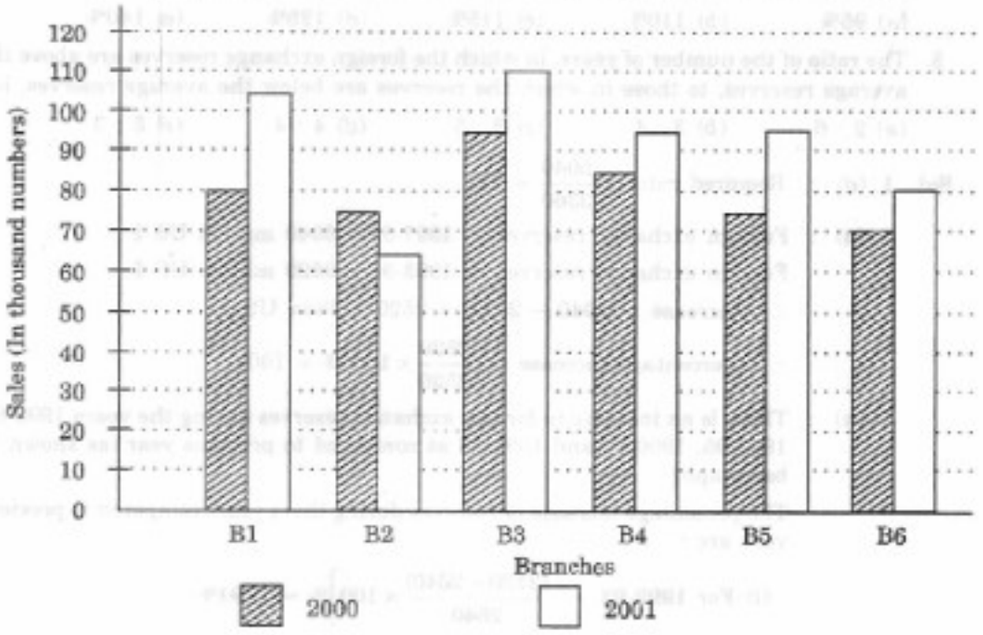
5. (c) : Average foreign exchange reserves over the given period = 3480 million US $.

The country had reserves above 3480 million US $ during the years 1992-93, 1996-97 and 1997-98 *i.e.*, for 3 years and below 3480 million US $ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 *i.e.*, for 5 years.

Hence, required ratio = 3 : 5.

Ex. 2. ***The bar-graph provided on next page gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph.*** **(Bank P.O. 2003)**

Sales of Books (in thousand numbers) from Six Branches — B1, B2, B3, B4, B5 and B6 of a Publishing Company in 2000 and 2001



1. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is :
   (a) 250 (b) 310 (c) 435 (d) 560 (e) 585
2. Total sales of branch B6 for both the years is what percent of the total sales of branch B3 for both the years ?
   (a) 68.54% (b) 71.11% (c) 73.17% (d) 75.55% (e) 77.26%
3. What is the average sale of all the branches (in thousand numbers) for the year 2000 ?
   (a) 73 (b) 80 (c) 83 (d) 88 (e) 96
4. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years ?
   (a) 2 : 3 (b) 3 : 5 (c) 4 : 5 (d) 5 : 7 (e) 7 : 9
5. What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000 ?
   (a) 75% (b) 77.5% (c) 82.5% (d) 85% (e) 87.5%

Sol. 1. (d) : Total sales of branches B1, B3 and B5 for both the years (in thousand numbers) = (80 + 105) + (95 + 110) + (75 + 95) = 560.

2. (c) : Required Percentage $= \left[\frac{(70+80)}{(95+110)} \times 100\right]\% = \left(\frac{150}{205} \times 100\right)\% = 73.17\%$.

3. (b) : Average sales of all the six branches (in thousand numbers) for the year 2000 $= \frac{1}{6} \times [80 + 75 + 95 + 85 + 75 + 70] = 80$.

4. (c) : Required ratio $= \frac{(75+65)}{(85+95)} = \frac{140}{180} = \frac{7}{9}$.

5. (e) : Average sales (in thousand numbers) of branches B1, B3 and B6 in 2000

$$= \frac{1}{3} \times (80 + 95 + 70) = \left(\frac{245}{3}\right).$$

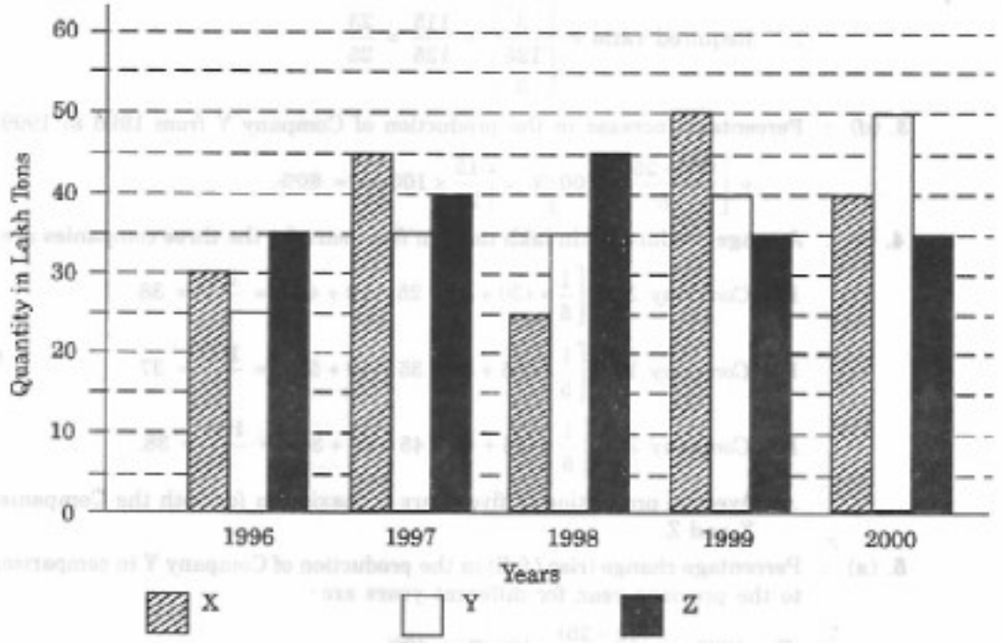
Average sales (in thousand numbers) of branches B1, B2 and B3 in 2001

$$= \frac{1}{3} \times (105 + 65 + 110) = \left(\frac{280}{3}\right).$$

$$\therefore \quad \text{Required Percentage} = \left[\frac{\left(\frac{245}{3}\right)}{\left(\frac{280}{3}\right)} \times 100\right]\% = \left(\frac{245}{280} \times 100\right)\% = 87.5\%.$$

Ex. 3. ***The bar graph provided below gives the data of the production of paper (in lakh tonnes) by three different companies X, Y and Z over the years. Study the graph and answer the questions that follow.*** (Bank P.O. 2001)

Production of Paper (in lakh tonnes) by Three Companies X, Y and Z over the Years



1. What is the difference between the production of Company Z in 1998 and Company Y in 1996 ?
   (a) 2,00,000 tons (b) 20,00,000 tens (c) 20,000 tons
   (d) 2,00,00,000 tons (e) None of these
2. What is the ratio of the average production of Company X in the period 1998-2000 to the average production of Company Y in the same period ?
   (a) 1 : 1 (b) 15 : 17 (c) 23 : 25 (d) 27 : 29 (e) None of these
3. What is the percentage increase in the production of Company Y from 1996 to 1999 ?
   (a) 30% (b) 45% (c) 50% (d) 60% (e) 75%
4. The average production for five years was maximum for which Company ?
   (a) X (b) Y (c) Z (d) X and Y both (e) X and Z both

5. For which of the following years, the percentage rise / fall in production from the previous year is the maximum for Company Y ?

(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 1997 and 2000

6. In which year was the percentage of production of Company Z to the production of Company Y the maximum ?

(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

Sol. 1. (b) : Required difference = [(45 – 25) × 1,00,000] tons = 20,00,000 tons.

2. (c) : Average production of Company X in the period 1998-2000

$$= \left[\frac{1}{3} \times (25 + 50 + 40)\right] = \left(\frac{115}{3}\right) \text{ lakh tons.}$$

Average production of Company Y in the period 1998-2000

$$= \left[\frac{1}{3} \times (35 + 40 + 50)\right] = \left(\frac{125}{3}\right) \text{ lakh tons.}$$

$$\therefore \text{ Required ratio} = \frac{\left(\frac{115}{3}\right)}{\left(\frac{125}{3}\right)} = \frac{115}{125} = \frac{23}{25}.$$

3. (d) : Percentage increase in the production of Company Y from 1996 to 1999

$$= \left[\frac{(40 - 25)}{25} \times 100\right]\% = \left(\frac{15}{25} \times 100\right)\% = 60\%.$$

4. (e) : Average production (in lakh tons) in five years for the three companies are :

$$\text{For } Company\ X = \left[\frac{1}{5} \times (30 + 45 + 25 + 50 + 40)\right] = \frac{190}{5} = 38$$

$$\text{For } Company\ Y = \left[\frac{1}{5} \times (25 + 35 + 35 + 40 + 50)\right] = \frac{185}{5} = 37$$

$$\text{For } Company\ Z = \left[\frac{1}{5} \times (35 + 40 + 45 + 35 + 35)\right] = \frac{190}{5} = 38.$$

∴ Average production of five years is maximum for both the Companies X and Z.

5. (a) : Percentage change (rise / fall) in the production of Company Y in comparison to the previous year, for different years are :

$$For\ 1997 = \left[\frac{(32 - 25)}{25} \times 100\right]\% = 40\%$$

$$For\ 1998 = \left[\frac{(35 - 35)}{25} \times 100\right]\% = 0\%$$

$$For\ 1999 = \left[\frac{(40 - 35)}{35} \times 100\right]\% = 14.29\%$$

$$For\ 2000 = \left[\frac{(50 - 40)}{40} \times 100\right]\% = 25\%$$

Hence, the maximum percentage rise / fall in the production of Company Y is for 1997.

6. (a) : The percentages of production of Company Z to the production of Company Z for various years are :

$$For\ 1996 = \left(\frac{35}{25}\times 100\right)\% = 140\%;\ For\ 1997 = \left(\frac{40}{35}\times 100\right)\% = 114.29\%;$$

$$For\ 1998 = \left(\frac{45}{35}\times 100\right)\% = 128.57\%;\ For\ 1999 = \left(\frac{35}{40}\times 100\right)\% = 87.5\%;$$

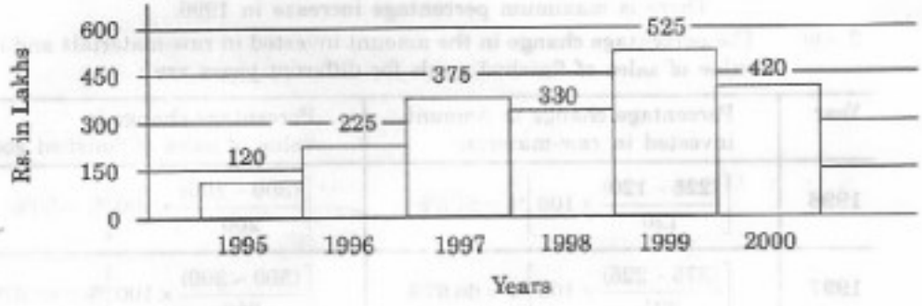
$$For\ 2000 = \left(\frac{35}{50}\times 100\right)\% = 70\%.$$

Clearly, this percentage is highest for 1996.

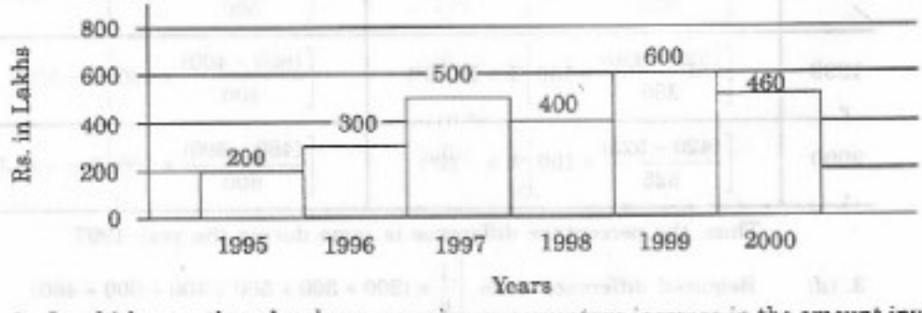
Ex. 4. ***Out of the two bar graphs provided below, one shows the amounts (in Lakh Rs.) invested by a Company in purchasing raw materials over the years and the other shows the values (in Lakh Rs.) of finished goods sold by the Company over the years. Study the two bar graphs and answer the questions based on them.***

Amount Invested in Raw Materials and the Value of Sales of Finished Goods for a Company over the Years

Amount Invested in Raw Materials (Rs. in Lakhs)



Value of Sales of Finished Goods (Rs. in Lakhs)



1. In which year, there has been a maximum percentage increase in the amount invested in Raw Materials as compared to the previous year ?
   (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
2. In which year, the percentage change (compared to the previous year) in the investment on Raw Materials is the same as that in the value of sales of finished goods ?
   (a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
3. What was the difference between the average amount invested in Raw Materials during the given period and the average value of sales of finished goods during this period ?
   (a) Rs. 62.5 lakhs (b) Rs. 68.5 lakhs (c) Rs. 71.5 lakhs
   (d) Rs. 77.5 lakhs (e) Rs. 83.5 lakhs

4. The value of sales of finished goods in 1999 was approximately what percent of the average amount invested in Raw Materials in the years 1997, 1998 and 1999 ?

(*a*) 33% (*b*) 37% (*c*) 45% (*d*) 49% (*e*) 53%

5. The maximum difference between the amount invested in Raw Materials and the value of sales of finished goods was during the year :

(*a*) 1995 (*b*) 1996 (*c*) 1997 (*d*) 1998 (*e*) 1999

Sol. 1. (*a*) : The percentage increase in the amount invested in raw-materials as compared to the previous year, for different years are :

For 1996 $= \left[\frac{(225-120)}{120} \times 100\right]\% = 87.5\%$

For 1997 $= \left[\frac{(375-225)}{225} \times 100\right]\% = 66.67\%$

For 1998 there is a decrease.

For 1999 $= \left[\frac{(525-330)}{330} \times 100\right]\% = 59.09\%$

For 2000 there is a decrease.

∴ There is maximum percentage increase in 1996.

2. (*b*) : The percentage change in the amount invested in raw-materials and in the value of sales of finished goods for different years are :

Year	Percentage change in Amount invested in raw-material	Percentage change in value of sales of finished goods
1996	$\left[\frac{(225-120)}{120} \times 100\right]\% = 87.5\%$	$\left[\frac{(300-200)}{200} \times 100\right]\% = 50\%$
1997	$\left[\frac{(375-225)}{225} \times 100\right]\% = 66.67\%$	$\left[\frac{(500-300)}{300} \times 100\right]\% = 66.67\%$
1998	$\left[\frac{(330-375)}{375} \times 100\right]\% = -12\%$	$\left[\frac{(400-500)}{500} \times 100\right]\% = -20\%$
1999	$\left[\frac{(525-330)}{330} \times 100\right]\% = 59.09\%$	$\left[\frac{(600-400)}{400} \times 100\right]\% = 50\%$
2000	$\left[\frac{(420-525)}{525} \times 100\right]\% = -20\%$	$\left[\frac{(460-600)}{600} \times 100\right]\% = -23.33\%.$

Thus, the percentage difference is same during the year 1997.

3. (*d*) : Required difference = Rs. $\left[\frac{1}{6} \times (200+300+500+400+600+460) - \frac{1}{6} \times (120+225+375+330+525+420)\right]$ lakhs

= Rs. $\left[\left(\frac{2460}{6}\right) - \left(\frac{1995}{6}\right)\right]$ lakhs = Rs. (410 − 332.5) lakhs = Rs. 77.5 lakhs.

4. (*d*) : Required percentage $= \left[\frac{600}{(375+330+525)} \times 100\right]\% = 48.78\% = 49\%.$

5. (c) : The differences between the amount invested in raw material and the value of sales of finished goods for various years are:

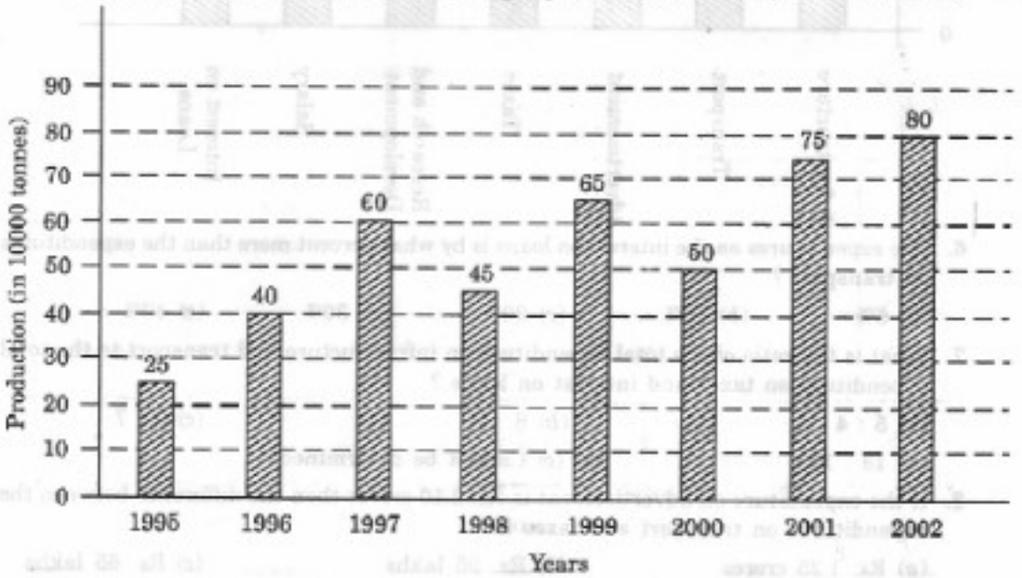
For 1995 = Rs. (200 – 120) lakhs = Rs. 80 lakhs.
For 1996 = Rs. (300 – 225) lakhs = Rs. 75 lakhs.
For 1997 = Rs. (500 – 375) lakhs = Rs. 125 lakhs.
For 1998 = Rs. (400 – 330) lakhs = Rs. 70 lakhs.
For 1999 = Rs. (600 – 525) lakhs = Rs. 75 lakhs.
For 2000 = Rs. (460 – 420) lakhs = Rs. 40 lakhs.

Clearly, maximum difference was during 1997.

EXERCISE 37

Directions (Questions 1 to 5) : ***Study the following bar-graph and answer the questions given below.*** **(Bank P.O. 2002)**

Production of Fertilizers by a Company (in 10000 tonnes) over the Years



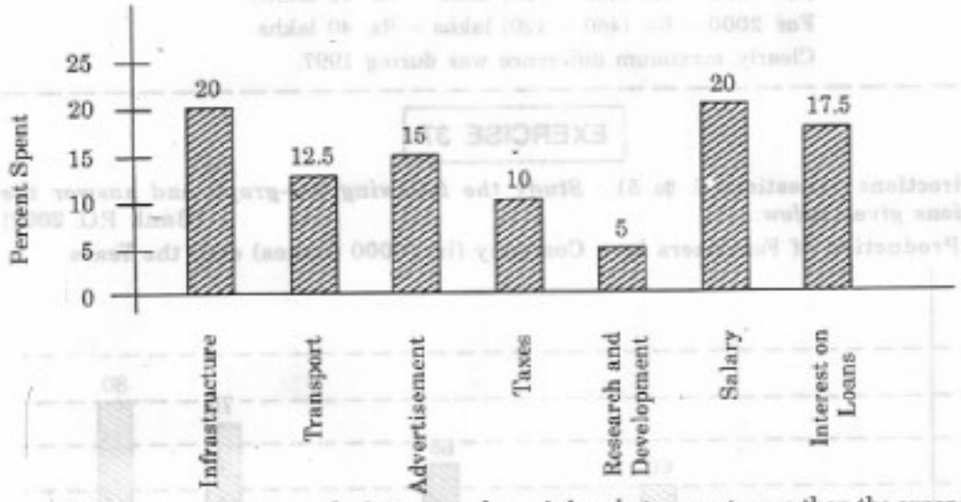
1. In how many of the given years was the production of fertilizers more than the average production of the given years ?
   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
2. The average production of 1996 and 1997 was exactly equal to the average production of which of the following pairs of years ?
   (a) 2000 and 2001 (b) 1999 and 2000 (c) 1998 and 2000
   (d) 1995 and 1999 (e) 1995 and 2001
3. What was the percentage decline in the production of fertilizers from 1997 to 1998?
   (a) $33\frac{1}{3}\%$ (b) 30% (c) 25% (d) 21% (e) 20%
4. In which year was the percentage increase in production as compared to the previous year the maximum ?
   (a) 2002 (b) 2001 (c) 1999 (d) 1997 (e) 1996

5. What was the percentage increase in production of fertilizers in 2002 compared to that in 1995 ?

(a) 320% (b) 300% (c) 220% (d) 200% (e) 150%

Directions (Questions 6 to 10) : ***The bar-graph given below shows the percentage distribution of total expenditures of a Company under various expense heads during 2003. Study the graph and answer the questions that follow :***

Percentage Distribution of Total Expenditures of a Company



6. The expenditures on the interest on loans is by what percent more than the expenditures on transport ?

(a) 5% (b) 10% (c) 20% (d) 30% (e) 40%

7. What is the ratio of the total expenditure on infrastructure and transport to the total expenditure on taxes and interest on loans ?

(a) 5 : 4 (b) 8 : 7 (c) 9 : 7
(d) 13 : 11 (e) Cannot be determined

8. If the expenditure on advertisement is Rs. 2.10 crores then the difference between the expenditures on transport and taxes is :

(a) Rs. 1.25 crores (b) Rs. 95 lakhs (c) Rs. 65 lakhs
(d) Rs. 35 lakhs (e) Rs. 25 lakhs

9. The total amount of expenditures of the Company is how many times the expenditure on research and development ?

(a) 27 (b) 20 (c) 18 (d) 8 (e) 5

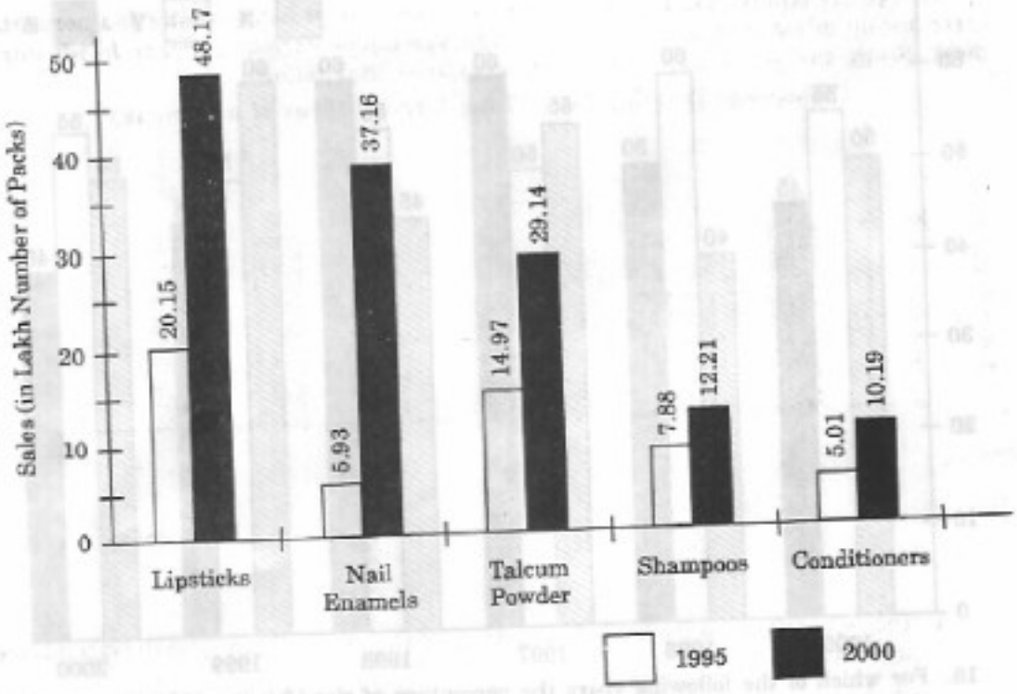
10. If the interest on loans amounted to Rs. 2.45 crores then the total amount of expenditure on advertisement, taxes and research and development is :

(a) Rs. 7 crores (b) Rs. 5.4 crores (c) Rs. 4.2 crores
(d) Rs. 3 crores (e) Rs. 2.4 crores

Directions (Questions 11 to 15) : ***A cosmetic company produces five different products. The sales of these five products (in lakh number of packs) during 1995 and 2000 are shown in the following bar-graph. The questions given below are based on this graph.***

(Bank P.O. 2001)

Sales (in lakh number of packs) of five different products of a Cosmetic Company during 1995 and 2000



11. The sales have increased by nearly 55% from 1995 to 2000 in the case of :
(*a*) Lipsticks (*b*) Nail enamels (*c*) Talcum powders
(*d*) Shampoos (*e*) Conditioners

12. During the period 1995-2000, the minimum rate of increase in sales is in the case of :
(*a*) Lipsticks (*b*) Nail enamels (*c*) Talcum powders
(*d*) Shampoos (*e*) Conditioners

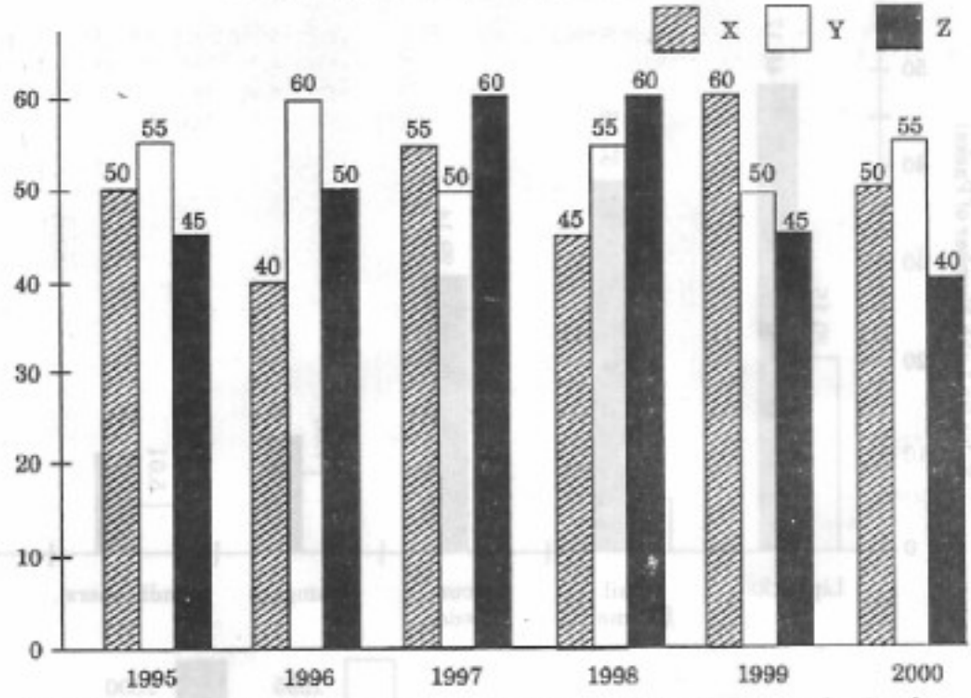
13. The sales of lipsticks in 2000 was by what percent more than the sales of nail enamels in 2000 ? (rounded off to the nearest integer)
(*a*) 33% (*b*) 31% (*c*) 28% (*d*) 22% (*e*) 21%

14. The sales of conditioners in 1995 was by what percent less than the sales of shampoos in 1995 ? (rounded off to the nearest integer)
(*a*) 57% (*b*) 36% (*c*) 29% (*d*) 25% (*e*) 19%

15. What is the approximate ratio of the sales of nail enamels in 2000 to the sales of Talcum powders in 1995 ?
(*a*) 7 : 2 (*b*) 5 : 2 (*c*) 4 : 3 (*d*) 2 : 1 (*e*) 5 : 3

Directions (Questions 16 to 20) : A soft-drink company prepares drinks of three different flavours — X, Y and Z. The production of the three flavours over a period of six years has been expressed in the bar-graph provided below. Study the graph and answer the questions based on it. (I.B.P.S. 2002)

Production of three different flavours of soft-drinks X, Y, Z by a Company over the years (in lakh bottles)



16. For which of the following years the percentage of rise / fall in production from the previous year is the maximum for the flavour Y ?

(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

17. For which flavour was the average annual production maximum in the given period ?

(a) X only (b) Y only (c) Z only (d) X and Y (e) X and Z

18. The total production of flavour Z in 1997 and 1998 is what percentage of the total production of flavour X in 1995 and 1996 ?

(a) 96.67% (b) 102.25% (c) 115.57% (d) 120% (e) 133.33%

19. What is the difference between the average production of flavour X in 1995, 1996 and 1997 and the average production of flavour Y in 1998, 1999 and 2000 ?

(a) 50,000 bottles (b) 80,000 bottles (c) 2,40,000 bottles
(d) 3,30,000 bottles (e) 5,00,000 bottles

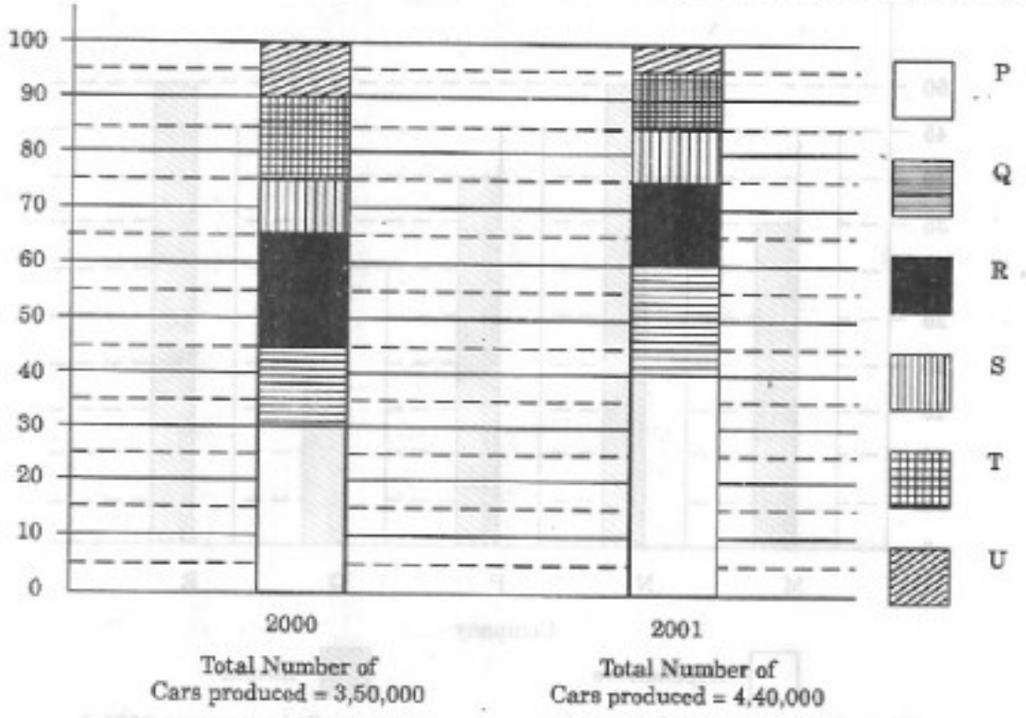
20. What was the approximate decline in the production of flavour Z in 2000 as compared to the production in 1998 ?

(a) 50% (b) 42% (c) 33% (d) 25% (e) 22.5%

Directions (Questions 21 to 25) : ***The bar-graph given below shows the percentage distribution of the total production of a car manufacturing company into various models over two years. Study the graph carefully and answer the questions that follow.***

(Bank P.O. 2001)

Percentage of Six different types of Cars manufactured by a Company over two years



Total Number of Cars produced = 3,50,000

Total Number of Cars produced = 4,40,000

21. Total number of cars of models P, Q and T manufactured in 2000 is :

(a) 2,45,000 (b) 2,27,500 (c) 2,10,000 (d) 1,92,500 (e) 1,57,500

22. For which model the percentage rise / fall in production from 2000 to 2001 was minimum ?

(a) Q (b) R (c) S (d) T (e) U

23. What was the difference in the number of Q type cars produced in 2000 and that produced in 2001 ?

(a) 35,500 (b) 27,000 (c) 22,500 (d) 17,500 (e) 16,000

24. If the percentage production of P type cars in 2001 was the same as that in 2000, then the number of P type cars produced in 2001 would have been :

(a) 1,40,000 (b) 1,32,000 (c) 1,17,000 (d) 1,05,000 (e) 97,000

25. If 85% of the S type cars produced in each year were sold by the Company, how many S type cars remained unsold ?

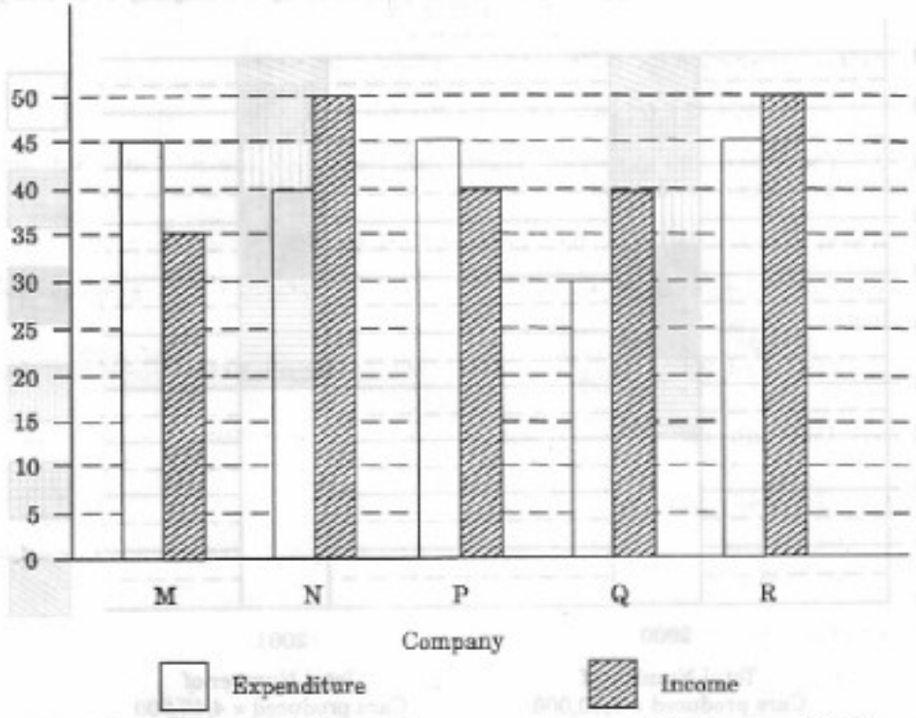
(a) 7650 (b) 9350 (c) 11,850 (d) 12,250 (e) 13,350

Directions (Questions 26 to 30) : ***The following bar-graph shows the Income and Expenditures (in million US $) of five Companies in the year 2001. The percent profit or loss of a Company is given by***

$$(Profit / Loss)\% = \frac{Income - Expenditure}{Expenditure} \times 100.$$

Study the graph and answer the questions that are based on it. (S.B.I.P.O. 2002)

Income and Expenditure (in million US $) of five Companies in the year 2001



26. Which Company earned the maximum percentage profit in the year 2001 ?
(*a*) M (*b*) N (*c*) P (*d*) Q (*e*) R

27. The Companies M and N together had a percentage profit / loss of :
(*a*) 12% loss (*b*) 10% loss (*c*) 10% profit
(*d*) 12% profit (*e*) There was no loss or profit

28. In 2001 what was the approximate percentage of profit / loss of all the five Companies taken together ?
(*a*) 5% profit (*b*) 6.5% profit (*c*) 4% loss (*d*) 7% loss (*e*) 10% profit

29. If the income of Company Q in 2001 was 10% more than its income in 2000 and the Company had earned a profit of 20% in 2000, then its expenditure in 2000 (in million US $) was :
(*a*) 28.28 (*b*) 30.30 (*c*) 32.32 (*d*) 34.34 (*e*) 36.36

30. For Company R, if the expenditure had increased by 20% in year 2001 from year 2000 and the Company had earned a profit of 10% in 2000, what was the Company's income in 2000 (in million US $) ?
(*a*) 35.75 (*b*) 37.25 (*c*) 38.5 (*d*) 41.25 (*e*) 42.75

ANSWERS

1. (*d*)	**2.** (*e*)	**3.** (*c*)	**4.** (*e*)	**5.** (*c*)	**6.** (*e*)	**7.** (*d*)	**8.** (*d*)	**9.** (*b*)
10. (*c*)	**11.** (*d*)	**12.** (*d*)	**13.** (*c*)	**14.** (*b*)	**15.** (*b*)	**16.** (*b*)	**17.** (*b*)	**18.** (*e*)
19. (*e*)	**20.** (*c*)	**21.** (*c*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)	**25.** (*c*)	**26.** (*d*)	**27.** (*e*)
28. (*a*)	**29.** (*b*)	**30.** (*d*)						

SOLUTIONS

1. Average production (in 10000 tonnes) over the given years

$= \frac{1}{8} \times (25 + 40 + 60 + 45 + 65 + 50 + 75 + 80) = 55$

$\therefore$ The productions during the years 1997, 1999, 2001 and 2002 are more than the average production.

2. Average production (in 10000 tonnes) of 1996 and 1997 $= \frac{40 + 60}{2} = 50$.

We shall find the average production (in 10000 tonnes) for each of the given alternative pairs :

(*a*) **2000 and 2001** $= \frac{50 + 75}{2} = 62.5$ (*b*) **1999 and 2000** $= \frac{65 + 50}{2} = 57.5$

(*c*) **1998 and 2000** $= \frac{45 + 50}{2} = 47.5$ (*d*) **1995 and 1999** $= \frac{25 + 65}{2} = 45$

(*e*) **1995 and 2001** $= \frac{25 + 75}{2} = 50$.

$\therefore$ The average production of 1996 and 1997 is equal to the average production of 1995 and 2001.

3. Required percentage $= \left[\frac{(45 - 60)}{60} \times 100\right]\% = -25\%$.

$\therefore$ There is a decline of 25% in production from 1997 to 1998.

4. The percentage increase in production compared to previous year for different years are :

In **1996** $= \left[\frac{(40 - 25)}{25} \times 100\right]\% = 60\%$; In **1997** $= \left[\frac{(60 - 40)}{40} \times 100\right]\% = 50\%$

In **1998** there is a decrease in production.

In **1999** $= \left[\frac{(65 - 45)}{45} \times 100\right]\% = 44.44\%$

In **2000** there is a decrease in production.

In **2001** $= \left[\frac{(75 - 50)}{50} \times 100\right]\% = 50\%$; In **2002** $= \left[\frac{(80 - 75)}{75} \times 100\right]\% = 6.67\%$

Clearly, there is maximum percentage increase in production in 1996.

5. Required percentage $= \left[\frac{(80 - 25)}{25} \times 100\right]\% = 220\%$.

6. Let the total amount of expenditures be Rs. x.

Then, the expenditure on interest on loans = Rs. (17.5% of x) = Rs. $\left(\frac{17.5}{100}x\right)$

and the expenditure on transport = Rs. (12.5% of x) = Rs. $\left(\frac{12.5}{100}x\right)$

$\therefore$ Difference between the two expenditures = Rs. $\left(\frac{17.5}{100}x - \frac{12.5}{100}x\right)$ = Rs. $\left(\frac{5x}{100}\right)$

and so, the required percentage $= \left[\frac{\left(\frac{5x}{100}\right)}{\left(\frac{12.5x}{100}\right)} \times 100\right]\% = 40\%$.

7. Let the total amount of expenditures be Rs. x.
Then, the total expenditure on infrastructure and transport
$= \text{Rs. } [(20 + 12.5)\% \text{ of } x] = \text{Rs. } (32.5\% \text{ of } x) = \text{Rs. } \left(\frac{32.5x}{100}\right)$
and total expenditure on taxes and interest on loans
$= \text{Rs. } [(10 + 17.5)\% \text{ of } x] = \text{Rs. } (27.5\% \text{ of } x) = \text{Rs. } \left(\frac{27.5x}{100}\right)$
$\therefore$ Required ratio $= \frac{\left(\frac{32.5x}{100}\right)}{\left(\frac{27.5x}{100}\right)} = 13 : 11.$

8. Let the total expenditure be Rs. x crores.
Then, 15% of $x = 2.10 \Rightarrow x = \left(\frac{2.10 \times 100}{15}\right) = 14.$
$\therefore$ Total expenditure = Rs. 14 crores
and so, the difference between the expenditures on transport and taxes
= Rs. [(12.5 – 10)% of 14] crores = Rs. (2.5% of 14) crores
= Rs. 0.35 crores = Rs. 35 lakhs.

9. Let the total expenditures be Rs. x
Then, the expenditure on Research and Development = Rs. (5% of x) = Rs. $\left(\frac{x}{20}\right)$.
$\therefore$ Ratio of the total expenditure to the expenditure on Research and Development
$= \frac{x}{\left(\frac{x}{20}\right)} = \frac{20}{1}.$
Thus, the total expenditure is 20 times the expenditure on Research and Development.

10. Let the total expenditure be Rs. x crores. Then, 17.5% of $x = 2.45 \Rightarrow x = 14$.
$\therefore$ Total expenditure = Rs. 14 crores
and so, the total expenditure on advertisement, taxes and research and development = Rs. [(15 + 10 + 5)% of 14] crores
= Rs. (30% of 14) crores = Rs. 4.2 crores.

11. The percentage increase from 1995 to 2000 for various products are :

Lipsticks $= \left[\frac{(48.17 - 20.15)}{20.15} \times 100\right]\% = 139.06$

Nail enamels $= \left[\frac{(37.76 - 5.93)}{5.93} \times 100\right]\% = 536.76\%$

Talcum powders $= \left[\frac{(29.14 - 14.97)}{14.97} \times 100\right]\% = 94.66\%$

Shampoos $= \left[\frac{(12.21 - 7.88)}{7.88} \times 100\right]\% = 54.95\% \approx 55\%$

Conditioners $= \left[\frac{(10.19 - 5.01)}{5.01} \times 100\right]\% = 103.39\%.$

12. As calculated in the Solution of Q. 11, the minimum rate of increase in sales from 1995 to 2000 is in the case of Shampoos.

13. Required percentage $= \left[\frac{(48.17 - 37.76)}{37.76} \times 100\right]\% = 27.57\% = 28\%$.

14. Required percentage $= \left[\frac{(7.88 - 5.01)}{7.88} \times 100\right]\% = 36.42\% \approx 36\%$.

15. Required ratio $= \frac{37.76}{14.97} \approx 2.5 = \frac{5}{2}$.

16. The percentage rise / fall in production from the previous year for flavour Y during various years are :

In **1996** $= \left[\frac{(60 - 55)}{55} \times 100\right]\% = 9.09\%$ (increase)

In **1997** $= \left[\frac{(60 - 50)}{60} \times 100\right]\% = 16.67\%$ (decrease)

In **1998** $= \left[\frac{(55 - 50)}{55} \times 100\right]\% = 10\%$ (increase)

In **1999** $= \left[\frac{(55 - 50)}{55} \times 100\right] = 9.09\%$ (decrease)

In **2000** $= \left[\frac{(55 - 50)}{50} \times 100\right]\% = 10\%$ (increase)

$\therefore$ Maximum change is decrease of 16.67% during 1997.

17. Average annual productions over the given period for various flavours are :

For *flavour X* $= \left[\frac{1}{6} \times (50 + 40 + 55 + 45 + 60 + 50)\right]$ lakh bottles $= 50$ lakh bottles.

For *flavour Y* $= \left[\frac{1}{6} \times (55 + 60 + 50 + 55 + 50 + 55)\right]$ lakh bottles

$= 54.17$ lakh bottles.

For *flavour Z* $= \left[\frac{1}{6} \times (45 + 50 + 60 + 60 + 45 + 40)\right]$ lakh bottles $= 50$ lakh bottles.

$\therefore$ Maximum average production is for flavour Y.

18. Required percentage $= \left[\frac{(60 + 60)}{(50 + 40)} \times 100\right]\% = \left(\frac{120}{90} \times 100\right)\% = 133.33\%$.

19. Average production of flavour X in 1995, 1996 and 1997 $= \left[\frac{1}{3} \times (50 + 40 + 55)\right]$

$= \left(\frac{145}{3}\right)$ lakh bottles.

Average production of flavour Y in 1998, 1999 and 2000 $= \left[\frac{1}{3} \times (55 + 50 + 55)\right]$

$= \left(\frac{160}{3}\right)$ lakh bottles.

$\therefore$ Difference $= \left(\frac{160}{3} - \frac{145}{3}\right) = \frac{15}{3} = 5$ lakh bottles $= 5{,}00{,}000$ bottles.

20. Percentage decline in the production of flavour Z in 2000 as compared to the production in 1998 $= \left[\frac{(60-40)}{60} \times 100\right]\% = \left(\frac{20}{60} \times 100\right)\% = 33.33\% \approx 33\%$.

21. We shall first determine the number of cars of each model produced by the Company during the two years :

In 2000 : Total number of cars produced = 3,50,000.

P = (30 – 0)% of 3,50,000 = 30% of 3,50,000 = 1,05,000

Q = (45 – 30)% of 3,50,000 = 15% of 3,50,000 = 52,500

R = (65 – 45)% of 3,50,000 = 20% of 3,50,000 = 70,000

S = (75 – 65)% of 3,50,000 = 10% of 3,50,000 = 35,000

T = (90 – 75)% of 3,50,000 = 15% of 3,50,000 = 52,500

U = (100 – 90)% of 3,50,000 = 10% of 3,50,000 = 35,000.

In 2001 : Total number of cars produced = 4,40,000.

P = (40 – 0)% of 4,40,000 = 40% of 4,40,000 = 1,76,000

Q = (60 – 40)% of 4,40,000 = 20% of 4,40,000 = 88,000

R = (75 – 60)% of 4,40,000 = 15% of 4,40,000 = 66,000

S = (85 – 75)% of 4,40,000 = 10% of 4,40,000 = 44,000

T = (95 – 85)% of 4,40,000 = 10% of 4,40,000 = 44,000

U = (100 – 95)% of 4,40,000 = 5% of 4,40,000 = 22,000.

Now, we shall solve the questions.

Total number of cars of models P, Q and T manufactured in 2000

= (105000 + 52500 + 52500) = 2,10,000.

22. Using the above calculation, the percentage change (rise / fall) in production from 2000 to 2001 for various models is :

For P $= \left[\frac{(176000-105000)}{105000} \times 100\right]\% = 67.62\%$, rise.

For Q $= \left[\frac{(88000-52500)}{52500} \times 100\right]\% = 67.62\%$, rise.

For R $= \left[\frac{(70000-66000)}{70000} \times 100\right]\% = 5.71\%$, fall.

For S $= \left[\frac{(44000-35000)}{35000} \times 100\right]\% = 25.71\%$, rise.

For T $= \left[\frac{(52500-44000)}{52500} \times 100\right]\% = 16.19\%$, fall.

For U $= \left[\frac{(35000-22000)}{35000} \times 100\right]\% = 37.14\%$, fall.

∴ Minimum percentage rise / fall in production is in the case of model R.

23. Required difference = 88000 – 52500 = 35500

(Using calculations in the Solution of Q. 21)

24. If the percentage production of P type cars in 2001 = percentage production of P type cars in 2000 = 30%

then, number of P type cars produced in 2001 = 30% of 440000 = 132000.

25. Number of S type cars which remained unsold in 2000 = 15% of 35000

and number of S type cars which remained unsold in 2001 = 15% of 44000

∴ Total number of S type cars which remained unsold
= 15% of (35000 + 44000) = 15% of 79000 = 11850.

26. The percentage profit / loss in the year 2001 for various companies are :

$For\ M = \left[\frac{(30-45)}{45}\times 100\right]\% = -33.33\%$ *i.e.* %Loss = 33.33%

$For\ N = \left[\frac{(50-40)}{40}\times 100\right]\% = 25\%$ *i.e.* %Profit = 25%

$For\ P = \left[\frac{(40-45)}{45}\times 100\right]\% = -11.11\%$ *i.e.* %Loss = 11.11%

$For\ Q = \left[\frac{(40-30)}{30}\times 100\right]\% = 33.33\%$ *i.e.* %Profit = 33.33%

$For\ R = \left[\frac{(50-45)}{45}\times 100\right]\% = 11.11\%$ *i.e.* %Profit = 11.11%

Clearly, the Company Q earned the maximum profit in 2001.

27. Total income of companies M and N together = (35 + 50) million US $
= 85 million US $

Total expenditure of companies M and N together = (45 + 40) million US $
= 85 million US $

∴ Percent Profit / Loss of companies M and N together

$$\%\text{Profit/Loss} = \left(\frac{85-85}{85}\times 100\right) = 0\%.$$

Thus, there was neither loss nor profit for companies M and N together.

28. Total income of all five companies = (35 + 50 + 40 + 40 + 50) = 215 million US $
Total expenditure of all five companies = (45 + 40 + 45 + 30 + 45)
= 205 million US $

$$\therefore\ \%\text{Profit} = \left[\frac{(215-205)}{205}\times 100\right]\% = 4.88\% = 5\%.$$

29. Let the income of Company Q in 2000 = x million US $

Then, income of Company Q in 2001 = $\left(\frac{110}{100}x\right)$ million US $

$$\therefore\ \frac{110}{100}x = 40 \Rightarrow x = \left(\frac{400}{11}\right).$$

i.e. income of Company Q in 2000 = $\left(\frac{400}{11}\right)$ million US $.

Let the expenditure of Company Q in 2000 be E million US $.

Then, $20 = \frac{\left[\left(\frac{400}{11}\right) - E\right]}{E}\times 100$ [∵ % Profit = 20%]

$$\Rightarrow\ 20 = \left[\left(\frac{400}{11E}\right) - 1\right]\times 100 \Rightarrow E = \frac{400}{11}\times\frac{100}{120} = 30.30.$$

∴ Expenditure of Company Q in 2000 = 30.30 million US $.

30. Let the expenditure of Company R in 2000 be x million US $.

Then, expenditure of Company R in 2001 = $\left(\frac{120}{100}x\right)$ million US $.

$\therefore \quad \frac{120}{100}x = 45 \Rightarrow x = 37.5$

i.e. expenditure of Company R in 2000 = 37.5 million US $.

Let the income of Company R in 2000 be I million US $.

Then, $10 = \frac{(I - 37.5)}{37.5} \times 100$ [$\because$ % profit in 2000 = 10%]

$\Rightarrow$ I − 37.5 = 3.75 $\Rightarrow$ I = 41.25

i.e. Income of Company R in 2000 = 41.25 million US $.

38. PIE-CHARTS

IMPORTANT FACTS AND FORMULAE

The **pie-chart** or a **pie-graph** is a method of representing a given numerical data in the form of sectors of a circle.

The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding value of the component of the data.

From geometry, we know that the area of the sector of a circle is proportional to the central angle.

So, the central angle of each sector must be proportional to the corresponding value of the component.

Since the sum of all the central angles is 360°, we have

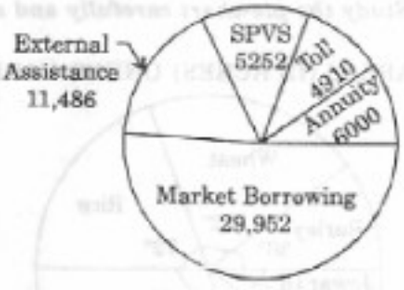
$$\text{Central angle of the component} = \left(\frac{\text{Value of the component}}{\text{Total value}} \times 360\right)^\circ$$

SOLVED EXAMPLES

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

Example 1. ***The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the questions that follow.***

SOURCES OF FUNDS TO BE ARRANGED BY NHAI
FOR PHASE II PROJECTS (IN CRORES RS.)



Total funds to be arranged for Projects (Phase II) = Rs. 57,600 crores.

1. Near about 20% of the funds are to be arranged through :
   (*a*) SPVS (*b*) External Assistance
   (*c*) Annuity (*d*) Market Borrowing
2. The central angle corresponding to Market Borrowing is :
   (*a*) 52° (*b*) 137.8% (*c*) 187.2° (*d*) 192.4°
3. The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is :
   (*a*) 2 : 9 (*b*) 1 : 6 (*c*) 3 : 11 (*d*) 2 : 5

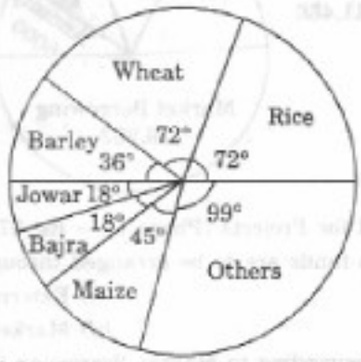
4. If NHAI could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowings to arrange for the shortage of funds ?
   (a) 4.5% (b) 7.5% (c) 6% (d) 8%
5. If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4910 crores ?
   (a) Rs. 6213 crores (b) Rs. 5827 crores (c) Rs. 5401 crores (d) Rs. 5216 crores

SOLUTION

1. **(b)** : 20% of the total funds to be arranged = Rs. (20% of 57600) crores
   = Rs. 11520 crores = Rs. 11486 crores.
   Rs. 11486 crores is the amount of funds to be arranged through External Assistance.
2. **(c)** : Central angle corresponding to Market Borrowing $= \left(\frac{29952}{57600} \times 360^\circ\right) = 187.2^\circ$.
3. **(b)** : Required ratio $= \frac{4910}{29952} = \frac{1}{6.1} \approx \frac{1}{6}$.
4. **(c)** : Shortage of funds arranged through External Assistance
   = Rs. (11486 – 9695) crores = Rs. 1791 crores.
   ∴ Increase required in Market Borrowings = Rs. 1791 crores.
   Percentage increase required $= \left(\frac{1791}{29952} \times 100\right)\% = 5.98\% \approx 6\%$.
5. **(c)** : Amount permitted = (Funds required from Toll for projects of Phase II)
   + (10% of these funds)
   = Rs. 4910 crores + Rs. (10% of 4910) crores
   = Rs. (4910 + 491) crores = Rs. 5401 crores

Example 2. ***The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.***

DISTRIBUTION OF AREAS (IN ACRES) UNDER VARIOUS FOOD CROPS



1. Which combination of three crops contribute to 50% of the total area under the food crops ?
   (a) Wheat, Barley and Jowar (b) Rice, Wheat and Jowar
   (c) Rice Wheat and Barley (d) Bajra, Maize and Rice
2. If the total area under jowar was 1.5 million acres, then what was the area (in million acres) under rice ?
   (a) 6 (b) 7.5 (c) 9 (d) 4.5

3. If the production of wheat is 6 times that of barley, then what is the ratio between the yield per acre of wheat and barley ?

(*a*) 3 : 2 (*b*) 3 : 1 (*c*) 12 : 1 (*d*) 2 : 3

4. If the yield per acre of rice was 50% more than that of barley, then the production of barley is what percent of that of rice ?

(*a*) 30% (*b*) $33\frac{1}{3}\%$ (*c*) 35% (*d*) 36%

5. If the total area goes up by 5%, and the area under wheat production goes up by 12%, then what will be the angle for wheat in the new pie-chart ?

(*a*) 62.4° (*b*) 76.8° (*c*) 80.6° (*d*) 84.2°

SOLUTION

1. (*c*) : The total of the central angles corresponding to the three crops which cover 50% of the total area, should be 180°. Now, the total of the central angles for the given combinations are :

(*i*) Wheat, Barley and Jowar = (72° + 36° + 18°) = 126°

(*ii*) Rice, Wheat and Jowar = (72° + 72° + 18°) = 162°

(*iii*) Rice, Wheat and Barley = (72° + 72° + 36°) = 180°

(*iv*) Bajra, Maize and Rice = (18° + 45° + 72°) = 135°

Clearly, (*iii*) is the required combination.

2. (*a*) : The area under any of the food crops is proportional to the central angle corresponding to that crop.

Let, the area under rice production be x million acres.

Then, $18 : 72 = 1.5 : x \Rightarrow x = \left(\frac{72 \times 1.5}{18}\right) = 6.$

Thus, the area under rice production = 6 million acres.

3. (*b*) : Let the total production of barley be T tonnes and let Z acres of land be put under barley production.

Then, the total production of wheat = (6T) tonnes.

Also, area under wheat production = (2Z) acres.

$$\left[\because \frac{\text{Area under Wheat production}}{\text{Area under Barley production}} = \frac{72^\circ}{36^\circ} = 2\right.$$

$$\left.\text{and therefore, Area under wheat} = 2 \times \text{Area under barley} = (2Z) \text{ acres}\right]$$

Now, yield per acre for wheat $= \left(\frac{6T}{2Z}\right)$ tonnes/ acre $= \left(\frac{3T}{Z}\right)$ tonnes/ acre

and yield per acre for barley $= \left(\frac{T}{Z}\right)$ tonnes/ acre.

$\therefore$ Required Ratio $= \left(\frac{3T/Z}{T/Z}\right) = 3 : 1.$

4. (*b*) : Let Z acres of land be put under barley production.

Then, $\frac{\text{Area under rice production}}{\text{Area under barley production}} = \frac{72^\circ}{36^\circ} = 2.$

$\therefore$ Area under Rice production = 2 × area under barley production = (2Z) acres.

Now, if p tonnes be the yield per acre of barley then, yield per acre of rice

$$= (p + 50\% \text{ of } p) \text{ tonnes} = \left(\frac{3}{2}p\right) \text{ tonnes.}$$

$\therefore$ Total production of rice = (yield per acre) × (area under production)

$$= \left(\frac{3}{2}p\right) \times 2Z = (3pZ) \text{ tonnes.}$$

And, Total production of barley = (pZ) tonnes.

$\therefore$ Percentage production of barley to that of rice = $\left(\frac{pZ}{3pZ} \times 100\right)\% = 33\frac{1}{3}\%$.

5. (b) : Initially, let t acres be the total area under consideration.

Then, area under wheat production initially was = $\left(\frac{72}{360} \times t\right)$ acres = $\left(\frac{t}{5}\right)$ acres.

Now, if the total area under consideration be increased by 5%, then the new value of the total area = $\left(\frac{105}{100}t\right)$ acres.

Also, if the area under wheat production be increased by 12%, then the new value of the area under wheat = $\left[\frac{t}{5} + \left(12\% \text{ of } \frac{t}{5}\right)\right]$ acres = $\left(\frac{112t}{500}\right)$ acres.

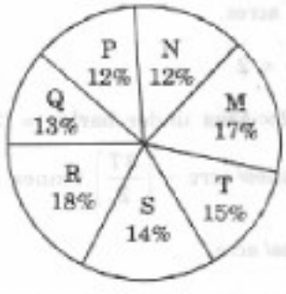
$\therefore$ Central angle corresponding to wheat in the new pie-chart

$$= \left[\frac{\text{Area under wheat (new)}}{\text{Total area (new)}} \times 360\right]^\circ = \left[\frac{\left(\frac{112t}{500}\right)}{\left(\frac{105t}{100}\right)} \times 360\right]^\circ = 76.8^\circ.$$

Example 3. ***The following pie-charts show the distribution of students of graduate and post-graduate levels in seven different institutes — M, N, P, Q, R, S and T in a town.*** **(Bank P.O. 2003)**

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES — M, N, P, Q, R, S AND T

Total Number of Students of Graduate Level = 27300



Total Number of Students of Post-Graduate Level = 24700

1. How many students of institutes M and S are studying at graduate level ?
   (a) 7516 (b) 8463 (c) 9127 (d) 9404
2. Total number of students studying at post-graduate level from institutes N and P is :
   (a) 5601 (b) 5944 (c) 6669 (d) 7004
3. What is the total number of graduate and post-graduate level students in institute R ?
   (a) 8320 (b) 7916 (c) 9116 (d) 8372
4. What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S ?
   (a) 14 : 19 (b) 19 : 21 (c) 17 : 21 (d) 19 : 14

5. What is the ratio between the number of students studying at post-graduate level from institute S and the number of students studying at graduate level from institute Q ?

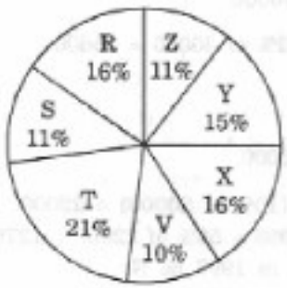
(a) 13 : 19 (b) 21 : 13 (c) 13 : 8 (d) 19 : 13

SOLUTION

1. (b) : Students of institute M at graduate level = 17% of 27300 = 4641.
Students of institute S at graduate level = 14% of 27300 = 3822.
∴ Total number of students at graduate level in institutes M and S
= 4641 + 3822 = 8463.
2. (c) : Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.
3. (d) : Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.
4. (d) : Required ratio = $\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{21 \times 24700}{14 \times 27300} = \frac{19}{14}$.
5. (d) : Required ratio = $\frac{(21\% \text{ of } 24700)}{(13\% \text{ of } 27300)} = \frac{21 \times 24700}{13 \times 27300} = \frac{19}{13}$.

Example 4. ***Study the following pie-chart and the table and answer the questions based on them.*** (S.B.I.P.O. 1999)

PROPORTION OF POPULATION OF SEVEN VILLAGES IN 1997



Village	% Population Below Poverty Line
X	38
Y	52
Z	42
R	51
S	49
T	46
V	58

1. Find the population of village S if the population of village X below poverty line in 1997 is 12160.

(a) 18500 (b) 20500 (c) 22000 (d) 26000

2. The ratio of population of village T below poverty line to that of village Z below poverty line in 1997 is :

(a) 11 : 23 (b) 13 : 11 (c) 23 : 11 (d) 11 : 13

3. If the population of village R in 1997 is 32000, then what will be the population of village Y below poverty line in that year ?

(a) 14100 (b) 15600 (c) 16500 (d) 17000

4. If in 1998, the population of villages Y and V increase by 10% each and the percentage of population below poverty line remains unchanged for all the villages, then find the population of village V below poverty line in 1998, given that the population of village Y in 1997 was 30000.

(a) 11250 (b) 12760 (c) 13140 (d) 13780

5. If in 1999, the population of village R increases by 10% while that of village Z reduces by 5% compared to that in 1997 and the percentage of population below poverty line remains unchanged for all the villages, then find the approximate ratio of population of village R below poverty line to the ratio of population of village Z below poverty line for the year 1999.

(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 5 : 4

SOLUTION

1. (c) : Let the population of village X be x.

Then, 38% of $x = 12160 \Rightarrow x = \frac{12160 \times 100}{38} = 32000$.

Now, if s be the population of village S, then

$16 : 11 = 32000 : s \Rightarrow s = \frac{11 \times 32000}{16} = 22000$.

2. (c) : Let N be the total population of all the seven villages.

Then, population of village T below poverty line = 46% of (21% of N)

and population of village Z below poverty line = 42% of (11% of N)

$\therefore$ Required ratio = $\frac{46\% \text{ of } (21\% \text{ of N})}{42\% \text{ of } (11\% \text{ of N})} = \frac{46 \times 21}{42 \times 11} = \frac{23}{11}$.

3. (b) : Population of village R = 32000 (given).

Let the population of village Y be y.

Then, $16 : 15 = 32000 : y \Rightarrow y = \frac{15 \times 32000}{16} = 30000$

$\therefore$ Population of village Y below poverty line = 52% of 30000 = 15600.

4. (b) : Population of village Y in 1997 = 30000 (given).

Let the population of village V in 1997 be v.

Then, $15 : 10 = 30000 : v \Rightarrow v = \frac{30000 \times 10}{15} = 20000$.

Now, population of village V in 1998 = 20000 + (10% of 20000) = 22000.

$\therefore$ Population of village V below poverty line in 1998 = 58% of 22000 = 12760.

5. (a) : Let the total population of all the seven villages in 1997 be N.

Then, population of village R in 1997 = 16% of $N = \frac{16}{100}N$

and population of village Z in 1997 = 11% of $N = \frac{11}{100}N$.

$\therefore$ Population of village R in 1999 = $\left\{\frac{16}{100}N + \left(10\% \text{ of } \frac{16}{100}N\right)\right\} = \frac{1760}{10000}N$

and population of village Z in 1999 = $\left\{\frac{11}{100}N - \left(5\% \text{ of } \frac{11}{100}N\right)\right\} = \frac{1045}{10000}N$

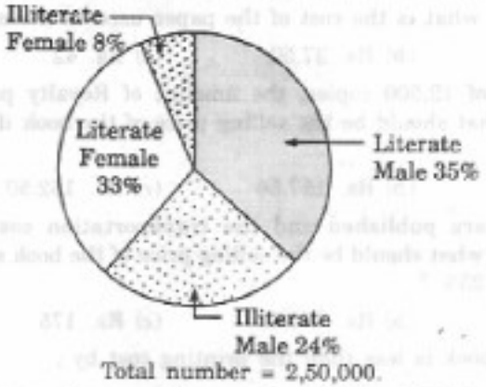
Now, population of village R below poverty line for 1999 = 51% of $\left(\frac{1760}{10000}N\right)$

and population of village Z below poverty line for 1999 = 42% of $\left(\frac{1045}{10000}N\right)$.

$\therefore$ Required ratio = $\frac{51\% \text{ of } \left(\frac{1760}{10000}N\right)}{42\% \text{ of } \left(\frac{1045}{10000}N\right)} = \frac{51 \times 1760}{42 \times 1045} = \frac{2}{1}$.

EXERCISE 38

1. The following pie-chart shows the percentage of Literate and Illiterate — Males and Females in a city. **(Bank P.O. 2003)**



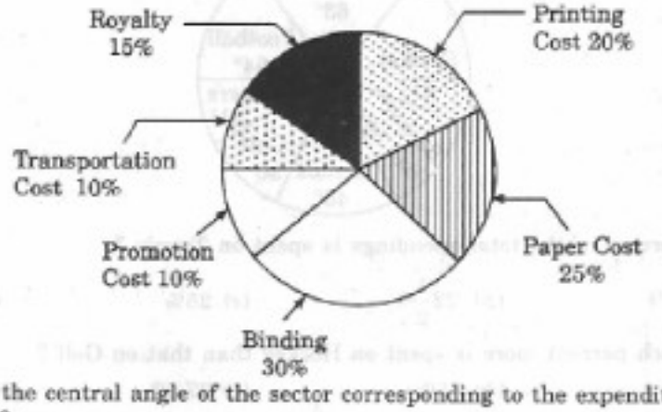
Total number = 2,50,000.

What is the difference between the number of Literate Males and Literate Females ?

(*a*) 75,000 (*b*) 1,500 (*c*) 5,000 (*d*) 500

Directions (Questions 2 to 10) : ***The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and answer the questions based on it.*** **(Bank P.O. 2002)**

VARIOUS EXPENDITURES (IN PERCENTAGE) INCURRED IN PUBLISHING A BOOK



2. What is the central angle of the sector corresponding to the expenditure incurred on Royalty ?

(*a*) 15° (*b*) 24° (*c*) 54° (*d*) 48°

3. Which two expenditures together have a central angle of 108° ?

(*a*) Binding Cost and Transportation Cost (*b*) Printing Cost and Paper Cost
(*c*) Royalty and Promotion Cost (*d*) Binding Cost and Paper Cost

4. If the difference between the two expenditures are represented by 18° in the pie-chart, then these expenditures possibly are :

(*a*) Binding Cost and Promotion Cost (*b*) Paper Cost and Royalty
(*c*) Binding Cost and Printing Post (*d*) Paper Cost and Printing Cost

5. If for an edition of the book, the cost of paper is Rs. 56250, then find the promotion cost for this edition.

(*a*) Rs. 20,000 (*b*) Rs. 22,500 (*c*) Rs. 25,500 (*d*) Rs. 28,125

6. If for a certain quantity of books, the publisher has to pay Rs. 30,600 as printing cost, then what will be the amount of royalty to be paid for these books ?

(a) Rs. 19,450 (b) Rs. 21,200 (c) Rs. 22,950 (d) Rs. 26,150

7. The price of the book is marked 20% above the C.P. If the marked price of the book is Rs. 180, then what is the cost of the paper used in a single copy of the book ?

(a) Rs. 36 (b) Rs. 37.50 (c) Rs. 42 (d) Rs. 44.25

8. For an edition of 12,500 copies, the amount of Royalty paid by the publisher is Rs. 2,81,250. What should be the selling price of the book if the publisher desires a profit of 5% ?

(a) Rs. 152.50 (b) Rs. 157.50 (c) Rs. 162.50 (d) Rs. 167.50

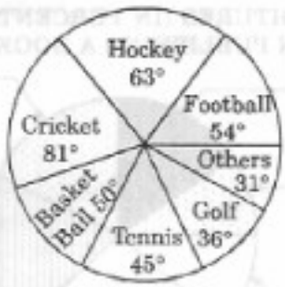
9. If 5500 copies are published and the transportation cost on them amounts to Rs. 82,500, then what should be the selling price of the book so that the publisher can earn a profit of 25% ?

(a) Rs. 187.50 (b) Rs. 191.50 (c) Rs. 175 (d) Rs. 180

10. Royalty on the book is less than the printing cost by :

(a) 5% (b) $33\frac{1}{3}\%$ (c) 20% (d) 25%

Directions (Questions 11 to 15) : ***The circle-graph given here shows the spendings of a country on various sports during a particular year. Study the graph carefully and answer the questions given below it.***



11. What percent of the total spendings is spent on Tennis ?

(a) $12\frac{1}{2}\%$ (b) $22\frac{1}{2}\%$ (c) 25% (d) 45%

12. How much percent more is spent on Hockey than that on Golf ?

(a) 27% (b) 35% (c) 37.5% (d) 75%

13. How much percent less is spent on Football than that on Cricket ?

(a) $22\frac{2}{9}\%$ (b) 27% (c) $33\frac{1}{3}\%$ (d) $37\frac{1}{2}\%$

14. If the total amount spent on sports during the year was Rs. 2 crores, the amount spent on Cricket and Hockey together was :

(a) Rs. 8,00,000 (b) Rs. 80,00,000 (c) Rs. 1,20,00,000 (d) Rs. 1,60,00,000

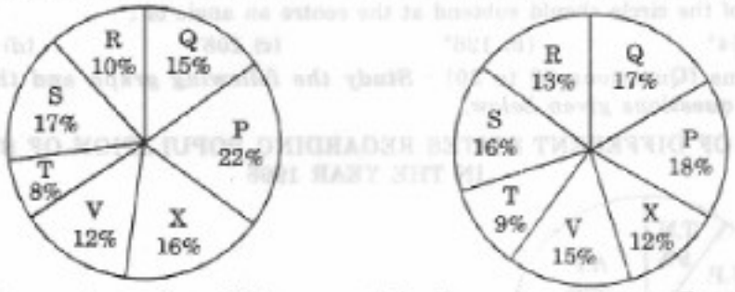
15. If the total amount spent on sports during the year be Rs. 1,80,00,000, the amount spent on Basketball exceeds that on Tennis by :

(a) Rs. 2,50,000 (b) Rs. 3,60,000 (c) Rs. 3,75,000 (d) Rs. 4,10,000

Directions (Questions 16 to 20) : ***Study the following graph carefully and answer the questions given below :*** **(Bank P.O. 2002)**

DISTRIBUTION OF CANDIDATES WHO WERE ENROLLED FOR MBA ENTRANCE EXAM AND THE CANDIDATES (OUT OF THOSE ENROLLED) WHO PASSED THE EXAM IN DIFFERENT INSTITUTES

Candidates Enrolled = 8550 **Candidates who Passed the Exam = 5700**



16. What percentage of candidates passed the Exam from institute T out of the total number of candidates enrolled from the same institute ?
(*a*) 50% (*b*) 62.5% (*c*) 75% (*d*) 80%

17. What is the ratio of candidates passed to the candidates enrolled from institute P ?
(*a*) 9 : 11 (*b*) 14 : 17 (*c*) 6 : 11 (*d*) 9 : 17

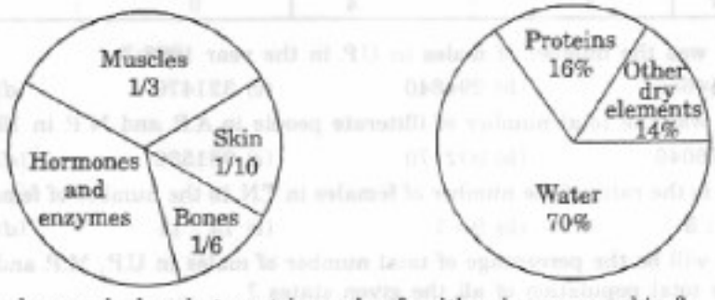
18. What is the percentage of candidates passed to the candidates enrolled for institutes Q and R together ?
(*a*) 68% (*b*) 80% (*c*) 74% (*d*) 65%

19. Which institute has the highest percentage of candidates passed to the candidates enrolled ?
(*a*) Q (*b*) R (*c*) V (*d*) T

20. The number of candidates passed from institutes S and P together exceeds the number of candidates enrolled from institutes T and R together by :
(*a*) 228 (*b*) 279 (*c*) 399 (*d*) 407

Directions (Questions 21 to 25) : ***Study the following pie-diagrams carefully and answer the questions given below it.***

PERCENTAGE COMPOSITION OF HUMAN BODY



21. In the human body, what part is made of neither bones nor skin ?
(*a*) $\frac{1}{40}$ (*b*) $\frac{3}{80}$ (*c*) $\frac{2}{5}$ (*d*) None of these

22. What is the ratio of the distribution of proteins in the muscles to that of the distribution of proteins in the bones ?
(*a*) 1 : 18 (*b*) 1 : 2 (*c*) 2 : 1 (*d*) 18 : 1

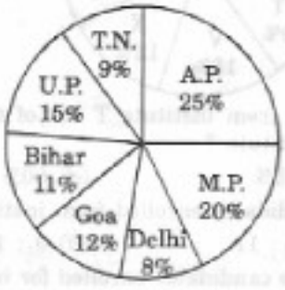
23. What will be the quantity of water in the body of a person weighing 50 kg ?
(a) 20 kg (b) 35 kg (c) 41 kg (d) 42.5 kg

24. What percent of the total weight of human body is equivalent to the weight of the proteins in skin in human body ?
(a) 0.016 (b) 1.6 (c) 0.16 (d) Data inadequate

25. To show the distribution of proteins and other dry elements in the human body, the arc of the circle should subtend at the centre an angle of :
(a) 54° (b) 126° (c) 108° (d) 252°

Directions (Questions 26 to 30) : ***Study the following graph and the table and answer the questions given below.***

DATA OF DIFFERENT STATES REGARDING POPULATION OF STATES IN THE YEAR 1998



Total Population of the given States = 3276000

States	Sex and Literacy wise Population Ratio					
	Sex			Literacy		
	M	—	F	Literate	—	Illiterate
A.P.	5	:	3	2	:	7
M.P.	3	:	1	1	:	4
Delhi	2	:	3	2	:	1
Goa	3	:	5	3	:	2
Bihar	3	:	4	5	:	1
U.P.	3	:	2	7	:	2
T.N.	3	:	4	9	:	4

26. What was the number of males in U.P. in the year 1998 ?
(a) 254650 (b) 294840 (c) 321470 (d) 341200

27. What was the total number of illiterate people in A.P. and M.P. in 1998 ?
(a) 876040 (b) 932170 (c) 981550 (d) 1161160

28. What is the ratio of the number of females in T.N. to the number of females in Delhi ?
(a) 7 : 5 (b) 9 : 7 (c) 13 : 11 (d) 15 : 14

29. What will be the percentage of total number of males in U.P., M.P. and Goa together to the total population of all the given states ?
(a) 25% (b) 27.5% (c) 28.5% (d) 31.5%

30. If in the year 1998, there was an increase of 10% in the population of U.P. and 12% in the population of M.P. compared to the previous year, then what was the ratio of populations of U.P. and M.P. in 1997 ?
(a) 42 : 55 (b) 48 : 55 (c) 7 : 11 (d) 4 : 5

ANSWERS

1. (c)	2. (c)	3. (a)	4. (d)	5. (b)	6. (c)	7. (b)	8. (b)	9. (a)
10. (d)	11. (a)	12. (d)	13. (c)	14. (b)	15. (a)	16. (c)	17. (c)	18. (b)
19. (b)	20. (c)	21. (d)	22. (c)	23. (b)	24. (b)	25. (c)	26. (b)	27. (d)
28. (d)	29. (c)	30. (a)						

SOLUTIONS

1. Difference = (35% of 2,50,000) – (33% of 2,50,000)
= (35% – 33%) of 2,50,000 = 2% of 2,50,000 = 5000.

2. Central angle corresponding to Royalty = (15% of 360)° = 54°.

3. Central angle of 108° = $\left(\frac{108}{360}\times 100\right)\%$ of the total expenditure
= 30% of the total expenditure.

From the pie-chart it is clear that :
Binding Cost + Transportation Cost = (20% + 10%) of the total expenditure
= 30% of the total expenditure.

∴ Binding Cost and Transportation Cost together have a central angle of 108°.

4. Central angle of 18° = $\left(\frac{18}{360}\times 100\right)\%$ of the total expenditure
= 5% of the total expenditure.

From the pie-chart it is clear that :
Out of the given combinations, only in combination (d) the difference is 5% *i.e.*
Paper Cost – Printing Cost = (25% – 20%) of total expenditure
= 5% of total expenditure.

5. Let the Promotion Cost for this edition be Rs. p.

Then, $25 : 10 = 56250 : p \Rightarrow p = \text{Rs.}\left(\frac{56250\times 10}{25}\right) = \text{Rs. } 22500.$

6. Let the amount of Royalty to be paid for these books be Rs. r.

Then, $20 : 15 = 30600 : r \Rightarrow r = \text{Rs.}\left(\frac{30600\times 15}{20}\right) = \text{Rs. } 22950.$

7. Clearly, marked price of the book = 120% of C.P.
Also, cost of paper = 25% of C.P.
Let the cost of paper for a single book be Rs. n.

Then, $120 : 25 = 180 : n \Rightarrow n = \text{Rs.}\left(\frac{25\times 180}{120}\right) = \text{Rs. } 37.50.$

8. Clearly, S.P. of the book = 105% of C.P.
Let the selling price of this edition (of 12500 books) be Rs. x.

Then, $15 : 105 = 281250 : x \Rightarrow x = \text{Rs.}\left(\frac{105\times 281250}{15}\right) = \text{Rs. } 1968750.$

∴ S.P. of one book = $\text{Rs.}\left(\frac{1968750}{12500}\right) = \text{Rs. } 157.50.$

9. For the publisher to earn a profit of 25%, S.P. = 125% of C.P.
Also Transportation Cost = 10% of C.P.
Let the S.P. of 5500 books be Rs. x
Then, $10 : 125 = 82500 : x \Rightarrow x = \text{Rs.}\left(\frac{125 \times 82500}{10}\right) = \text{Rs. } 1031250.$

$\therefore$ S.P. of one book $= \text{Rs.}\left(\frac{1031250}{5500}\right) = \text{Rs. } 187.50.$

10. Printing Cost of book = 20% of C.P.
Royalty on book = 15% of C.P.
Difference = (20% of C.P.) − (15% of C.P.) = 5% of C.P.

$\therefore$ Percentage difference $= \left(\frac{\text{Difference}}{\text{Printing Cost}} \times 100\right)\%$

$= \left(\frac{5\% \text{ of C.P.}}{20\% \text{ of C.P.}} \times 100\right)\% = 25\%.$

Thus, Royalty on the book is 25% less than the Printing Cost.

11. Percentage of money spent on Tennis $= \left(\frac{45}{360} \times 100\right)\% = 12\frac{1}{2}\%.$

12. Let the total spendings on sports be Rs. x. Then,

Amount spent on Golf $= \text{Rs.}\left(\frac{36}{360} \times x\right) = \text{Rs. } \frac{x}{10}.$

Amount spent on Hockey $= \text{Rs.}\left(\frac{63}{360} \times x\right) = \text{Rs. } \frac{7}{40}x.$

Difference $= \text{Rs.}\left(\frac{7}{40}x - \frac{x}{10}\right) = \text{Rs. } \frac{3x}{40}.$

$\therefore$ Required Percentage $= \left[\left(\frac{3x/40}{x/10}\right) \times 100\right]\% = 75\%.$

13. Let the total spendings on sports be Rs. x. Then,

Amount spent on Cricket $= \text{Rs.}\left(\frac{81}{360} \times x\right) = \text{Rs.}\left(\frac{9}{40}x\right).$

Amount spent on Football $= \text{Rs.}\left(\frac{54}{360} \times x\right) = \text{Rs.}\left(\frac{3}{20}x\right).$

Difference $= \text{Rs.}\left(\frac{9}{40}x - \frac{3}{20}x\right) = \text{Rs. } \frac{3}{40}x.$

$\therefore$ Required Percentage $= \left[\left(\frac{3x/40}{9x/40}\right) \times 100\right]\% = 33\frac{1}{3}\%.$

14. Amount spent on Cricket and Hockey together

$= \text{Rs.}\left[\frac{(81+63)}{360} \times 2\right]$ crores $= \text{Rs. } 0.8$ crores $= \text{Rs. } 8000000.$

15. Amount spent on Basketball exceeds that on Tennis by :

$\text{Rs.}\left[\frac{(50-45)}{360} \times 18000000\right] = \text{Rs. } 250000.$

16. Required percentage $= \left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550} \times 100\right)\% = \left(\frac{9 \times 5700}{8 \times 8550} \times 100\right)\% = 75\%.$

17. Required ratio $= \left(\frac{18\% \text{ of } 5700}{22\% \text{ of } 8550}\right) = \left(\frac{18 \times 5700}{22 \times 8550}\right) = \frac{6}{11}$.

18. Candidates passed from institutes Q and R together
$= [(13\% + 17\%) \text{ of } 5700] = 30\% \text{ of } 5700.$

Candidates enrolled from institutes Q and R together
$= [(15\% + 10\%) \text{ of } 8550] = 25\% \text{ of } 8550.$

$\therefore$ Required Percentage $= \left(\frac{30\% \text{ of } 5700}{25\% \text{ of } 8550} \times 100\right)\% = \left(\frac{30 \times 5700}{25 \times 8550} \times 100\right)\% = 80\%.$

19. The percentage of candidates passed to candidates enrolled can be determined for each institute as under :

(*i*) $P = \left[\left(\frac{18\% \text{ of } 5700}{22\% \text{ of } 8550}\right) \times 100\right]\% = \left[\frac{18 \times 5700}{22 \times 8550} \times 100\right]\% = \left[\frac{18 \times 2}{22 \times 3} \times 100\right]\% = 54.55\%.$

(*ii*) $Q = \left[\left(\frac{17\% \text{ of } 5700}{15\% \text{ of } 8550}\right) \times 100\right]\% = 75.56\%.$

(*iii*) $R = \left[\left(\frac{13\% \text{ of } 5700}{10\% \text{ of } 8550}\right) \times 100\right]\% = 86.67\%.$

(*iv*) $S = \left[\left(\frac{16\% \text{ of } 5700}{17\% \text{ of } 8550}\right) \times 100\right]\% = 62.75\%.$

(*v*) $T = \left[\left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550}\right) \times 100\right]\% = 75\%.$

(*vi*) $V = \left[\left(\frac{15\% \text{ of } 5700}{12\% \text{ of } 8550}\right) \times 100\right]\% = 83.33\%.$

(*vii*) $X = \left[\left(\frac{12\% \text{ of } 5700}{16\% \text{ of } 8550}\right) \times 100\right]\% = 50\%.$

Highest of these is 86.67% corresponding to institute R.

20. Required difference $= [(16\% + 18\%) \text{ of } 5700] - [(8\% + 10\%) \text{ of } 8550]$
$= [(34\% \text{ of } 5700) - (18\% \text{ of } 8550)] = (1938 - 1539) = 399.$

21. Part of the body made of neither bones nor skin $= 1 - \left(\frac{1}{6} + \frac{1}{10}\right) = \frac{11}{15}$.

22. Required ratio $= \frac{16\% \text{ of } \frac{1}{3}}{16\% \text{ of } \frac{1}{6}} = \frac{6}{3} = \frac{2}{1}$.

23. Quantity of water in the body of a person weighing 50 kg = (70% of 50) kg = 35 kg.

24. Let the body weight be x kg.

Then, weight of skin protein in the body $= \left[16\% \text{ of } \left(\frac{1}{10} \text{ of } x\right)\right]$ kg $= \left(\frac{16}{1000}x\right)$ kg

$\therefore$ Required percentage $= \left[\frac{\left(\frac{16}{1000}x\right)}{x} \times 100\right]\% = 1.6\%.$

25. Percentage of proteins and other dry elements in the body = (16% + 14%) = 30%

$\therefore$ Central angle corresponding to proteins and other dry elements together
$= 30\% \text{ of } 360^\circ = 108^\circ.$

26. Number of males in U.P. $= \left[\frac{3}{5} \text{ of } (15\% \text{ of } 3276000)\right] = \frac{3}{5} \times \frac{15}{100} \times 3276000 = \mathbf{294840}$.

27. No. of illiterate people in A.P. $= \left[\frac{7}{9} \text{ of } (25\% \text{ of } 3276000)\right] = 637000$.

No. of illiterate people in M.P. $= \left[\frac{4}{5} \text{ of } (20\% \text{ of } 3276000)\right] = 524160$.

Total number $= (637000 + 524160) = 1161160$.

28. Required ratio $= \dfrac{\frac{4}{7} \text{ of } (9\% \text{ of } 3276000)}{\frac{3}{5} \text{ of } (8\% \text{ of } 3276000)} = \dfrac{\left(\frac{4}{7} \times 9\right)}{\left(\frac{3}{5} \times 8\right)} = \left(\frac{4}{7} \times 9 \times \frac{5}{3} \times \frac{1}{8}\right) = \frac{15}{14}$.

29. Number of males in U.P. $= \left[\frac{3}{5} \text{ of } (15\% \text{ of } N)\right] = \frac{3}{5} \times \frac{15}{100} \times N = 9 \times \frac{N}{100}$

where $N = 3276000$.

Number of males in M.P. $= \left[\frac{3}{4} \text{ of } (20\% \text{ of } N)\right] = \frac{3}{4} \times \frac{20}{100} \times N = 15 \times \frac{N}{100}$.

Number of males in Goa $= \left[\frac{3}{8} \text{ of } (12\% \text{ of } N)\right] = \frac{3}{8} \times \frac{12}{100} \times N = 4.5 \times \frac{N}{100}$.

$\therefore$ Total number of males in these three states $= (9 + 15 + 4.5) \times \frac{N}{100} = \left(28.5 \times \frac{N}{100}\right)$.

$\therefore$ Required Percentage $= \left[\dfrac{\left(28.5 \times \frac{N}{100}\right)}{N} \times 100\right]\% = \mathbf{28.5\%}$.

30. Let x be the population of U.P. in 1997. Then,

Population of U.P. in 1998 $= 110\%$ of $x = \frac{110}{100} \times x$.

Also, let y be the population of M.P. in 1997. Then,

Population of M.P. in 1998 $= 112\%$ of $y = \frac{112}{100} \times y$.

Ratio of populations of U.P. and M.P. in 1998 $= \dfrac{\left(\frac{110}{100} \times x\right)}{\left(\frac{112}{100} \times y\right)} = \frac{110x}{112y}$.

From the pie-chart, this ratio is $\frac{15}{20}$.

$\therefore \quad \frac{110x}{112y} = \frac{15}{20} \Rightarrow \frac{x}{y} = \frac{15}{20} \times \frac{112}{110} = \frac{42}{55}$.

Thus, ratio of populations of U.P. and M.P. in 1997 $= x : y = 42 : 55$.

39. LINE-GRAPHS

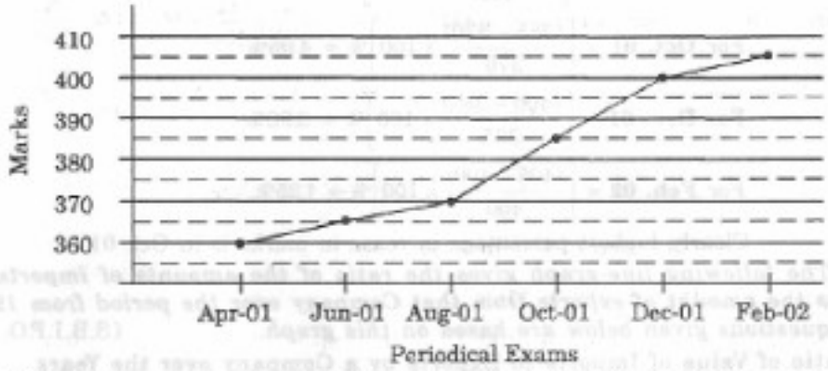
This section comprises of questions in which the data collected in a particular discipline are represented by specific points joined together by straight lines. The points are plotted on a two-dimensional plane taking one parameter on the horizontal axis and the other on the vertical axis. The candidate is required to analyse the given information and thereafter answer the given questions on the basis of the analysis of data.

SOLVED EXAMPLES

Ex. 1. ***In a school the periodical examinations are held every second month. In a session during Apr. 2001 – Mar. 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below. Study the graph and answer the questions based on it.*** (S.B.I.P.O. 2003)

MARKS OBTAINED BY A STUDENT IN SIX PERIODICAL EXAMS HELD IN EVERY TWO MONTHS DURING THE YEAR IN THE SESSION 2001-02

Maximum Total Marks in each Periodical Exam = 500



1. The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in Apr. 01 ?
   (*a*) 110% (*b*) 112.5% (*c*) 115% (*d*) 116.5% (*e*) 117.5%
2. What are the average marks obtained by the student in all the periodical exams during the session ?
   (*a*) 373 (*b*) 379 (*c*) 381 (*d*) 385 (*e*) 389
3. What is the percentage of marks obtained by the student in the periodical exams of Aug. 01 and Oct. 01 taken together ?
   (*a*) 73.25% (*b*) 75.5% (*c*) 77% (*d*) 78.75% (*e*) 79.5%
4. In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams ?
   (*a*) None (*b*) Jun. 01 (*c*) Oct. 01 (*e*) Feb. 02 (*e*) None of these
5. In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams ?
   (*a*) Jun. 01 (*b*) Aug. 01 (*c*) Oct. 01 (*d*) Dec. 01 (*e*) Feb. 02

Sol. Here it is clear from the graph that the student obtained 360, 365, 370, 385, 400 and 405 marks in periodical exams held in Apr. 01, Jun. 01, Aug. 01, Oct. 01, Dec. 01 and Feb. 02 respectively.

1. (*b*) : Required percentage $= \left(\frac{405}{360} \times 100\right)\% = 112.5\%.$

2. (*c*) : Average marks obtained in all the periodical exams

$= \frac{1}{6} \times [360 + 365 + 370 + 385 + 400 + 405] = 380.83 \approx 381.$

3. (*b*) : Required percentage $= \left[\frac{(370 + 385)}{(500 + 500)} \times 100\right]\% = \left(\frac{755}{1000} \times 100\right)\% = 75.5\%.$

4. (*a*) : As is clear from the graph, the total marks obtained in periodical exams, go on increasing. Since, the maximum marks for all the periodical exams are same, it implies that the percentage of marks also goes on increasing. Thus, in none of the periodical exams, there is a fall in percentage of marks compared to the previous exam.

5. (*c*) : Percentage increase in marks in various periodical exams compared to the previous exams are :

For **Jun. 01** $= \left[\frac{(365 - 360)}{360} \times 100\right]\% = 1.39\%$

For **Aug. 01** $= \left[\frac{(370 - 365)}{365} \times 100\right]\% = 1.37\%$

For **Oct. 01** $= \left[\frac{(385 - 370)}{370} \times 100\right]\% = 4.05\%$

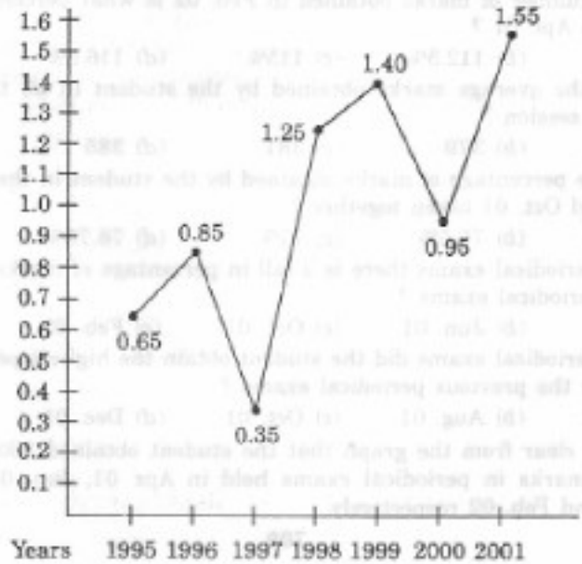
For **Dec. 01** $= \left[\frac{(400 - 385)}{385} \times 100\right]\% = 3.90\%$

For **Feb. 02** $= \left[\frac{(405 - 400)}{400} \times 100\right]\% = 1.25\%.$

Clearly, highest percentage increase in marks is in Oct. 01.

Ex. 2. ***The following line-graph gives the ratio of the amounts of imports by a Company to the amount of exports from that Company over the period from 1995 to 2001. The questions given below are based on this graph.*** (S.B.I.P.O. 2001)

Ratio of Value of Imports to Exports by a Company over the Years



1. In how many of the given years were the exports more than the imports ?
(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

2. The imports were minimum proportionate to the exports of the Company in the year :
(a) 1995 (b) 1996 (c) 1997 (d) 2000 (e) 2001

3. If the imports of the Company in 1996 was Rs. 272 crores, the exports from the Company in 1996 was :
(a) Rs. 370 crores (b) Rs. 320 crores (c) Rs. 280 crores
(d) Rs. 275 crores (e) Rs. 264 crores

4. What was the percentage increase in imports from 1997 to 1998 ?
(a) 72 (b) 56 (c) 28 (d) None of these (e) Data inadequate

5. If the imports in 1998 was Rs. 250 crores and the total exports in the years 1998 and 1999 together was Rs. 500 crores, then the imports in 1999 was :
(a) Rs. 250 crores (b) Rs. 300 crores (c) Rs. 357 crores
(d) Rs. 420 crores (e) None of these

Sol. 1. (d) : The exports are more than the imports implies that the ratio of value of imports to exports is less than 1.

Now, this ratio is less than 1 in the years 1995, 1996, 1997 and 2000.

Thus, there are four such years.

2. (c) : The imports are minimum proportionate to the exports implies that the ratio of the value of imports to exports has the minimum value.

Now, this ratio has a minimum value of 0.35 in 1997, *i.e.*, the imports are minimum proportionate to the exports in 1997.

3. (b) : Ratio of imports to exports in the year 1996 = 0.85.

Let the exports in 1996 = Rs. x crores.

Then, $\frac{272}{x} = 0.85 \Rightarrow x = \frac{272}{0.85} = 320.$

$\therefore$ Exports in 1996 = Rs. 320 crores.

4. (e) : The graph gives only the ratio of imports to exports for different years. To find the percentage increase in imports from 1997 to 1998, we require more details such as the value of imports or exports during these years. Hence, the data is inadequate to answer this question.

5. (d) : The ratio of imports to exports for the years 1998 and 1999 are 1.25 and 1.40 respectively.

Let the exports in the year 1998 = Rs. x crores.

Then, the exports in the year 1999 = Rs. $(500 - x)$ crores.

$\therefore \quad 1.25 = \frac{250}{x} \Rightarrow x = \frac{250}{1.25} = 200$ [Using ratio for 1998]

Thus, the exports in the year 1999 = Rs. (500 – 200) crores = Rs. 300 crores.

Let the imports in the year 1999 = Rs. y crores.

Then, $1.40 = \frac{y}{300} \Rightarrow y = (300 \times 1.40) = 420.$

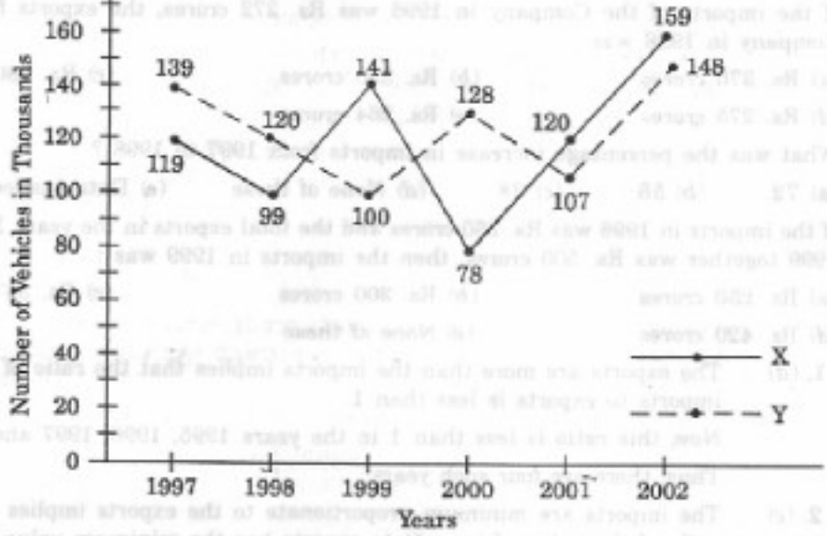
$\therefore$ Imports in the year 1999 = Rs. 420 crores.

Ex. 3. ***Study the following line-graph and answer the questions based on it.***

(R.B.I. 2003)

Number of Vehicles Manufactured by Two Companies over the Years

(Number in Thousands)



1. What is the difference between the total productions of the two Companies in the given years ?

   (a) 19000 (b) 22000 (c) 26000 (d) 28000 (e) 29000

2. What is the difference between the numbers of vehicles manufactured by Company Y in 2000 and 2001 ?

   (a) 50000 (b) 42000 (c) 33000 (d) 21000 (e) 13000

3. What is the average number of vehicles manufactured by Company X over the given period ? (rounded off to the nearest integer)

   (a) 119333 (b) 113666 (c) 112778 (d) 111223 (e) None of these

4. In which of the following years, the difference between the productions of Companies X and Y was the maximum among the given years ?

   (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

5. The production of Company Y in 2000 was approximately what percent of the production of Company X in the same year ?

   (a) 173 (b) 164 (c) 132 (d) 97 (e) 61

Sol. From the line-graph it is clear that the productions of Company X in the years 1997, 1998, 1999, 2000, 2001 and 2002 are 119000, 99000, 141000, 78000, 120000 and 159000 respectively and those of Company Y are 139000, 120000, 100000, 128000, 107000 and 148000 respectively.

1. (c) : Total production of Company X from 1997 to 2002

   = 119000 + 99000 + 141000 + 78000 + 120000 + 159000 = 716000.

   and total production of Company Y from 1997 to 2002

   = 139000 + 120000 + 100000 + 128000 + 107000 + 148000 = 742000.

   Difference = 742000 − 716000 = 26000.

2. (d) : Required difference = 128000 − 107000 = 21000.

3. (*a*) : Average number of vehicles manufactured by Company X

$$= \frac{1}{6} \times (119000 + 99000 + 141000 + 78000 + 120000 + 159000) = 119333.$$

4. (*d*) : The difference between the productions of Companies X and Y in various years are :

For 1997 = (139000 – 119000) = 20000;
For 1998 = (120000 – 99000) = 21000;
For 1999 = (141000 – 100000) = 41000;
For 2000 = (128000 – 78000) = 50000;
For 2001 = (120000 – 107000) = 13000;
For 2002 = (159000 – 148000) = 11000.

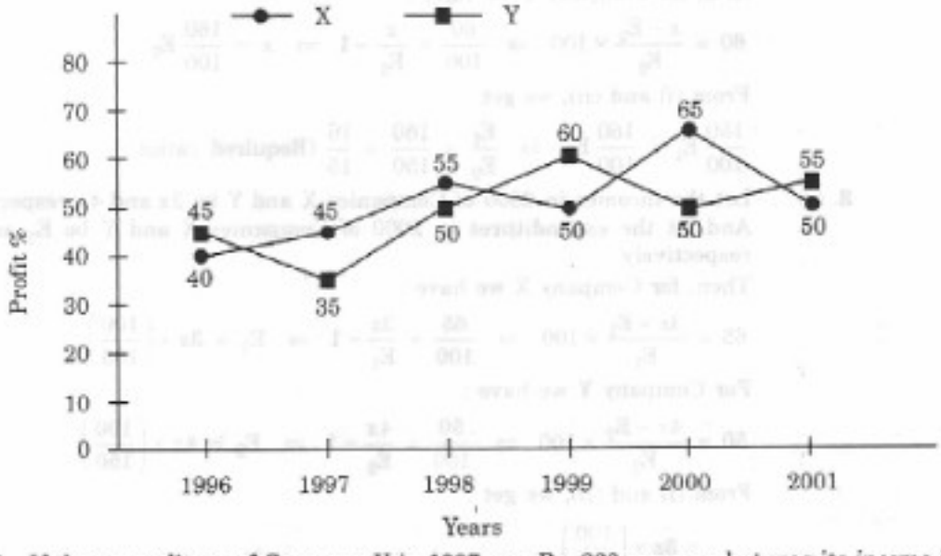
Clearly, maximum difference was in 2000.

5. (*b*) : Required percentage $= \left(\frac{128000}{78000} \times 100\right)\% \approx 164\%$.

Ex. 4. ***The following line-graph gives the percent profit earned by two Companies X and Y during the period 1996 – 2001. Study the line-graph and answer the questions that are based on it.*** **(NABARD, 2002)**

Percentage Profit Earned by Two Companies X and Y over the Given Years

$$\%\text{Profit / Loss} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



1. If the expenditure of Company Y in 1997 was Rs. 220 crores, what was its income in 1997 ?
   (*a*) Rs. 312 crores (*b*) Rs. 297 crores (*c*) Rs. 283 crores
   (*d*) Rs. 275 crores (*e*) Rs. 261 crores
2. If the incomes of the two Companies were equal in 1999, then what was the ratio of expenditure of Company X to that of Company Y in 1999 ?
   (*a*) 6 : 5 (*b*) 5 : 6 (*c*) 11 : 6 (*d*) 16 : 15 (*e*) 15 : 16
3. The incomes of the Companies X and Y in 2000 were in the ratio of 3 : 4 respectively. What was the respective ratio of their expenditures in 2000 ?
   (*a*) 7 : 22 (*b*) 14 : 19 (*c*) 15 : 22 (*d*) 27 : 35 (*e*) 33 : 40

4. If the expenditures of Companies X and Y in 1996 were equal and the total income of the two Companies in 1996 was Rs. 342 crores, what was the total profit of the two Companies together in 1996 ? (Profit = Income – Expenditure)

(*a*) Rs. 240 crores (*b*) Rs. 171 crores (*c*) Rs. 120 crores
(*d*) Rs. 102 crores (*e*) None of these

5. The expenditure of Company X in the year 1998 was Rs. 200 crores and the income of Company X in 1998 was the same as its expenditure in 2001. The income of Company X in 2001 was :

(*a*) Rs. 465 crores (*b*) Rs. 385 crores (*c*) Rs. 335 crores
(*d*) Rs. 295 crores (*e*) Rs. 255 crores

Sol. 1. (*b*) : Profit percent of Company Y in 1997 = 35.

Let the income of Company Y in 1997 be Rs. x crores.

Then, $35 = \frac{x-220}{220} \times 100 \Rightarrow x = 297.$

$\therefore$ Income of Company Y in 1997 = Rs. 297 crores.

2. (*d*) : Let the incomes of each of the two Companies X and Y in 1999 be Rs. x. And let the expenditures of Companies X and Y in 1999 be E_1 and E_2 respectively.

Then, for Company X we have :

$$50 = \frac{x-E_1}{E_1} \times 100 \Rightarrow \frac{50}{100} = \frac{x}{E_1} - 1 \Rightarrow x = \frac{150}{100} E_1 \quad ...(i)$$

Also, for Company Y we have :

$$60 = \frac{x-E_2}{E_2} \times 100 \Rightarrow \frac{60}{100} = \frac{x}{E_2} - 1 \Rightarrow x = \frac{160}{100} E_2 \quad ...(ii)$$

From (*i*) and (*ii*), we get :

$$\frac{150}{100} E_1 = \frac{160}{100} E_2 \Rightarrow \frac{E_1}{E_2} = \frac{160}{150} = \frac{16}{15} \text{ (Required ratio).}$$

3. (*c*) : Let the incomes in 2000 of Companies X and Y be $3x$ and $4x$ respectively. And let the expenditures in 2000 of Companies X and Y be E_1 and E_2 respectively.

Then, for Company X we have :

$$65 = \frac{3x-E_1}{E_1} \times 100 \Rightarrow \frac{65}{100} = \frac{3x}{E_1} - 1 \Rightarrow E_1 = 3x \times \left(\frac{100}{165}\right) \quad ...(i)$$

For Company Y we have :

$$50 = \frac{4x-E_2}{E_2} \times 100 \Rightarrow \frac{50}{100} = \frac{4x}{E_2} - 1 \Rightarrow E_2 = 4x \times \left(\frac{100}{150}\right) \quad ...(ii)$$

From (*i*) and (*ii*), we get :

$$\frac{E_1}{E_2} = \frac{3x \times \left(\frac{100}{165}\right)}{4x \times \left(\frac{100}{150}\right)} = \frac{3 \times 150}{4 \times 165} = \frac{15}{22} \text{ (Required ratio).}$$

4. (*d*) : Let the expenditures of each of the Companies X and Y in 1996 be Rs. x crores. And let the income of Company X in 1996 be Rs. z crores so that the income of Company Y in 1996 = Rs. $(342 - z)$ crores.

Then, for Company X we have :

$$40 = \frac{z-x}{x} \times 100 \Rightarrow \frac{40}{100} = \frac{z}{x} - 1 \Rightarrow x = \frac{100z}{140} \quad ...(i)$$

Also, for Company Y we have :

$$45 = \frac{(342 - z) - x}{x} \times 100 \Rightarrow \frac{45}{100} = \frac{(342 - z)}{x} - 1 \Rightarrow x = \frac{(342 - z) \times 100}{145} \quad ...(ii)$$

From (*i*) and (*ii*), we get :

$$\frac{100z}{140} = \frac{(342 - z) \times 100}{145} \Rightarrow z = 168.$$

Substituting $z = 168$ in (*i*), we get : $x = 120$.

∴ Total expenditure of Companies X and Y in 1996 = $2x$ = Rs. 240 crores.

Total income of Companies X and Y in 1996 = Rs. 342 crores.

∴ Total profit = Rs. (342 – 240) crores = Rs. 102 crores.

5. (*a*) : Let the income of Company X in 1998 be Rs. x crores.

Then, $55 = \frac{x - 200}{200} \times 100 \Rightarrow x = 310$.

∴ Expenditure of Company X in 2001

= Income of Company X in 1998 = Rs. 310 crores.

Let the income of Company X in 2001 be Rs. z crores.

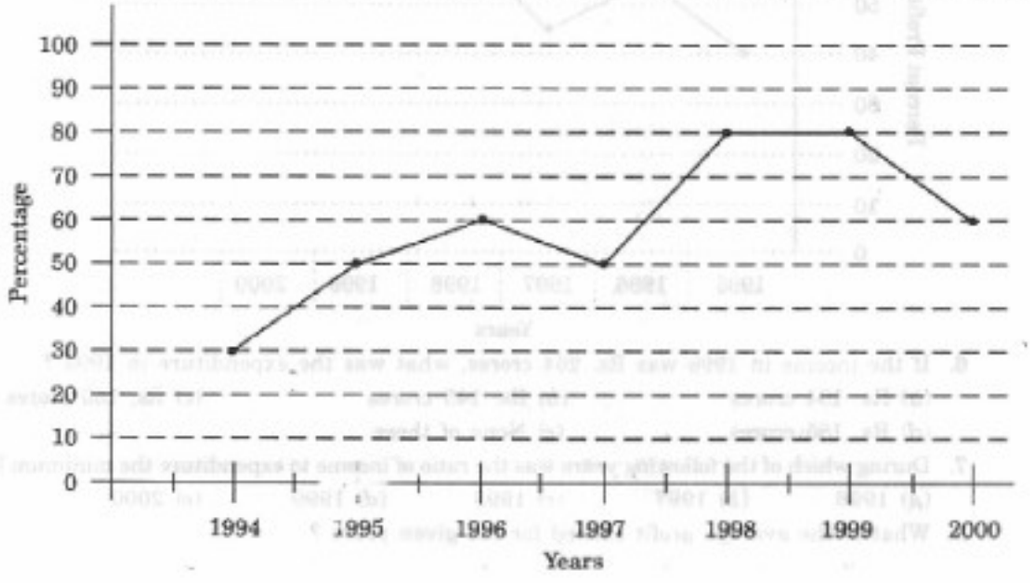
Then, $50 = \frac{z - 310}{310} \times 100 \Rightarrow z = 465$.

∴ Income of Company X in 2001 = Rs. 465 crores.

EXERCISE 39

Directions (Questions 1 to 5) : ***The following line-graph gives the percentage of the number of candidates who qualified an examination out of the total number of candidates who appeared for the examination over a period of seven years from 1994 to 2000. Study the graph and answer the questions based on it.*** **(Bank P.O. 2000)**

Percentage of Candidates Qualified to Appeared in an Examination Over the Years

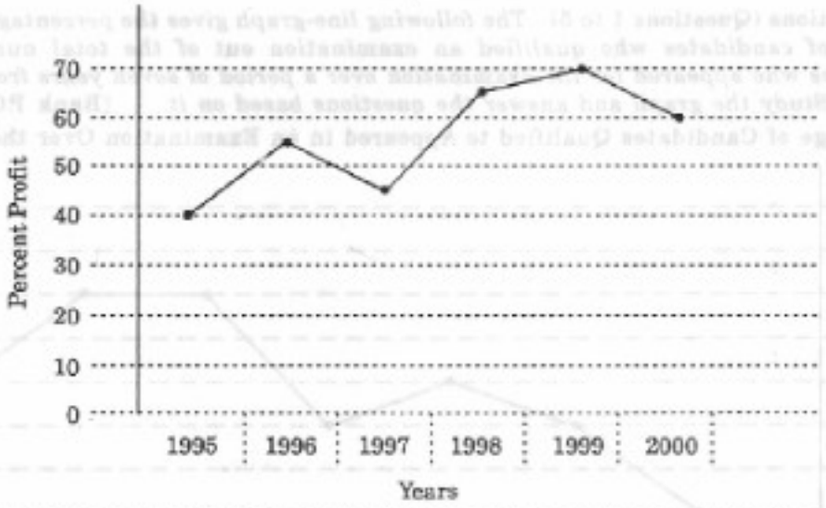


1. The difference between the percentages of candidates qualified to appeared was maximum in which of the following pairs of years ?
   (*a*) 1994 and 1995 (*b*) 1997 and 1998 (*c*) 1998 and 1999
   (*d*) 1999 and 2000 (*e*) 1994 and 1997
2. In which pair of years was the number of candidates qualified, the same ?
   (*a*) 1995 and 1997 (*b*) 1995 and 2000 (*c*) 1998 and 1999
   (*d*) 1996 and 2000 (*e*) Data inadequate
3. If the number of candidates qualified in 1998 was 21200, what was the number of candidates appeared in 1998 ?
   (*a*) 32000 (*b*) 28500 (*c*) 26500 (*d*) 25000 (*e*) 24500
4. If the total number of candidates appeared in 1996 and 1997 together was 47400, then the total number of candidates qualified in these two years together was :
   (*a*) 34700 (*b*) 32100 (*c*) 31500
   (*d*) None of these (*e*) Data inadequate
5. The total number of candidates qualified in 1999 and 2000 together was 33500 and the number of candidates appeared in 1999 was 26500. What was the number of candidates appeared in 2000 ?
   (*a*) 24500 (*b*) 22000 (*c*) 20500 (*d*) 19000 (*e*) 18500

Directions (Questions 6 to 13) : ***The following line-graph gives the annual percent profit earned by a Company during the period 1995-2000. Study the line-graph and answer the questions that are based on it.*** (R.B.I. 2003)

Percent Profit Earned by a Company Over the Years

$$\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



6. If the income in 1998 was Rs. 264 crores, what was the expenditure in 1998 ?
   (*a*) Rs. 104 crores (*b*) Rs. 145 crores (*c*) Rs. 160 crores
   (*d*) Rs. 185 crores (*e*) None of these
7. During which of the following years was the ratio of income to expenditure the minimum ?
   (*a*) 1996 (*b*) 1997 (*c*) 1998 (*d*) 1999 (*e*) 2000
8. What is the average profit earned for the given years ?
   (*a*) $50\frac{2}{3}$ (*b*) $55\frac{5}{6}$ (*c*) $60\frac{1}{6}$ (*d*) 335 (*e*) None of these

9. During which year the ratio of percentage profit earned to that in the previous year is the minimum ?
(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

10. If the expenditures in 1996 and 1999 are equal, then the approximate ratio of the incomes in 1996 and 1999 respectively, is :
(a) 1 : 1 (b) 2 : 3 (c) 9 : 10
(d) 13 : 14 (e) Cannot be determined

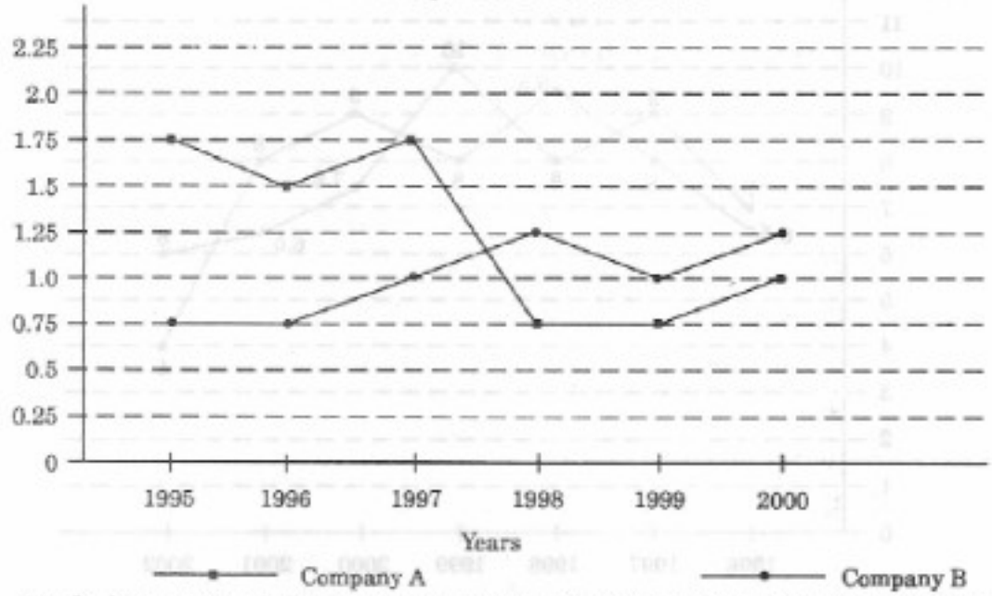
11. If the expenditure in 2000 is 25% more than the expenditure in 1997, then the income in 1997 is what percent less than the income in 2000 ?
(a) 22.5% (b) 25% (c) 27.5% (d) 31.25% (e) 32.5%

12. If the profit in 1999 was Rs. 4 crores, what was the profit in 2000 ?
(a) Rs. 4.2 crores (b) Rs. 6.6 crores (c) Rs. 6.8 crores
(d) Cannot be determined (e) None of these

13. In which year is the expenditure minimum ?
(a) 2000 (b) 1997 (c) 1996
(d) Cannot be determined (e) None of these

Directions (Questions 14 to 18) : *Answer the questions based on the line-graph given below.* **(Bank P.O. 2003)**

Ratio of Exports to Imports (in terms of money in Rs. crores) of Two Companies Over the Years



14. In how many of the given years were the exports more than the imports for Company A ?
(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

15. In which year(s) was the difference between imports and exports of Company B the maximum ?
(a) 2000 (b) 1996 (c) 1998 and 2000
(d) Cannot be determined (e) None of these

16. If the exports of Company A in 1998 were Rs. 237 crores, what was the amount of imports in that year ?
(a) Rs. 189.6 crores (b) Rs. 243 crores (c) Rs. 281 crores
(d) Rs. 316 crores (e) None of these

17. If the imports of Company A in 1997 were increased by 40 percent, what would be the ratio of exports to the increased imports ?

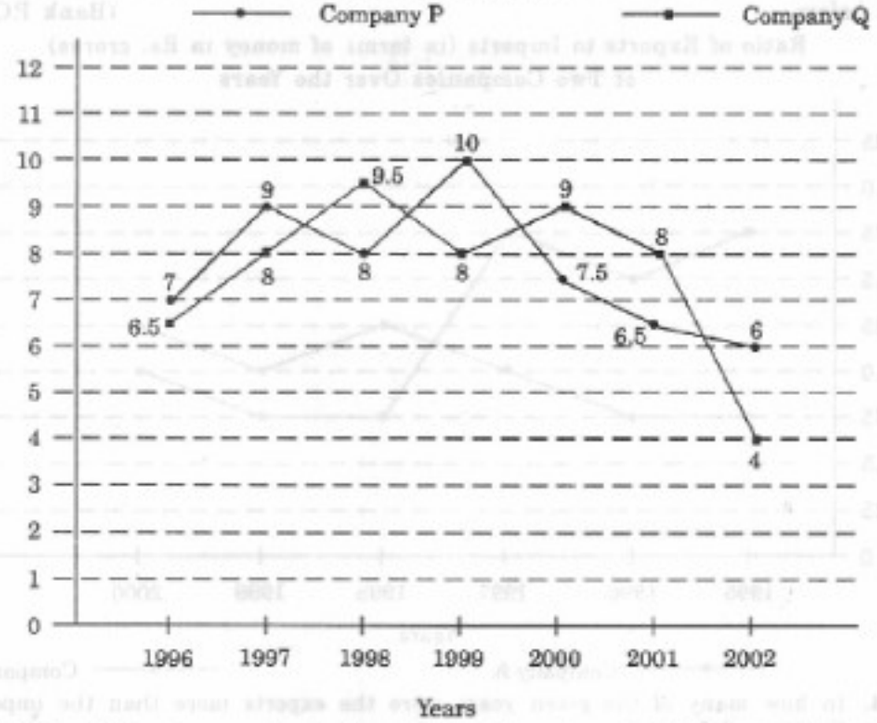
(a) 1.20 (b) 1.25 (c) 1.30
(d) None of these (e) Cannot be determined

18. In 1996, the export of Company A was double that of Company B. If the imports of Company A during the year was Rs. 180 crores, what was the approximate amount of imports of Company B during that year ?

(a) Rs. 190 crores (b) Rs. 210 crores (c) Rs. 225 crores
(d) Cannot be determined (e) None of these

Directions (Questions 19 to 23) : ***Two different finance companies declare fixed annual rate of interest on the amounts invested with them by investors. The rate of interest offered by these companies may differ from year to year depending on the variation in the economy of the country and the banks' rate of interest. The annual rate of interest offered by the two Companies P and Q over the years are shown by the line-graph provided below. Answer the questions based on this graph.*** (Bank P.O. 2003)

ANNUAL RATE OF INTEREST OFFERED BY TWO FINANCE COMPANIES OVER THE YEARS



19. If two different amounts in the ratio 8 : 9 are invested in Companies P and Q respectively in 2002, then the amounts received after one year as interests from Companies P and Q are respectively in the ratio :

(a) 2 : 3 (b) 3 : 4 (c) 6 : 7 (d) 4 : 3 (e) 9 : 8

20. In 2000, a part of Rs. 30 lakhs was invested in Company P and the rest was invested in Company Q for one year. The total interest received was Rs. 2.43 lakhs. What was the amount invested in Company P ?

(a) Rs. 9 lakhs (b) Rs. 11 lakhs (c) Rs. 12 lakhs
(d) Rs. 14 lakhs (e) Rs. 18 lakhs

21. A sum of Rs. 4.75 lakhs was invested in Company Q in 1999 for one year. How much more interest would have been earned if the sum was invested in Company P ?
(*a*) Rs. 19,000 (*b*) Rs. 14,250 (*c*) Rs. 11,750 (*d*) Rs. 9500 (*e*) Rs. 7500

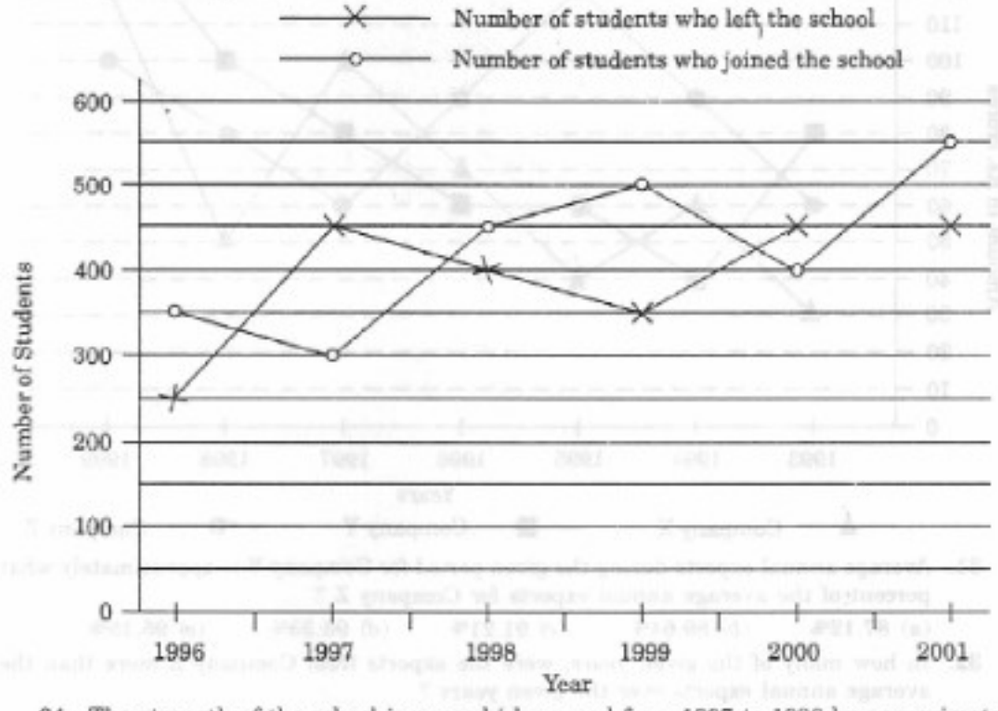
22. An investor invested a sum of Rs. 12 lakhs in Company P in 1998. The total amount received after one year was reinvested in the same Company for one more year. The total appreciation received by the investor on his investment was :
(*a*) Rs. 2,96,200 (*b*) Rs. 2,42,000 (*c*) Rs. 2,25,600
(*d*) Rs. 2,16,000 (*e*) Rs. 2,03,500

23. An investor invested Rs. 5 lakhs in Company Q in 1996. After one year, the entire amount along with the interest was transferred as investment to Company P in 1997 for one year. What amount will be received from Company P, by the investor ?
(*a*) Rs. 5,94,550 (*b*) Rs. 5,80,425 (*c*) Rs. 5,77,800
(*d*) Rs. 5,77,500 (*e*) Rs. 5,75,075

Directions (Questions 24 to 30) : ***Study the following line-graph which gives the number of students who joined and left the school in the beginning of year for six years, from 1996 to 2001.***

Initial strength of the school in 1995 = 3000.

The questions given below the graph are based on this line-graph. (S.B.I.P.O. 2001)



24. The strength of the school increased / decreased from 1997 to 1998 by approximately what percent ?
(*a*) 1.2% (*b*) 1.7% (*c*) 2.1% (*d*) 2.4% (*e*) 2.6%

25. The number of students studying in the school during 1999 was :
(*a*) 2950 (*b*) 3000 (*c*) 3100 (*d*) 3150 (*e*) 3200

26. During which of the following pairs of years, the strength of the school was same ?
(*a*) 1999 and 2001 (*b*) 1998 and 2000 (*c*) 1997 and 1998
(*d*) 1996 and 2000 (*e*) 1999 and 2000

27. The number of students studying in the school in 1998 was what percent of the number of students studying in the school in 2001 ?
(a) 92.13% (b) 93.75% (c) 96.88% (d) 97.25% (e) 99%

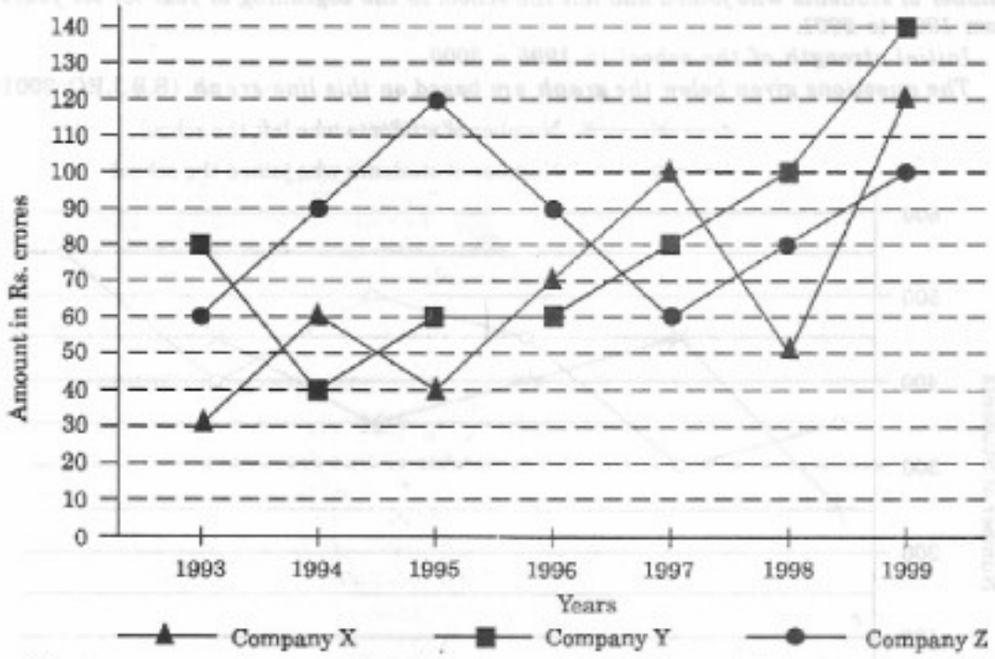
28. Among the given years, the largest number of students joined the school in the year :
(a) 1996 (b) 1998 (c) 1999 (d) 2000 (e) 2001

29. For which year, the percentage rise / fall in the number of students who left the school compared to the previous year is maximum ?
(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

30. The ratio of the least number of students who joined the school to the maximum number of students who left the school in any of the years during the given period is :
(a) 7 : 9 (b) 4 : 5 (c) 3 : 4 (d) 9 : 11 (e) 2 : 3

Directions (Questions 31 to 35) : ***Study the following graph and answer the questions based on it.*** **(S.B.I.P.O. 2000)**

Exports from Three Companies Over the Years (in Crore Rs.)



31. Average annual exports during the given period for Company Y is approximately what percent of the average annual exports for Company Z ?
(a) 87.12% (b) 89.64% (c) 91.21% (d) 93.33% (e) 95.15%

32. In how many of the given years, were the exports from Company Z more than the average annual exports over the given years ?
(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

33. What was the difference between the average exports of the three Companies in 1993 and the average exports in 1998 ?
(a) Rs. 15.33 crores (b) Rs. 18.67 crores (c) Rs. 20 crores
(d) Rs. 22.17 crores (e) Rs. 25 crores

34. In which year was the difference between the exports from Companies X and Y the minimum ?
(a) 1994 (b) 1995 (c) 1996 (d) 1997 (e) None of these

35. For which of the following pairs of years the total exports from the three Companies together are equal ?
(*a*) 1995 and 1998 (*b*) 1996 and 1998 (*c*) 1997 and 1998
(*d*) 1995 and 1996 (*e*) 1993 and 1994

ANSWERS

1. (*b*)	**2.** (*e*)	**3.** (*c*)	**4.** (*e*)	**5.** (*c*)	**6.** (*c*)	**7.** (*b*)	**8.** (*b*)	**9.** (*b*)
10. (*a*)	**11.** (*c*)	**12.** (*d*)	**13.** (*d*)	**14.** (*b*)	**15.** (*d*)	**16.** (*d*)	**17.** (*b*)	**18.** (*b*)
19. (*d*)	**20.** (*e*)	**21.** (*d*)	**22.** (*c*)	**23.** (*b*)	**24.** (*b*)	**25.** (*d*)	**26.** (*d*)	**27.** (*b*)
28. (*e*)	**29.** (*a*)	**30.** (*e*)	**31.** (*d*)	**32.** (*c*)	**33.** (*c*)	**34.** (*c*)	**35.** (*d*)	

SOLUTIONS

1. The differences between the percentages of candidates qualified to appeared for the given pairs of years are :
For *1994 and 1995* = 50 – 30 = 20; For *1997 and 1998* = 80 – 50 = 30;
For *1998 and 1999* = 80 – 80 = 0; For *1999 and 2000* = 80 – 60 = 20;
For *1994 and 1997* = 50 – 30 = 20.
Thus, the maximum difference is between the years 1997 and 1998.

2. The graph gives the data for the percentage of candidates qualified to appeared and unless the absolute values of number of candidates qualified or candidates appeared is known we cannot compare the absolute values for any two years. Hence, the data is inadequate to solve this question.

3. Let the number of candidates appeared in 1998 be x.
Then, 80% of x = 21200 $\Rightarrow$ $x = \dfrac{21200 \times 100}{80}$ = 26500 (required number).

4. The total number of candidates qualified in 1996 and 1997 together, cannot be determined until we know at least, the number of candidates appeared in any one of the two years 1996 or 1997 or the percentage of candidates qualified to appeared in 1996 and 1997 together. Hence, the data is inadequate.

5. The number of candidates qualified in 1999 = 80% of 26500 = 21200.
$\therefore$ Number of candidates qualified in 2000 = 33500 – 21200 = 12300.
Let the number of candidates appeared in 2000 be x.
Then, 60% of x = 12300 $\Rightarrow$ $x = \dfrac{12300 \times 100}{60}$ = 20500.

6. Let the expenditure in 1998 be Rs. x crores.
Then, $65 = \dfrac{264 - x}{x} \times 100 \Rightarrow \dfrac{65}{100} = \dfrac{264}{x} - 1 \Rightarrow x = \dfrac{264 \times 100}{165} = 160.$
$\therefore$ Expenditure in 1998 = Rs. 160 crores.

7. It is given that : $\%\,\text{Profit} = \dfrac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$
$\Rightarrow \dfrac{\%\,\text{Profit}}{100} = \dfrac{\text{Income}}{\text{Expenditure}} - 1 \Rightarrow \dfrac{\text{Income}}{\text{Expenditure}} = \dfrac{\%\,\text{Profit}}{100} + 1.$
From this it is clear that the ratio of income to expenditure is minimum for the year in which the % profit has the minimum value. Since, out of the given years (*i.e.*, out of 1996, 1997, 1998, 1999 and 2000), the Company has the minimum % profit in the year 1997, so the minimum ratio of income to expenditure is in the year 1997.

8. Average percent profit earned for the given years

$$= \frac{1}{6} \times [40 + 55 + 45 + 65 + 70 + 60] = \frac{335}{6} = 55\frac{5}{6}.$$

9. The ratio of percentage profit earned to that in the previous year, for different years are :

For 1996 $= \frac{55}{40} = 1.38$; **For 1997** $= \frac{45}{55} = 0.82$; **For 1998** $= \frac{65}{45} = 1.44$;

For 1999 $= \frac{70}{65} = 1.08$; **For 2000** $= \frac{60}{70} = 0.86$.

Clearly, this ratio is minimum for 1997.

10. Let the expenditure in 1996 = expenditure in 1999 = x.

Also, let the incomes in 1996 and 1999 be I_1 and I_2 respectively.

Then, for the year 1996, we have :

$$55 = \frac{I_1 - x}{x} \times 100 \Rightarrow \frac{55}{100} = \frac{I_1}{x} - 1 \Rightarrow I_1 = \frac{155x}{100} \quad ...(i)$$

And, for the year 1999, we have :

$$70 = \frac{I_2 - x}{x} \times 100 \Rightarrow \frac{70}{100} = \frac{I_2}{x} - 1 \Rightarrow I_2 = \frac{170x}{100} \quad ...(ii)$$

From (i) and (ii), we get :

$$\frac{I_1}{I_2} = \frac{\left(\frac{155x}{100}\right)}{\left(\frac{170x}{100}\right)} = \frac{155}{170} = \frac{0.91}{1} = 9 : 10.$$

11. Let the expenditure in 1997 be x.

Then, expenditure in 2000 $= x + (25\% \text{ of } x) = \frac{5}{4}x$.

Also, let the incomes in 1997 and 2000 be I_1 and I_2 respectively.

Then, for the year 1997, we have :

$$45 = \frac{I_1 - x}{x} \times 100 \Rightarrow \frac{45}{100} = \frac{I_1}{x} - 1 \Rightarrow I_1 = \frac{145x}{100} = 1.45x.$$

Also, for the year 2000, we have :

$$60 = \frac{\left(I_2 - \frac{5}{4}x\right)}{\left(\frac{5}{4}x\right)} \times 100 \Rightarrow \frac{60}{100} = \frac{4I_2}{5x} - 1 \Rightarrow I_2 = \frac{160}{100} \times \frac{5x}{4} = 2x.$$

Difference between the two incomes $= (2x - 1.45x) = 0.55x$.

$\therefore$ Percentage by which I_1 is less than $I_2 = \left(\frac{0.55x}{2x} \times 100\right)\% = 27.5\%$.

12. From the line-graph we obtain information about the percentage profit only. To find the profit in 2000 we must have the data for the income or expenditure in 2000. Therefore, the profit for 2000 cannot be determined.

13. The line-graph gives the comparison of percent profit for different years but the comparison of the expenditures is not possible without more data. Therefore, the year with minimum expenditure cannot be determined.

14. The exports are more than the imports in those years for which the exports to imports ratio is more than 1. For Company A, such years are 1995, 1996 and 1997. Thus, during these 3 years, the exports are more than the imports for Company A.

15. We shall try to find the difference between the imports and exports of Company B for various years one by one :

For 1995 : We have

$\frac{E}{I} = 0.75$ (where E = amount of exports and I = amount of imports in 1995)

$\Rightarrow$ $E = 0.75I$ $\therefore$ $I - E = I - 0.75I = 0.25I$.

Thus, the difference between the imports and exports of Company B in 1995 is dependent on the amount of imports of Company B in 1995.

Similarly, the difference for other years can be determined only if the amount of imports for these years are known. Since the imports or exports for various years are not known, the differences between imports and exports for various years cannot be determined.

16. Let the amount of imports of Company A in 1998 be Rs. x crores.

Then, $\frac{237}{x} = 0.75 \Rightarrow x = \frac{237}{0.75} = 316$.

$\therefore$ Amount of imports of Company A in 1998 = Rs. 316 crores.

17. In 1997 for Company A we have :

$\frac{E}{I} = 1.75$ *i.e.*, $E = 1.75I$ ...(*i*)

[where E = amount of exports and I = amount of imports of Company A in 1997]

Now, the required imports $I_1 = I + 40\%$ of $I = 1.4I$.

$\therefore$ Required ratio $= \frac{E}{I_1} = \frac{1.75I}{1.4I} = 1.25$.

18. In 1995 for Company A we have :

$\frac{E_A}{I_A} = 1.75$ ...(*i*) [where E_A = amount of exports and I_A = amount of imports of Company A in 1995]

In 1995 for Company B we have :

$\frac{E_B}{I_B} = 0.75$ ...(*ii*) [where E_B = amount of exports and I_B = amount of imports of Company B in 1995]

Also, we have $E_A = 2E_B$ ...(*iii*)

Substituting I_A = Rs. 180 crores (given) in (*i*), we get

E_A = Rs. (180 × 1.75) crores = Rs. 315 crores.

Using E_A = Rs. 315 crores in (*iii*), we get : $E_B = \frac{E_A}{2}$ = Rs. $\left(\frac{315}{2}\right)$ crores.

Substituting E_B = Rs. $\left(\frac{315}{2}\right)$ crores in (*ii*), we get :

$I_B = \frac{E_B}{0.75}$ = Rs. $\left(\frac{315}{2 \times 0.75}\right)$ crores = Rs. 210 crores.

i.e., amount of imports of Company B in 1995 = Rs. 210 crores.

19. Let the amounts invested in 2002 in Companies P and Q be Rs. $8x$ and Rs. $9x$ respectively.

Then, interest received after one year from Company P

= Rs. (6% of $8x$) = Rs. $\frac{48}{100}x$

and interest received after one year from Company Q

$$= \text{Rs. } (4\% \text{ of } 9x) = \text{Rs. } \frac{36}{100}x.$$

$$\therefore \quad \text{Required ratio} = \frac{\left(\frac{48}{100}x\right)}{\left(\frac{36}{100}x\right)} = \frac{4}{3}.$$

20. Let Rs. x lakhs be invested in Company P in 2000, then amount invested in Company Q in 2000 = Rs. $(30 - x)$ lakhs.

Total interest received from the two Companies after 1 year

$$= \text{Rs. } [(7.5\% \text{ of } x) + (9\% \text{ of } (30 - x))] \text{ lakhs} = \text{Rs. } \left[2.7 - \left(\frac{1.5x}{100}\right)\right] \text{ lakhs.}$$

$$\therefore \quad \left[2.7 - \left(\frac{1.5x}{100}\right)\right] = 2.43 \implies x = 18.$$

i.e., amount invested in Company P = Rs. 18 lakhs.

21. Difference = Rs. [(10% of 4.75) − (8% of 4.75)] lakhs

= Rs. (2% of 4.75) lakhs = Rs. 0.095 lakhs = Rs. 9500.

22. Amount received from Company P after one year (*i.e.*, in 1999) on investing Rs. 12 lakhs in it = Rs. [12 + (8% of 12)] lakhs = Rs. 12.96 lakhs.

Amount received from Company P after one year on investing Rs. 12.96 lakhs in the year 1999 = Rs. [12.96 + (10% of 12.96)] lakhs = Rs. 14.256 lakhs.

Appreciation received on investment during the period of two years

= Rs. (14.256 − 12) lakhs = Rs. 2.256 lakhs = Rs. 2,25,600.

23. Amount received from Company Q after one year on investment of Rs. 5 lakhs in the year 1996 = Rs. [5 + (6.5% of 5)] lakhs = Rs. 5.325 lakhs.

Amount received from Company P after one year on investment of Rs. 5.325 lakhs in the year 1997 = Rs. [5.325 + (9% of 5.325)] lakhs = Rs. 5.80425 lakhs = Rs. 5,80,425.

Questions 24 to 30 :

Before solving the questions, we shall analyse the graph :

From the graph it is clear that :

In 1996 : Number of students left = 250 and number of students joined = 350.

In 1997 : Number of students left = 450 and number of students joined = 300.

In 1998 : Number of students left = 400 and number of students joined = 450.

In 1999 : Number of students left = 350 and number of students joined = 500.

In 2000 : Number of students left = 450 and number of students joined = 400.

In 2001 : Number of students left = 450 and number of students joined = 550.

Therefore, the numbers of students studying in the school (*i.e.*, strength of the school) in various years :

In 1995 = 3000 (given); **In 1996** = 3000 − 250 + 350 = 3100;

In 1997 = 3100 − 450 + 300 = 2950; **In 1998** = 2950 − 400 + 450 = 3000;

In 1999 = 3000 − 350 + 500 = 3150; **In 2000** = 3150 − 450 + 400 = 3100;

In 2001 = 3100 − 450 + 550 = 3200.

Now, we shall solve the questions.

24. Percentage increase in the strength of the school from 1997 to 1998

$$= \left[\frac{(3000 - 2950)}{2950} \times 100\right]\% = 1.69\% = 1.7\%.$$

25. As calculated above, the number of students studying in the school during 1999 = 3150.

26. As calculated above, in the years 1996 and 2000 the strength of the school was same *i.e.*, 3100.

27. Using the calculations above we have :

$$\text{Required percentage} = \left(\frac{3000}{3200} \times 100\right)\% = 93.75\%.$$

28. As calculated above, the largest number of students (*i.e.*, 550) joined the school in the year 2001.

29. The percentage rise / fall in the number of students who left the school (compared to the previous year) during various years are :

$$\textbf{For 1997} = \left[\frac{(450-250)}{250} \times 100\right]\% = 80\% \text{ (rise)};$$

$$\textbf{For 1998} = \left[\frac{(450-400)}{450} \times 100\right]\% = 11.11\% \text{ (fall)};$$

$$\textbf{For 1999} = \left[\frac{(400-350)}{400} \times 100\right]\% = 12.5\% \text{ (fall)};$$

$$\textbf{For 2000} = \left[\frac{(450-350)}{350} \times 100\right]\% = 28.57\% \text{ (rise)};$$

$$\textbf{For 2001} = \left[\frac{(450-450)}{450} \times 100\right]\% = 0\%.$$

Clearly, the maximum percentage rise / fall is for 1997.

30. Using the calculations above we get :

$$\text{Required ratio} = \frac{300}{450} = \frac{2}{3}.$$

Questions 31 to 35 :

Analysis of the graph : From the graph it is clear that

(*i*) The amount of exports of Company X (in crore Rs.) in the years 1993, 1994, 1995, 1996, 1997, 1998 and 1999 are 30, 60, 40, 70, 100, 50 and 120 respectively.

(*ii*) The amount of exports of Company Y (in crore Rs.) in the years 1993, 1994, 1995, 1996, 1997, 1998 and 1999 are 80, 40, 60, 60, 80, 100 and 140 respectively.

(*iii*) The amount of exports of Company Z (in crore Rs.) in the years 1993, 1994, 1995, 1996, 1997, 1998 and 1999 are 60, 90, 120, 90, 60, 80 and 100 respectively.

31. Average annual exports (in Rs. crore) of Company Y during the given period

$$= \frac{1}{7} \times (80 + 40 + 60 + 60 + 80 + 100 + 140) = \frac{560}{7} = 80.$$

Average annual exports (in Rs. crore) of Company Z during the given period

$$= \frac{1}{7} \times (60 + 90 + 120 + 90 + 60 + 80 + 100) = \left(\frac{600}{7}\right).$$

$$\therefore \quad \text{Required percentage} = \left[\frac{80}{\left(\frac{600}{7}\right)} \times 100\right]\% = 93.33\%.$$

32. Average annual exports of Company Z during the given period

$$= \text{Rs.} \left[\frac{1}{7} \times (60 + 90 + 120 + 90 + 60 + 80 + 100)\right] \text{ crores} = \text{Rs.} \left(\frac{600}{7}\right) \text{ crores}$$

$$= \text{Rs. } 85.71 \text{ crores.}$$

From the analysis of graph the exports of Company Z are more than the average annual exports of Company Z (*i.e.*, Rs. 85.71 crores) during the years 1994, 1995, 1996 and 1999, *i.e.*, during 4 of the given years.

33. Average exports of the three Companies X, Y and Z in 1993

$$= \text{Rs.}\left[\frac{1}{3}\times(30+80+60)\right]\text{ crores} = \text{Rs.}\left(\frac{170}{3}\right)\text{ crores.}$$

Average exports of the three Companies X, Y and Z in 1998

$$= \text{Rs.}\left[\frac{1}{3}\times(50+100+80)\right]\text{ crores} = \text{Rs.}\left(\frac{230}{3}\right)\text{ crores.}$$

$$\text{Difference} = \text{Rs.}\left[\left(\frac{230}{3}\right)-\left(\frac{170}{3}\right)\right]\text{ crores} = \text{Rs.}\left(\frac{60}{3}\right)\text{ crores} = \text{Rs. 20 crores.}$$

34. The differences between the exports from the Companies X and Y during various years are :

In 1993 = Rs. (80 – 30) crores = Rs. 50 crores;
In 1994 = Rs. (60 – 40) crores = Rs. 20 crores;
In 1995 = Rs. (60 – 40) crores = Rs. 20 crores;
In 1996 = Rs. (70 – 60) crores = Rs. 10 crores;
In 1997 = Rs. (100 – 80) crores = Rs. 20 crores;
In 1998 = Rs. (100 – 50) crores = Rs. 50 crores;
In 1999 = Rs. (140 – 120) crores = Rs. 20 crores.

Clearly, the difference is minimum in the year 1996.

35. Total exports of the three Companies X, Y and Z together, during various years are :

In 1993 = Rs. (30 + 80 + 60) crores = Rs. 170 crores.
In 1994 = Rs. (60 + 40 + 90) crores = Rs. 190 crores.
In 1995 = Rs. (40 + 60 + 120) crores = Rs. 220 crores.
In 1996 = Rs. (70 + 60 + 90) crores = Rs. 220 crores.
In 1997 = Rs. (100 + 80 + 60) crores = Rs. 240 crores.
In 1998 = Rs. (50 + 100 + 80) crores = Rs. 230 crores.
In 1999 = Rs. (120 + 140 + 100) crores = Rs. 360 crores.

Clearly, the total exports of the three Companies X, Y and Z together are same during the years 1995 and 1996.