CS 102: Tierce - 2 Examination

Solutions & Marking Policy

(Online Exam held on April 24, 2020)

1. (5 points) Let H be a hash table where the collision is resolved by linear probing. Suppose the size of H is M and we have to perform $n \leq M/2$ insertions. What is the worst case time complexity (in Big-Oh notations) to insert n distinct keys?

   (**Hint**: Think about ocurrence of collisions.)

   Solution: As $n \leq M/2$, load factor does not exceed 1/2. However, collision may occur with every insertion (with each key colliding). So with ith insertion there may be $i-1$ collisions. Therefore total number of collisions for n insertions will be

   $$\sum_{0}^{n-1} i = n(n-1)/2 = O(n^2)$$

   Hence, the worst case time complexity is O(n^2).

   +3 for correct time complexity.

   +2 for correct explanation.

2. (5 points) Given adjacency representation of a directed graph $G = (V, E)$. How long will it take to compute the out degree of every vertex? How long will it take to compute the indegree of every vertex?

   Solution: Computation of out degree is simple. Just walk through each adjacency list once to compute outdegree of corresponding vertex. So the time complexity is $O(|V| + |E|)$.

   For computing in-degree we need to keep a counter for each vertex and walk through each adjacency list increasing counter assoicated with a vertex each time the vertex is encountered. The time complexity is again $O(|V| + |E|)$.

   +3 for correct complexity.

   +2 for the corect explanation.

3. (5 points) Consider the hashing function

   $$h(x, i) = (x + i^2) \mod 16, \quad i = 0, 1, 2, \ldots.$$

   for hashing with open addressing that uses quadratic probing for collision resolution on a table of size 16. What is the earliest probe number where a collision will occur?

   (**Hint**: In hashing with open addressing, the elements are directly inserted into table slots (no chaining).)

   Solution: If a slot is not occupied, there will be no collision on the first probe, the slot gets filled. However, quadratic probe sequence probes slots at distances i^2 from the first slot. Since $(x + 16)$ mod $16 = x$, the earliest probe number where collision occurs is $i^2 = 16$ or $i = 4$.

   The answer $i = 4$ or probe number 5 will be accepted.

   +3 for correct value of probe number.

   +2 for correct explanation.

4. (5 points) Give adjacency representation of a complete binary tree of 7 vertices assuming the vertices are numbered from 1 to 7 as in a binary min-heap. Remember that a min-heap is a heap where the minimum element stored at the root.

Solution: The solution is given below:

$$1 \longrightarrow 2 \longrightarrow 3$$

$$2 \longrightarrow 1 \longrightarrow 4 \longrightarrow 5$$

$$3 \longrightarrow 1 \longrightarrow 6 \longrightarrow 7$$

$$4 \longrightarrow 2$$

$$5 \longrightarrow 2$$

$$6 \longrightarrow 3$$

$$7 \longrightarrow 3$$

+5 for correct solution.

0 for incorrect solution.

5. (2 points) Suppose we need to put 1000 items in a hash table of size 250 using hashing with chaining. What is the value of the load factor α? What is the average number of accesses for a successful search?

Solution: The load factor is $1000/250 = 4$. So, average number of accesses is $= 1+\alpha/2 = 1+4/2 = 3$.

+ 2 for correct solution.

0 for incorrect solutin.

6. (5 points) Given below is an adjacency list representation of a graph.

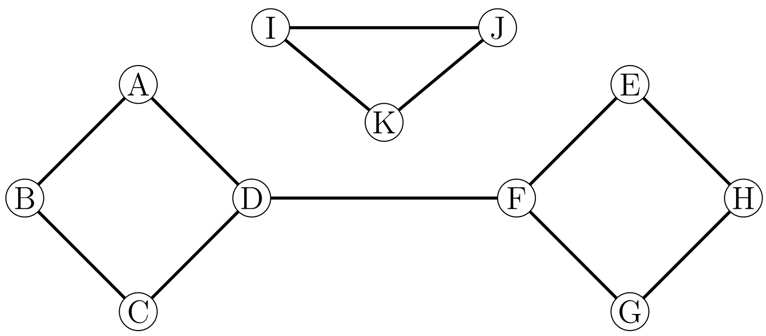
A→ B→ D, B→ A→ C, C→ B→ D, D→ A→ C,→ F E→ F→ H,

F→ D→ E→ G, G→ F→ H, H→ E→ G, I→ J→ K, J→ I→ K,

K→ I→ J,

Draw the graph. Is the graph disconnected? If yes, what is the minimum number of edges that should be inserted to make it connected and which are those?

Solution: Graph is drawn below:



The graph is disconnected. Insertion of just one edge can make it connected. We can use any of the edge that connects I, or J or K to any of the eight other vertices A, B, C, D, E, F, G, H.

+ 5 correct solution.

0 for incorrect solution.

7. (5 points) Consider a set of K keys being hashed to m slots, where $|K| = (n-1) * m + 1$. Show that there is a subset of $K_s \subset K$ of size n such that all its elements are hashed into the same slot, and prove that the worst-case time for any dictionary operation is $\Theta(n)$ when hashing with chaining is used.

Solution: Since, $|K| = (n-1) * m + 1$ and size of the hash table is m by pigeon hole principle after each slot of the table get $n-1$ keys one extra key is left out. So, this key must be placed into one of the m slots. Since, each slot already has $n-1$ keys, putting 1 extra key to any slot will lead to that slot now getting n keys. If we use hashing with chaining, the chain for the slot with n keys will need $\Theta(n)$ in an average for any dictionary operatiom.

+ 5 correct solution with explanation.

0 for incorrect solution.

8. (3 points) Suppose we need to have 1500 items in a hash table using hashing with chaining. The requirement for our application is that the average number of accesses for a successful search should be 2.0. What is the array size should we allocate for the hash table?

Solution: The load factor $\alpha = 1500/m$. For hashing with chaining the average number of accesses for successful search is $1 + \frac{\alpha}{2}$. Setting $1 + \frac{\alpha}{2} = 2$, we find $\alpha = 2$. But we know that $\alpha = \frac{1500}{m}$. So, $m = 750$.

+3 for correct solution.

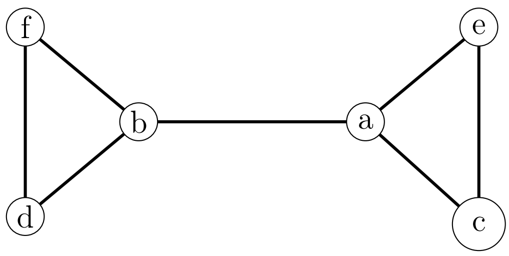
0 for incorrect solution.

9. (4 points) Given below is the adjacency matrix representation of a graph.

	a	b	c	d	e	f
a	0	1	1	0	1	0
b	1	0	0	1	0	1
c	1	0	0	0	1	0
d	0	1	0	0	0	1
e	1	0	1	0	0	0
f	0	1	0	1	0	0

Draw the graph. Is the graph connected? If yes, what is the minimum number of edges to be deleted to increase the number of connected components of the graph by 1. Identify those edges.

Solution: The graph is given below.



It is a connected graph. Deletion of edge (a, b) will make it disconnected.

+4 for correct solution.

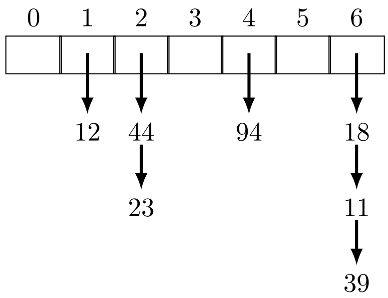
0 for incorrect solution.

10. (5 points) Draw a 7-slot hash table using following hash functions

$$h(x) = (2 * x + 5) \mod 7$$

to hash keys 12, 44, 18, 23, 94, 11, and 39 assuming that collisions are handled by chaining.

Solution:



Explanation: 12, 44, 18, 23, 94, 11, 39 will be mapped respectively as follows:

- 2*12+5 = 29 mod 7 = 1
- 2*44+5 = 93 mod 7 = 2
- 2*18+5 = 41 mod 7 = 6
- 2*23+5 = 51 mod 7 = 2
- 2*94+5 = 193 mod 7 = 4
- 2*11+5 = 27 mod 7 = 6
- 2*39+5 = 83 mod 7 = 6

Therefore, the elements and corresponding slots would be:

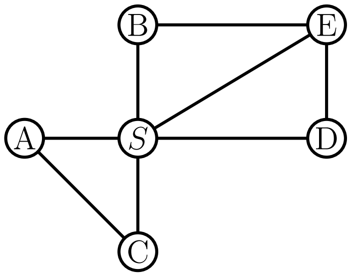
Element	Slot No.
12	1
44, 23	2
94	4
18, 11, 39	6

+5 for correct solution with explanation.

+4 for correct solution without explanation.

0 for incorrect solution.

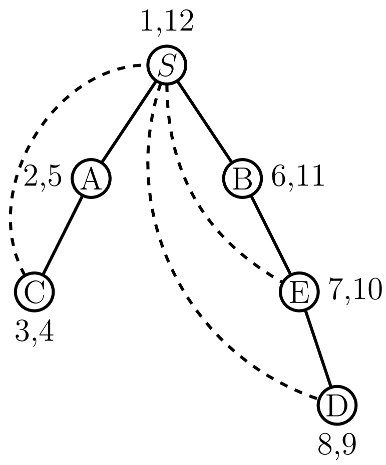
11. (6 points) Consider the following graph. Assume that the adjacency list of each vertex is ordered alphabetically. Now perform a DFS with S as start vertex.



Give the start time of DFS for each each vertex, where, start time is the instance when the vertex is visited first while performing DFS. Also give the finish time of DFS for each vertex, where finish time is the instance when a vertex is visited for the last by DFS.

(**Hint**: Time starts at unit 1 when DFS starts and increases by 1 unit each time on visiting a vertex. As you know, a vertex may be visited several times during a DFS.)

Solution:



+6 for correct solution of DFS tree with correct start and correct finish time.

+4 for correct solution of DFS with correct start time.

0 otherwise.

12. (5 points) The hash function

$$h(x, i) = (2 * x + 5 + i) \mod 7, i = 0, 1, \ldots$$

is used for open address hashing with linear probing. After inserting elements 12, 44, 13, 18, 23, 94, and 39 the hash table is shown the figure below.

0	1	2	3	4	5	6
23	12	44	39	94	18	13

There are some mistakes in the table. Draw the correct hash table.

Solution: Except for 12 and 44 all elements are in wrong slots. The correct hash table is:

0	1	2	3	4	5	6
39	12	44	13	23	94	18

I the given sequence of insertions

- In first probe, i.e. $i = 0$, 12 goes to slot 1, 44 goes to slot 2, 13 goes to slot 3, 18 goes to slot 6.
- for the next element 23, probe $i = 0$ give slot 2. Since 2 and 3 are both occupied it will go into slot 4 in probe $i = 2$.
- for the next element 94, probe $i = 0$ gives slot 4. Since it is occupied 94 inserted into slot 5 by probe $i = 1$.
- For the next element 39, probe $i = 0$ gives slot 4, but slot 4 is occupied, next slot 5 is occupied, so is 6, so in probe $i = 3$, 39 gets inserted into slot 0.

+5 for all correct solution with explanation.

+3 for correct solution without explanation.

0 otherwise.