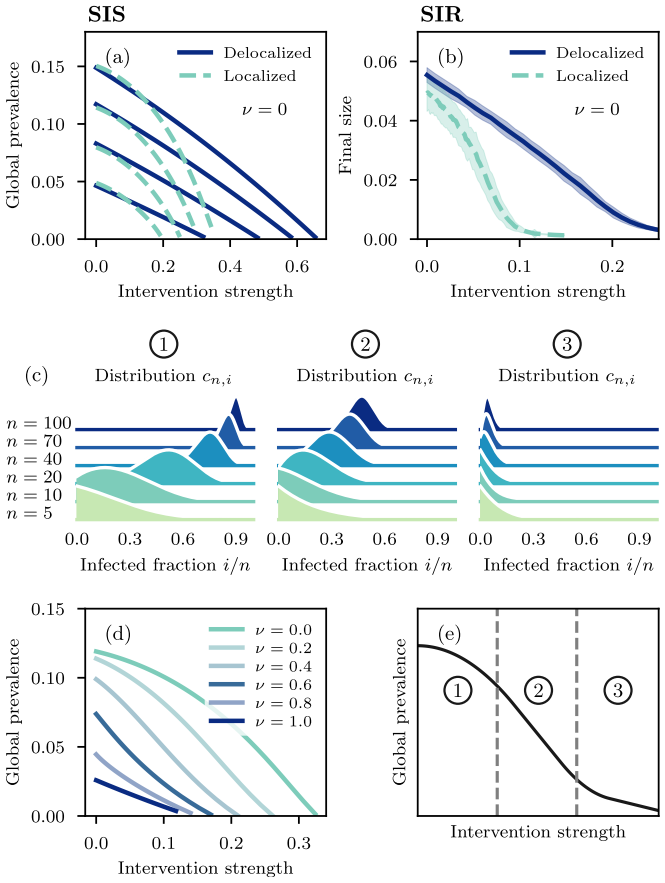

FIG. 4. **Global impact of structural interventions in different localization regimes of SIS and SIR dynamics.** (a) We compare the global prevalence of epidemics against the intervention strength [Eq. (9)] using the same networks as in Fig. 3. We use $\beta \in \{0.004, 0.005, 0.006, 0.007\}$ and $\beta \in \{0.07, 0.09, 0.11, 0.13\}$ for the delocalized and localized regimes. (b) A similar behavior is observed for the final size (fraction of recovered nodes at $t \to \infty$) in simulations of the SIR dynamics [18]. We generate 1000 networks of size $N = 10^6$ and run 10^4 SIR simulations on each network, starting with a single random infected node, with $\beta = 0.0036$ in the delocalized regime and $\beta = 0.083$ in the localized regime. We only kept macroscopic outbreaks, with a final size over 10^{-3}. Solid and dashed lines are sample means and the shaded regions represent twice the standard deviation. (c) Three examples of $c_{n,i}$ distributions for (1) strongly localized, (2) weakly localized, and (3) delocalized epidemics, using ν equal 0, 0.6 and 1 respectively. We used networks with $\gamma_m = \gamma_n = 3.5$ and $\beta = 0.3/\langle k \rangle$, where $\langle k \rangle$ is the average weighted degree before interventions. (d) We look at the impact of interventions for intermediate localization regimes by varying ν with the same networks as in (c). (e) Schematic representation of how the impact of interventions varies as it passes through different localization regimes.

Let us now look at the global impact of these interventions in Fig. 4. It is important to note that while we have assumed a SIS dynamics, our results hold for a SIR dynamics as well [see the similarity of Fig. 4(a-b)], which is generally a more realistic model for epidemics. The reason is that smeared phase transitions occur for this type of process as well [13].

In Fig. 4(a-b), we observe that the global impact of interventions in the delocalized regime is again similar to mass-action models—the prevalence decreases approximately linearly with the intervention strength. In contrast, in the localized regime, the intervention has a rapid non-linear effect in reducing the global prevalence as its strength is ramped up.

In both Fig. 3(b) and Fig. 4(a-b), it is surprising that the removal of the largest groups does not produce the largest decrease in prevalence, since these are the ones in which we expect most nodes to be infected. Even less clear is what drives the sudden collapse and if we should expect this behavior for all localized epidemics.

To clarify the situation, we need to explore the regimes between the delocalized and the *strongly* localized regimes displayed in Fig. 3(b) and 4(a-b). We break down these regimes in three parts in Fig. 4(c) to simplify the discussion: (1) strongly localized, (2) weakly localized, and (3) delocalized. In the strongly localized regime (1), large groups have a high prevalence, and act as independent entities that are barely affected by interventions elsewhere [as in Fig. 3(b)]. In the weakly localized regime (2), the disease still thrives in large groups, but they are not isolated from one another—interventions in one group now affect the others as well. This is the regime where interventions are most effective. Finally, in the delocalized regime (3), groups act as a whole, but the infection does not thrive in any of them, leading to a lower effectiveness of targeted interventions.

The easiest way to interpolate between the two extremes is to tune the group interaction strength through ν. As shown in Fig. 4(d), the initial impact of interventions changes as ν is increased. With $\nu = 0.6$ for instance, the epidemic is weakly localized, and now the removal of the largest groups produce the largest decrease in prevalence. Further interventions become eventually less effective, as the intervention itself causes the epidemic to shift to a delocalized regime.

More generally, the non-linear decrease of the global prevalence as a function of the intervention strength is explained by transitions between different localization regimes, as illustrated in Fig. 4(e). The sudden collapse for interventions on strongly localized epidemics is thus the result of a shift to a weakly localized regime, where interventions become much more effective.

Altogether, the lesson from Fig. 3 and 4 is that, just as we take heterogeneity of individual risks into account when preferentially vaccinating individuals, we should take heterogeneity of group risks into account when designing interventions. While pathogens operate at the scale of individuals, epidemics themselves interact with our entire social network, which has a modular, hierarchical, higher-order structure. Since we expect real epidemics to experience localization effects, we should aim to leverage their sudden collapse when designing structural interventions, and account for this emergent feature in our models.

Over the last few years, dynamics on higher-order representation of networks have shown time and time again that intuition built from simpler models does not always hold in more