Assuming that prevalence (x%) is a predictor of achievement, the achievement score (p) for a practice on the logit scale can be expressed though a simple regression as:

Logit$(p) = a + \beta x$, where a is the intercept and β is the regression coefficient quantifying the association between x and p.

Achievement on the logit scale for the same practice, if prevalence increases by 1%, becomes:

Logit$(p_{new}) = a + \beta(x + 1)$

The difference gives the change in achievement (on the logit scale) for a 1% increase in prevalence (beta values in a linear regression model): Logit(p_{new}) − Logit$(p) = \beta$

Achievement on the logit scale is related to a particular value of percentage achievement (as defined by the logit transformation):

$$\text{Logit}(p) = \ln\left(\frac{p}{1-p}\right) \quad (1)$$

Similarly,

$$\text{Logit}(p_{new}) = \ln\left(\frac{p_{new}}{1-p_{new}}\right)$$

To work backwards to get a change in percentage achievement, we assume that percentage achievement at prevalence (x+1)% is equal to the percentage achievement at prevalence x% plus a constant c, the change in percentage achievement per 1% increase in prevalence.

$$\left.\begin{array}{l} p_{new} = p + c \\ \text{Logit}(p_{new}) - \text{Logit}(p) = \beta \end{array}\right\} \overset{(1)}{\Rightarrow} \ln\left(\frac{p+c}{1-(p+c)}\right) - \ln\left(\frac{p}{1-p}\right) = \beta$$

Solving for c, we obtain:

$$c = \left[exp(-\beta)\left(\frac{1-p}{p}\right) + 1\right]^{-1} - p$$

To obtain c, we need to assume an anchor value for p, and the average achievement score across clinical units or practices is as good an assumption as any.

Fig 3 | Back-transformation explained

at improving performance, satisfaction, or safety in a low achieving setting only.

We would therefore argue that choice of an anchor score is largely at the discretion of researchers, who should use the research aims to inform their choice. However, the mean or median scores should be suitable in most scenarios and a priori justification would be needed for alternative anchor choices. Another approach could be to present back-transformed results obtained using several different anchor scores to stimulate discussion around this issue, although attention needs to be given to the interpretation of the group of results. It should also be noted that transformed scores, like percentage scores, do not account for the difficulty in meeting a specific indicator and that investigators should be careful with comparisons across indicators of varying difficulty levels. In these cases, the anchor score can be chosen to reflect the inherent difficulty for an indicator, although the relation between the anchor score and difficulty is not intuitive.

To aid researchers with use of these methods, we have made available an Excel workbook with the trans-formation and back-transformation formulas, given different anchor scores (available from the corresponding author on request or from his personal website (www.statanalysis.co.uk/files/logit_transformation.xlsx)).

Discussion

We have demonstrated the use of empirical logit transformation for the analysis of performance, satisfaction, or safety indicators that are subject to ceiling or floor effects. We have argued the benefits of this method, algebraically described the processes, provided guidance on interpretation, and have made available a simple tool to aid researchers in using the method. These methods have broad applicability in health services research, but can also be applied in other settings, for example, citizen satisfaction with urban services[16] or hotel websites.[17]

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Table 1 | Quantification of back-transformed effects of a 1% increase in diabetes prevalence, at various anchors

Increase in prevalence (%)	Effect of predictor (prevalence) on logit scale (95% CI)	Anchor achievement score (p)	Back-transformed effect of predictor on achievement score (absolute difference c) (95% CI)
1	−0.031 (−0.041 to −0.021)	0.9500	−0.0015 (−0.0020 to −0.0010)
		0.9245 (median)	−0.0022 (−0.0029 to −0.0015)
		0.7500	−0.0059 (−0.0078 to −0.0040)
		0.5000	−0.0077 (−0.0102 to −0.0052)
		0.2500	−0.0058 (−0.0076 to −0.0039)
		0.0500	−0.0015 (−0.0019 to −0.0010)