JAYPEE INSTITUTE OF INFORMATION TECHNOLOGY
Electronics and Communication Engineering
Digital Signal Processing (15B11EC413)
Tutorial Sheet: III

Q1. [CO1] Compute the DFT of the following sequences

a) x[n] = [1,0,-1,0]

b) x[n] = [j, 0, j, 1]

c) x [n] = [1,1,1,1,1,1,1,1]

d) x[n] = cos (0.25πn), n =0,1,…,7.

e) $x[n] = 0.9^{n}$, n =0,1,…,7.

Sol. 1.

a) $N = 4$ therefore $w_4 = e^{-j2\pi/4} = -j$. Therefore

$X[k] = \sum_{n=0}^{3} x[n]\,(-j)^{nk} = 1 - (-j)^{2k}, \quad k = 0, 1, 2, 3$. This yields

$$X = [0, 2, 0, 2]$$

b) Similarly, $X[k] = j + j\,(-j)^{2k} + (-j)^{3k} = j + j\,(-1)^{k} + j^{k}, \quad k = 0, \ldots, 3$, which yields

$$X = [1 + 2j,\ j,\ -1 + 2j,\ -j]$$

c) $N = 8$ and $w_8 = e^{-j2\pi/8} = e^{-j\pi/4}$. Therefore, applying the geometric sum we obtain

$$X[k] = \sum_{n=0}^{7} (w_8)^{nk} = \begin{cases} \frac{1-(w_8)^{8k}}{1-(w_8)^{k}} = 0 & \text{when } k \neq 0 \\ 8 & \text{when } k = 0 \end{cases}$$

since $(w_N)^{N} = (e^{-j2\pi/N})^{N} = 1$. Therefore

$$X = [8, 0, 0, 0, 0, 0, 0, 0]$$

d) Again $N = 8$ and $w_8 = e^{-j\pi/4}$. Therefore

$$X[k] = \frac{1}{2}\left(\sum_{n=0}^{7} e^{j\frac{\pi}{4}n} e^{-j\frac{\pi}{4}nk} + \sum_{n=0}^{7} e^{-j\frac{\pi}{4}n} e^{-j\frac{\pi}{4}nk}\right) =$$

$$= \frac{1}{2}\left(\sum_{n=0}^{7} e^{j\frac{\pi}{4}(k-1)n} + \sum_{n=0}^{7} e^{j\frac{\pi}{4}(-k-1)n}\right)$$

Applying the geometric sum we obtain

$$X = [0,\ 4,\ 0,\ 0,\ 0,\ 0,\ 4]$$

e) $X[k] = \sum_{n=0}^{7} 0.9^n e^{-j\frac{\pi}{4}nk} = \frac{1-(0.9)^8}{1-0.9\,e^{-j\frac{\pi}{4}k}}$, for $k = 0, \ldots, 7$ Substituting numerically we obtain

$$X = [5.69,\ 0.38 - 0.67j,\ 0.31 - 0.28j,\ 0.30 - 0.11j,\ 0.30,\ 0.30 + 0.11j,\ 0.31 + 0.28j,\ 0.38 + 0.67j]$$

Q2. [CO1] Two finite sequences x [n] = { x[0],x[1],x[2],x[3] }and h [n] = { h[0], h[1], h[2], h[3] } have DFT 's given by X(k) = DFT {x} = [1, j, -1,-j] and H(k) = [0, 1+ j, 1, 1- j]
Using the properties of the DFT, compute the following

a) DFT of { x[3] ,x[0], x[1], x[2] }

b) DFT of {h[0] ,-h[1], h[2], -h[3] }

c) DFT of (h[n] ⊗ x[n]), where ⊗ denotes the circular convolution.

d) DFT { x[0], h[0], x[1], h[1], x[2], h [2], x[3], h[3] }

a) DFT $\{[x[3], x[0], x[1], x[2]]\}$ = DFT $\{x[(n-1)_4]\} = w_4^{-k} X[k]$ with $w_4 = e^{-j2\pi/4} = -j$ Therefore DFT $\{x[3], x[0], x[1], x[2]\} = (-j)^k [1, j, -1, -j] = [1, 1, 1, 1]$

b) DFT $\{[h[0], -h[1], h[2], -h[3]]\}$ =

DFT $\{(-1)^n h[n]\}$ = DFT $\{e^{-j2(2\pi/4)n} h[n]\} = H[(k-2)_4] = [1, 1-j, 0, 1+j]$

c) DFT $\{h \otimes x\} = H[k]\, X[k] = [0, -1+j, -1, -1-j]$

d) Let $y = [x[0], h[0], x[1], h[1], x[2], h[2], x[3], h[3]]$, with length $N = 8$. Therefore its DFT is

$$Y[k] = \sum_{n=0}^{7} y[n]\, w_8^{nk} = \sum_{m=0}^{3} y[2m]\, w_8^{2mk} + \sum_{m=0}^{3} y[2m+1]\, w_8^{(2m+1)k}$$

$$Y[k] = X[k] + w_8^k H[k], \text{ for } k = 0, \ldots, 7$$

This yields

$$Y = \left[1, j + (1+j)e^{-\frac{j\pi}{4}}, -1-j, -j + (1-j)e^{-\frac{3j\pi}{4}}, 1, j + (1+j)e^{\frac{3j\pi}{4}}, -1+j, -j + (1-j)e^{\frac{j\pi}{4}}\right]$$

Q3. [CO1] Let x be a finite sequence with DFT X = DFT{ x} = [0, 1+ j, 1 , 1- j], Using the properties of the DFT , determine the DFT's of the following

(a) $y[n] = e^{j(\pi/2)n} x[n]$

(b) $y[n] = \cos(\pi/2\, n)\, u[n]$

(c) $y[n] = x[(n-1)_4]$

(d) $y[n] = [\,0,0,1,0\,] \otimes x[n]$, where $\otimes$ denotes the circular convolution.

a) Since $e^{j(\pi/2)n} x[n] = e^{j(2\pi/4)n} x[n]$ then DFT $\{e^{j(\pi/2)n} x[n]\} = X[(k-1)_4] = [1-j, 0, 1+j, 1]$

b) In this case $y[n] = \frac{1}{2} e^{j(2\pi/4)n} x[n] + \frac{1}{2} e^{-j(2\pi/4)n} x[n]$ and therefore its DFT is $\frac{1}{2} X[(k-1)_4] + \frac{1}{2} X[(k+1)_4] = \frac{1}{2}[1-j, 0, 1+j, 1] + \frac{1}{2}[1+j, 1, 1-j, 0]$. Putting things together we obtain

$$Y = [1, \tfrac{1}{2}, 1, \tfrac{1}{2}]$$

c) DFT $\{x[(n-1)_4] = e^{-j(2\pi/4)k} X[k] = (-j)^k X[k]$ and therefore

$$Y = [0, 1-j, -1, 1+j]$$

d) DFT $\{x[(n-2)_4]\} = (-j)^{2k} X[k] = (-1)^k X[k] = [0, -1, j, 1, -1+j]$

Q4. [CO1]

(a) Compute N point DFT of $x[n] = 3\,\delta[n]$

(b) Compute N point DFT of x[n] = 7 δ [n -n_o]

(c) Verify Parseval's Theorem's for the sequence x[n] = $(1/4)^n$ u(n)

(a)

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{j2\Pi kn}{N}}$$

$$= \sum_{n=0}^{N-1} 3\delta(n) e^{\frac{j2\Pi kn}{N}}$$

$$= 3\delta(0) \times e^{0} = 1$$

$$x(k) = 3, 0 \leq k \leq N-1 \quad \text{... Ans.}$$

(b)

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{j2\Pi kn}{N}}$$

Substituting the value of x(n),

$$\sum_{n=0}^{N-1} 7\delta(n - n_0) e^{-\frac{j2\Pi kn}{N}}$$

$$= e^{-kj14\Pi kn_0/N}$$

(c)

Solution – $\sum_{-\infty}^{\infty} |x_1(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X_1(e^{j\omega})|^2 d\omega$

L.H.S $\sum_{-\infty}^{\infty} |x_1(n)|^2$

$$= \sum_{-\infty}^{\infty} x(n)x^*(n)$$

$$= \sum_{-\infty}^{\infty} (\frac{1}{4})^{2n} u(n) = \frac{1}{1-\frac{1}{16}} = \frac{16}{15}$$

R.H.S. $X(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e-j\omega} = \frac{1}{1-0.25\cos\omega+j0.25\sin\omega}$

$$\Longleftrightarrow X^*(e^{j\omega}) = \frac{1}{1-0.25\cos\omega-j0.25\sin\omega}$$

Calculating, $X(e^{j\omega}).X^*(e^{j\omega})$

$$= \frac{1}{(1-0.25\cos\omega)^2+(0.25\sin\omega)^2} = \frac{1}{1.0625-0.5\cos\omega}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1.0625-0.5\cos\omega} d\omega$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1.0625-0.5\cos\omega} d\omega = 16/15$$

Q5. [CO1] Perform the periodic (circular) convolution of two sequences

$h[n]$ = {1,3,-1,-2} and $x[n]$ = {1,2,0,-1} using DFT and IDFT .

Q6. [CO1] Let x[n] = IDFT{X(k)} for n, k=0,1,…,N-1. Determine the relationship between x[n] and the following IDFT's.

a) IDFT { X*(k) }
b) IDFT (X [$(-k)_N$] }
c) IDFT { Re{X[k] }
d) IDFT { Ime{X[k] }}

a) $\text{IDFT}\{X^*[k]\} = \frac{1}{N}\sum_{k=0}^{N-1} X^*[k]\, W_N^{-nk} = \left(\frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{nk}\right)^* = x^*[(-n)_N]$

b) $\text{IDFT}\{X[(-k)_N]\} = \frac{1}{N}\sum_{k=0}^{N-1} X[(-k)_N]\, W_N^{-nk} = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{nk} = x[(-n)_N]$

c) $\text{IDFT}\{\text{Re}\{X[k]\}\} = \frac{1}{2}\,\text{IDFT}\{X[k]\} + \frac{1}{2}\,\text{IDFT}\{X^*[k]\} = \frac{1}{2}\,x[n] + \frac{1}{2}\,x^*[(-n)_N]$

d) $\text{IDFT}\{\text{Im}\{X[k]\}\} =$
$\frac{1}{2j}\,\text{IDFT}\{X[k]\} - \frac{1}{2j}\,\text{IDFT}\{X^*[k]\} = \frac{1}{2j}\,x[n] - \frac{1}{2j}\,x^*[(-n)_N]$

Q7. [CO1] Let x[n] be an infinite periodic sequence with period N .This sequence is input to a BIBO stable LTI system with impulse response h[n] , $-\infty \langle\, n \,\langle\, \infty$. Say how you can use the DFT to determine the output of the system?

Call $x = [x[0], \ldots, x[N-1]]$ one period of the signal, and let $X = \text{DFT}\{x\}$. Then the whole infinite sequence $x[n]$ can be expressed as $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{k2\pi}{N}n}$. Therefore the output becomes

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, H\left(\frac{k2\pi}{N}\right) e^{j\frac{k2\pi}{N}n}$$

We can see that the output sequence $y[n]$ is also periodic, with the same period N given by

$$y[n] = \text{IDFT}\{H\left(\tfrac{k2\pi}{N}\right) X[k]\}, \text{ for } n,\ k = 0, \ldots, N-1$$

Q8. [CO1] Let x[n] be a periodic signal with one period given by $[1,-2,\underset{n=0}{3},-4,5,-6]$ with the zero index as shown. It is the input to the LTI System with impulse response $h[n] = 0.8^{|n|}$. Determine one period of the output sequence y[n].

The input signal is periodic with period $N = 6$ and it can be written as

$$x[n] = \frac{1}{6} \sum_{k=0}^{5} X[k]\, e^{jk\frac{\pi}{3}n}, \text{ for all } n,$$

where

$$X[k] = \text{DFT}\{[3, -4, 5, -6, 1, -2]\} =$$
$$[-3.0,\ 3.0 + j\,1.7321,\ -3.0 - j\,5.1962,\ 21.0,\ -3.0 + j\,5.1962,\ 3.0 - j\,1.7321]$$

The frequency response of the system is given by

$$H(\omega) = \text{DTFT}\{0.8^n\, u[n] + 1.25^n\, u[-n-1]\} = \frac{e^{j\omega}}{e^{j\omega}-0.8} - \frac{e^{j\omega}}{e^{j\omega}-1.25}$$

Therefore the response to the periodic signal $x[n]$ becomes

$$y[n] = \frac{1}{6} \sum_{k=0}^{5} H[k]\, X[k]\, e^{jk\frac{\pi}{3}n}, \text{ for all } n,$$

with

$$H[k] = H(\omega)\,|_{\omega=\mathrm{k}\pi/3} = \frac{e^{jk\frac{\pi}{3}}}{e^{jk\frac{\pi}{3}}-0.8} - \frac{e^{jk\frac{\pi}{3}}}{e^{jk\frac{\pi}{3}}-1.25}, \ k = 0, \ \ldots, 5$$

Therefore one period of the output signal is determined as

$$y[n] = \text{IDFT}\{H[k]\, X[k]\}, \ n = 0, \ \ldots, 5,$$

where

$$H[k] = [9.0,\ 0.4286,\ 0.1475,\ 0.1111,\ 0.1475,\ 0.4286]$$

Computing the IDFT we obtain

$$y[n] = [-3.8301,\ -4.6079,\ -3.8160,\ -5.4650,\ -4.6872,\ -4.5938],\ n = 0, \ \ldots, 5$$

Q9. [CO1] A narrowband signal is sampled at 8.0 kHz and we take the DFT of 16 points as follows. Determine the best estimate of the frequency of the sinusoid, its possible range of values and an estimate of the amplitude:

X (k) = { 0.4889, 4.0267- j24.6698, 2.0054 - j5.0782, 1.8607 - j2.8478, 1.8170 - j1.8421,

1.7983 – j1.2136, 1.7893- j0.7471, 1.7849 –j0.3576, 1.7837 , 1.7849- j0.3576, 1.7893 +

j0.7471, 1.7983 +j 1.2136, 1.8170+ j 1.842, 1.8607+j2.8478, 2.0054+j5.0782, 4.0267+j24.6698 }

Looking at the DFT we see that there is a peak for $k = 1$ and $k = 15$, withe the respective values being $X[1] = 4.0267 - j24.6698$, and $X[15] = 4.0267 + j24.6698$. The magnitude is $|X[1]| = |X[15]| = 25.0259$. Therefore the frequency is within the interval
$(k_0 - 1)\ \frac{F_s}{N} < F < (k_0 + 1)\ \frac{F_s}{N}$

with $k_0 = 1$. $F_s = 8$ kHz and $N = 16$. This yield an estimated frequency
$0 < F < 2 \times \frac{8}{16}$ kHz $= 1$ kHz.

Q10. [CO1]

A real signal x(t) is sampled at 8.0 kHz and we store 256 samples x[0],x[1], ..., x[255]. The magnitude of the DFT X[k] has two sharp peaks at k=15 and k=241. What can you say about the signal?

The signal has a dominant frequency component in the range $14 \times \frac{8}{256} < F < 16 \times \frac{8}{256}$ kHz, that is to say 0.4375 kHz $< F < 0.500$ kHz.