Uniform Distribution

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$$f(x) = \begin{cases} \dfrac{1}{b-a}, & \text{for } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

and the cumulative distribution function is

$$F(x) = \begin{cases} 0, & \text{if } x < a \\ \dfrac{x-a}{b-a}, & \text{if } a \le x \le b \\ 1, & \text{if } b < x \end{cases}$$

The expected value and variance of a uniform random variable X are computed as follows:

$$E[X] = \frac{a+b}{2} \quad \textbf{(5.20)}$$

$$\mathrm{Var}[X] = \frac{(b-a)^2}{12} \quad \textbf{(5.21)}$$

35. The time required to play a game of Battleship™ is uniformly distributed between 20 and 60 minutes.

a. Find the expected value and variance of the time to complete the game.

b. What is the probability of finishing within 30 minutes?

c. What is the probability that the game would take longer than 40 minutes?

Uniform (20, 60)

(loc, loc+scale)
loc=20, loc+scale=60
scale=40

Suppose that sales revenue, X, for a product varies uniformly each week between $a = \$1{,}000$ and $b = \$2{,}000$. The density function is $f(x) = 1/(2{,}000 - 1{,}000) = 1/1{,}000$ and is shown in Figure 5.15. Note that the area under the density function is 1, which you can easily verify by multiplying the height by the width of the rectangle.

Suppose we wish to find the probability that sales revenue will be less than $x = \$1{,}300$. We could do this in two ways. First, compute the area under the density function using geometry, as shown in Figure 5.16. The area is $(1/1{,}000)(300) = 0.30$. Alternatively, we could use formula (5.19) to compute $f(1{,}300)$:

$$F(1{,}300) = (1{,}300 - 1{,}000)/(2{,}000 - 1{,}000) = 0.30$$

In either case, the probability is 0.30.

Now suppose we wish to find the probability that revenue will be between \$1,500 and \$1,700. Again, using geometrical arguments (see Figure 5.17), the area of the rectangle between \$1,500 and \$1,700 is $(1/1{,}000)(200) = 0.2$. We may also use formula (5.17) and compute it as follows:

$$\begin{aligned} P(1{,}500 \le X \le 1{,}700) &= P(X \le 1{,}700) - P(X \le 1{,}500) \\ &= F(1{,}700) - F(1{,}500) \\ &= \frac{(1{,}700 - 1{,}000)}{(2{,}000 - 1{,}000)} - \frac{(1{,}500 - 1{,}000)}{(2{,}000 - 1{,}000)} \\ &= 0.7 - 0.5 = 0.2 \end{aligned}$$