<u>Logical Reasoning and Inequalities (18B12MA312)</u>

Case Study Based Project

On

“Mathematical Puzzles and its Applications”

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INDEX

INTRODUCTION

A puzzle is a game, problem, or piece of information about any facts, clues, etc., that testsa person's ingenuity or knowledge. A person is required to put pieces or the given information together in a logical way to arrive at the answer. Puzzles check the intellect, common sense, knowledge, awareness, logic and reasoning ability of a person. Crossword puzzles, word-search puzzles, number puzzles, relational puzzles, and logic puzzles are different genres of puzzles. Theacademic study of puzzles is **enigmatology**.

It helps to improve memory, solving puzzles help reinforce existing connections between our brain cells. It improves our problem-solving skills, improved visual-spatial reasoning, increased IQ, lowers stress level. It also helps to increase our attention to the details of given problem.

It is an interesting way to learn many mathematical concepts because solving a mathematical puzzle is close to solving a mathematical problem. It is a powerful way to develop the culture of mathematical thinking, based on algorithms, logic, abstraction and spatial images forpupils.

Thus, the main purpose of the study of Mathematical Puzzles is to determine methodological features of puzzles in teaching mathematics. To model the methodical system of teaching mathematics with the use of new didactic tools like puzzles for additional mathematical knowledge.

METHODOLOGY

Mathematical puzzles are sometimes used to motivate students in teaching elementary school math problem solving techniques. Creative thinking – or "thinking outside the box" – often helps to find the solution. Research shows that children who play with puzzles are better able to imagine what something would look like if it were changed, such as rotated or flipped. These spatial skills support children's understanding of math and science and have been shown to predict children's success in the STEM disciplines

Our research methodology is focused on puzzles as a tool for teaching students in general and extra mathematics education, with the goal of increasing students' interest in mathematics and developing their mathematical skills such as ability to think logically, ability to powerful abstract thinking, combinatorial abilities, abilities to spatial imagination and manipulation with spatial images, ability to critical thinking, mathematical memory as a specific ability of mathematical activity.

a)Puzzles as a tool for teaching Mathematics

Classical methods of teaching mathematics have a clear structure of the lesson steps. The introduction of new elements changes the type of lesson and the lesson becomes innovative. However, in the structure of classical lesson of mathematics you can find a place for puzzles as a tool for development mathematical abilities. This, in our opinion, may occur at one of three stages of the lesson.

At the *actualization of knowledge stage*, one can organize the work with pupils aimed to prepare them to study new material. Thus, for example, in studying the characteristics of divisibility in the 6-th grade, we can use arithmetic puzzles. They will not only aid in the development of logic, combinatorial abilities, critical thinking, and mathematical memory, but will also aid in the development of logic, combinatorial abilities, and critical thinking.. In addition, the puzzles allow the application of tables of addition and multiplication.

b)Using Puzzles in Cfct-Tips System

In conducting non-traditional math lessons, we focus on the implementation of the CFCT- TIPS system (continuous formation of creative thinking based on the theory of inventive problem solving), developed by Professor Zinovkina (2008). Here the author describes the subsystems of creative pre-school, school, secondary vocational, higher and postgraduate education. The structure of these subsystems includes the stage of puzzle solving as one of the main ones. There is a so-called creative math lesson in *school education*.

It should also be noted that CFCT-TIPS system is intended to include puzzles as a separate stage of lesson in the educational process even in preschool education. Our manual “Exciting game with Sovionok” for pre-school education (Zinovkina, Gorev, & Utomov, 2015) presents a system of such lessons. Puzzles are used in each lesson, composing a system of increasingly complex tasks embodied in real objects. We should also note that puzzles at the creative lesson may be used at the stage of motivation (“Meeting with a miracle”), since many of the object puzzles are unique ideas based on physical or mathematical principles that have always struck the imagination of the people, and not only at pre-school age. For example, at the motivation stage, children can be demonstrated wire or jigsaw puzzles made of wood or plastic, which have the idea of ingenious assembly or hinge puzzle like Rubik’s Cube. All this creates a strong interest to the study of mathematics with the effect of a miracle at the same time, which later can lead pupils to the serious study of puzzles as mathematical problems.

a) Puzzles as a Didactic Tool in Additional Mathematical Education

Basic structural unit of additional mathematical education for pupils is a *mathematical club* (elective, studio). During club activities we may consider puzzles as separate math problems and a series of puzzles. For example, the lesson can be devoted to the study of Mobius band properties or flexagons, topological puzzles with rope and buttons, or be constructed on effects of impossible figures and optical illusions. For children of a younger age, it is recommended that a series of classes be devoted to tangible puzzles such as "Tangram," pentomino, "Towers of Hanoi," tracing figures with one stroke, and so on. It allows students to concentrate on the puzzle's problem and comprehend its structure. And these features enable students to improve not only their knowledge of the puzzle's mathematical essence, but also their logical and combinatorial abilities, their capacity for abstract thinking and functioning with spatial representations, their critical thinking, and their mathematical memory. It should be noted that our mathematical club program “Lessons of developmental mathematics”, worked out and tested for pupils of 3-6-th grades (Gorev et al., 2017), presents a technological model of club lessons by modules. Each module comprises units, based on solving puzzles.

In the article (Gorev & Novoselova, 2017), the process of adding the produced technical model in supplementary mathematical education for 5-6th grade students is discussed, and the didactic support of the technological model is offered in the books "Lessons of developmental mathematics. "Mathematical club assignments for 5-6th graders" (Gorev & Utemov, 2014) and "Twenty difficult riddles by Sovionok" (Gorev & Utemov, 2014). (Gorev & Utomov, 2015).

Another important component of additional mathematical education are mathematical competitions. Puzzles can become their integral part as specific tasks. So, we have successfully used puzzle challenges in mathematical battles, team competitions, such as “Mathematical Carousel”, “Mathematical skirmish”, “Mathematical auction” and others. However, separate competitions based on puzzles may also be organized.

Another area of additional mathematical education for schoolchildren is the work of the summer mathematical camp. Kirov city lyceum № 21 has organized the camp “Mathematics. Creation. Intellect” for pupils of 5-10-th grades since 2001. It is aimed first of all to develop mathematical abilities of pupils by various means: from unusual tasks to project activities. In the 5-6-th grades, the work with puzzles is performed in two main directions.

Table 1. Project clubs and workshops for students of 5-6 grades

Name of the club	***Project objectives***	***Final product***
Do not believe your eyes	To explore and present geometric illusions, non-existent in the real-world figures that can only be pictured	Album with illusions
Hot cockles	Explore the history and features of the game “hot cockles” and create your own games like this one	Set of games
Magic Tetris	Consider different types of polyominoes (domino trimino, pentamino with square, triangular, hexagonal elements) and tasks associated with them	Sets of polyominoes, wall newspaper
Supercubes	Explore the history of the emergence and creation of supercube, different versions of the game with it and putting figures together	“City” of the supercube’s parts
Finding Minotaur	Explore the history of legends and myths associated with labyrinths, their types and algorithms of escape from labytinths	Labyrinths, wall newspaper
3D tasks	To study and learn how to assemble a variety of three-dimensional object puzzles	Set of puzzles and wall newspaper about them
Flexagons	To study different types of flexagons, the history of their origin and creation	Sets of flexagons and wall newspaper about them

RESEARCH

The research had three stages. At the first stage, the state of the problem was revealed inthe theory and practice of teaching students in basic and additional mathematical education,

implemented in primary and basic education. For this purpose, the study and analysis of psychological, pedagogical and methodological literature on the research problem, observation and analysis of the experience of teachers of mathematics and primary school were carried out in order to explore possible ways of including puzzles in the process of teaching mathematics and the likely organizational forms of organizing such activities with children and adolescents for effective continuous formation of personality, who possess basic types of mathematical abilities: logical thinking, abstraction, combination, operation of spatial images, criticality of thinking, mathematical memory. At the second stage, there were developed methodical approaches to the implementation of the didactic tools based on puzzles and the mechanisms for its implementation within the framework of the summer mathematical camp, as well as during the academic year in the classroom and extra-curricular activities. Discussion of their implementation was carried out and continues to be carried out during the work of seminars and round tables with teachers of mathematics in the region, reports at conferences and seminars of various levels, which leads to a consistent improvement of the methods of working with puzzles in the practice of teaching mathematics to children and adolescents. In parallel with the second, the third stage is realized, during which the experienced teaching is conducted on the proposed methodological aspects on the basis of lyceum No. 21 of the city of Kirov.

40 pupils of Kirov city lyceum № 21 aged 11 - 12 years (5-6-th grades) took part in the experiment. They were divided into two groups of 20 pupils who participated in all stages of the experiment. Pupils of 5-a class were Group A in the 2015 / 2016. They studied in the lyceum from the first grade and during the education process in 4-th grade they attended a mathematical club where the method “Lessons of developmental mathematics” worked out by the authors (Gorev et al., 2017) was used and puzzles were one of the main didactic tools. Pupils of 5-g class were Group B. They didn’t attend this mathematical club.

Diagnostics at all stages of the experiment was carried out using a battery of five tests in oreder to determine the level of cognitive development (intelligence quotient IQ): tests for abstract thinking (AT); tests to determine the mathematical abilities (MT); perceptual tests (PT); verbal tests (VT); tests to assess the ability to spatial-visual thinking (SV). Brief designations of tests are shown in tables and diagrams. Input diagnostics on IQ tests was conducted at the beginning of the 5th year of study in 2015 / 2016. Table 2 and Figure 1 show its results in a proportion of the number of pupils who have reached a certain level of certain types of abilities formation, and the average score (the conversion tables are presented in the book (Eysenk & Evans, 1998)) for each test category. Scoring was carried out on a scale for 11 year-old children; the average score is rounded up to the nearest whole number by the accepted rules of rounding.

Table 2. Results at the beginning of the experiment 2015 / 2016, 5-th grade (14/09/2015)

	Group A (20)					Group B (20)				
	AT	MT	PT	VT	SV	AT	MT	PT	VT	SV
Perfect	0	0	0	0	0	0	0	0	0	0
Excellent	1	3	2	1	0	1	2	1	0	0
Very good	3	4	2	3	4	3	4	2	4	3
Good	6	7	4	10	8	4	6	6	8	6
Average	5	3	8	3	5	7	4	7	6	7
Satisfactory	5	3	4	3	3	5	4	4	2	4
The average score of the group	**60**	**43**	**58**	**40**	**50**	**54**	**40**	**57**	**40**	**46**

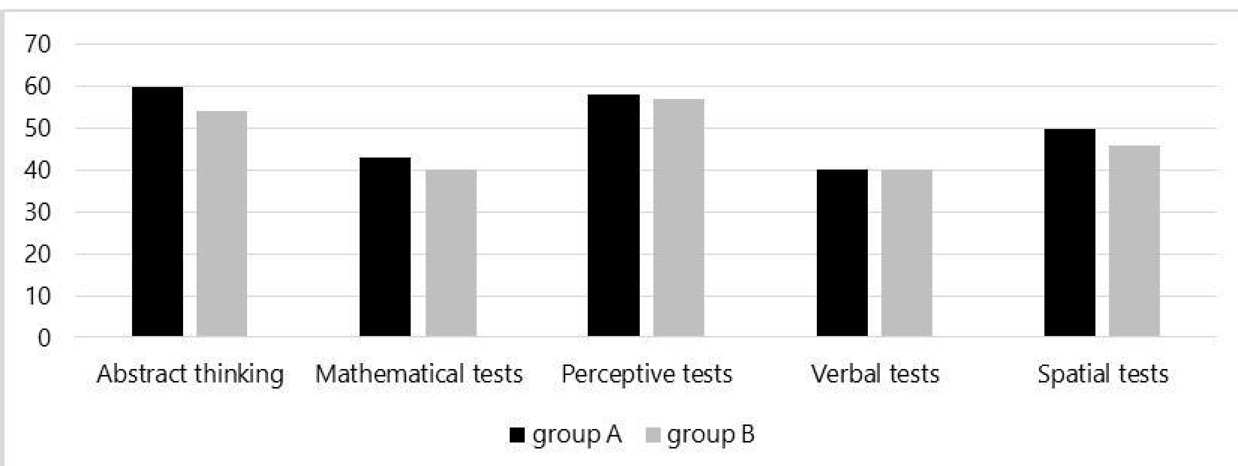

Figure 1. Comparison of average scores in the groups at the beginning of the experiment

As we can see from the chart, there is little difference in the results of perceptual and verbal tests. This is likely due to the fact that the pupils of the 5-th grade after 4 years of primary school have little distinction in perception and verbal presentation of information. The difference in solving mathematical tests, abstract tests and tests on spatial-visual thinking refers to the results of the work with group A in the previous period with the use of didactic tools, based on puzzles.

During the next academic year both groups had the same teacher and the same textbooks. Pupils of 5-a class attended a mathematical club based on the method "Lessons of developmental mathematics" (1 hour a week), where puzzles were used as a didactic tool. During the first two weeks of June, all students in both classrooms were urged to attend a mathematical camp titled "Mathematics. Creation. Intelligence." **Table 3** shows IQ results at the beginning of the shift (i.e. at the end of the analyzed academic year). Scoring was carried out on a scale for 12 year old pupils.

Table 3. Results at the beginning of summer shift in 2015/2016, 5-th grade (01/06/2016)

	Group A (20)					Group B (20)				
	AT	**MT**	**PT**	**VT**	**SV**	**AT**	**MT**	**PT**	**VT**	**SV**
Perfect	0	0	0	0	0	0	0	0	0	0
Excellent	1	3	2	4	1	1	2	1	1	1
Very good	5	5	3	4	6	4	5	2	5	5
Good	9	7	6	6	7	7	7	7	8	6
Average	3	3	6	4	4	7	4	7	6	7
Satisfactory	2	2	3	2	2	5	4	4	2	4
The average score of the group	**66**	**50**	**63**	**47**	**56**	**54**	**41**	**55**	**42**	**46**

Let us compare the results of each group. They are presented in **Table 4** and in **Figures 2** and **3**.

Table 4. Comparison of the results of three group works

	Group A (20)					Group B (20)				
	AT	**MT**	**PT**	**VT**	**SV**	**AT**	**MT**	**PT**	**VT**	**SV**
Average score at the beginning of the school year	60	43	58	40	50	54	40	57	40	46
Average score at the beginning of the shift	66	50	63	47	56	54	41	55	42	46

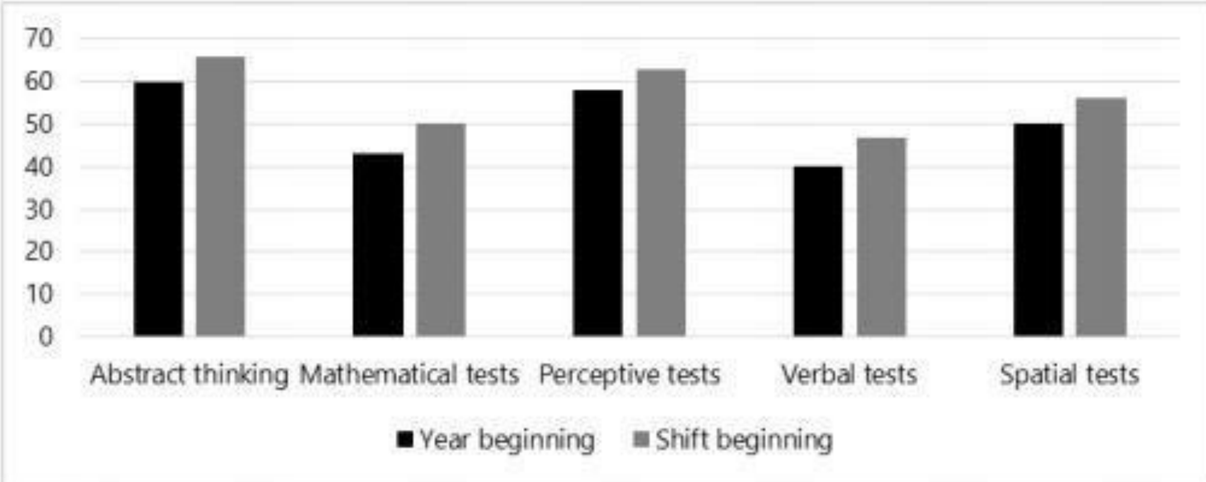

Figure 2. Comparison of average scores in group A

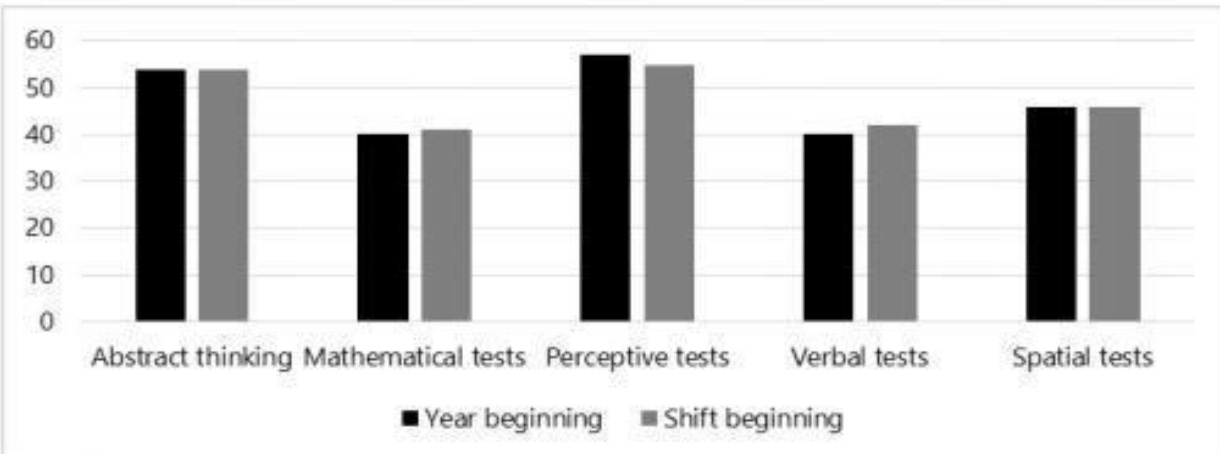

Figure 3. Comparison of average scores in group B

Let us note that group A had a significant increase in all indicators. We explain it by the fact that pupils of group A were taught by the program “Lessons of developmental mathematics”, and pupils of group B didn’t have this opportunity.Potentially both groups initially showed good capacity for intellectual activity, which was confirmed by the results of the check. Let us pay attention to the fact that the same type of materials (different options) were offered for tests. However, the children did not purposefully solve the test tasks during the learning process. Instead of them different types of puzzles were used, the solution of which was very similar to certain tasks in IQ tests.Thus, we came to the conclusion that the proposed methodology “Lessons of developmental mathematics”, which uses didactic tools based on puzzles, improves mathematical abilities of pupils: ability to think logically, ability to abstract thinking, combinatorial abilities, abilities at spatial representation and manipulation with spatial images, critical thinking abilities. Moreover, the influence of other teaching tools undoubtedly has an influence on the results. However, we have not seen such growth in similar classes earlier, when our worked out methodological model of usingpuzzles in the educational process has not been used.

APPLICATIONS

Sudoku is a logic-based, combinatorial number-placement puzzle. In classic Sudoku, the objective is to fill a 9 × 9 grid with digits so that each column, each row, and each of the nine 3 × 3 sub grids that compose the grid (also called "boxes", "blocks", or "regions") contain all of the digits from 1 to 9. The puzzle setter provides a partially completed grid, which for a well-posed puzzle has a single solution. French newspapers featured variations of the Sudoku puzzles in the 19th century, and the puzzle has appeared since 1979 in puzzle books under the name Number Place. Sudoku keeps your brain active and reduces the risk of Alzheimer's, a most common cause of dementia that affects a person's thinking and behavioural skills. Stimulates your mind: The game works on your logical thinking process as you are absorbed in solving a puzzle and eventually improve your number skills.

The Tangram is a dissection puzzle consisting of seven flat polygons, called tans, which are put together to form shapes. The objective is to replicate a pattern (given only an outline) generally found in a puzzle book using all seven pieces without overlap. Alternatively, the tans can be used to create original minimalist designs that are either appreciated for their inherent aesthetic merits or as the basis for challenging others to replicate its outline. Research hints that it can boost arithmetic performance, too. When Yi Ling Cheng and Kelly Mix asked kids, aged 6-8, to perform a series of tangram-like spatial tasks, the practice session seemed to prime the brain for mathematics. Teachers argue that playing with tangrams may help kids classify shapes, develop positive feelings about geometry, gain a stronger grasp of spatial relationships, develop an understanding of how geometric shapes can be decomposed, hone spatial rotation skills, acquire a precise vocabulary for manipulating shapes (e.g., “flip,” “rotate”).

The Rubik's Cube is a 3-D combination puzzle originally invented in 1974 by Hungarian sculptor and professor of architecture Ernő Rubik. Originally called the Magic Cube, the puzzle was licensed by Rubik to be sold by Pentangle Puzzles in the UK in 1978, and then by Ideal Toy Corp in 1980 via businessman Tibor Laczi and Seven Towns founder Tom Kremer. The cube was released internationally in 1980 and became one of the most recognized icons in popular culture. It won the 1980 German Game of the Year special award for Best Puzzle. As of January 2009, 350 million cubes had been sold worldwide, making it the world's bestselling puzzle game and bestselling toy. Benefits of solving a Rubik’s cube are Improves patience, Improves problem-solving skills, Keeps your mind active, Improves speed, Improves agility, Improves reflexes, Enhances concentration and configuration.

Surprisingly, the research on insight problem solving has been dominated by a small number of puzzles, dubbed "classic" by Chu and Macgregor (2011). Nine Dots, Cheap Necklace, Eight Coins, Ten-Coin Triangle, Mutilated Checkerboard, Socks, and Water Lilies are among the algorithmic puzzles available, as well as a number of additional verbal, spatial, and mathematical brain teasers. Newer additions to this limited repertoire include matchstick arithmetic with Roman numerals, compound remote associates (CRAs), and rebus puzzles. Despite the advantages of this enlargement of the insight issue universe, it's worth noting that CRAs and rebus puzzles, as well as other linguistic problems like anagrams and brainteasers based on alternative meanings of words in statements, all have the same flaw: their solutions require a native-speaker command of a language used.

CONCLUSION

In the course of studying various points of view on the development of mathematical abilities in children and adolescents, methodological approaches based on the teaching possibilitiesof a new teaching tool - mathematical puzzles - have been developed and applied in educational practice.Testing of various puzzles as a didactic tool in general and additional education of schoolchildren and preschool children for the development of their mathematical abilities allowed them to build methodological approaches to their inclusion in the process of teaching mathematics.As a result of studies and experienced teaching undertaken over the last three years by the creativegroup of mathematics and primary school teachers, methodological approaches have been established and significant concepts on the use of puzzles in the educational process have been recognized. Practical use of puzzles as a didactic tool allows to observe pupils’ progress in learningthe subject.

Using puzzles as a tool improves pupils’ results in mastering program material. Monitoringof pupils’ learning results shows positive dynamics. When solving learning and extracurricular tasks students demonstrate creative thinking, initiative, resourcefulness, activity, ability to emotional perception of math problems and arguments, take responsibility for the choice of solvingmethod and answer. So, puzzles may be the means to achieve not only learning, but also personalresults. It should be also noted that during the last two years a school museum of entertaining sciences has been actively filled up with exhibits created by pupils, among them is a huge numberof puzzles. Thus, the direct experience of puzzles used in additional education lets us come to theconclusion that their harmonious combination with the lessons activity provides realization of basicrequirements for the results of mastering the basic educational program of basic general education,formulated in national educational standard. The indicators of this are participation and victories of our pupils in contests and Olympiads of various levels, and also the fact that students themselvesare initiators of new mathematical activity based on solving and creating puzzles.

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