Course Name: Digital Hardware Design
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Finite State Machine-1

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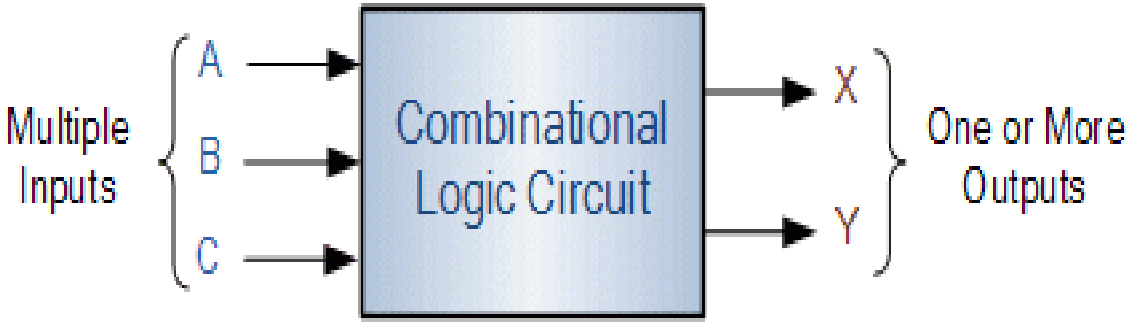
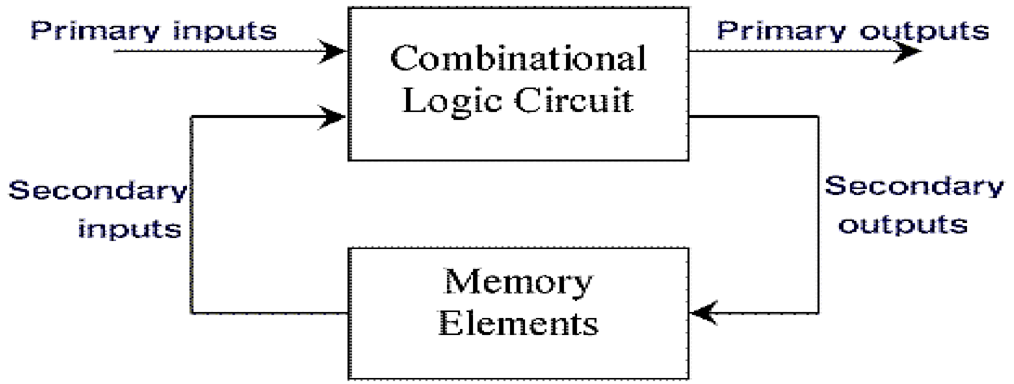
Introduction(Cont.)

- **Logic circuits can be classified into two types:**
  - **Combinational (No memory)**
    - **Outputs are based on the combination of inputs.**
  - **Sequential (With memory)**
    - **Outputs are based on memory (stored data) and inputs.**



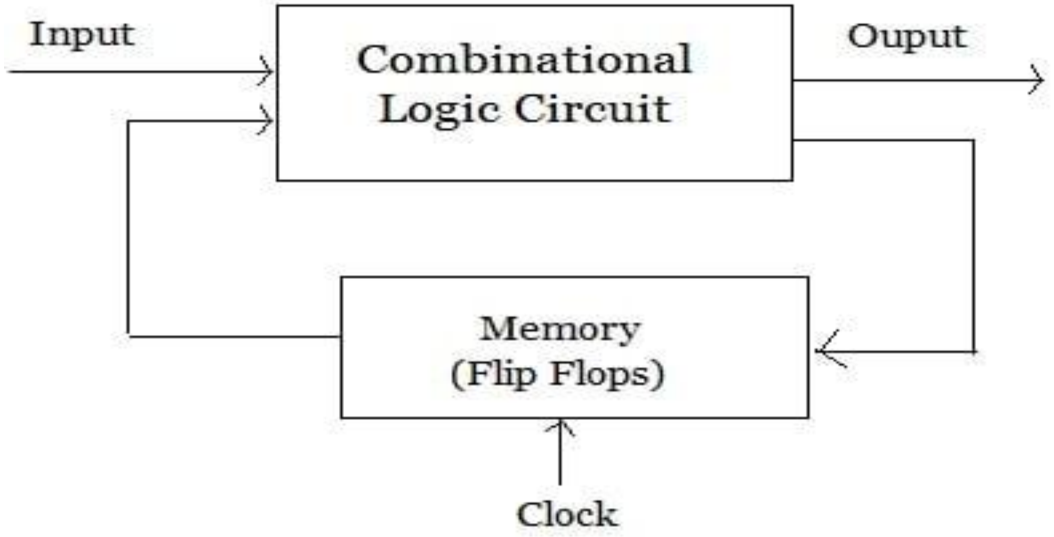
Output = f(input)



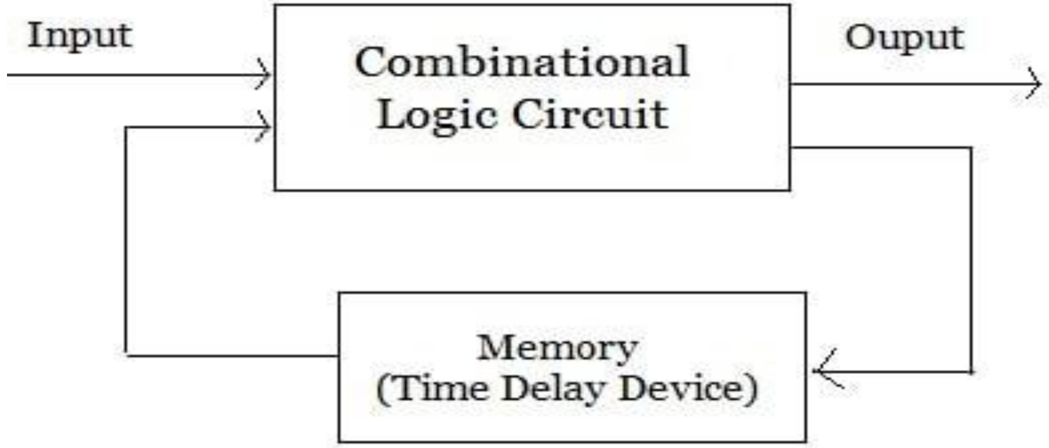
Primary O/P =f(primary I/Ps + Secondary I/Ps

Types of Sequential Circuits

- Synchronous Sequential Circuits
  - Behaviour depends on the input at a discrete time
  - Use clock pulses in the inputs of memory elements are called clocked sequential circuits.

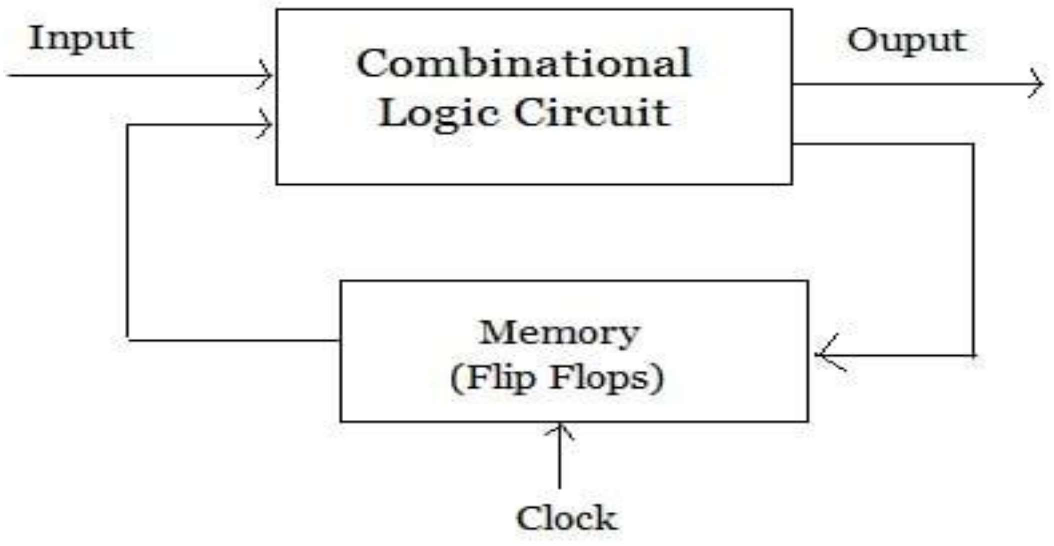


- Asynchronous Sequential Circuits
  - Output depends on the sequence of input changes called asynchronous sequential circuits.



State Machine

- Synchronous Sequential Circuits
  - As its present O/P depends on present and previous sequence of inputs, its functioning cannot be explained by a simple truth table.
  - State tables and state diagram are normally preferred to describe the working.
  - Necessary to know the internal state of the circuit.



- Internal state is determined by the type of latches, flip flops, counters, shift registers, or memories used in the circuit.

State Machine

Few Definitions:

Input variable – All external input variables (A).

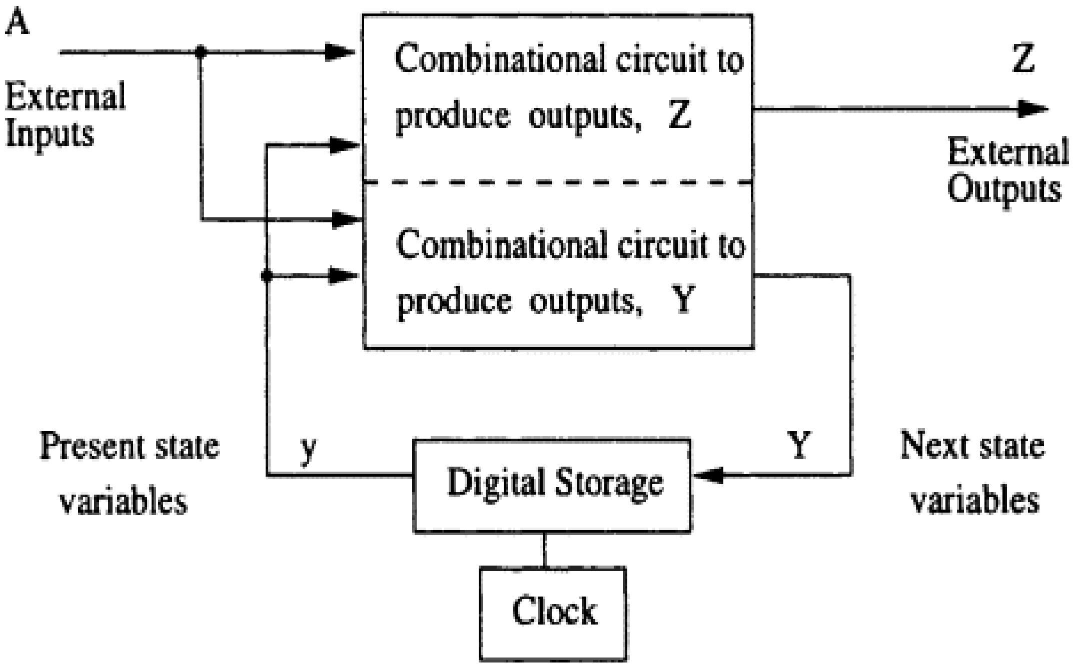
Output variable – All external output variables (Z).

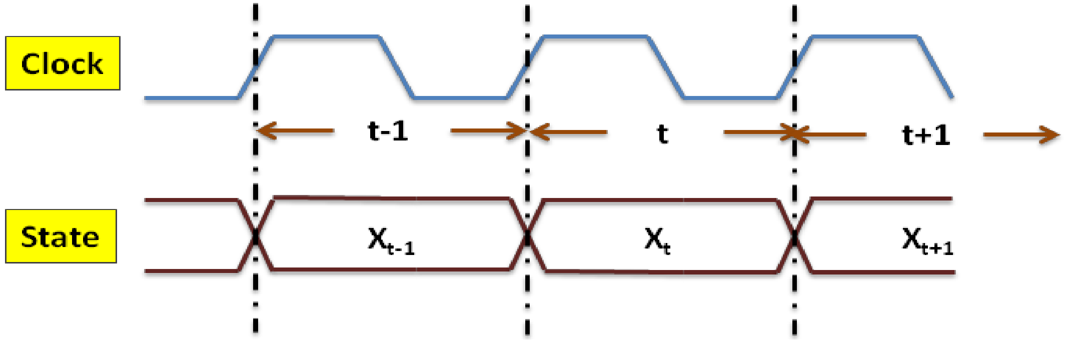
State variable – output of memory defines the state of sequential machine and known as state variables(y).

Excitation variable – defined as the inputs to the memory(Y). It excites the memory to change its state.

State – defined by Q outputs.

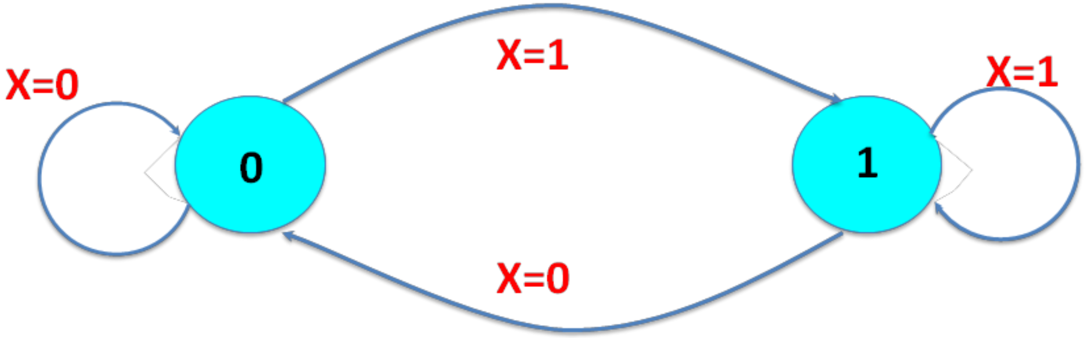
Present state and Next State –state variables at time 't', is a **present state. At** time 't+1' after the clock edge application is called a **next state**. Present state and next states are used to prepare state table of the state machine.





State Diagram and Table

- Circuit with two stable states '0' and '1' and an external input X.
- If input X =0 then the circuit remains in the state '0'.
- If input X becomes 1, then the circuit makes a transition from state 0 to state 1.
- Draw the state diagram and table.

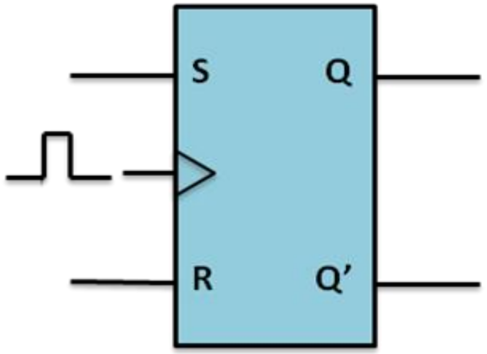


Present State (Qn)	Next State (Qn+1)	
	X=0	X=1
0	0	1
1	0	1

Sequential Storage Units

- A flip-flop (FF) (SR, JK, D, T) is a simple memory storage component that stores the binary information.
- FFs follow a sequence of internal states.
- Characteristic table defines the state of each FF as a function of inputs and previous state.

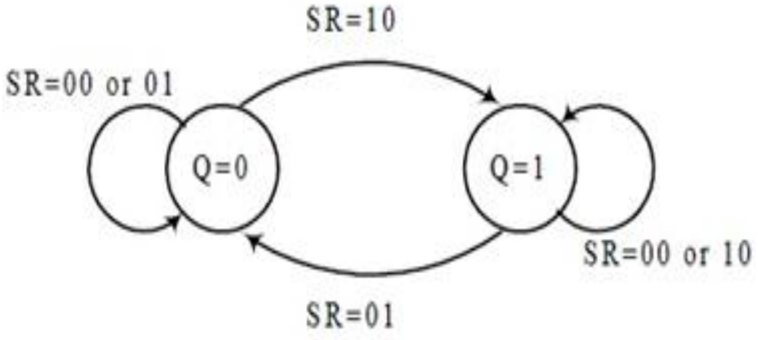
SR flip flop



Characteristic Table

Q	S	R	Q(t+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	X
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	X

State Diagram



Excitation Table

Qn	Qn+1	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

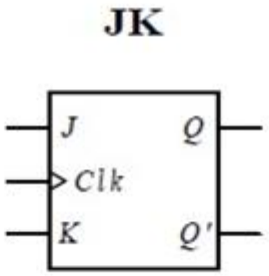
Characteristic Equation

$$Q(t+1) = S + R'Q$$

Sequential Storage Units

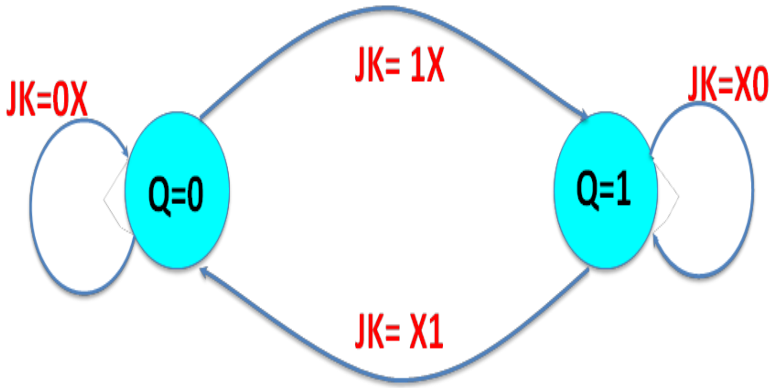
- Reset state (Q=0) and JK inputs are 0X, FF remains in the same state.
- If JK inputs are 1X then FF makes 0 to 1 state transition.
- Set state (Q=1) and JK inputs are X0, FF remains in the same state. B
- If JK inputs are X1, then FF makes 1 to 0 state transitions.

JK flip flop



J	K	Q	Qnext
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

State Diagram



Excitation Table

Present State	Next State	Inputs	
Q	Q(t+1)	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Characteristic Equation

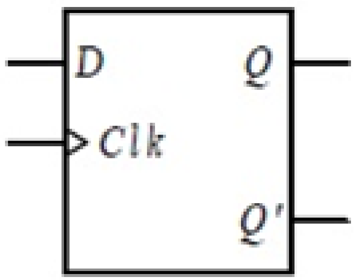
$$
\begin{aligned}
Q_{next} &= J'K'Q + JK' + JKQ' \\
&= J'K'Q + JK'Q + JK'Q' + JKQ' \\
&= K'Q(J'+J) + JQ'(K'+K) \\
&= K'Q + JQ'
\end{aligned}
$$

Sequential Storage Units

- Reset state when Q=0 and and D input is 0. FF in same state.
- But if D input is 1 then FF makes 0 to 1 state transition.
- Set state when Q=1 and D input is 1, FF in same state.
- But if D input is 0, then FF makes 1 to 0 state transitions

D flip flop

D

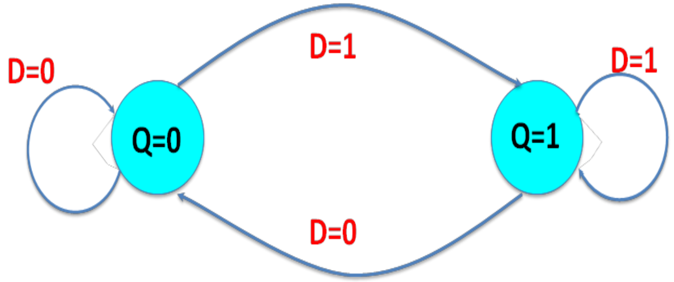


D	Q	Qnext
0	×	0
1	×	1

Excitation Table

Present State	Next State	Inputs
Q	Q(t+1)	D
0	0	0
0	1	1
1	0	0
1	1	1

State Diagram



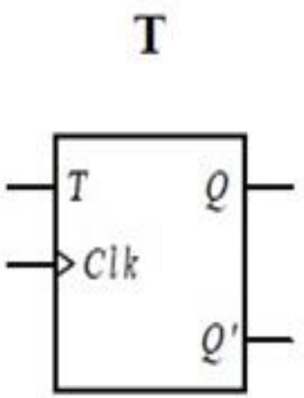
Characteristic Equation

$$Q(t+1) = D$$

Sequential Storage Units

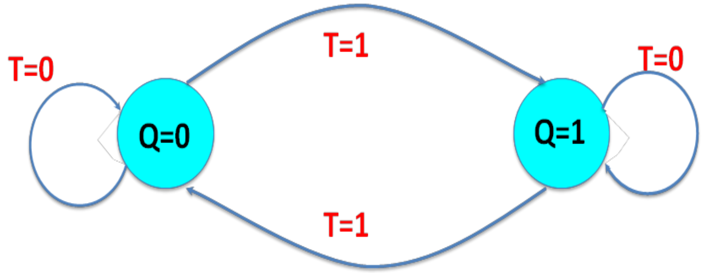
- Reset state when Q=0 and and T input is 0. FF in same state.
- But if T input is 1 then FF makes 0 to 1 state transition.
- Set state when Q=1 and T input is 0, FF in same state.
- But if T input is 1, then FF makes 1 to 0 state transitions

T flip flop



T	Q	Qnext
0	0	0
0	1	1
1	0	1
1	1	0

State Diagram



Excitation Table

Present State	Next State	Inputs
Q	Q(t+1)	T
0	0	0
0	1	1
1	0	1
1	1	0

Characteristic Equation

$$Q(t+1) = T \text{ XOR } Q$$
$$= T'Q + TQ'$$

Finite State Machine

- A state machine is a behavioral model. It consists of a finite number of states and is therefore also called a Finite-State Machine (FSM).

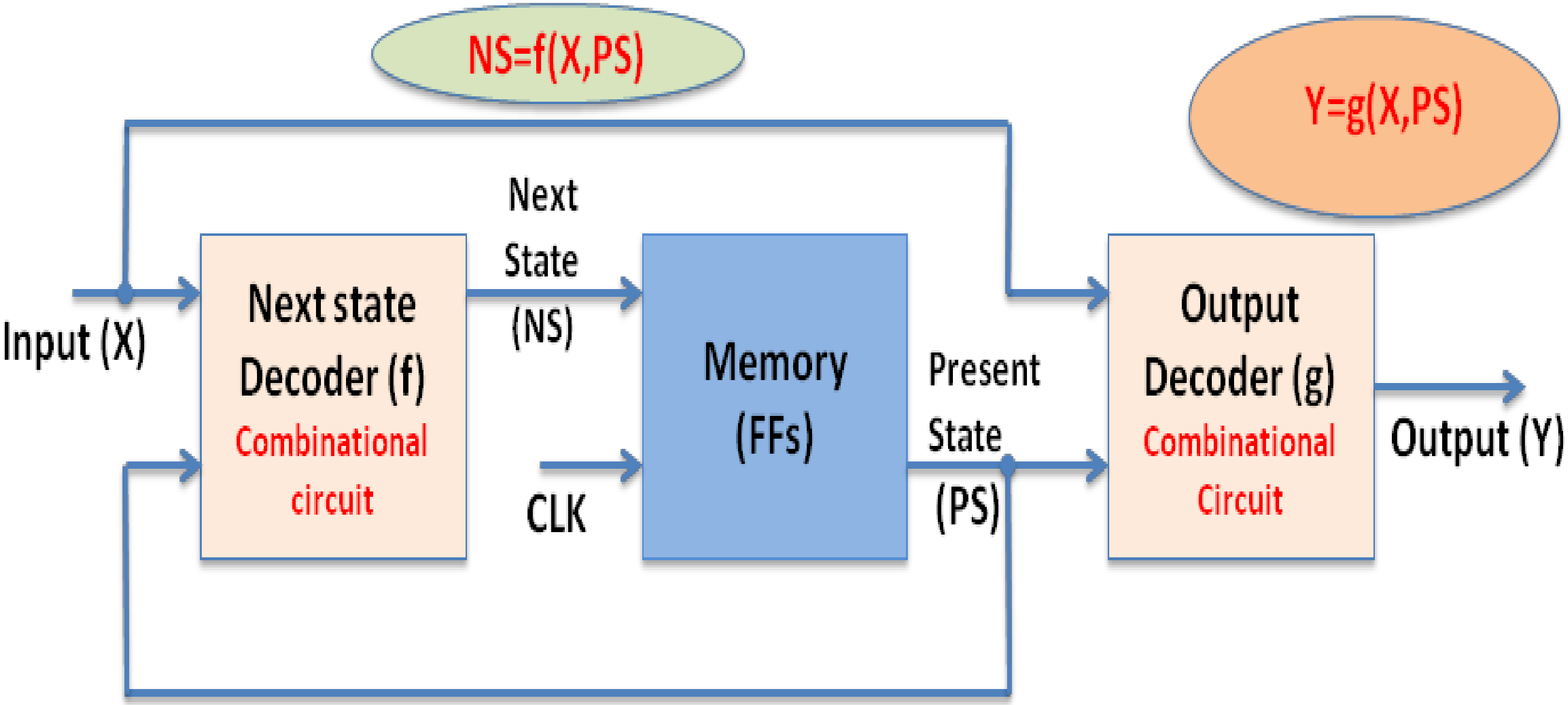
- Based on the current state and a given input the machine performs state transitions and produces outputs.

- The outputs and the next state are both functions on the applied inputs and the present state.

Types of FSM

Mealy Machine

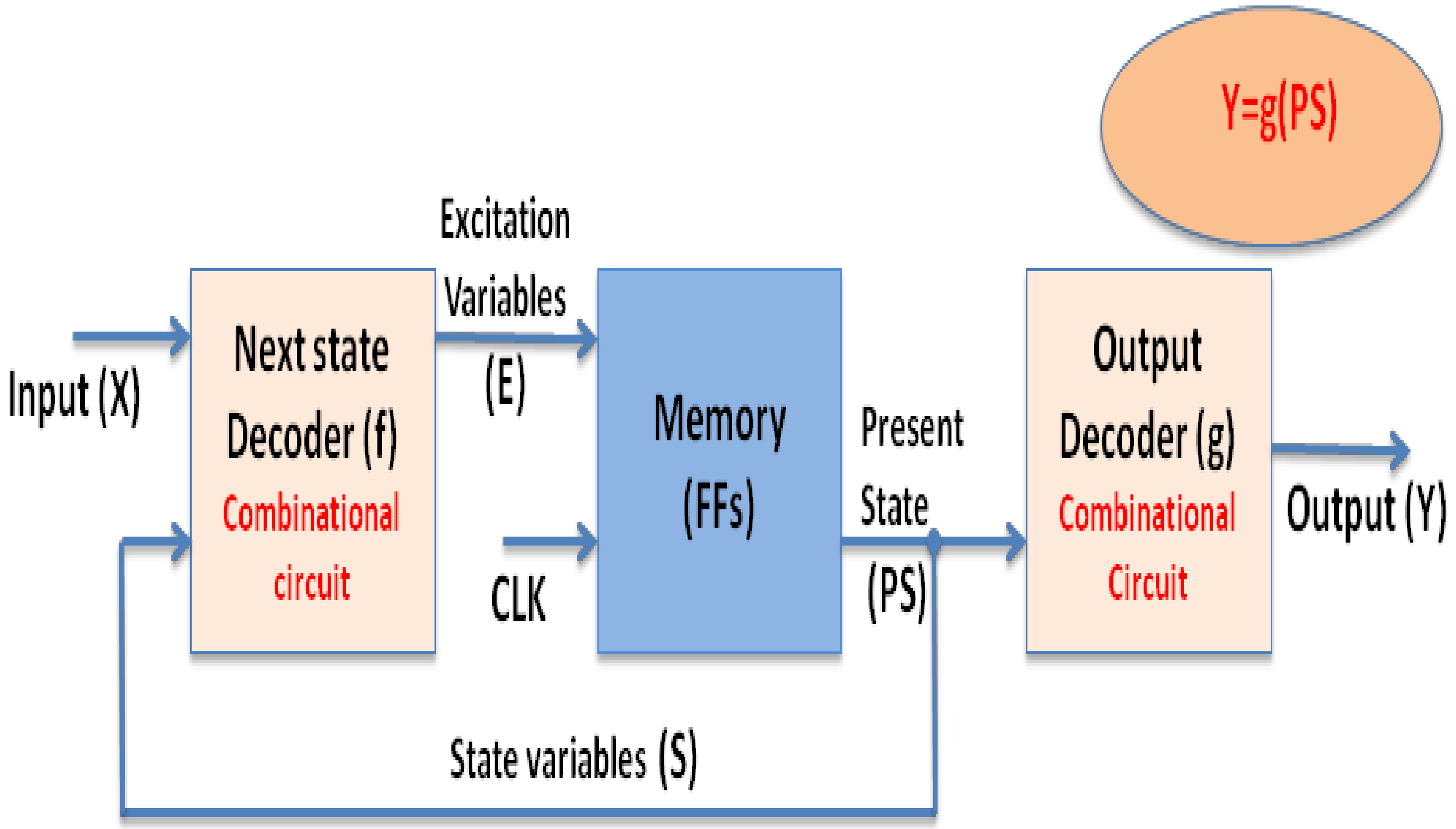
- Three major blocks, next state decoder, memory and output decoder.
- The output is a function of present input conditions (X) and the present state (PS) and defined as Y= g(X,PS).



Types of FSM

Moore Machine

- its output is strictly a function of the present state (PS) of the sequential circuit and is defined as Y=g(PS).



State Equations

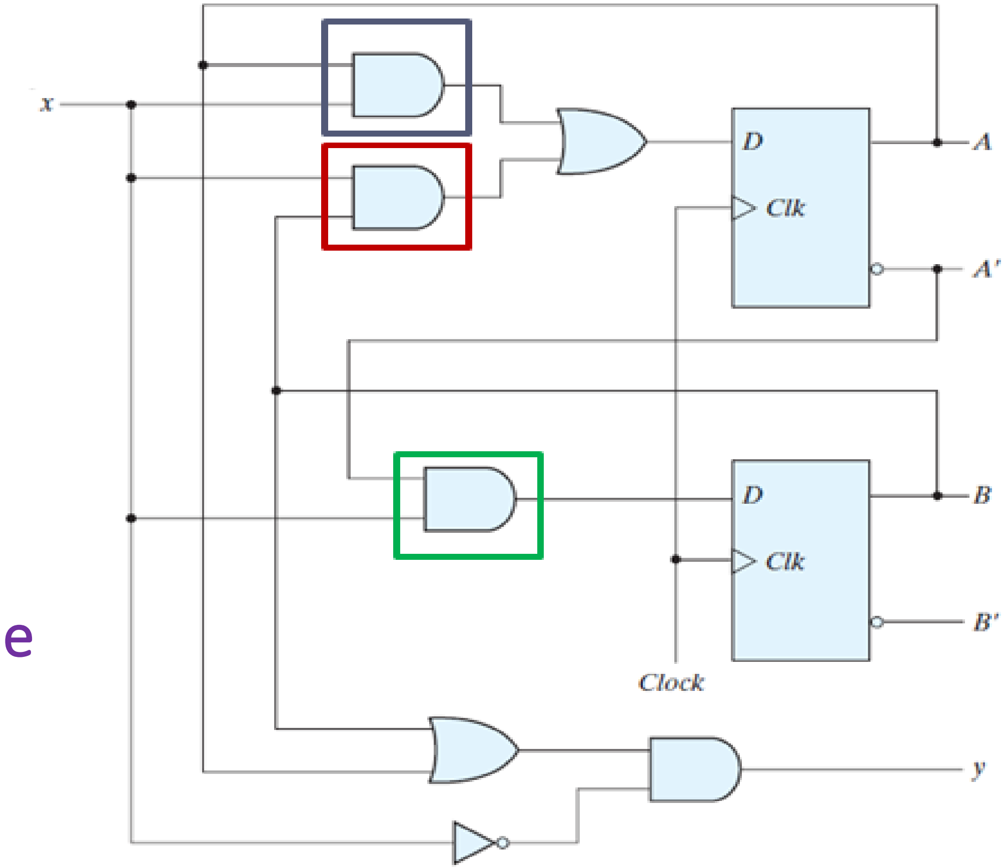
State Equations: The algebraic equations through which clocked sequential circuits are described. It is used to describe the next state as a function of present state.

Example:

The circuit has input x, output y and 2 flip flops A and B.

$$A(t+1) = A(t)x + B(t)x$$

$$B(t+1) = A'(t)x$$



A state equation specifies the condition for a FF state transition.

State Equations

- For simplicity (t) will be omitted from the present state. Hence the state equation become:

$$A(t+1) = Ax + Bx$$

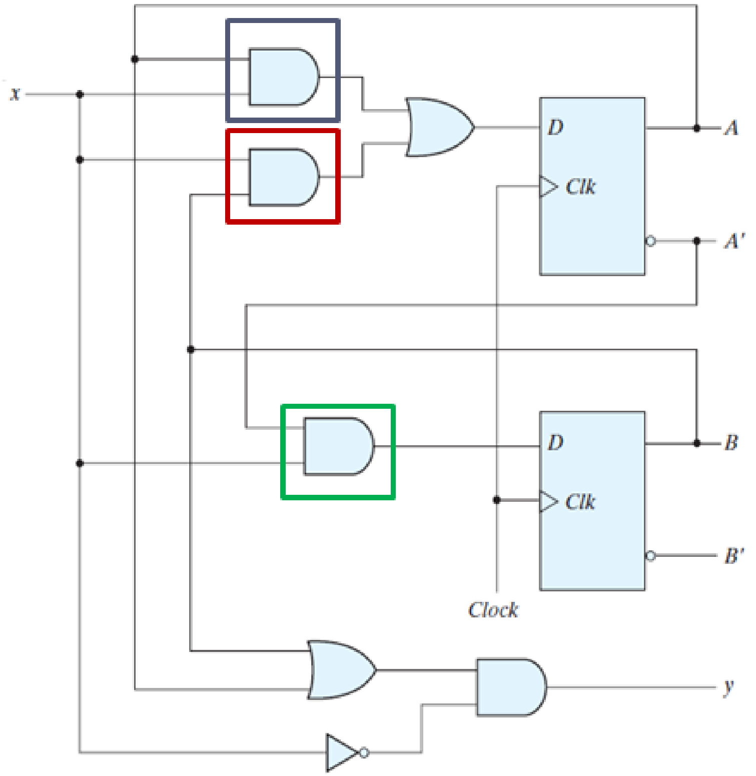
$$B(t+1) = A'x$$

- Consequently, the output is:

$$y(t) = x'(t)(A(t) + B(t))$$

- Becomes

$$y = x'(A + B)$$



State Table

The time sequence of the inputs, outputs and FF states are listed in a table called ***state table*** or ***transition table*** .

2 FFs with one input x will make 8 entries in transition table

State Table

Present State		Input	Next State		Output
A	***B***	***x***	***A***	***B***	***y***
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

$$y = x'(A+B)$$

State Table

Second Form of the State Table

Present State		Next State				Output	
		$x = 0$		$x = 1$		$x = 0$	$x = 1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

In general, for a sequential circuit with m FF's, n inputs, and o outputs, the state table has:

- 2^{m+n} rows.
- m next state columns
- o output columns.

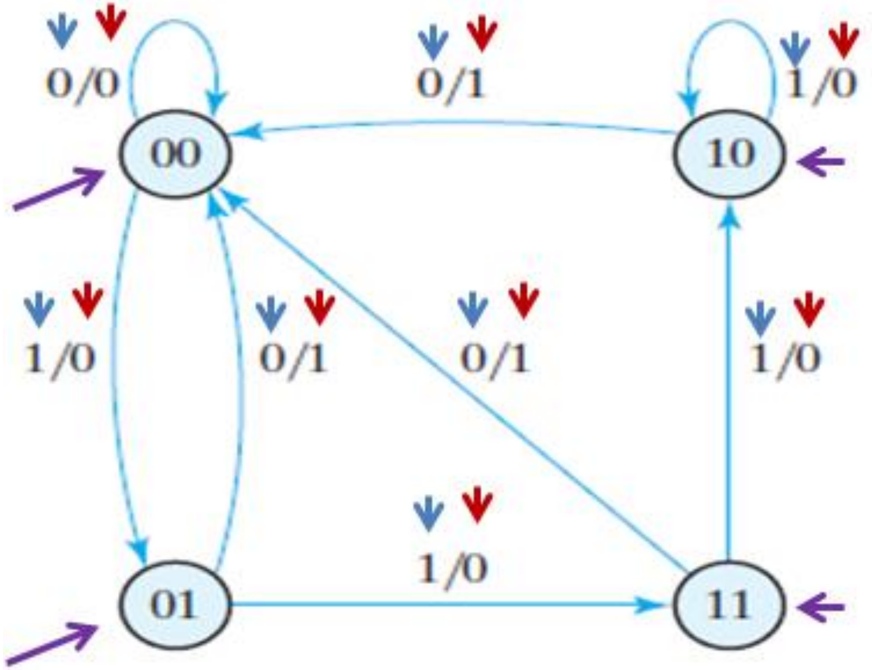
State Diagram

- State diagram is a graphical presentation of the information available in a state table
- The state of the FF’s is represented by a binary number inside a circle.
- The clock triggered transitions are represented by labeled directed lines connecting the circles. The labels are two numbers separated by a slash:
  - The number before the slash is the input value during the present state.
  - The second number after the slash is the output during the present state with the given input .

State Diagram

Second Form of the State Table

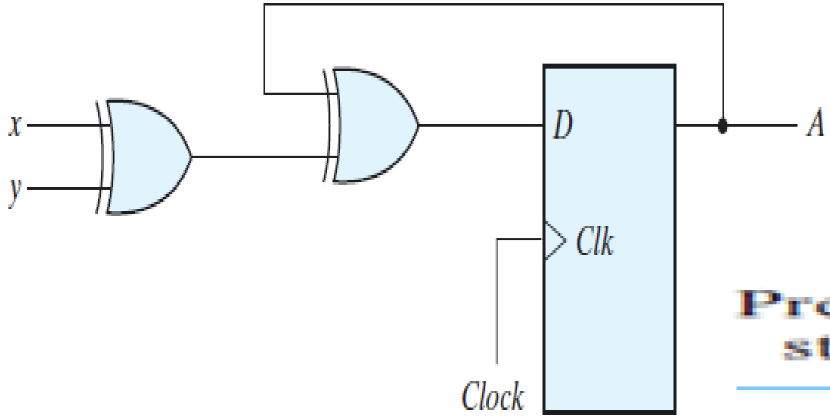
Present State		Next State				Output	
		$x = 0$		$x = 1$		$x = 0$	$x = 1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0



Final Steps

Circuit diagram → Equations – State table → State diagram

Example 2



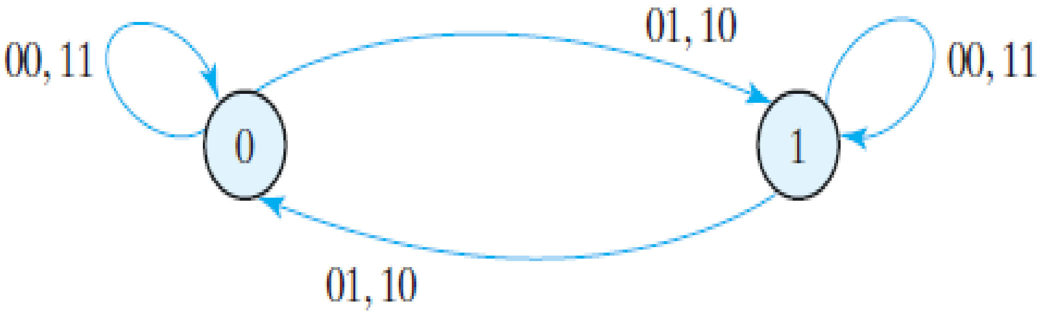
Circuit diagram

$$A(t+1) = A \oplus x \oplus y$$

Present state		Inputs		Next state
A		x	y	A
0	1	0	0	0
0	2	0	1	1
0	3	1	0	1
0	4	1	1	0
1	5	0	0	1
1	6	0	1	0
1	7	1	0	0
1	8	1	1	1

State table

Example 2



State diagram

Present state		Inputs		Next state
A		x	y	A
0	1	0	0	0
0	2	0	1	1
0	3	1	0	1
0	4	1	1	0
1	5	0	0	1
1	6	0	1	0
1	7	1	0	0
1	8	1	1	1

State table

More on FSM