AE 308 : Control Theory | Project

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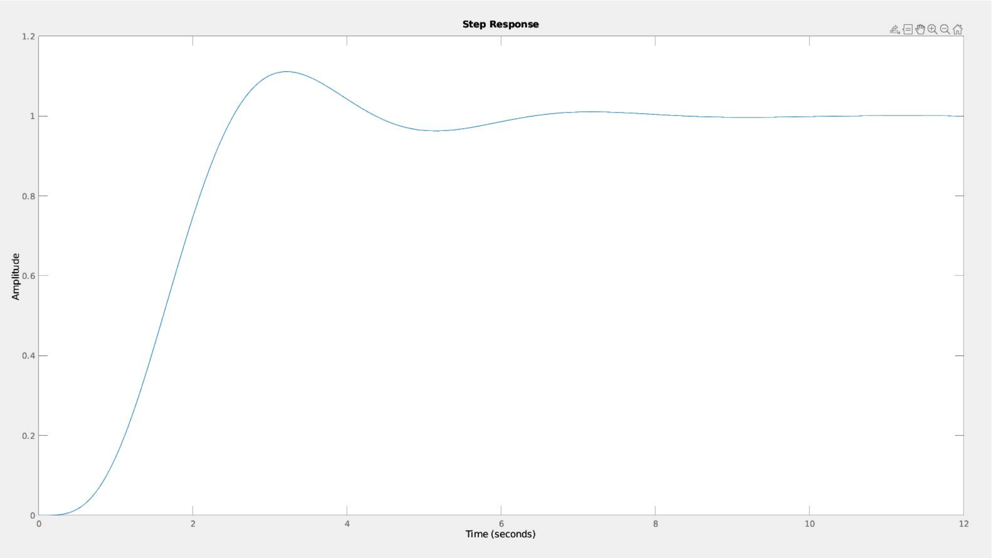
A self-guided automatic vehicle employed on shop floor to transport materials from one place to another has transfer function as :

$$G(s) = \frac{10}{s(s+3)(s2+2s+5)}$$

Ramp error constant of the plant (K_v) = 0.6667 | Other performance parameters for unit-step input :




```
     RiseTime: 1.4001
TransientTime: 5.8407
 SettlingTime: 5.8407
  SettlingMin: 0.9114
  SettlingMax: 1.1113
    Overshoot: 11.1311
   Undershoot: 0
         Peak: 1.1113
     PeakTime: 3.2236
```

Control Objectives:

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Need to design controller/compensator which can fulfill the following requirements:

A) Improvement in Ramp error constant of the plant (**K_v**) by **factor of 30**

i.e,

K_v (required) = 0.6667 x 30 = 20

B) Percentage Overshoot $\leq$ 11%

i.e,

Damping ratio $\geq$ 0.5749

As, 0.11 = $e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$

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Steps involved in designing **Lag Compensator** which satisfy the control objectives:

1. **Satisfying K_v requirement by multiplying G(s) by 30 :** New G(s) = G1(s) = 30 x G(s)

2. **Finding phase margin for required damping ratio (=0.5749):**

$$\text{Phase margin} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}$$

we add 10 deg to the phase margin in order to compensate for the phase angle contribution of the lag compensator. So we get **PM = 67.4714 deg**.
So, Phase angle = PM - 180 = **-112.5286 deg**

3. **Finding the new required Gain Crossover Frequency:**
Using Bode Plot of G1(s) we get the corresponding frequency at phase angle, **= 0.514 rad/sec (w_{GCO})**

So, since the Magnitude(G1 $)_{0.514}$ = 31.9 db, therefore we need the lag compensator to provide, **-31.9 db attenuation** at w = 0.514 rad/sec

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4. **Finding compensator parameters using Bode Plot:**

$$G_{Lag}(s) = K_c \frac{\beta(Ts+1)}{(\beta Ts+1)} = K_c \frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\beta T}\right)}$$

a. First draw the high-frequency asymptote at −31.9 dB
b. Arbitrarily select the higher break frequency to be about one decade below the phase-margin frequency (0.0514 rad/sec)
c. Starting at the intersection of this frequency with the lag compensator's high-frequency asymptote, draw a −20 dB/decade line until 0 dB is reached.
d. The lower break frequency is found to be 0.00130606 rad/sec.
e. So we get, lag compensator transfer function as:

$$G_c(s) = \frac{0.025409728(s + 0.0514)}{(s + 0.00130606)}$$

where, gain of the compensator is **0.025409728** to yield a dc gain = 1

Bode Plot of G1(s):

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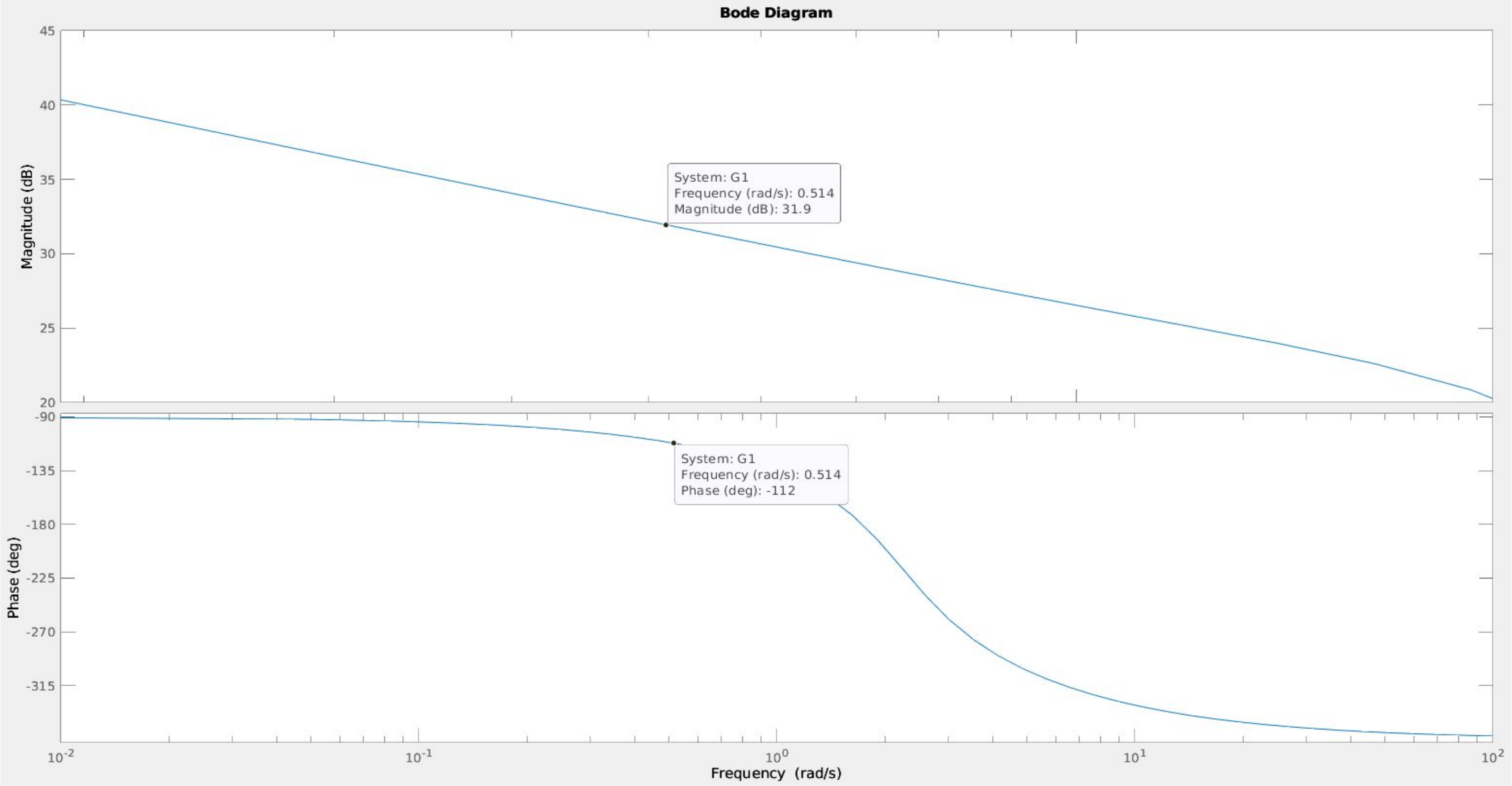

Bode Plots:

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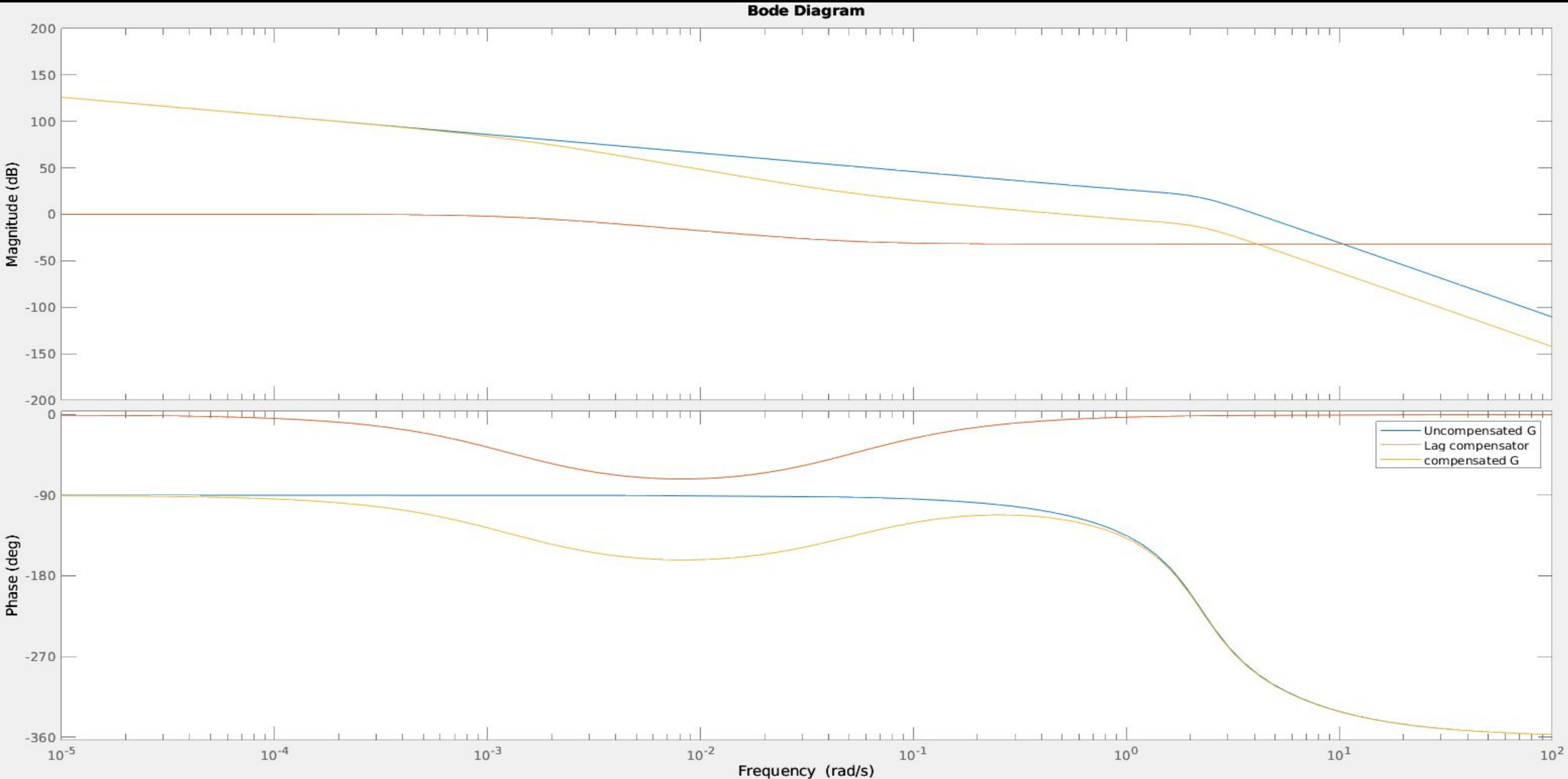

Unit Step response of Compensated System:

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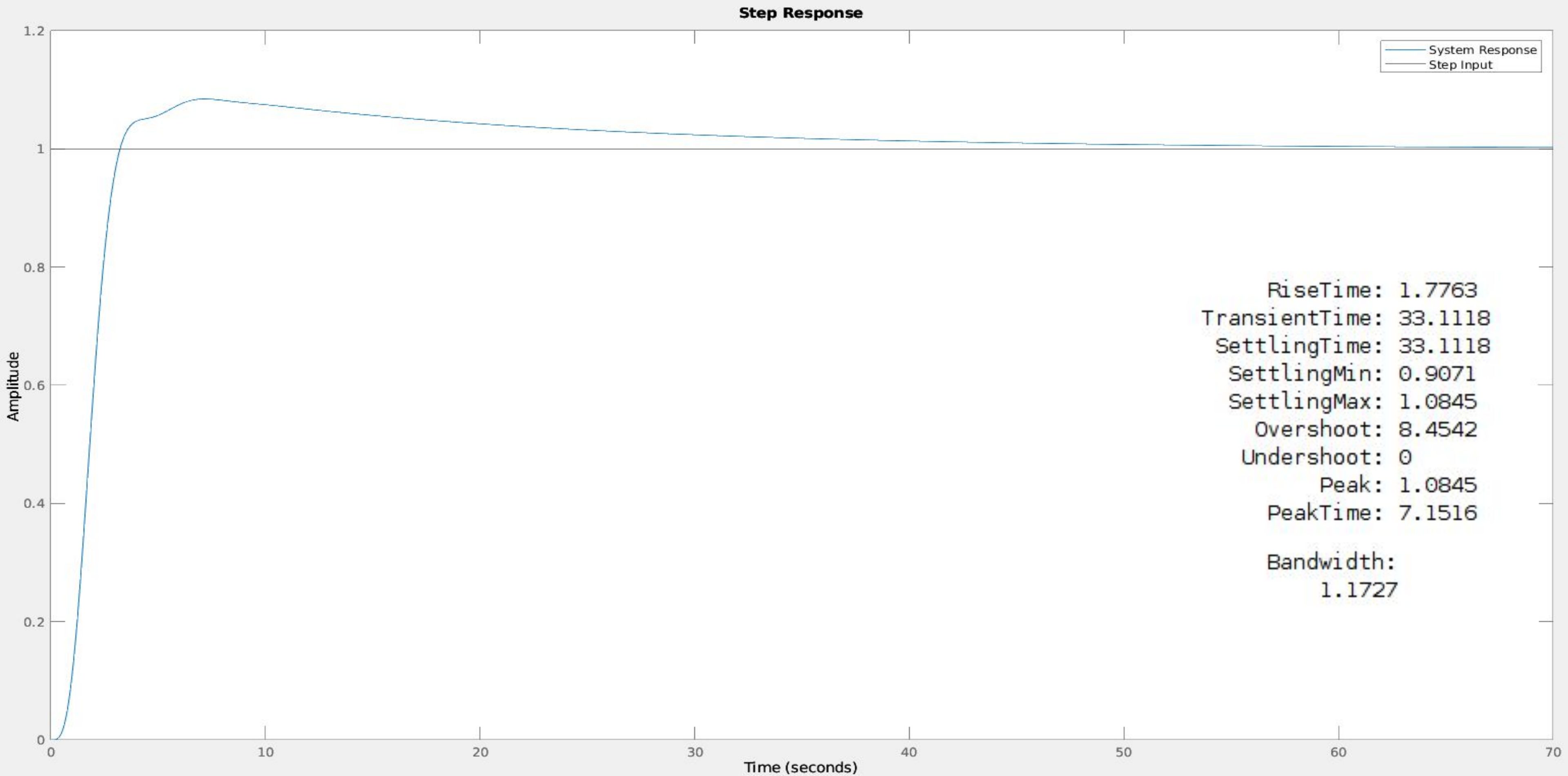

Conclusion:

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- Successfully designed Lag Compensator which can satisfy the required specifications :
    - K_v = 20
    - Peak Overshoot $\leq$ 11%

- Resulting Settling Time = 33.1118 seconds
- Bandwidth = 1.1727

- Relation between Peak Overshoot and Closed loop Resonant Peaks:
    - Magnitudes of Resonant Peaks = $[\, 2\zeta\sqrt{(1 - \zeta^2)}\,]^{-1}$
    - Overshoot Peaks = $\exp(\, -\zeta\pi/\sqrt{(1 - \zeta^2)}\,) + 1$

So, we can say Resonant peak frequency corresponds to the peak overshoot in the time domain transient response for certain values of damping ratio (ζ). So the resonant peaks and overshoot are correlated to each other.