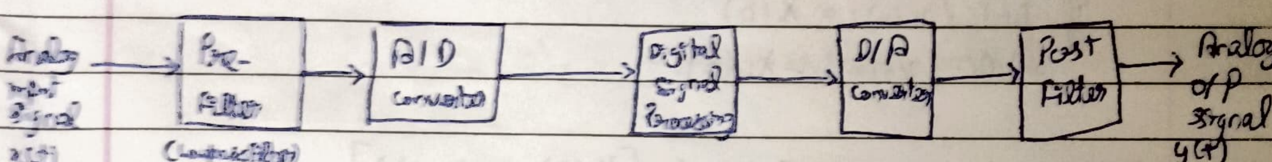
Digital Signal Processing

*** Signal Processing ->**

Analyze, Modify and Synthesize

i) Analog Signal Processing
ii) Digital Signal Processing

*** Block diagram - DSP ->**



*** Application - DSP ->**

-> Speech and Audio
-> Image and Video
-> Military and Space
-> Biomedical and Healthcare
-> Consumer Electronics.

*** Disadvantage of DSP ->**

-> Limited Speed of Operation.

DTFT ->

$$x\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}k\cdot n}$$

IDFT ->

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}\cdot k\right) e^{j\frac{2\pi}{N}kn}$$

*** Advantages - DSP ->**

-> Flexible in operation
-> Accurate results
-> Stable System.
-> Data Storage - less expensive
-> Low Cost

*** Properties of DFT ->**

i) Circular Time Shift -> $x(n) \Rightarrow X(k)$

$x(n-n_0) \Rightarrow X(k)\, e^{-j\frac{2\pi}{N}kn_0}$

2) Circular Convolution ->

If DFT $\{x(n)\} = X(k)$

then DFT $\{y(n) = x_1(n) \, Ⓝ \, x_2(n)\} = Y(k) = X_1(k) \cdot X_2(k)$

$\circledast \Leftrightarrow Ⓝ$ -> Circular Convolution.

3) Multiplication or Modulation ->

If DFT $\{x_1(n)\} = X_1(k)$
and DFT $\{x_2(n)\} = X_2(k)$

then DFT $\{x_1(n) \cdot x_2(n)\} = \frac{1}{N} [X_1(k) \, Ⓝ \, X_2(k)]$

4) Parseval's Theorem ->

If DFT $\{x_1(n)\} = X_1(k)$
And DFT $\{x_2(n)\} = X_2(k)$
then, $\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k) \cdot X_2^*(k)$

5) Circular Correlation Property ->

If DFT $\{x(n)\} = X(k)$
and DFT $\{y(n)\} = Y(k)$

then DFT $\{r_{xy}(l)\} = R_{xy}(k) = X(k) \cdot Y^*(k)$

6) Symmetry Property →

$x(n) \to X(k)$ (complex value)

$$x(n) = x_R(n) + j\, x_I(n)$$

$$X(k) = X_R(k) + j X_I(k)$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} \quad , \quad 0 \le k \le N-1$$

$$X_R(k) + j X_I(k) = \sum_{n=0}^{N-1} [x_R(n) + j x_I(n)]\, e^{-j\frac{2\pi}{N}nk}$$

$$x_R(n) = \frac{1}{N} \sum_{k=0}^{N-1} \left[X_R(k) \cos\frac{2\pi}{N}kn - X_I(k) \sin\frac{2\pi}{N}kn\right]$$

$$x_I(n) = \frac{1}{N} \sum_{k=0}^{N-1} \left[X_R(k) \sin\frac{2\pi}{N}kn + X_I(k) \cos\frac{2\pi}{N}kn\right]$$

(i) Real and Even Sequence →

$$x(n) = x_R(n) \quad ; \quad x_I(n) = 0$$

$$X(k) = \sum_{n=0}^{N-1} x(n) \cos\frac{2\pi}{N}kn$$

(ii) Real and Odd Sequence →

$$x(n) = x_R(n) \; ; \quad x_I(n) = 0$$

$$X(k) = \sum_{n=0}^{N-1} x(n) \sin\frac{2\pi}{N}kn$$

(iii) Purely Imaginary →

$$x(n) = j x_I(n), \; x_R(n) = 0$$

$$X_R(k) + j X_I(k) = \sum_{n=0}^{N-1} x_I(n) \sin\frac{2\pi}{N}kn + j \sum_{n=0}^{N-1} x_I(n) \cos\frac{2\pi}{N}kn$$

$$X_R(k) = \sum_{n=0}^{N-1} x(n) \sin\frac{2\pi}{N}kn$$

$$X_I(k) = \sum_{n=0}^{N-1} x(n) \cos\frac{2\pi}{N}kn$$

7) Duality ->

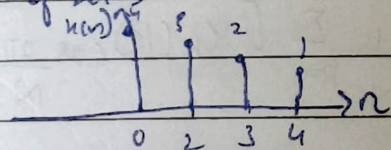
If $x(n) \xleftrightarrow[DFT]{N} X(k)$

then $X(n) \xleftrightarrow[DFT]{N} N\left[x((-k))_N\right] = N\,x(N-k)$

8.) Circular Symmetry ->

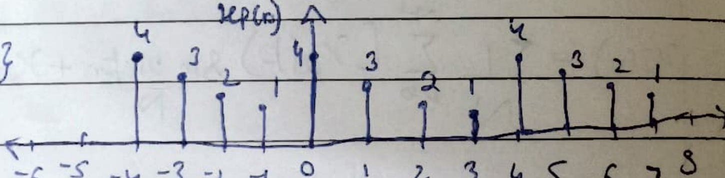
$DFT\{x(n)\} = X(k)$ then $DFT\{x_p(n)\} = X(k)$

$x_p(n) \Rightarrow$ Periodic representation of $x(n)$



① $x(n) = \{\underset{\uparrow}{4},3,2,1\}$

• $x_p(n) = \{\underset{\uparrow}{4},3,2,1\}$



② $x(n) = \{4,3,\underset{\uparrow}{2},1\}$

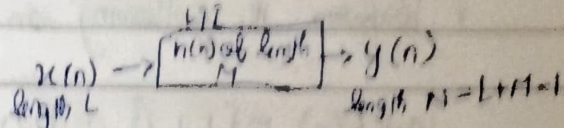
$x_p(n) = \{\underset{\uparrow}{2},1,4,3\}$

Method

Circular convolution

* Linear filtering using DFT and IDFT ->

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k) \quad -(1)$$



$$Y(\omega) \overset{FT}{=} F\left\{\sum_{k=-\infty}^{\infty} h(k)\,x(n-k)\right\} \quad -(2)$$

a) Linear convolution between h(n)

Convolution Property of F.T.

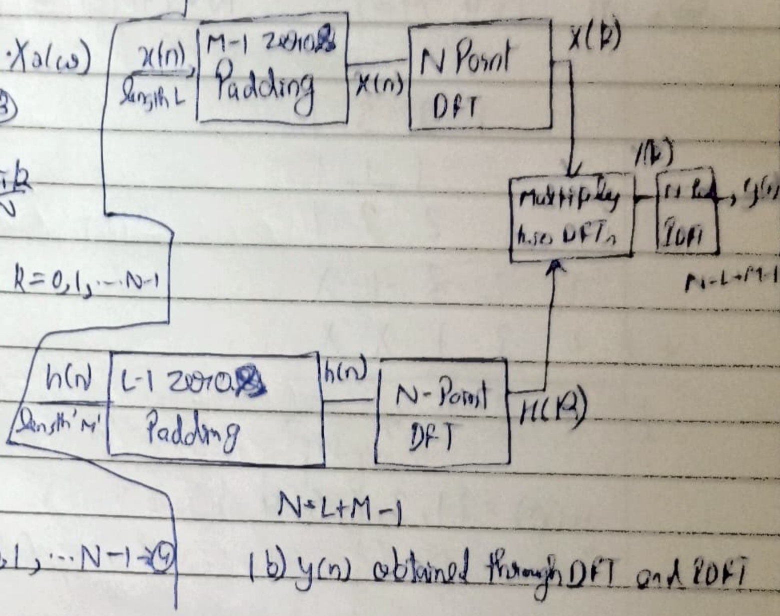
$$F\{x_1(n) * x_2(n)\} = X_1(\omega)\cdot X_2(\omega)$$

$$(2) \Rightarrow Y(\omega) = H(\omega)\cdot X(\omega) \rightarrow (3)$$

w.r.t. $Y(k) = Y(\omega)\Big|_{\omega = \frac{2\pi k}{N}}$

$$X(k) = X(\omega)\Big|_{\omega = \frac{2\pi k}{N}} \quad k = 0, 1, \ldots N-1$$

$$H(k) = H(\omega)\Big|_{\omega = \frac{2\pi k}{N}}$$



$$(3) \Rightarrow Y(k) = X(k)\cdot H(k);\ k = 0, 1, \ldots N-1 \rightarrow (4)$$

b) y(n) obtained through DFT and IDFT

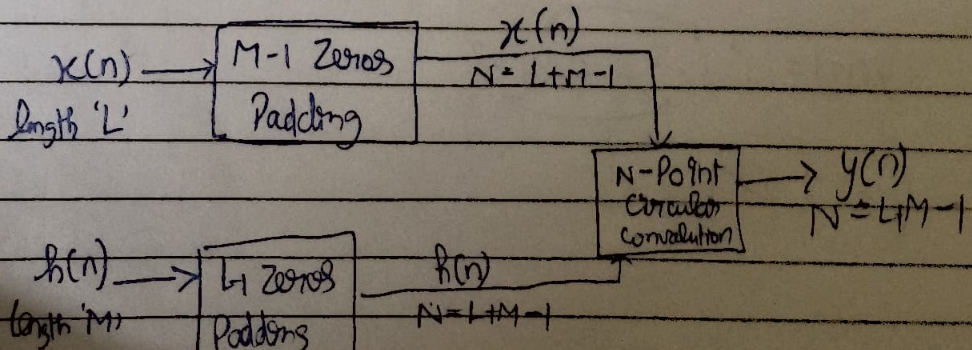
$$y(n) = IDFT\{Y(k)\} = IDFT\{X(k)\cdot H(k)\} \rightarrow (5)$$

Linear Convolution using Circular Convolution ->

$$Y(k) = X(k)\cdot H(k)$$

from Circular Convolution : $x_1(n)$ Ⓝ $x_2(n) = X_1(k)\cdot X_2(k)$

$$y(n) = x(n) \text{ Ⓝ } h(n)$$



• Perform the following on the seq. $x(n) = \{1, 2, 3, 1\}$ and $h(n) = \{1, 1, 1\}$; (i) Linear Convolution (ii) Circular Convolution (iii) Linear Convolution using circular convolution.

Sol→ (i) $L = 4,\ M = 3$ $\therefore N = L + M - 1 = 6$

```
          1   2   3   1
              1   1   1
      ------------------
          1   2   3   1
      1   2   3   1   x
  +1  2   3   1   x   x
  ----------------------
  1   3   6   6   4   1
```

$y(n) = \{1, 3, 6, 6, 4, 1\}$

(ii) ~~Linear Convolution using~~ Circular Convolution →

$h(n) = \{1, 1, 1, 0\}$

Matrix of $x(n)$

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 6 \\ 6 \end{bmatrix}, \quad y(n) = \{5, 4, 6, 6\}$$

$6 - 4 = 2$

$y(n) = \{1, 3, 6, 6, 4, 1\}$

$y(n) = \{5, 4, 6, 6\}$

Linear Convolution using circular conv. ->

$$N = L + M - 1, \quad M-1 \text{ zeros to } x(n)$$
$$L-1 \text{ zeros to } h(n)$$

$M-1 \Rightarrow 3-1 = 2$

$L-1 \Rightarrow 4-1 = 3$

$x(n) = \{1, 2, 3, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0\}$

Matrix form of $x(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

$y(n) = [1, 3, 6, 6, 4, 1]$

$\{1, 2, 0\}, \{2, 1, 0\}$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$2+0+\cdot$

$4+1$

* Discrete Time Fourier Transforms -> / Fast Fourier Transform

← Discrete Fourier Series ->

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{jk\omega_0 n}, \text{ for all } n$$

where $X(k)$ is the coefficients and ω_0 is fundamental digital frequency

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-jk\omega_0 n} \quad \text{for all } k.$$

$$\omega_0 = 2\pi/N$$

-> Properties ->

1) Linearity ->

$$DFS[x_1(n)] = X_1(k) \text{ and}$$
$$DFS[x_2(n)] = X_2(k)$$
then $$DFS[a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1(k) + a_2 X_2(k)$$

2) Time Shifting ->

$$DFS(x(n)) = X(k)$$
then $$DFS[x(n-m)] = e^{-j(2\pi/N)mk} X(k)$$

3) Periodic Convolution ->

$$DFS[x_1(n)] = X_1(k) \text{ and}$$
$$DFS[x_2(n)] = X_2(k)$$
then $$DFS[x_1(n) * x_2(n)] = X_1(k)\,(X_2(k))$$

← Discrete Time Fourier Transform ->

DFT $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}, \quad k \Rightarrow 0 \leq k \leq N-1$$

IDFT $$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega \text{ or } x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$

$$0 \leq n \leq N-1$$

→ Properties of DFT →

1) Shifting Property :-

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

$$x'(n) = \begin{cases} x_p'(n), & 0 \le n \le N-1 \\ 0, & \text{otherwise.} \end{cases}$$

Can be obtain from $x(n)$ by circular shift

$$x'(n) = x(n-k, (\text{mod } N))$$

2.) Time Reversal of a Sequence →

$$x(n) \xleftrightarrow[N]{DFT} X(k), \text{ then}$$

$$x(-n, (\text{mod } N)) = x(N-n) \xleftrightarrow[N]{DFT} X(-k, (\text{mod } N)) = X(N-k)$$

Hence, when the N-point sequence in time is reversed, it is equivalent to reversing the DFT values.

3.) Circular Time Shift →

$$x(n) \xleftrightarrow[N]{DFT} X(k), \text{ then}$$

$$x(n-l, (\text{mod } N)) \xleftrightarrow[N]{DFT} X(k)\, e^{-j2\pi kl/N}$$

Shifting of the sequence by l units in the time-domain is equivalent to multiplication of $e^{-j2\pi kl/N}$ in the freq. domain.

4) Circular Correlation ->

$$x(n) \xleftrightarrow[N]{DFT} X(k), \quad y(n) \xleftrightarrow[N]{DFT} Y(k), \text{ then}$$

$$\tilde{r}_{xy}(l) \xleftrightarrow[N]{DFT} \tilde{R}_{xy}(k) = X(k)\, Y^*(k)$$

$$\tilde{r}_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((n-l, (\text{mod } N)))$$

5) Multiplication of Two Sequence ->

$$x_1(n) \xleftrightarrow[N]{DFT} X_1(k) \text{ and } x_2(n) \xleftrightarrow[N]{DFT} X_2(k), \text{ then}$$

$$x_1(n)\, x_2(n) \xleftrightarrow[N]{DFT} \frac{1}{N} X_1(k) \, Ⓝ \, X_2(k)$$

* Fast Fourier Transform ->

It is an algorithm that efficiently computes the Discrete Fourier Transform. The DFT of a sequence $\{x(n)\}$ of length N is given by a complex-valued sequence $\{X(k)\}$.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}, \quad 0 \le k \le N-1$$

Let W_N be the complex valued phase factor, which is an Nth root of unity expressed by

$$W_N = e^{-j2\pi/N}$$

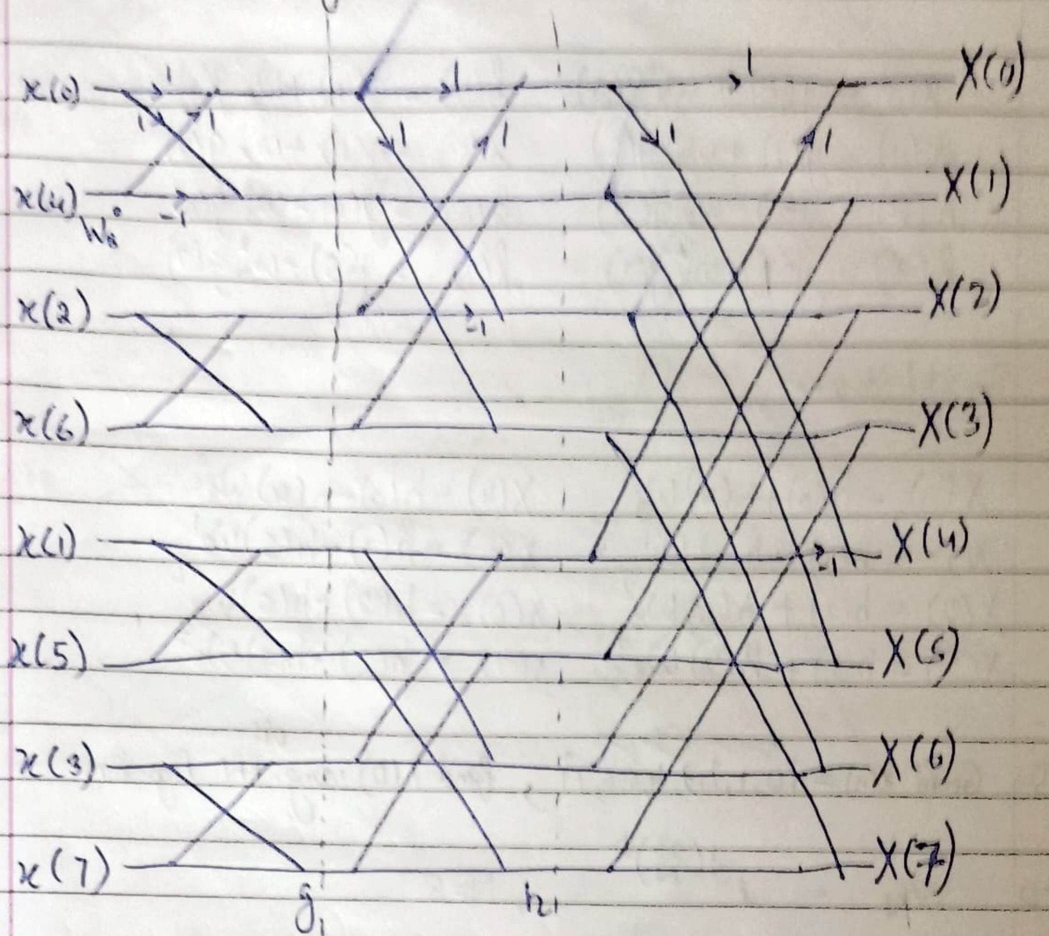
Hence, X(k) becomes,

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \quad 0 \le k \le N-1$$

Similarly for IDFT.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-nk}, \quad 0 \le n \le N-1$$

* Decimation In time Algorithm ->



I st

$W_8^0 = 1$

II nd

$W_8^0 = 1$

$W_8^2 = -j$

III rd

$W_8^0 = 1$

$W_8^1 = \frac{1}{\sqrt{2}} \cdot -j \cdot \frac{1}{\sqrt{2}}$, $W_8^2 = -j$, $W_8^3 = -\frac{1}{\sqrt{2}}$,

III nd

$W_8^0 = 1$, $W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$W_8^2 = -j$; $W_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

For Stage ->1:

$g(0) = x(0) + x(4)$ $\quad$ $g(4) = x(1) + x(5)$

$g(1) = x(0) - x(4)$ $\quad$ $g(5) = x(1) - x(5)$

$g(2) = x(2) + x(6)$ $\quad$ $g(6) = x(3) + x(7)$

$g(3) = x(2) - x(6)$ $\quad$ $g(7) = x(3) - x(7)$

For 2nd Stage ->

$h(0) = g(0) + W_8^0 g(2)$ $\quad$ $h(4) = g(4) + W_8^0 g(6)$

$h(1) = g(1) + W_8^2 g(3)$ $\quad$ $h(5) = g(5) + W_8^2 g(7)$

$h(2) = g(0) - W_8^0 g(2)$ $\quad$ $h(6) = g(4) - W_8^0 g(6)$

$h(3) = g(1) - W_8^2 g(3)$ $\quad$ $h(7) = g(5) - W_8^2 g(7)$

For 3rd Stage ->

$X(0) = h(0) + h(4) W_8^0$ $\quad$ $X(4) = h(0) - h(4) W_8^0$

$X(1) = h(1) + h(5) W_8^1$ $\quad$ $X(5) = h(1) - h(5) W_8^1$

$X(2) = h(2) + h(6) W_8^2$ $\quad$ $X(6) = h(2) - h(6) W_8^2$

$X(3) = h(3) + h(7) W_8^3$ $\quad$ $X(7) = h(3) - h(7) W_8^3$

Q-> Given $x(n) = \{0,1,2,3,4,5,6,7\}$, find $X(k)$ using DIT FFT Algorithm.

Sol-> $W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$, $N=8$

$W_8^0 = 1$, $\quad W_8^2 = e^{-j\left(\frac{2\pi}{8}\right)2} = -j$

$W_8^1 = e^{-j(2\pi/8)}$, $\quad W_8^3 = e^{-j\left(\frac{2\pi}{8}\right)3} = -0.707 - 0.707j$

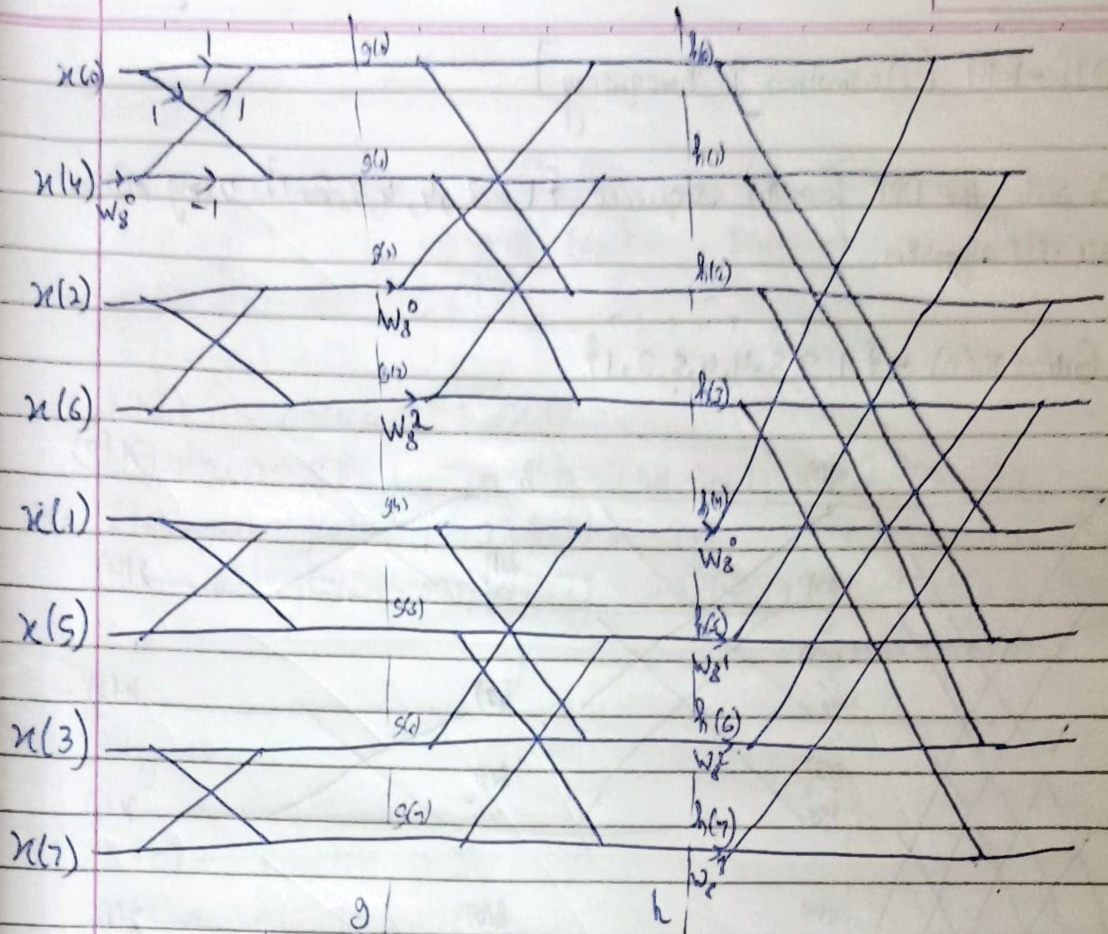
$= \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$= 0.707 - 0.707j$

$X(k) = \{28, -4+9.656j, -4+4j, -4+1.656j, -4, -4-j1.656, -4-4j, -4-j9.656\}$

* Note ->

The direct computation of DFT requires $2N^2$ evaluations of trigonometric functions, $4N^2$ real multiplications and $4N(N-1)$ real additions while using FFT algorithm it becomes $\frac{N}{2}\log_2 N$



I st	II nd	III rd
$W_8^0 = 1$	$W_8^0 = 1$	$W_8^0 = 1$
	$W_8^2 = -j$	$W_8^1 = 0.707 - 0.707j$
		$W_8^2 = -j$; $W_8^3 = -0.707 - 0.707j$

Stage → 1	Stage → 2	Stage → 3
$g(0) = x(0) + W_8^0 x(4) = 4$	$h(0) = g(0) + g(2) = 12$	$X(0) = h(0) + h(4) = 28$
$g(1) = x(0) - x(4) = -4$	$h(1) = g(1) + W_8^2 g(3) = -4 + j4$	$X(1) = h(1) + W_8^1 h(5) = -4 + 9.656j$
$g(2) = x(2) + x(6) = 8$	$h(2) = g(0) - g(2) = 4$	$X(2) = h(2) + W_8^2 h(6)$
$g(3) = x(2) - x(6) = -4$	$h(3) = g(1) - W_8^2 g(3) = -4 - 4j$	$= -4 + 4j$
$g(4) = x(1) + x(5) = 6$	$h(4) = g(4) + g(6) = 16$	$X(3) = h(3) + W_8^3 h(7)$
$g(5) = x(1) - x(5) = -4$	$h(5) = g(5) + g(7) W_8^2 = -4 + j4$	$= -4 + 1.656j$
$g(6) = x(3) + x(7) = 10$	$h(6) = g(4) - g(6) = -4$	
$g(7) = x(3) - x(7) = -4$	$h(7) = g(5) - W_8^2 g(7) = -4 - 4j$	$X(4) = h(0) - h(4) = -4$
		$X(5) = h(1) - W_8^1 h(5)$
		$= -4 - j1.656$
		$X(6) = h(2) - W_8^2 h(6)$
		$= -4 - 4j$
		$X(7) = h(3) - W_8^3 h(7)$
		$= -4 - 9.656$

$-4 + 4j$

$\cancel{-4 + 4j + (0.707 - 0.707j)16}$

$1 - 3j + (0.707 - 0.707j)10$

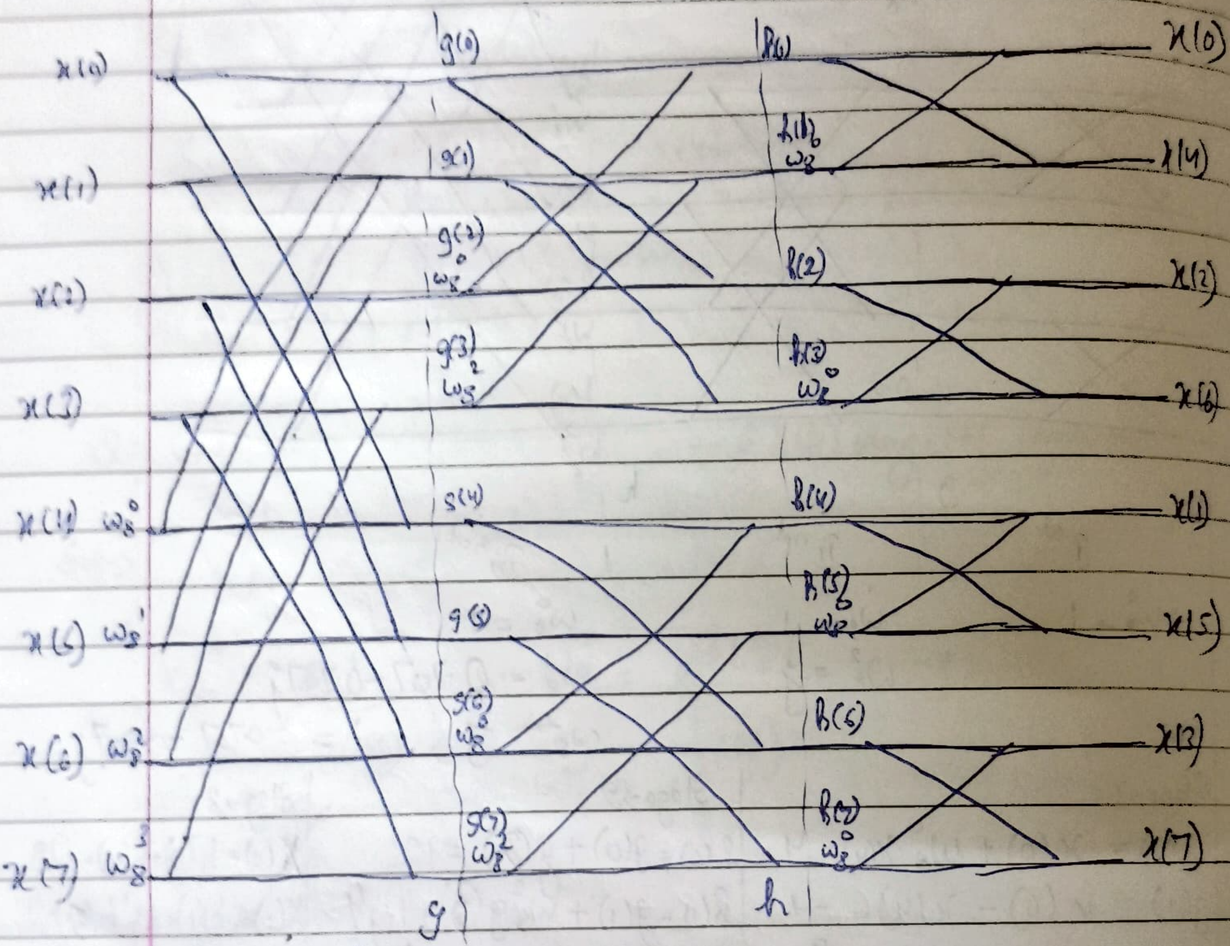
$=$

$6.07 - 9$

* DIF - FFT [Decimation In Frequency]

- Compute the DFT for the sequence {1, 2, 3, 4, 4, 3, 2, 1} using radix-2 DIF-FFT algorithm.

Sol: Given $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ (indices 0 1 2 3 4 5 6 7)



$W_8^0 = 1, \quad W_8^2 = -j$

$W_8^1 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j, \quad W_8^3 = -0.707 - 0.707j$

$= 0.707 - 0.707j$

Stage - 1

$g(0) = x(0) + x(4) = 5$

$g(1) = x(1) + x(5) = 5$

$g(2) = x(2) + x(6) = 5$

$g(3) = x(3) + x(7) = 5$

$g(4) = (x(0) - x(4)) W_8^0 = -3$

$g(5) = (x(1) - x(5)) W_8^1 = (-1)(0.707-0.707j) = -0.707 + 0.707j$

$g(6) = (x(2) - x(6)) W_8^2 = (1)(-j) =$

$g(7) = (x(3) - x(7)) W_8^3 = 3(-0.707-0.707j)$

$= (-2.121 - 2.121j)$

Stage → 2

$h(0) = g(0) + g(2) = 10$
$h(1) = g(1) + g(3) = 10$
$h(2) = g(0) - g(2) = 0$
$h(3) = g(1) - g(3) = 0$
$h(4) = g(4) + g(6) = -3-j$
$h(5) = g(5) + g(7) = (-0.707+0.707j - 2.121 - 2.121j) = (-2.828 - 1.414j)$
$h(6) = g(4) - g(6) = (-3+j)(W_8^2) = -3+j$
$h(7) = g(5) - g(7) = -0.707+0.707j + 2.121 + 2.121j = 1.414 + 2.828j\,(W_8^2)$
$= +2.828 - 1.414j$

Stage → 3

$X(0) = h(0)+h(1) = 20$
$X(1) = h(0)-h(1) = 0$

$X(2) = h(2)+h(3) = 0$
$X(3) = h(2)-h(3) = 0$

$X(4) = h(4)+h(5) = -5.828-2.414j$
$X(5) = h(4)-h(5) = -3-j+2.828+1.414j = -0.172-0.414j$

$X(6) = h(6)+h(7) = (-3+j)+(2.828-1.414j) = -0.172-0.414j$
$X(7) = h(6)-h(7) = -3+j-2.828+1.414j = -5.828-1.414j$

* Composite Radix FFT →

$m \to$ governs no. of stages
$N_1 \to$ governs no. of terms in each stage.

$N = 6 \} \to$ Comp. Radix
$N = 12$

$m \quad N_1$

$N = 2 \times 3$

$N = 4 \times 3$

$$X(k) = \sum_{n=0}^{N_1-1} x(nm) W_N^{nmk} + \sum_{n=0}^{N_1-1} x(mn+1) W_N^{(nm+1)k}$$

$$+ \sum \cdots + \sum_{n=0}^{N_1-1} x(nm_1+m_1-1) W_N^{(nm_1+m_1-1)k}$$

Composite Radix flowgraph →

$N = 6,$

$\Rightarrow N = 2 \cdot 3$ or $N = 3 \cdot 2$

Case→ for $N = 2 \cdot 3$
$m = 2$
$N_1 = 3$

$$X(k) = \sum_{n=0}^{N_1-1} x(nm) W_N^{nmk} + \sum_{n=0}^{N_1-1} x(mn+1) W_N^{(nm+1)k} + \cdots + \sum_{n=0}^{N_1-1} x(mn+m-1) W_N^{(nm+m-1)k}$$

Case-1

$$X(k) = \sum_{n=0}^{2} x(2n) W_6^{2nk} + \sum_{n=0}^{2} x(2n+1) W_6^{(2n+1)k}$$

Case-2 $m = 3, N_1 = 2$

$$X(k) = \sum_{n=0}^{1} x(3n) W_6^{3nk} + \sum_{n=0}^{1} x(3n+1) W_6^{(3n+1)k} + \sum_{n=0}^{1} x(3n+2) W_6^{(3n+2)k}$$

For Case-2:

$$X(k) = \sum_{n=0}^{2} x(2n) W_6^{2nk} + \sum_{n=0}^{2} x(2n+1) W_6^{(2n+1)k}$$

$$= \sum_{n=0}^{2} x(2n) W_6^{2nk} + \sum_{n=0}^{2} x(2n+1) W_6^{2nk} \cdot W_6^{k}$$

$$= \sum_{n=0}^{2} x(2n) W_6^{2nk} + \left[\sum_{n=0}^{2} x(2n+1) W_6^{2nk} \right] W_6^{k}$$

$$= \underbrace{\sum_{n=0}^{2} x(2n) W_3^{nk}}_{3\text{pt DFT}} + W_6^{k} \underbrace{\left[\sum_{n=0}^{2} x(2n+1) W_3^{kn} \right]}_{3\text{pt DFT}}$$

$$e^{-j\frac{2\pi}{6}(2n+1)k}$$

$$e^{-j\frac{2\pi}{6}2nk} \cdot e^{-j\frac{2\pi}{6}k}$$

$$W_6^{2nk} = e^{-j\frac{2\pi}{6}2nk} = e^{-j\frac{2\pi}{3}nk} = W_3^{nk}$$

$$X_1(k) = \sum_{n=0}^{2} x(2n) W_3^{nk} \quad (k=0,1,2)$$

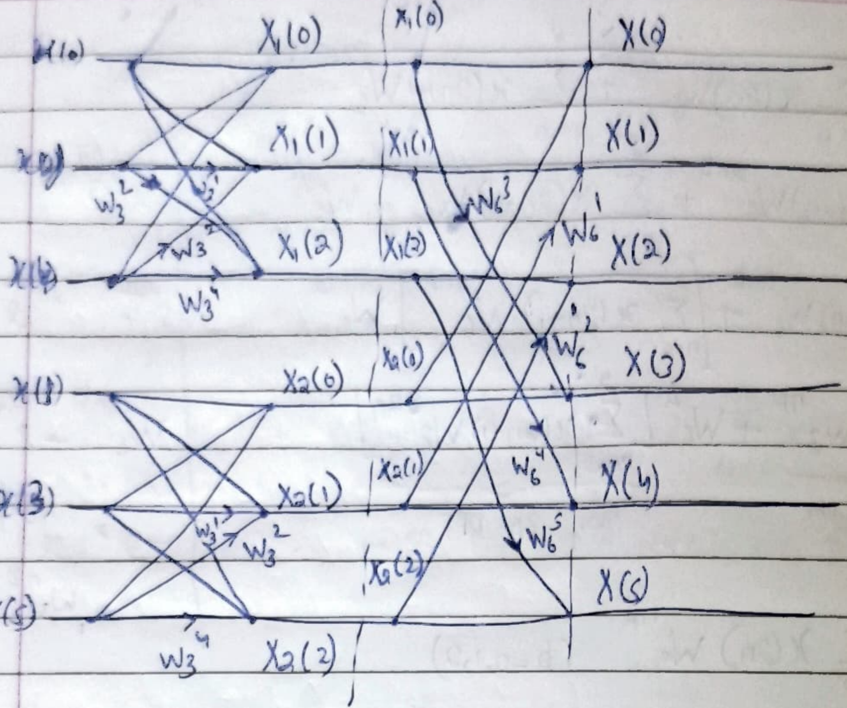
$$= x(0) W_3^{0} + x(2) W_3^{k} + x(4) W_3^{2k}$$

$$X_1(0) = x(0) W_3^0 + x(2) W_3^0 + x(4) W_3^0$$
$$X_1(1) = x(0) W_3^0 + x(2) W_3^1 + x(4) W_3^2$$
$$X_1(2) = x(0) W_3^0 + x(2) W_3^2 + x(4) W_3^4$$

$$X_2(k) = \sum_{n=0}^{2} x(2n+1) W_3^{nk} \quad \{k=0,1,2\}$$

$$X_2(k) = x(1) W_3^0 + x(3) W_3^{k} + x(5) W_3^{2k}$$

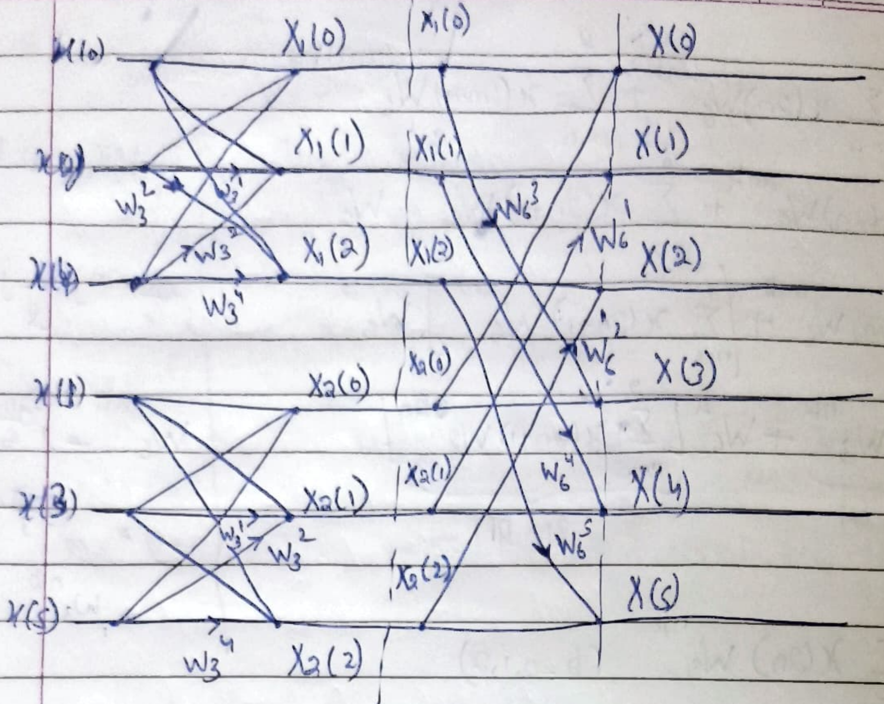
$$X_2(0) = x(1) W_3^0 + x(3) W_3^0 + x(5) W_3^0$$
$$X_2(1) = x(1) W_3^0 + x(3) W_3^1 + x(5) W_3^2$$
$$X_2(2) = x(1) W_3^0 + x(3) W_3^2 + x(5) W_3^4$$



$$X(k) = X_1(k) + W_6^{k} X_2(k)$$

$$X(0) = X_1(0) + W_6^0 X_2(0)$$
$$X(1) = X_1(1) + W_6^1 X_2(1)$$
$$X(2) = X_1(2) + W_6^2 X_2(2)$$

$$X(3) = X_1(0) + W_6^3 X_2(0)$$
$$X(4) = X_1(1) + W_6^4 X_2(1)$$
$$X(5) = X_1(2) + W_6^5 X_2(2)$$



$$X(k) = X_1(k) + W_6^{k} X_2(k)$$

$$X(0) = X_1(0) + W_6^{0} X_2(0)$$
$$X(1) = X_1(1) + W_6^{1} X_2(1)$$
$$X(2) = X_1(2) + W_6^{2} X_2(2)$$

$$X(3) = X_1(0) + W_6^{3} X_2(0)$$
$$X(4) = X_1(1) + W_6^{4} X_2(1)$$
$$X(5) = X_1(2) + W_6^{5} X_2(2)$$

Q) Develop the DIT FFT algorithm for decomposing the DFT for N = 12 and draw the flow diagram.

$$N = 12 = 3 \cdot 4 \quad ; \quad m = 3, \; N_1 = 4$$

Sol-> $$X(k) = \sum_{n=0}^{N_1-1} x(nm) W_N^{nmk} + \sum_{n=0}^{N_1-1} x(nm+1) W_N^{(nm+1)k} + \dots + \sum_{n=0}^{N_1-1} x(nm+m-1) W_N^{(nm+m-1)k}$$

$$X(k) = \sum_{n=0}^{3} x(3n) W_N^{3nk} + \sum_{n=0}^{3} x(3n+1) W_N^{(3n+1)k} + \sum_{n=0}^{3} x(3n+2) W_N^{(3n+2)k}$$

$$= \sum_{n=0}^{3} x(3n) W_N^{3nk} + \left[\sum_{n=0}^{3} x(3n+1) W_N^{3nk}\right] W_N^{k} + \left[\sum_{n=0}^{3} x(3n+2) W_N^{3nk}\right] W_N^{2k}$$

$$= X_1(k) + W_N^{k} X_2(k) + W_N^{2k} X_3(k)$$

$$X_1(k) = \sum_{n=0}^{3} x(3n) W_N^{3nk} \quad , k = 0,1,2,3$$

$$X_1(k) = x(0) W_{12}^{0} + x(3) W_{12}^{3k} + x(6) W_{12}^{6k} + x(9) W_{12}^{9k}$$

$$X_1(0) = x(0) W_{12}^{0} + x(3) W_{12}^{0} + x(6) W_{12}^{0} + x(9) W_{12}^{0}$$
$$X_1(1) = x(0) W_{12}^{0} + x(3) W_{12}^{3} + x(6) W_{12}^{6} + x(9) W_{12}^{9}$$
$$X_1(2) = x(0) W_{12}^{0} + x(3) W_{12}^{6} + x(6) W_{12}^{12} + x(9) W_{12}^{18}$$
$$X_1(3) = x(0) W_{12}^{0} + x(3) W_{12}^{9} + x(6) W_{12}^{18} + x(9) W_{12}^{27}$$

$$X_2(k) = \sum_{n=0}^{3} x(3n+1) W_N^{3nk} = x(1) W_{12}^{0} + x(4) W_{12}^{3k} + x(7) W_{12}^{6k} + x(10) W_{12}^{9k}$$

$$X_2(0) = x(1) W_{12}^{0} + x(4) W_{12}^{0} + x(7) W_{12}^{0} + x(10) W_{12}^{0}$$
$$X_2(1) = x(1) W_{12}^{0} + x(4) W_{12}^{3} + x(7) W_{12}^{6} + x(10) W_{12}^{9}$$
$$X_2(2) = x(1) W_{12}^{0} + x(4) W_{12}^{6} + x(7) W_{12}^{12} + x(10) W_{12}^{18}$$
$$X_2(3) = x(1) W_{12}^{0} + x(4) W_{12}^{9} + x(7) W_{12}^{18} + x(10) W_{12}^{27}$$

$$X_3(k) = \sum_{n=0}^{3} x(3n+2) W_N^{3nk} = x(2) W_N^{0} + x(5) W_N^{3k} + x(8) W_{12}^{6k} + x(11) W_{12}^{9k}$$

$$X_3(0) = x(2) W_{12}^{0} + x(5) W_{12}^{0} + x(8) W_{12}^{0} + x(11) W_{12}^{0}$$
$$X_3(1) = x(2) W_{12}^{0} + x(5) W_{12}^{3} + x(8) W_{12}^{6} + x(11) W_{12}^{9}$$
$$X_3(2) = x(2) W_{12}^{0} + x(5) W_{12}^{6} + x(8) W_{12}^{12} + x(11) W_{12}^{18}$$
$$X_3(3) = x(2) W_{12}^{9} + x(5) W_{12}^{9} + x(8) W_{12}^{18} + x(11) W_{12}^{27}$$

$$X(k) = X_1(k) + W_{12}^{k} X_2(k) + W_{12}^{2k} X_3(k)$$

$$X(0) = X_1(0) + W_{12}^{0} X_2(0) + W_{12}^{0} X_3(0)$$
$$X(1) = X_1(1) + W_{12}^{1} X_2(1) + W_{12}^{2} X_3(1)$$
$$X(2) = X_1(2) + W_{12}^{2} X_2(2) + W_{12}^{4} X_3(2)$$
$$X(3) = X_1(3) + W_{12}^{3} X_2(3) + W_{12}^{6} X_3(3)$$

$$X(4) = X_1(0) + W_{12}^{4} X_2(0) + W_{12}^{8} X_3(0)$$
$$X(5) = X_1(1) + W_{12}^{5} X_2(1) + W_{12}^{10} X_3(1)$$
$$X(6) = X_1(2) + W_{12}^{6} X_2(2) + W_{12}^{12} X_3(2)$$
$$X(7) = X_1(3) + W_{12}^{7} X_2(3) + W_{12}^{14} X_3(3)$$

$$X(8) = X_1(0) + W_{12}^{8} X_2(0) + W_{12}^{16} X_3(0)$$
$$X(9) = X_1(1) + W_{12}^{9} X_2(1) + W_{12}^{18} X_3(1)$$
$$X(10) = X_1(2) + W_{12}^{10} X_2(2) + W_{12}^{20} X_3(2)$$
$$X(11) = X_1(3) + W_{12}^{11} X_2(3) + W_{12}^{22} X_3(3)$$

$$x(t) \cos 2\pi f_c(t)$$

$$x(t) \frac{1}{2}$$