CS 102: QUIZ-2

Full time for this examination is 45 mts
Mention your name and roll number in each page of the paper in the space provided.
Answer the questions in the spaces provided on the question sheets.
Provide appropriate (but brief) explanation wherever needed.

Name: Soultions

Roll No: Solutions

(Feb 05, 2020)

1. (10 points) Mark True/False
   (a) (1 point) (**TRUE**/**FALSE**) For the division method, the most appropriate choice of table size m is an exact power of 2.
   (b) (1 point) (**TRUE**/**FALSE**) Hash table can be considered as a data structure in which keys are mapped to array positions by a hash function.
   (c) (1 point) (**TRUE**/**FALSE**) The storage requirement for a hash table is $O(n)$, where n is the actual number of keys used.
   (d) (1 point) (**TRUE**/**FALSE**) In the field $(Z_{10}, +, *)$, the multiplicative inverse of 2 is 5, where both + and * are modulo 10 operations.
   (e) (1 point) (**TRUE**/**FALSE**) The set $M(2, R)$ of all invertible 2×2 matrices with matrix multiplication is a commutative group.
   (f) (1 point) (**TRUE**/**FALSE**) Picking of a vector of random values $\langle a_0, a_1, \cdots, a_{r-1}\rangle$ is equivalent to picking a random hash function from the family of universal hash functions assuming each key as a r-digit number in base-m.
   (g) (1 point) (**TRUE**/**FALSE**) Ring is a non-commutative additive group where each nonzero element has a multiplicative inverse.
   (h) (1 point) (**TRUE**/**FALSE**) In closed hasing collision resolution by linear probing leads to primary clustering.
   (i) (1 point) (**TRUE**/**FALSE**) In closed hash the hash table is expanded by $m/2$ each time when the number of keys exceeds $m + 1$.
   (j) (1 point) (**TRUE**/**FALSE**) In closed hasing collision resolution by quadratic probing leads to primary clustering.

(a) False

(b) True

(c) True

(d) False

(e) False

(f) True

(g) False

(h) True

(i) False

(j) False

2. (4 points) Tick the correct answer for each of the following questions from among the possible choices.
   (a) (1 point) In a hash table, an element with key k is mapped to slot:
   (i) k (ii) $\log k$ (iii) $h(k)$ (iv) k^2
   (b) (1 point) In which of the following hash function, consecutive keys map to consecutive hash values?
   (i) Division method (ii) Multiplication method (iii) Folding method (iv) Mid-square method
   (c) (1 point) For unsuccessful search the number of links traveresed in hashing with chaining is:
   (i) $1{+}2\alpha$ (ii) $\log(1+\alpha)$ (iii) $1+\alpha$ (iv) $1+\log\alpha$
   (d) (1 point) Suppose we place m items in a hash table with an array size of s. What is the correct formula for the load factor?
   (i) $s+m$ (ii) $s-m$ (iii) $m-s$ (iv) m/s

(a) (iii)
(b) (i)
(c) (iii)
(d) (iv)

3. (6 points) Calculate hash values for the keys 5678, 321 and 34567 for hash table of 100 locations using:
   (a) (3 points) Folding method.
   (b) (3 points) Mid-square method

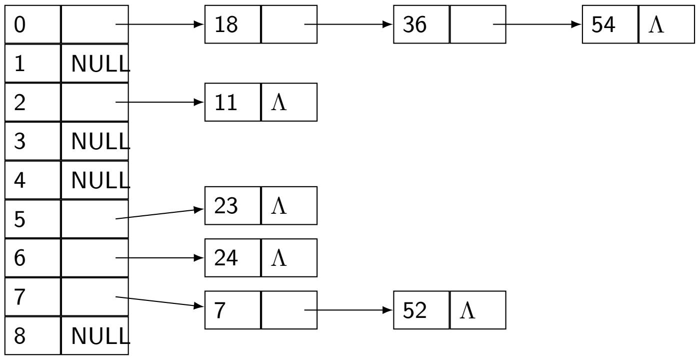
(a) Since 100 locations available in the table, we break the keys in two-digit parts. So,

key	5678	321	34567
Parts	56 and 78	32 and 1	34 56 and 7
Sum	134	33	97
Hash value	34	33	97

(b) The hash table has 100 memory locations whose indices vary from 0 to 99. This means that only two digits are needed to map the key to a location in the hash table, so $r = 2$. The hash value calculations are shown in the table below.

key k	5678	321	34567
k^2	32239684	103041	1194877489
$h(k)$	39	30	87

4. (5 points) Insert the keys 7, 24, 18, 52, 36, 54, 11, and 23 in a chained hash table of 9 memory locations. Use $h(k) = k \mod 9$.



5. (5 points) Suppose that we hash a string $s[1..r]$ of r characters into m slots by treating it as a base-128 number and then using the division method. We can easily represent the number m as a 32-bit computer word, but the string of r characters, treated as a base-128 number, takes many words. So we designed a three line code snippet which allows us to compute the hash value without using more than a constant number of words of storage outside the string itself. Two critical parts of that code has been blanked out. Provide appropriate replacement for those blanks. Also briefly explain the correctness of your answer.

```
sum = ___A_____;
for i = 1 to r
    sum = (___B___ + s[i]) % m
```

The code will be:

```
sum = 0;
for i = 1 to r
    sum = (sum * 128 + s[i]) % m
```

Since a string of r is treated as a base-128 we cannot consider the entire string at one go as it will take many words. The variable `sum` is used to accumulate value of the string considering each character at a time from left to right. Multiplying `sum` with 128 promotes the previous character of string's positional value. Next we add the current character. However to ensure that value require more than one 32-bit word, we reduce it by applying mod operation after the multiplication.

6. (6 points) Let $\mathcal{H}$ be a class of hash functions in which each $h \in \mathcal{H}$ maps the universe U of keys to $\{0, 1, ..., m-1\}$. Consider $\mathcal{H}$ to be 2-universal, if for every fixed pair (x, y) of keys where $x \neq y$, and for any h chosen uniformly at random from $\mathcal{H}$, the pair $\langle h(x), h(y)\rangle$ is equally likely to be any of the m^2 pairs of elements from $\{0, 1, ..., m-1\}$. (The probability is taken only over the random choice of the hash function.)

Show that, if $\mathcal{H}$ is 2-universal, then it is universal.

Hint: You have to show that

$$Pr_{h\in\mathcal{H}}[h(x) = h(y)] = \frac{1}{m}$$

Since $\langle h(x), h(y)\rangle$ has equal probability to be any of m^2 pair of elements, it implies that for a fixed $i \in U$:

$$Pr[\langle h(x), h(y)\rangle = \langle i, i\rangle] = \frac{1}{m^2}$$

There are exactly m ways in which $h(x)$ and $h(y)$ can collide, i.e., $h(x) = h(y) = i$ for $i \in \{0, 1, 2, \ldots, m-1\}$. Hence,

$$Pr_{h\in\mathcal{H}}[h(x) = h(y)] = \frac{m}{m^2} = \frac{1}{m}$$

Therefore, by definition $\mathcal{H}$ is universal.

7. (4 points) Work out the steps to determine multiplicative inverse of 16 mod 395.

$$\begin{aligned} 395 &= 16 * 24 + 11 \\ 16 &= 11 + 5 \\ 11 &= 2 * 5 + 1 \end{aligned}$$

So, we have

$$\begin{aligned} 1 &= 11 - 2 * 5 = 11 - 2 * (16 - 11) \\ &= 3 * 11 - 2 * 16 = 3 * (395 - 24 * 16) - 2 * 16 \\ &= 3 * 395 - 72 * 16 - 2 * 16 \\ &= 3 * 395 + (-74) * 16 \\ &= 3 * 395 + 321 * 16 = 321 * 16 \end{aligned}$$

Hence, 321 is the multiplicative inverse of 16 mod 395.

Space for Rough Work