Finding Outer Corners of the Cannon

Motivation: One way of changing the launch angle of the cannon is to click and hold the cannon and dragging the cannon up or down. However, when I was trying to create this code, I realised that the program took the cannon to only be the inside rectangular region. That is, if a user clicked on the border of the cannon, the program did not recognise this as a successful click on the cannon. So, the following method lets us find the outer corners of the cannon, given the inner corners of the cannon.

Important Note To match the coordinate system followed by HTML and JavaScript, right and downwards are considered to be positive directions. That is, a point M that is to the right of a point N will have a larger x coordinate and a point M that is below a point N will have a larger y coordinate.

Consider the following diagram:

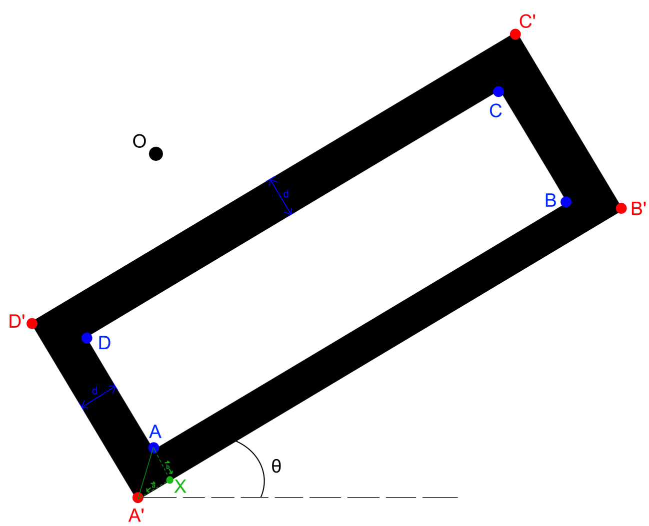

Figure 1: Overview of the whole cannon

Recall that points A, B, C and D are known points, while A', B', C' and D' are the points we are looking for.

Point X is constructed such that $AX \perp A'X$. Let

$$\vec{a} = \overrightarrow{XA'}$$

and

$$\vec{b} = \overrightarrow{AX}.$$

Notice that $\overrightarrow{AA'} = \vec{b} + \vec{a}$. Since the thickness of the border is constant at any region we can define

$$d = |\vec{a}| = |\vec{b}|.$$

Consider the following diagram:

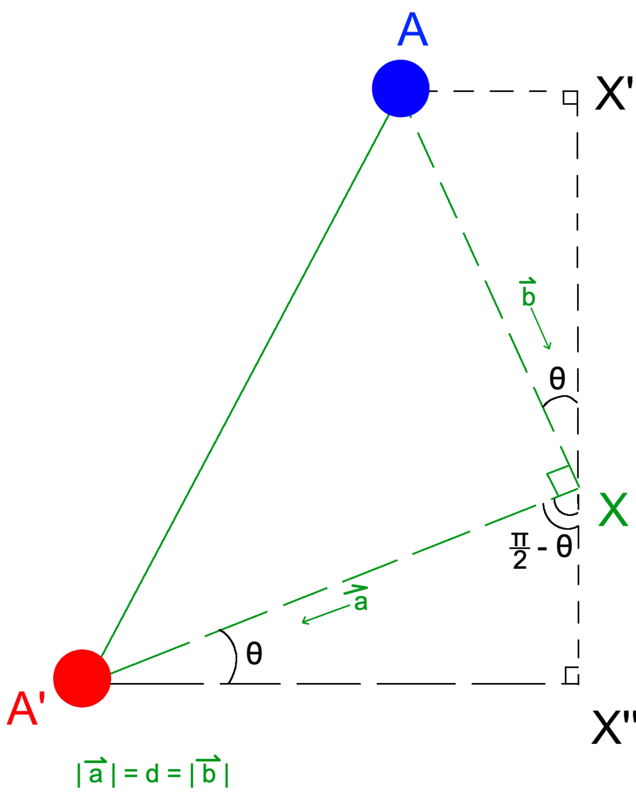

Figure 2: Focusing on the point A'

Notice that $\angle XA'X'' = \theta$ since $A'X''$ is a horizontal line and $A'X$ represents a segment of the long edge of the cannon. Therefore, $\angle A'XX'' = \frac{\pi}{2} - \theta$ by the internal angle sum of a triangle and $\angle AXX' = \theta$ by the angle of a straight line.

Breaking $\vec{a}$ and $\vec{b}$ into their horizontal and vertical components,

$$\overrightarrow{XX''} = \begin{pmatrix} 0 \\ d\sin\theta \end{pmatrix}$$

and

$$\overrightarrow{X''A'} = \begin{pmatrix} -d\cos\theta \\ 0 \end{pmatrix},$$

while

$$\overrightarrow{AX'} = \begin{pmatrix} d\sin\theta \\ 0 \end{pmatrix}$$

and

$$\overrightarrow{X'X} = \begin{pmatrix} 0 \\ d\cos\theta \end{pmatrix}.$$

Therefore, we can conclude that

$$\vec{a} = \overrightarrow{XX''} + \overrightarrow{X''A'} = \begin{pmatrix} -d\cos\theta \\ d\sin\theta \end{pmatrix}$$

and

$$\vec{b} = \overrightarrow{AX'} + \overrightarrow{X'X} = \begin{pmatrix} d\sin\theta \\ d\cos\theta \end{pmatrix}.$$

Now considering Figure 1 again, we can find the position vectors of A', B', C' and D'.

$$\overrightarrow{OA'} = \overrightarrow{OA} + \vec{b} + \vec{a},$$

$$\overrightarrow{OB'} = \overrightarrow{OB} + \vec{b} - \vec{a},$$

$$\overrightarrow{OC'} = \overrightarrow{OC} - \vec{b} - \vec{a}$$

and

$$\overrightarrow{OD'} = \overrightarrow{OD} - \vec{b} + \vec{a}.$$