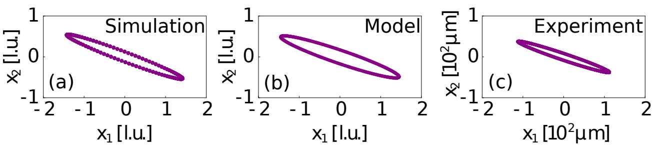

FIG. 3. **Configuration space of the dumbbell.** The ellipse results from the phase shift in the oscillations of the beads $x_1(t)$ and $x_2(t)$ around the swimmer geometric center. (a) LB simulations and (b) corresponding analytics ($\omega = 1.57 \times 10^{-3}$), and (c) experiments ($\omega = 15.7\,\mathrm{s}^{-1}$, 500-793$\mu$m)

ertheless, the time-averaged flow is dipolar (Fig. 1(b)), and the swimmer can be described as a puller in the investigated range of parameters. Secondly, knowing the phase-shift in the individual oscillations also allows us to cast the expression of the swimming speed into

$$\overline{U} \propto A_1 A_2 \sin \Delta\phi, \tag{4}$$

where A_i are the amplitudes of oscillation of the beads (see [30] (Sect. II.B)). Both of these effects are consistent with the phase shift in the dynamics and flow fields generated in the simulations reported in [21], where the dynamics of a dumbbell was investigated as a function of the fluid Reynolds number.

In conclusion, we used magneto-capillary swimmers and lattice Boltzmann simulations to provide the basis for a minimal theoretical model for swimming on the mesoscale. We show that there exists a dynamical regime where the swimmer inertia can be harnessed for self-propulsion in the low $\mathrm{Re_f}$ regime. Indeed, by including an asymmetry in coasting time in the design of the mesoswimmer, it is able to break the time-symmetry of the generated flow field. The swimming velocity then is related to the area of the trajectory drawn in the configuration space. This area is a measure of the nonreciprocity of the dynamics [1, 3] and is typically used to demonstrate the *scallop theorem*. The latter is, for the mesoswimmers, fulfilled by an intrinsic property of swimmer parts, namely their inertia that together mimic an independent degree of freedom. The analysis performed herein thus shows that the transition from microswimmers to mesoswimmers may occur through a delicate balance of viscous damping and inertial relaxation. At higher Reynolds numbers, naturally, the inertia of the fluid will couple to the coasting of the swimmer and dominate the dynamics. The analysis provided herein, however, may help to understand the emergence of this complex interplay.

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