

# [Statistics - day 4]

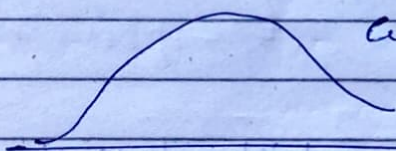
## Topics:-

- ① central limit theorem
- ② Probability
- ③ Permutation and Combination
- ④ Covariance, Pearson Correlation, Spearman Rank correlation.

## ① central limit theorem:-

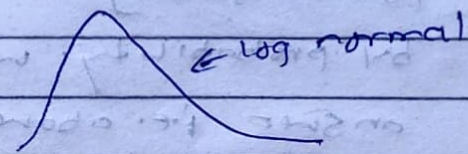
Size of sample =  $n$

no. of samples =  $m$



Gaussian

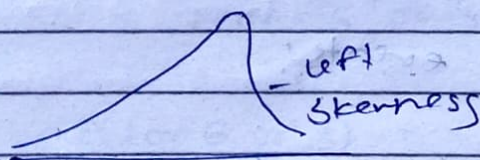
$$\rightarrow S_1 = \{x_1, x_2, x_3, \dots\} = \bar{x}_1$$



log normal

$$\rightarrow S_2 = \{x_1, x_2, x_3, \dots\} = \bar{x}_2$$

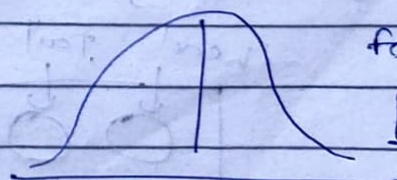
$$\rightarrow S_3 = \{x_1, x_2, x_3, \dots\} = \bar{x}_3$$



left skewness

If we draw the mean of all samples

it shows Gaussian distribution



for value

$$[n \geq 30]$$



## (2) Probability:

measure of the likelihood of an event.

e.g. → (1) Tossing a fair coin

$$P(H) = 0.5$$

$$P(T) = 0.5$$

(2) Rolling a dice

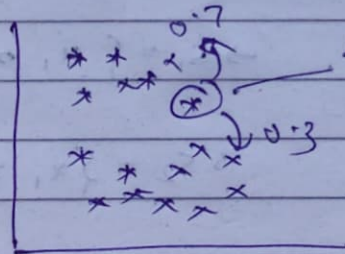
$$P(1) = \frac{1}{6}$$

$$P(2) = \frac{1}{6}$$

... etc.

why it is used:

suppose we have



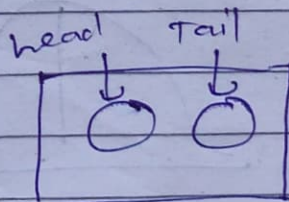
from which does this point belong

by probability we can answer i.e. above having Probability = 0.7

(3) mutual exclusive events:

→ The events mutually exclusive if they cannot occur at the same time.

e.g. ↑  
tossing a coin



if head comes then tail cannot occur



(2) Rolling a dice

if 1 come in dice then other numbers cannot occur.

(1) Picking randomly a card from a deck of cards the events "heart" and "king" can be selected.   
 non-mutual exclusive

Problems:-

(1) What is the Probability of coin landing on heads or tails

↓

Addition rule for mutual exclusive events.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) \\ &= 0.5 + 0.5 \\ &= 1 \end{aligned}$$

(2) What is probability of getting 1 or 6 or 3 while rolling a dice

$$\begin{aligned} P(1 \text{ or } 6 \text{ or } 3) &= P(1) + P(6) + P(3) \\ \frac{1}{6} + \frac{1}{6} + \frac{1}{6} &= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} \end{aligned}$$



Non mutual exclusive Event!

events are non mutual exclusive if they occur at same time

Bag of Marbles = 10 red, 6 Green  
 $\downarrow$  3 (R & G)

when picking randomly from a bag of marbles what is the probability of choosing a marble that is red and green?

Addition rule:-

$$\begin{aligned} \textcircled{1} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{13}{19} + \frac{9}{19} - \frac{3}{19} \\ &= \frac{19}{19} \\ &= 1 \end{aligned}$$

$\textcircled{2}$

$$P(\text{heart or queen}) = P(\text{heart}) + P(\text{queen}) - P(\text{heart and queen})$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{8}{26} = \frac{4}{13}$$



Multiplication Rule:

① Dependent event:- if they affect one another then it is dependent event.

Bag of marble =  $\begin{cases} 0000-2 \rightarrow 4 \text{ white} \\ 000 \rightarrow 3 \text{ yellow} \end{cases}$

if 1 white marble selected then

$$\boxed{P(W) = \frac{4}{7}} \quad \text{and} \quad \boxed{\text{total} = 7 - 1 = 6}$$

then,

$$P(W) = \frac{3}{6}$$

example: what is the probability of rolling a "5" and then a "3" with a normal 6 sided dice?

→ Independent

$$P(1) = \frac{1}{6} \quad P(2) = \frac{1}{6} \quad P(3) = \frac{1}{6} \quad P(5) = \frac{1}{6}$$

$$\begin{aligned} P(A \text{ and } B) &= P(A) \times P(B) \\ &= \frac{1}{6} \times \frac{1}{6} \end{aligned}$$

$$P(A \text{ and } B) = \frac{1}{36}$$



Permutation :-

in how many ways 5 chocolates

= { Dairy milk, kit kat, milky bar, sneakers, 5 star }

$$= 5 \times 4 \times 3$$

$$= \boxed{60 \text{ way}} \leftarrow \text{Permutation}$$

formula is :-

$${}^n P_r = \frac{n!}{(n-r)!} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

✓

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

$$= 60$$

Combination :-

Repetition is not allowed:

formula :-

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{5!}{3!(2!)}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2} = \frac{20}{2}$$

$$\boxed{{}^n C_r = 10 \text{ ways}}$$



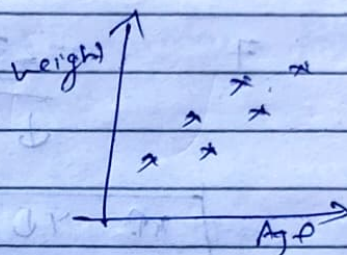
# \*) Covariance <sup>used in</sup> feature selection

X	Y
Age	weight
12	40
13	45
15	48
17	60
18	62

looking  
at data

Age ↑ weight ↑
Age ↓ weight ↓

quantify the  
relationship  
x & y using  
mathematical  
question



$$\bar{x} = 15$$

$$\bar{y} = 51$$

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

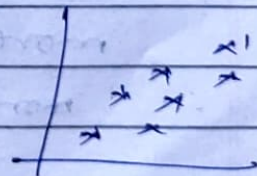
$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$\boxed{\text{Cov}(x, x) = \text{var}(x)}$$

+) Covariance

x ↑	y ↑
x ↓	y ↓



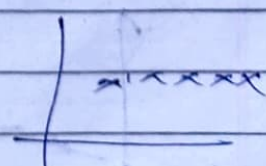
= Covariance

x ↑	y ↓
x ↓	y ↑



o - covariance

no relation
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$x$	$y$
10	4
8	6
7	8
6	10

$$\text{Cov}(x, y) = -4$$

$$= \frac{(10 - 7.75)(4 - 7) + (8 - 7.75)(6 - 7) + (7 - 7.75)(8 - 7) + (6 - 7.75)(10 - 7)}{3}$$

$$7.75 \quad 7 \quad = -3.25$$

↓

$x \uparrow$	$y \downarrow$
$x \downarrow$	$y \uparrow$

Pearson Correlation Coefficient (-1 to 1)

why to use this?

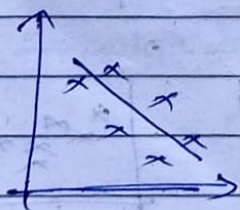
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because covariance doesn't provide

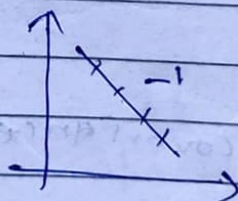
Scale

$$r(x, y) = \frac{\text{Cov}(x, y)}{\sigma_x \cdot \sigma_y}$$

more the value towards +1  
more the correlated is

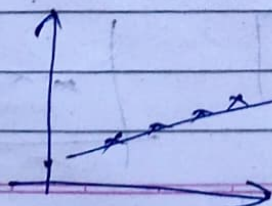


-1 to 0

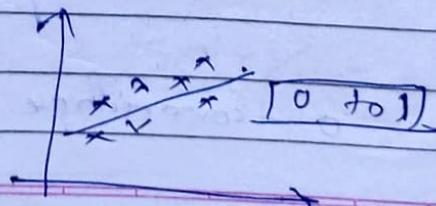


$x \downarrow$   $y \uparrow$

$x \uparrow$   $y \downarrow$



+1 to 0



0 to 1



# (\*) Spearman Rank Correlation:

$$r_s = \frac{\text{Cov}(R(x), R(y))}{\sigma(R(x)) \cdot \sigma(R(y))}$$

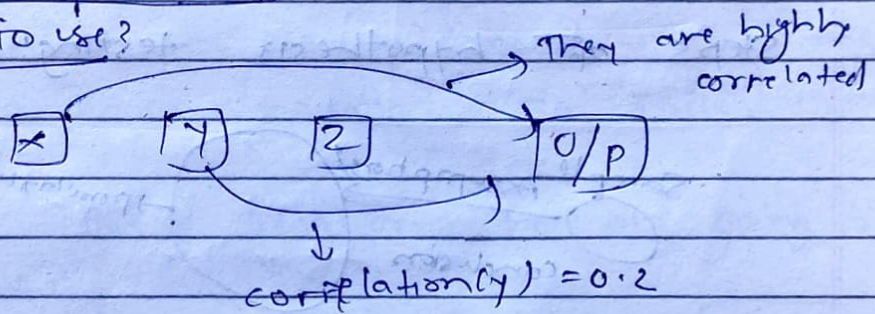
x	y
10	4
8	6
7	8
6	10

R(x)	R(y)
4	1
3	2
2	3
1	4

→ Ascending

→ Spearman Rank Correlation

Why to use?



then we can drop this

and x and z are 95% correlated

then we can drop (2)

