Course era

Let n be any odd prime. If we divide any n by 4, we get n=4k+r where 0≤r≤4 i.e., r=0,1,2, eithern=4korn=4k+1 or n=4k+2orn=4k+3 Clearly, 4n is never prime and 4n+2=2(2n+1) cannot be prime unless n=0 (since, 4 and 2 cannot be factors of an odd

5. Prove that for any Integer n. at least one of the Integers n.n+2n+4 is dhisible by 3.

let n be any positive integer and b-3 n-3q-r where q Is the quotient and is the remainder 0 <r-3 so the remainders may be 0.1 and 2 son may be in the form of 3q, 3q-1.3q-2 CASE-1 IF N-30 n-4-30-4-2-30-2 here n is only divisible by 3 CASE 2 If n=3q-1 n-4-30-5 n-2-30-3 here only n+2 is divisible by 3 CASE 3 IF n=30+2n+2+3q-4n-4-30-2-4-3q-6 here only n-4 is divisible by 3 HENCE IT IS JUSTIFIED THAT ONE AND ONLY ONE AMONG nun-21-4 IS DIVISIBLE BY 3 IN EACH CASE.

there are infinitely many pairs of "owin primes", pairs of primes separated by 2. such as 3 and 5. 11 and 13. or 71 and 73. Prove that the only prime triple (le. three primes, each 2 from the next is 3.5.7.

Proof: By contradiction.

Assume Sn, n-2, n-45 are prime. (in in mathbb(N) n t35)

For any Siris, at least one of Sin, n-2. n-45 is divisible by 3. since Sn-1 \equiv n-4 \equiv 11pmod(3)5.

So the prime triple Sin, n 2 n 45 is always expressed as the three consecutive numbers Sin, n+1. n-25. Contradiction.

$35 is the only prime that is a multiple of 3. Hence the

only prime triple is $35.75.

This proves the staternent. Fiblacksquares

7. Prove that for any natural number 2 + 2 ^ 2 + 2 ^ 2 + ...+ 2 ^ n = 2 ^ (n + 1) - 2

Proof: By Induction on ins

For the case 5n = 15 is true, since the left side $2-1- 25 and the right side is $2(1-1)-2-4-2-25.

Assume the identity holds for n. Add Sn-15 to both sides of 52+2*2+2^ 4 3+...+ 2 ^ 4 * n + 2 ^ 4 * (n + 1) * 0.25 . 1.8.52*2^ ^ 2*2^ ^ 3*...+ 2 ^ n + 2 ^ (n + 1) * 2 ^ ((m + 1) * 1) - 23 $2-22-23-2^n 20n-1)-2(n-1)-13-2- 2^ * (n - 1 - 1) - 25 by algebra.

which is the identity wich Sn 15 in place of n.

Hence by the principle of mathematical induction, the Identity hollds for all Sns. Siblacksquares

8. Prove (from the definition of a limit of a sequence) that if the sequence (@n) tends to limit & as n → oo, then for any fixed number M > 0, the sequence (Man) tends to the limit ML.

Proof:

Let $\varepsilon > 05 be given.

Then since Sa_ns converges to $LS, we need to find an $\bar{n} \in \mathbb(N)$ such that $) a_n-L< \frac{\varepsilon)(M), \forall(n) \ge \bar{n}$.

So for all sns, we will find

$| Ma_n-ML|M|a_n-L| <M\frac{\varepsilon)(M) = warepsilons

Thus $\(Ma_n\)^\infty_{n=1}$ tends to the limit SMILS. $\blacksquares

9. Given an infinite collection A. n = 1.2 ... Intervals of the real line, their intersection is defined to be cap n = 1 ^ (∞) ,A n =\ x|(forall n)(x in A n)\ Give an example of a family of intervals An, n = 1, 2,.... such that A n + 1 subset A n for all n and bigcap n = 1 ^ (∞) A n = emptyset. Prove that your example has the stated property.

Example: SA_(n) = (0, \frac{1)(n)), n = 1,2,...$

Proof:

First, show $(\forall(n))(A_{n+1} \subset A_(n))$.

Let n be an arbitrary natural number. Then $A_(n) =

(0. \frac{1}{n})$ and SA(n + 1) = 0 \frac{1}{n+1}]

SA(n + 1) subset A (n)5, i.e. $(0, \frac{1)(n+1)) \subset (0, \frac{1)(n))$.

Let x denote an arbitraly element of 5(0, \frac{1} (n + 1))5. Then 50 lt x \t \frac{1}{n+1), 0 \lt x \lt \frac{1} (n)$, since $\frac(1)(n+1) \t \frac(1)(n)$.

So every element of $A_ (n + 1) * 5 an element SA_(n)5.

Thus $(\forall(n))(A_{n+1} \subset A_{n})$.

Next show $\bigcap_{n=1}^{\infty) A_n = \emptysets.

We need to find an empty intersection such that $A_(n) \supset A_{n+1} \supset A_(n+2)... \supset A_{n \rightarrow \infty)s for all sns.

SA_(n), n \rightarrow \infty, Mim \limits_{n\rightarrow \infty) A (n) = 05, so if $n \rightarrow \infty, (0, \frac(1) (n)) = (0, 0)$.

$(0, 0)$ is an empty set. Thus $\bigcap_{n=1}^{\infty) A_n = \emptysets. $\blacksquares

Proot:

First, show $1(01) \subset \bigcap_{n=1}^{\infty) A_{n}S.

For every positive integer, 50$ is an element of $(\frac{-1)(n), \frac{1)(n))$, so $0 \in \bigcap_{n=1}^{\infty) (\frac{-1)(n), \frac{1}{n})$.

$0 \in \(01)$, so every element of $\(01)$ is in $\bigcap_{n=1}^{\infty) (\frac(-1)(n). \frac{1}{n})$ Hence, $\(01) \subset \bigcap_{n=1}^{\infty) (\frac{-1) (n), \frac{1)(n))$ by the definition of subset.

Next show bigcap_{n=1}^{\infty) A_(n) \subset \(01)$.

Let x be an element of S\bigcap_{n=1}^{\infty} A_{n}$. Then $x \in (\frac(-1)(n), \frac(1)(n))$ for every positive integer.

Assume that $x \neq 05.

Then $/x/ \gt 05 and there is a positive insteger SNS such that 50 \t \frac{1}{N} \t \frac{1}{x)$ by archimedean principle.

Hence, Sx \notin (\frac{(-1)(n), \frac{1}{n})$, but this

leads to a contracition, so Sx = 05.

Therefore 0 is the only element of

The sum of any 5 consecutive integers is, in fact, evenly divisible by 5!

To show this let's call the first integer: n

Then, the next four integers will be:

n+1, n+2, n+3 and n+4

Adding these five integers together gives:

n+n+1+n+2+n+3+n+4= n+n+n+n+n+1+2+3+4 In+in+in+in+in+1+2+3+4=
(1+1+1+1+1)+(1+2+3+4)

5n+10

5n+(5×2)=

5(n+2)

If we divide this sum of any 5 consecutive integers by 5 we get:

5(n+2)/5-

n+2

Because n was originally defined as an integer n+2 is

also an integer.

Therefore, the sum of any five consecutive integers is evenly divisible by 5 and the result is an integer with no remainder.