Solution 02

We have

$$\left(\frac{W}{L}\right)_n = \frac{C_{load}}{\tau_{PHL}\, \mu_n C_{ox} (V_{DD} - V_{T,n})}\left[\frac{2V_{T,n}}{V_{DD}-V_{T,n}} + \ln\left(\frac{4(V_{DD}-V_{T,n})}{V_{DD}+V_{T0}} - 1\right)\right]$$

$$\frac{W_n}{L_n} = 7.9.$$

$$\left(\frac{W}{L}\right)_p = \frac{C_{load}}{\tau_{PLH}\, \mu_p C_{ox} (V_{DD} - |V_{T,p}|)}\left[\frac{2|V_{T,p}|}{V_{DD}-|V_{T,p}|} + \ln\left(\frac{4(V_{DD}-|V_{T,p}|)}{V_{DD}} - 1\right)\right]$$

$$\Rightarrow \frac{W_p}{L_p} = 25.2$$

During the transition from $2V \rightarrow 0.5V$, nMOS operates in the linear region.

$$\Rightarrow C_{load}\frac{dV_{out}}{dt} = \frac{1}{2}\mu_n C_{ox}\frac{W_n}{L_n}\left[2(V_{DD}-V_{T0,n})V_{out} - V_{out}^2\right]$$

Integrating, we get

$$t_{delay} = 0.35\times 10^{-9} = -2C_{load}\int_{V_{out}=2}^{V_{out}=0.5}\frac{dV_{out}}{\mu_n C_{ox}\frac{W_n}{L_n}\left[2(V_{DD}-V_{T0,n})V_{out} - V_{out}^2\right]}$$

$$\Rightarrow t_{delay} = \frac{-C_{load}}{\mu_n C_{ox}\frac{W_n}{L_n}(V_{DD}-V_{T0,n})}\ln\left(\frac{V_{out}}{2(V_{DD}-V_{T0,n})-V_{out}}\right)\Bigg|_2^{0.5}$$

$$\Rightarrow \boxed{\frac{W_n}{L_n} = 6.1}$$

We take the larger $\left(\frac{W_n}{L_n}\right)$ ratio to satisfy timing constraint. $\Rightarrow$ $\underline{\underline{W_n = 4.7\,\mu m}}$. (Given $L_n = 0.6\,\mu m$)

Also,

$$\frac{V_{T0,n} + \sqrt{\frac{1}{k_R}}\left(V_{DD} + V_{T0,p}\right)}{1 + \sqrt{\frac{1}{k_R}}} = 1.5$$

$$k_R = \frac{\mu_n C_{ox}\,(W_n/L_n)}{\mu_p C_{ox}\,(W_p/L_p)} = 0.51$$

$\Rightarrow W_p/L_p = 31.$ Since the larger ratio will satisfy both the timing and the V_{th} constraints, $W_p/L_p = 31$ is taken/considered.

$\Rightarrow$ $\underline{\underline{W_p = 18.6\,\mu m}}$ for $L_p = 0.6\,\mu m$.

Solution 2.

(1) From In to out 1

$$\tau_{in\_out1} = [RC + (R+2R)\,2C + (R+2R+R)\,2C + R\cdot(C+C+3C)] = 20RC.$$

(2) From In to Out 2

$$\tau_{in\_out2} = [RC + (R+R)C + (R+R+R)C + (R+R+R+R)\cdot 3C] + [R\cdot(C + \cancel{2C} + 2C)]$$

$$= 22RC.$$

Solution (3)

Given:

$$\tau_{PLH} = \tau_{PHL}$$

$$\Rightarrow \frac{C_{LOAD}}{k_p(V_{DD} - |V_{T,p}|)}\left[\frac{2|V_{T,p}|}{V_{DD} - |V_{T,p}|} + \ln\left(\frac{4V_{DD} - |V_{T,p}|}{V_{DD}} - 1\right)\right]$$

$$= \frac{C_{LOAD}}{k_n(V_{DD} - V_{T,n})}\left[\frac{2V_{T,n}}{V_{DD} - V_{T,n}} + \ln\left(\frac{4V_{DD} - V_{T,n}}{V_{DD}} - 1\right)\right]$$

$$\Rightarrow \frac{1}{k_p(2.1)}\left[(0.38 + 2.36)\right] = \frac{1}{k_n(2.1)}\left[0.38 + 2.36\right]$$

$$\Rightarrow \frac{k_n}{k_p} = 1$$

$$\Rightarrow k_n' \frac{W_n}{L_n} = k_p' \frac{W_p}{L_p}$$

Given $L_n = L_p$.

$$\Rightarrow \frac{k_n'}{k_p'} = \frac{W_p}{W_n} = \frac{115}{30}$$

$$\Rightarrow \frac{W_p}{W_n} = 3.83.$$

Sol. 4. nMOS works in saturation from $t=0$ to $t=t_{sat}$.

Also, $\frac{C\,dv_{out}}{dt} = -I_D = -I_{Dsat} = -\frac{1}{2} k_n (V_{DD} - V_{T,n})^2$

Integrating $\int_{t=0}^{t_{sat}} dt = \int_{3.3}^{2.5} \frac{-C}{I_D} \cdot dv_{out}$

$\Rightarrow t_{sat} = \frac{0.8 \times 300}{2} = 120ps$

k_n can be calculated as:

$k_n = 2I_{Dsat} / (V_{DD} - V_{T,n})^2 = 1.64 \times 10^{-3}\ A/V^2$

For the linear region, we have

$C \frac{dv_{out}}{dv_{in}} = -I_{Dlin} = -\frac{1}{2} k_n \left[2(V_{DD} - V_{T,n})V_{out} - V_{out}^2 \right]$

$\Rightarrow \int_{t=t_{sat}}^{t=t_{delay}} dt = -2C \int_{V_{out}=2.5V}^{V_{out}=1.65V} \frac{dV_{out}}{k_n\left[2(V_{DD} - V_{T,n})V_{out} - V_{out}^2\right]}$

$\Rightarrow t_{delay} - t_{sat} = 133ps$

$\Rightarrow t_{delay} = t_{sat} + 133ps$

$t_{delay} = 253ps$