The Scallop Theorem and Swimming at the Mesoscale

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By synergistically combining modeling, simulation and experiments, we show that there exists a regime of self-propulsion in which the inertia in the fluid dynamics can be separated from that of the swimmer. This is demonstrated by the motion of an asymmetric dumbbell that, despite deforming in a reciprocal fashion, self-propagates in a fluid due to a non-reciprocal Stokesian flow field. The latter arises from the difference in the coasting times of the two constitutive beads. This asymmetry acts as a second degree of freedom, recovering the scallop theorem at the mesoscopic scale.

The time-reversibility and linearity of the Stokes equation require microswimmers to deform in a non-reciprocal fashion in order to swim, a rule known as the *scallop theorem* [1]. Many strategies in the Stokesian regime, requiring at least two degrees of freedom for successful propulsion, have been intensively investigated in the past decades [2–10]. This provided a fundamental understanding of the underlying dynamics as reflected by the emergence of several technological applications [11–16].

A natural way to break down the *scallop theorem* is by introducing inertia. This is commonly achieved by the inertial dynamics of the fluid [17, 18], here characterized by the Reynolds number $\mathrm{Re_f} = \rho_f L\overline{U}/\eta$ (ρ_f and η being the fluid density and viscosity, L the swimmer body length, and $\overline{U}$ the average swimming speed). For example, this can be achieved by using steady streaming [19–21], generation of vortices [22], or turbulent flows [23]. The possibilities for the exploitation of the swimmer's own inertia, however, are still subject to debate [24]. Interestingly, recent experiments and simulations have shown that mesoscopic structures, i.e. 100μm up to 1cm in scale, display coasting effects [25–27], while generating fluid flows with a time-reversible behavior [28, 29]. Those observations point to the possible existence of a swimming regime at low-$\mathrm{Re_f}$ where the inertia of the so-called mesoswimmer dominates and generates the motion, a hypothesis which warrants further investigations.

A minimal mesoswimmer that can verify this hypothesis is an asymmetric dumbbell consisting of two different interacting beads, driven in a force free manner (Fig. 1). The Reynolds number $\mathrm{Re_s}$ of such a swimmer is set by its bead density ρ_s, its bead size a, and its beating frequency ω such as $\mathrm{Re_s} = \rho_s a^2\omega/\eta$. This design possesses only one internal degree of freedom which leads to a reciprocal deformation, and therefore cannot swim without the help of inertia [19, 20, 24]. Assuming that the Reynolds number of the fluid $\mathrm{Re_f} \ll 1$ and of the swimmer $\mathrm{Re_s} \sim 1$, the flow should be dominated by the fluid viscosity while the propulsion mechanism should be related to the coasting time of the swimmer, which we define as $\tau = m/(6\pi\eta a)$,

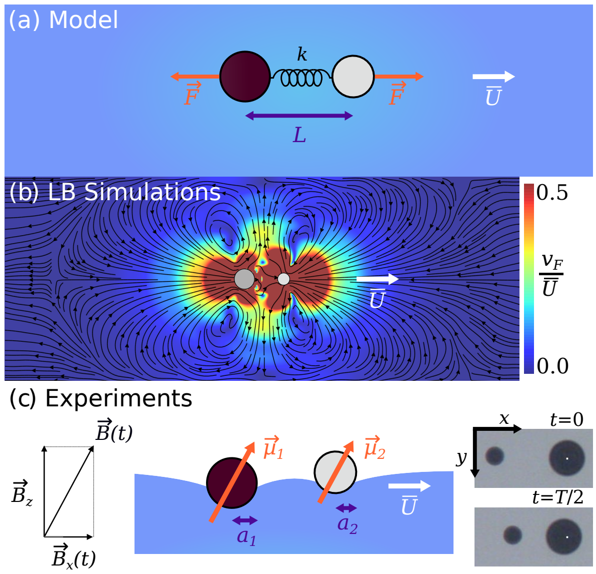

FIG. 1. **Asymmetric swimming dumbbell.** (a) The model assumes two spheres of density ρ_s with radii a_1 and a_2 at a distance L connected by a linear spring with constant k. The device is submerged in a fluid of viscosity η and driven by sinusoidal forces of same amplitudes F and frequencies ω acting in opposite directions. (b) An equivalent system is addressed by lattice Boltzmann simulations ($a_1 = 5$, $a_2 = 8$, $\eta = 1/6$, $k = 1/50$, $L = 28$, $\rho_f = 1$, and $\rho_s = 8$, all expressed in lattice units *l.u.*). The simulation box is discretized by $400 \times 160 \times 160$ lattice nodes. The background shows the flow field averaged over one cycle of the external sinusoidal forcing ($F = 0.1$, $\omega = 1.57 \times 10^{-3}$). (c) A magneto-capillary dumbbell is made of two ferromagnetic beads (magnetic moments $\vec{\mu}_i$, radii $a_i \in (397, 500, 793)\,\mu$m, and density $7830\,\mathrm{kg/m^3}$), pinned at the water/air interface. The two beads separation L is about 1400μm, and sets by the balance of capillary attraction and magnetic dipole repulsion. The device is driven by an external magnetic field $\vec{B}(t) = B_z\vec{e}_z + (B_0 + b\sin(\omega t))\,\vec{e}_x$, which induces small oscillations of the beads. Snapshots are from the beginning of the cycle and half way through.

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