KLE Technological University

DEPARTMENT OF MATHEMATICS

Mathematics Lab Assessment No. 5

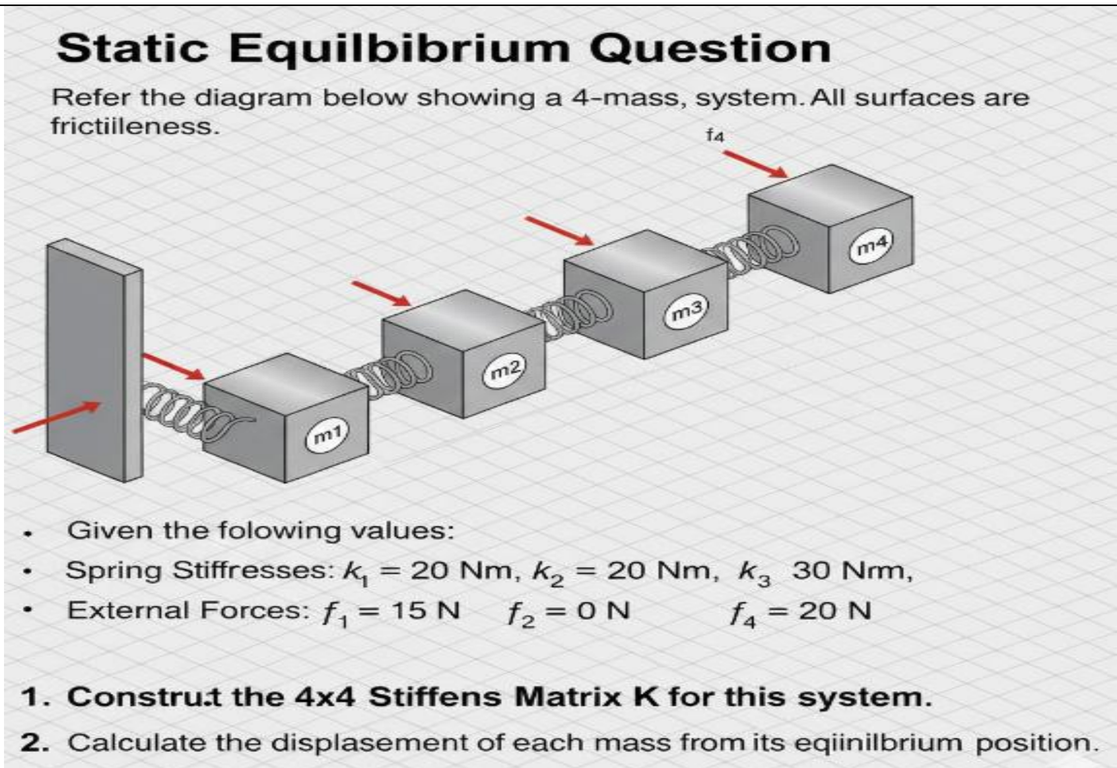
Name: Anup Wadekar

Division: A

SRN: 01FE24BAR008

Date: 07-1-2025

Question



- Spring stiffnesses:
  $k_1 = 20$ N/m, $k_2 = 20$ N/m, $k_3 = 30$ N/m, $k_4 = 30$ N/m
- External forces:
  $f_1 = 15$ N, $f_2 = 0$, $f_3 = 0$, $f_4 = 20$ N

Solution	i) Identify the parameters and mathematical concept **System Parameters** • Number of masses: 4 (m_1, m_2, m_3, m_4) • Springs: Linear elastic springs • Motion: One-dimensional (horizontal) • Surface: Frictionless • Unknown displacements: $\{u\} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$ **External Force Vector** $\{F\} = \begin{bmatrix} 15 \\ 0 \\ 0 \\ 20 \end{bmatrix}$ N **Mathematical Concept** • Static equilibrium of spring–mass systems • Linear algebra and matrix formulation • Governing equation: $[K]\{u\} = \{F\}$ Where $[K]$is the global stiffness matrix.

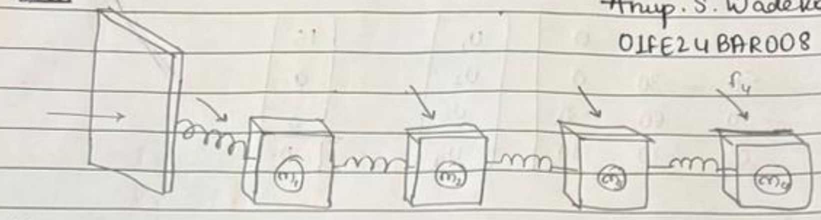
ii) Solve analytically

KLE TECHNOLOGICAL UNIVERSITY, Bhoomaraddi Campus, Hubballi

Page No.:
Date :

~~Given~~

Anup. S. Wadekar
01FE24BAR008



Given

$K_1 = 20\,N/m$.	$F_1 = 15\,N$
$K_2 = 20\,N/m$.	$F_2 = 0N$
$K_3 = 30\,N/m$.	$F_3 = 0N$
$K_4 = 30\,N/m$.	$F_4 = 20\,N$

unknown displacements

U_1, U_2, U_3, U_4.

force equilibrium eqn.

mass m_1

$$F_1 = k_1 U_1 + k_2(U_1 - U_2). \quad -(1)$$

mass m_2

$$F_2 = k_2(U_2 - U_1) + k_3(U_2 - U_3) \quad -(2)$$

mass m_3

$$F_3 = k_3(U_3 - U_2) + k_4(U_3 - U_4) \quad -(3)$$

mass m_4 (free end

$$F_4 = k_4(U_4 - U_3) \quad -(4)$$

Stiffness matrix

$$\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$\begin{bmatrix} 40 & -20 & 0 & 0 \\ -20 & 50 & -30 & 0 \\ 0 & -30 & 60 & -30 \\ 0 & 0 & -30 & 30 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 0 \\ 20 \end{bmatrix}$$

Solve eqn.

eqn ④

$30(U_4 - U_3) = 20$

$U_4 = U_3 + 0.667$

eqn ③

$-30U_2 + 60U_3 - 30(U_3 + 0.667) = 0$

$-30U_2 + 30U_3 = 20$

$U_3 = U_2 + 0.667$.

eqn ②

$-20U_1 + 50U_2 - 30(U_2 + 0.667) = 0$

$-20U_1 + 20U_2 = 20 \Rightarrow U_2 = U_1 + 1$.

eqn ①

$40U_1 - 20(U_1 + 1) = 15$

$20U_1 = 35 \Rightarrow U_1 = 1.75$.

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 1.75 \\ 2.75 \\ 3.42 \\ 4.08 \end{bmatrix}$$

	iii) GeoGebra Screenshot / Program Execution

Anup. S. Wadekar
01FE24BAR008.

```
import numpy as np

#spring stifness (N/m).
k1, k2, k3, k4 = 20, 20, 30, 30.

# Global stifness matrix.
k = np.array([
 [k1+k2 , -k2 , 0, 0 ],
 [ -k2 , k2+k3 , -k3, 0 ],
 [ 0, -k3 , k3+k4 , -k4],
 [0 , 0, -k4 , k4 ]
], dtype =float)

# force vector (N)
f = np.array ([15, 0, 0,20 ], dtype =float).

#solve for displacements
x= np. linalg .solve (k,f).

Print("stiffness matrix k:\n",k)
Print ("\n force vector f: \n", f)
Print ("\n displacement x(m):\n",x)
```

output

Stiffness matrix k:

[[40, -20, 0, 0],
[-20, 50, -30, 0],
[0, -30, 60, -30],
[0, 0, -30, 30]]

force vector f:

[15, 0, 0, 20]

Displacement x (m):

[1.75, 2.75 3.416 4.083]

iv) Results and analysis from the graph

- Displacement increases progressively from $m_1$to m_4
- Maximum displacement occurs at the free end due to applied force f_4
- The system exhibits linear elastic behavior
- Static equilibrium condition is satisfied
- Results validate the stiffness matrix formulation

Final Answer

$$\boxed{\{u\} = \begin{bmatrix} 1.750 \\ 2.750 \\ 3.420 \\ 4.083 \end{bmatrix} \text{ m}}$$