KLE Technological University

DEPARTMENT OF MATHEMATICS

Mathematics Lab Assessment No. 1

Name: Anup Wadekar
Division: S
SRN:01FE24BAR008
Date: 06-10-2025

Question

Problem on Springs #1

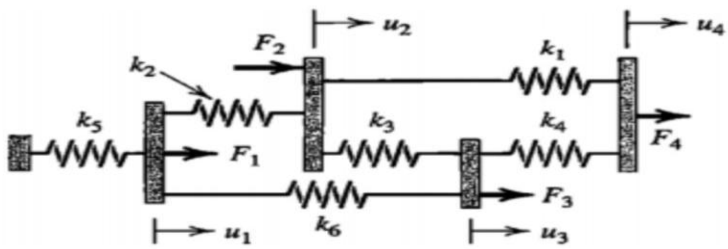


Fig 1: Springs and blocks arrangement

Springs and blocks arrangement is shown in Fig 1. The displacement of springs allowed in horizontal direction only and the blocks are considered as rigid and connected linear springs with stiffness. Write the system of equilibrium of equations for the applied forces $F_1=2F_2=20$ kN, $F_3=F_4=30$ kN. Evaluate the displacement fields for the applied loads

$k_1 = 100\ N/m \quad k_2 = 200\ N/m \quad k_3 = 300\ N/m$

$k_4 = 500\ N/m \quad k_5 = 400\ N/m \quad k_6 = 150\ N/m$

Solution

i) Identify the parameters and mathematical concept

Parameters Identified

- **Spring stiffness constants:** $k_1, k_2, k_3, k_4, k_5, k_6$(N/m)
- **External forces:** F_1, F_2, F_3, F_4
- **Displacement variables:** u_1, u_2, u_3, u_4
- **Degrees of Freedom (DOF):** 4
- **System type:** One-dimensional linear spring–block system

Mathematical Concept Used

- Linear elasticity (Hooke's law)
- Multi-degree-of-freedom system
- Matrix method
- Linear algebra (simultaneous equations)

Governing Equation

$$[K]\{u\} = \{F\}$$

ii) Solve analytically

Given data

$k_1 = 100\ N/m \quad k_2 = 200 N/m$.

$k_3 = 300\ N/m \quad k_4 = 500\ N/m$.

$k_5 = 400 N/m \quad k_6 = 150 N/m$

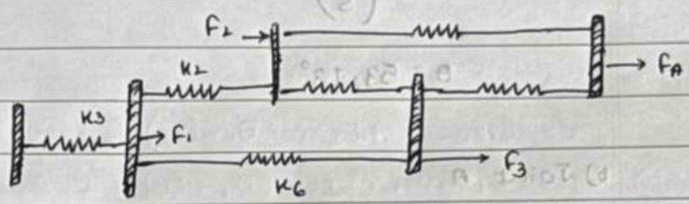
Applied force

$F_1 = 20 \quad F_2 = 10$

$F_3 = 30 \quad F_4 = 30$.

Displacement Vector

$$\{U\} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$



node wise Equilibrium eqn.

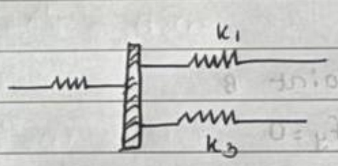
Node 1, $F_1 = (k_2 + k_5 + k_6) U_1 - k_2 U_2 - k_6 U_3$

node 2, $F_2 = -k_2 U_1 + (k_1 + k_2 + k_3) U_2 - k_3 U_3 - k_1 U_4$

node 3, $F_3 = -k_6 U_1 - k_3 U_2 + (k_3 + k_4 + k_6) U_3 - k_4 U_4$.

node 4, $F_4 = -k_1 U_2 - k_4 U_3 + (k_1 + k_4) U_4$

Global stiffness matrix



$U_1 = 0.225$.

$U_2 = 0.402$.

$U_3 = 0.455$.

$U_4 = 0.497$

Final displacement Vector

$$\{U\} = \begin{bmatrix} 0.225 \\ 0.402 \\ 0.455 \\ 0.497 \end{bmatrix}$$

	iii) GeoGebra Screenshot / Program Execution

```python
import numpy as np

# Stiffness constants
k1 = 100
k2 = 200
k3 = 300
k4 = 500
k5 = 400
k6 = 150

# Construct the global stiffness matrix (4x4)
K = np.array([
    [(k2 + k5 + k6),     -k2,            -k6,                0],
    [-k2,          (k1 + k3 + k2),     -k3,                -k1],
    [-k6,         -k3,       (k3 + k4 + k6),         -k4],
    [0,           -k1,       -k4,          (k1 + k4)]
], dtype=float)

# External force vector (4x1)
b = np.array([20, 20, 30, 30], dtype=float)

# Solve for displacements x1, x2, x3, x4
x = np.linalg.solve(K, b)

# Print results
print("Global Stiffness Matrix [K]:")
print(K)

print("\nExternal Force Vector {b}:")
print(b)

print("\nDisplacement {x} (m):")
print(x)
```

iv) Results and analysis from the graph

```
Global Stiffness Matrix [K]:
[[ 750. -200. -150.    0.]
 [-200.  600. -300. -100.]
 [-150. -300.  950. -500.]
 [   0. -100. -500.  600.]]

External Force Vector {b}:
[20. 20. 30. 30.]

Displacement {x} (m):
[0.25       0.45964467 0.50380711 0.5464467 ]
```