

The electronic complexity of the ground-state of the FeMo cofactor of nitrogenase as relevant to quantum simulations

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We report that a recent active space model of the nitrogenase FeMo cofactor, proposed in the context of quantum simulations, is not representative of the electronic structure of the FeMo cofactor ground-state. Although quantum resource estimates, outside of the cost of adiabatic state preparation, will not be much affected, conclusions should not be drawn from the complexity of classical simulations of the electronic structure of this system in this active space. We provide a different model active space for the FeMo cofactor that contains the basic open-shell qualitative character, which may be useful as a benchmark system for making classical and quantum resource estimates.

The process of nitrogen fixation, namely that of converting atmospheric dinitrogen to a reduced form, such as ammonia, which can then be metabolized by biological species, is essential to life on this planet^{1–4}. The industrial Haber-Bosch process to produce fertilizer from the endothermic reaction $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$ is very energy intensive, requiring a careful balance of high temperatures and high pressures to achieve efficient catalysis. In contrast, natural bacteria and archaea carry out nitrogen fixation under ambient conditions through nitrogenases. At the molecular level, the nitrogenase enzyme, an agglomeration of a homodimer Fe protein and the MoFe protein (in its most common Mo containing form), catalyzes the nitrogen bond-breaking process via a family of 3 metallic cofactors: the $[\text{Fe}_4\text{S}_4]$ iron cubane, the $[\text{Fe}_8\text{S}_7]$ P cluster, and the $[\text{MoFe}_7\text{S}_9\text{C}]$ FeMo cofactor (FeMoco), with FeMoco serving as the site of nitrogen reduction. The contrast between the conditions of biological nitrogen fixation and the Haber-Bosch process is an enduring source of fascination for chemists.

In the search to unravel the secrets of biological nitrogen fixation, the first stage is to understand the structure of the enzyme itself. After many decades, we now possess atomic scale resolution structures of nitrogenase, including all cofactors^{5,6}. However, the electronic structure of the cofactors, and in particular the large P cluster and FeMo cofactor, remains poorly understood. This is due to the complexity of tackling the multiple transition metal ions with their multiple charge states and complicated spin-couplings. Even though the qualitative electronic structure is believed to be captured using only the valence active space of the metals and bridging S ligands which provides a great reduction of the problem size (to, for example, 103 electrons in 71 orbitals in the case of FeMoco considering the Fe 3d, S 3p, Mo 4d, and the interstitial C 2s2p for the $[\text{MoFe}_7\text{S}_9\text{C}]$ core assuming a total charge -1), no satisfactory classical many-electron simulation within this valence active space has yet been performed. Because of the need for tangible objectives for quantum simulations of electronic structure, these metallic cofactors have thus been suggested as an interesting target for future quantum simulators. Ref. 7 provides a pedagogical discussion of

the chemical questions that must be considered when elucidating a complex reaction such as nitrogen fixation, as well as concrete resource estimates resulting from 54 electron in 54 orbital (54e,54o), and 65 electron in 57 orbital (65e,57o) models of the FeMoco cluster.

Although the focus of Ref. 7 was the quantum resource estimates for this problem, it is natural to ask whether a classical calculation of the electronic structure of FeMoco at the level described in Ref. 7 is feasible. For this reason, we report that the active space in Ref. 7 does not actually contain the representative features of the electronic structure of the FeMoco ground-state that make its classical simulation difficult. Consequently, if taken out of context, it provides a misleading characterization of the classical complexity of obtaining the low-energy states. In fact, as shown in Fig. 1 for the (54e,54o) model of Ref. 7, we can obtain accurate ground-state energies ($S = 0$) using standard classical algorithms such as coupled cluster theory⁸, variational density matrix renormalization group^{9–13}, and the semistochastic heatbath configuration interaction (SHCI) method^{14–16} (a recent variant of selected configuration interaction plus perturbation theory methods). The lowest DMRG variational energy and the extrapolated SHCI energy agree to within 5 mE_h or about 0.6 mE_h per metal center (comparable to the 1 mE_h accuracy in relative energies usually considered to be chemical accuracy). Note that only modest resources were required for these calculations and higher accuracy, e.g. more variational determinants in selected CI or larger bond dimensions in DMRG, is very feasible.

As we have mentioned, the reason for the simplicity of the classical simulations is not from the intrinsic electronic structure of the FeMo cofactor but due to the active space in Ref. 7. In FeMoco, the Fe and Mo ions are expected to be in the (II), (III), or (IV) formal oxidation states^{17–22}, which leads to approximately 35 open shells (singly filled orbitals) depending on the charge state of the cluster. The prevalence of Fe(II) and Fe(III) oxidation states is supported experimentally by the Fe Mössbauer spectrum^{17–19}, and can be seen in direct theoretical calculations of smaller pieces of the FeMo cofactor, such as the $[\text{Fe}_2\text{S}_2]$ or $[\text{Fe}_4\text{S}_4]$

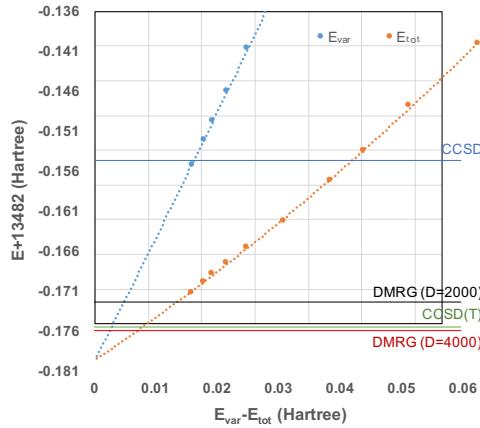


FIG. 1. SHCI variational and total energies for progressively decreasing cutoffs (dots) along with quadratic extrapolations (dotted curves) of the (54e,54o) model of FeMoco in Ref. 7 (the estimated error in the extrapolated energy is about 2 mHa); variational DMRG results at bond dimension $D = 2000$ and $D = 4000$; CCSD and CCSD(T) energies. All calculations are for the $S = 0$ state.

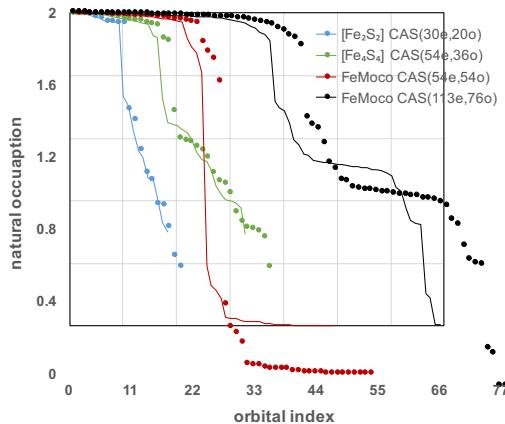


FIG. 2. Natural occupations obtained with DMRG for $S = 0$ state of a $[\text{Fe}_2\text{S}_2]$ complex with CAS(30e,20o) and $D = 8000$, $S = 0$ state of a $[\text{Fe}_4\text{S}_4]$ complex with CAS(54e,36o) and $D = 4000$, $S = 0$ state of FeMoco with CAS(54e,54o) reported in Ref. 7 and $D = 2000$, and the $S = 3/2$ state of FeMoco with CAS(113e,76o) constructed in this work and $D = 2000$. In contrast to the other models, the CAS(54e,54o) ground-state has no open shells.

clusters^{23,24}. However, the one-body density matrix in the FeMo cofactor model of Ref. 7 has no open shells, as seen from the eigenvalues of the one-body density matrix (Figure 2). A related point is that the coefficient of the dominant (natural orbital) determinant in SHCI is very large (0.67) indicating that the wavefunction has mainly single or few determinantal character, which is not possible for a low-spin system with many open shells. A large determinant weight has also been observed in Ref. 25 (in fact they observed an even larger determinant weight, probably because of using a smaller number of variational determinants). As shown in Figure 1, the CCSD(T) energy is also within a few mE_h of

the variational DMRG and extrapolated SHCI energies, confirming the single reference nature of this problem.

Although the electronic structure of the ground state within the active space of Ref. 7 is qualitatively incorrect, we nonetheless believe that the quantum resource estimates in Ref. 7, e.g. for a Trotter step, that are the primary focus of the paper, are probably reasonable and the main conclusions in that work are thus unaffected. This is because the cost of the Trotter step relies primarily on the magnitude and number of the Hamiltonian matrix elements which does not vary much with different choices of valence active space of similar size. The character of the ground-state affects the efficiency of adiabatic state preparation in the quantum algorithm, but this is left as an open problem in Ref. 7.

Nonetheless, it seems desirable to have a more qualitatively reasonable active space for future studies. For this purpose, we attach a valence active space Hamiltonian²⁶ of the FeMo cofactor constructed from all Fe 3d, S 3p, Mo 4d, and C 2s2p orbitals in the $[\text{MoFe}_7\text{S}_9\text{C}]$ core, as well as some bonding ligand orbitals. The active orbitals were obtained by first performing high-spin unrestricted Kohn-Sham calculations with the B3LYP functional^{27–29} and the TZP-DKH³⁰ basis for Fe, S, and Mo, and the def2-SVP basis³¹ for the other atoms (C, H, O, and N) using a structure in Ref. 22, and then split-localizing the unrestricted natural orbitals. This results in an active space model with 113 electrons in 76 orbitals. The detailed composition is shown in Table I and some selected localized orbitals are shown in Figure 3. The dimension of the full configuration interaction (FCI) space is on the order of $O(10^{35})$ for the spin $S = 3/2$ ground state^{17,18} in this FeMo active space. We have performed preliminary DMRG calculations to check the qualitative features of the active space. As shown in Figure 2, the natural occupation numbers obtained with a DMRG solution ($D = 2000$) for $S = 3/2$ show a large number of singly occupied orbitals, which demonstrates that this active space captures the open-shell character of FeMo in sharp contrast with the previous model⁷. While we emphasize that a detailed and chemically meaningful study on FeMo should consider many other factors, such as the convergence of the environment representation, different protonations, etc., we conclude that the active space Hamiltonian we provide contains at least a qualitative model of the open-shell character and low energy states of the cofactor. We hope this will be useful in future quantum (or classical) estimates of the complexity of FeMo cofactor electronic structure.

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TABLE I. Composition of the active space with 76 orbitals for FeMoco.

group	orbital	orbital index
left cubane		
1	left end	1, 2
2	Fe1 3d	3, 4, 5, 6, 7
3	S 3p	8, 9, 10, 11, 12, 13, 14, 15, 16
4	Fe2 3d	17, 18, 19, 20, 21
5	Fe3 3d	22, 23, 24, 25, 26
6	Fe4 3d	27, 28, 29, 30, 31
central part		
7	S 3p, C 2s2p	32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
right cubane		
8	Fe5 3d	45, 46, 47, 48, 49
9	Fe6 3d	50, 51, 52, 53, 54
10	Fe7 3d	55, 56, 57, 58, 59
11	S 3p	60, 61, 62, 63, 64, 65, 66, 67, 68
12	Mo8 4d	69, 70, 71, 72, 73
13	right end	74, 75, 76

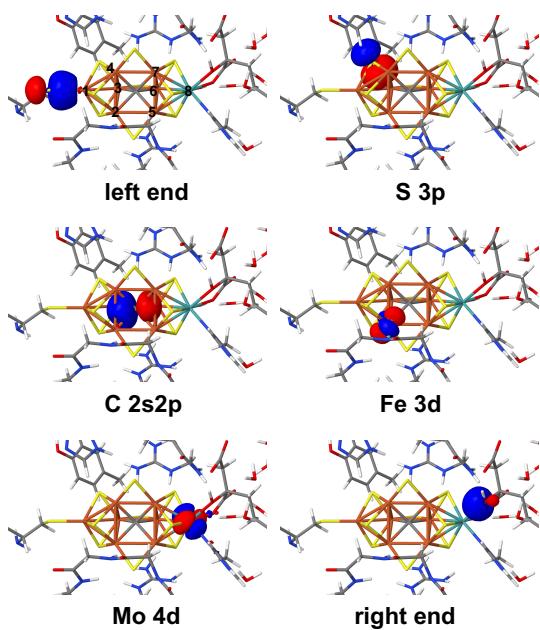


FIG. 3. Illustration of some selected active orbitals for FeMoco in the active space model CAS(113e,76o) constructed in this work.

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