**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Let the prob of not meeting commitment be P(E). We have to calculate the z-score first for the given scenario Given : μ = 45 , σ = 8 , time = 60 – 10 = 50 Minutes Z-Score at 50 => (time – mean time)/std dev => (50-45)/8 = 0.625 Corresponding probability from Z-table = 0.7324 P(E) = 1 – 0.7324 = 0.2676 (Answer = Option B)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False Explanation: Consider the following Normal Distribution Graph. The range between ages 38 and 44 is within one standard deviation from the mean. This means that it contains about 34.1% of 400 approx. = 136 people. Going beyond age 44 will result in about 16% approx. = 64 people which is less than the former. Therefore, the answer is False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True Explanation: Finding the corresponding probability at age 30 gives approx. 36 people.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: iid stands for independent, identically distributed random variables. A good example is a succession of throws of a fair coin. As per the question, consider X1 and X2 be the outcomes of two die rolls. They iid normal random variables. Then X1+X2 is the sum of the numbers on the two dice and 2X1 is twice the number on the first die. These don't have the same distribution - for example, X1+X2 can be odd, and 2X1 is always even.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: In case of 0.99 symmetric prob, to get symmetry about mean => (1-0.99)/2 = 0.005 z-score corresponding to the value is -2.57. To find the a,b values => 20x(-2.57) +/- 100 would give approx. (48.6, 151.4)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: According sum of normal random variables rules, we can add up the profits. Annual\_profit ~ N(5+7, 32 + 42 ) => N(12, 5 2 ) Rupee Range = [99008103.48, 980991896.52] Rupee Range ~ 99MillionRupees to 980MillionRupees (Answer)

Code:

import scipy.stats as ss

mean, std, p = 12, 5, 0.95

mean = mean\*(10\*\*6)\*45

std = std\*(10\*\*6)\*45

print([round(x,2) for x in ss.norm.interval(alpha=p, loc=mean, scale=std)])

Output: [99008103.48, 980991896.52]

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: We already have the upper and lower range of the Annual\_profit. We can calculate the 5th percentile using python. 5 th percentile of profit = 143.1 Million Rupees (Answer)

Code:

import scipy.stats as ss

mean, std, p = 12, 5, 0.05

mean = mean\*(10\*\*6)\*45

std = std\*(10\*\*6)\*45 #to get z-score and rupee\_value

y = ss.scoreatpercentile([99008103.48, 980991896.52], 5)

print(y)

Output: 143107293.132 ~ approx. 143.1Million

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Division 1 will have larger probability for making a loss. (Answer)