Sentence Meanings and Truth

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Recap

- Meaning as part of a larger enterprise: semiotics
- Linguistic signs are usually in the **symbolic** mode
- Problems of circularity, exactness and context
- Sentence Meaning: Semantics
- Relations: contradiction, entailment

Logic and Truth

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- Principles of argument and inference:
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1. a. If Anu left work early then she is in the gym.
   b. Anu left work early

   ---

   c. She is in the gym

- Steps a and b are the premises (separated by a line)
- Step 3 is the conclusion
- If a and b are true, then c is guaranteed to be true

- **Modus tollens**

1. a. If Anu has arrived then she is in the pub.
   b. Anu is not in the pub

   ---
   c. Anu has not arrived

If a and b (premises) are true, then the conclusion c must be true

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- Truth conditions: the facts of the world that would be required to make a sentence true or false
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  - If you understand the meaning of a sentence, then it implies knowing under which circumstances it will be true or false.

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  - i) Translate sentences in natural language to formulae in mathematical logic
  - ii) Associate these formulae with meanings
- The logical language is **unambiguous**, allowing us to express association between logical expressions and meaning

Logical semantics

- Logical semantics is the study of meaning with the aid of mathematical logic
- Mathematical logic allows us to use notation ('metalanguage') for the investigation of meaning
- When we convert statements from ordinary language into logical language, we are forced to examine them with more care

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- A proposition will either evaluate to `True` or `False`

Example

Susan volunteers on Monday
Susan does not volunteer on Monday.

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Translation into Propositional Letters

p
¬ p
('it is not the case that p'/'not p')

p	**¬ p**
1	0
0	1

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- A succinct way of describing the truth effects of negation

- *Susan volunteers on Monday and Wednesday* can be translated as:-
- p: Susan volunteers on Monday
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p	q	p$\wedge$ q
1	1	1
1	0	0
0	1	0
0	0	0

- We can use **Boolean connectives** such as $\neg$ 'not', $\wedge$ 'and' $\vee$ 'or' to combine larger expressions from smaller ones
- A propositional letter standing alone is an **atomic formula** and when combined with the help of connectives, it forms a **complex formula**
- So far, we've seen examples of `CONJUNCTION` and `NEGATION`

Disjunction ‘inclusive or’

- *Susan volunteers on Monday or Wednesday* can be translated as:-
- p: Susan volunteers on Monday
- q: Susan volunteers on Wednesday
- $\vee$ represents disjunction (‘or’) and we can write the above as $p \vee q$
- Note that this is an `INCLUSIVE DISJUNCTION`- when both disjuncts are true, the statement is true

p	q	p$\vee$ q
1	1	1
1	0	1
0	1	1
0	0	0

- If you ask your friend : *Are you free today or tomorrow?*
- She responds with a *No*- that means she is not available today *and* not available tomorrow
- $\neg$ [P $\vee$ Q] is the same as [$\neg$P $\wedge$ $\neg$Q]
- But if you ask : *Are you free today and tomorrow* and she says *No*
- It means either she isn't free today or she isn't free tomorrow (or both)
- $\neg$ [P $\wedge$ Q] is the same as [$\neg$ P $\vee$ $\neg$ Q]
- These are De Morgan's Laws

Disjunction 'exclusive or'

- *Susan volunteers on Monday or Wednesday* can be translated as:-
- p: Susan volunteers on Monday
- q: Susan volunteers on Wednesday
- On the other hand, if you conclude that Susan volunteers on Monday or Wednesday – *but not both* then we have an `EXCLUSIVE` interpretation
- Natural language *or* can be exclusive e.g. *I'll visit him today or tomorrow*
- The interpretation is inclusive in the case of *I don't like cats or dogs*

Material implication

- Defining the semantics of conditional statements, statements of the form *if A then B*
- We represent this using the connective $\rightarrow$
- A formula of the form p $\rightarrow$ q will be as follows:-
- It is only false when p is true and q is false and true otherwise

p	q	p $\rightarrow$ q
1	1	1
1	0	0
0	1	1
0	0	1

- A conditional sentence is always of the form ‘if A then B,’ where A is called the `ANTECEDENT` and B is called the `CONSEQUENT`.
- If it’s sunny, then it’s warm
  1. It’s sunny and it’s warm
  2. It’s sunny and it’s not warm
  3. It’s not sunny and it’s warm
  4. It’s not sunny and it’s not warm
- If it’s not sunny, then whether it’s warm is irrelevant- we are only interested in situations where it’s sunny
- The only situation where it may be falsified is when the antecedent is true and consequent is false (i.e. second row in the truth table)

Bidirectional

- Some conditionals can hold in both directions, e.g.
  1. If today is Monday then yesterday was Sunday
  2. If yesterday was Sunday then today is Monday
- To express this, we have the biconditional written as $\leftrightarrow$
- In order for p $\leftrightarrow$ to be true, p and q must have the same truth value

p	q	p $\leftrightarrow$ q
1	1	1
1	0	0
0	1	0
0	0	1

- To sum up, propositional logic gives us propositonal letters e.g. P, Q, R which are atomic formulas
- Complex formulae can be created by using connectives,

  including the connective $\neg$

Conjunction	$\wedge$
Disjunction	$\vee$
Conditional	$\rightarrow$
Biconditional	$\leftrightarrow$

Causal relations in *if .. then*

- As we know, the expression p $\rightarrow$ q is only false when p (antecedent) is true and q (consequent) is false
- Does it always match with our ordinary use of conditional sentences?
- May not work when there is a **causal** connection between the antecedent clause (*if*-clause) and the consequent (*then*-clause)

If Patricia goes to the party, then John will go too.
Here, John is going *because* of Patricia

- If Patricia goes to the party, but John doesn't, then it will evaluate to false (p=1, q=0)
- But due to the *causal* implication, if Patricia doesn't go (p=0) then John **won't** go
- However, even though the antecedent is false, the statement is true (no matter the truth value of the consequent)
- Material implication captures only part of what we mean by *if ... then*

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

Counterfactuals

- In counterfactuals, the antecedent is overtly signalled to be false
- *If wishes were money, then we'd all be rich*

1. If Janis Joplin were alive today, she would drive a Mercedes-Benz. `False`
2. If Janis Joplin were alive today, she would metabolize food. *True*

- The material implication does not *simultaneously* allow for p=0 and q=0 to be `True` and p=0 and q=0 to be `False`

Source: Stanford Encyclopedia of Philosophy

References

John Saeed Semantics. Ch 4