CGS698 Assignment-1

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Part-1

1.1 a) Sample space $\Omega = \{HH, HT, TH, TT\}$

b) Event space consists of all possible events that can occur.

No. of possible events $= 2^4 = 16$.

So, the event space:

$\{\phi, \{HH\}, \{HT\}, \{TH\}, \{TT\}, \{HH, HT\},$
$\{HH, TH\}, \{HH, TT\}, \{HT, TH\}, \{HT, TT\},$
$\{TH, TT\}, \{HH, HT, TH\}, \{HH, HT, TT\},$
$\{HH, TH, TT\}, \{HT, TH, TT\},$
$\{HH, HT, TH, TT\}\}$.

c) i) $P(HH) = P(HT) = P(TH) = P(TT)$

$P(HH) + P(HT) + P(TH) + P(TT) = 1$

So,

$$\boxed{P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}}$$

ii) P(Atleast one head) =

$P(\{HH, HT, TH\})$

$= \boxed{P(HH) + P(HT) + P(TH) = \frac{3}{4}}$

iii) $P(\text{Exactly one head}) = P(\{HT, TH\})$

$= \boxed{P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}}$

Part-2.

2.1. $$f(k, n, p) = \frac{n!}{k(n-k)!} p^k (1-p)^{n-k}$$

$k = 45, \quad n = 50, \quad p = 0.9$

$$f(45, 50, 0.9) = \frac{50!}{45!\,5!} (0.9)^{45} (0.1)^{5}$$

$$\boxed{f(45, 50, 0.9) = 0.185}$$

2.2

$$f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad \lambda = 10$$

a) $k = 0, \quad \lambda = 10$

$f(0, 10) = e^{-10} = 4.54 \times 10^{-5}$ $\boxed{f(0,10) = 4.54 \times 10^{-5}}$

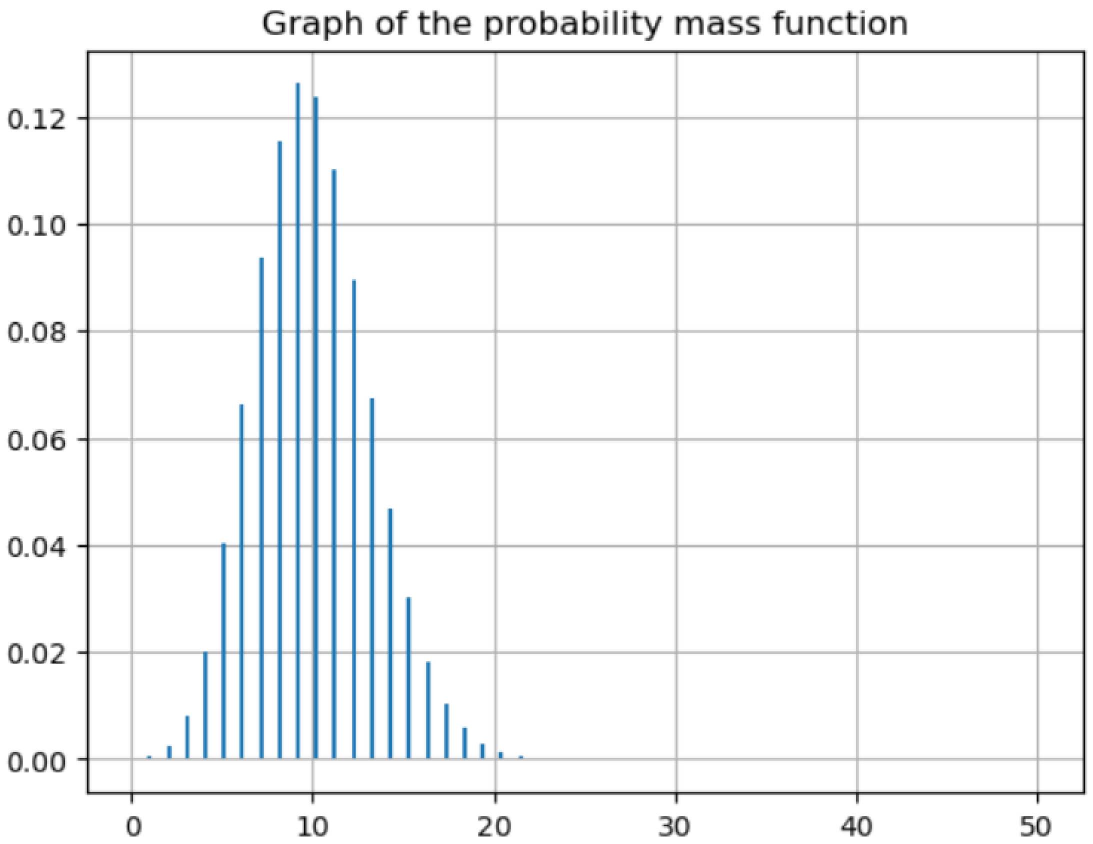
b) $7 < k < 10 \quad k = 8, 9$

$f(8, 10) + f(9, 10)$

$$= \frac{10^8 e^{-10}}{8!} + \frac{10^9 e^{-10}}{9!}$$

$$\boxed{= 0.237}$$

Part 2: Discrete random variables (graph 2.2 (c))



Part - 3

3.1 $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

a) $x = 0, \mu = 1, \sigma = 1$

$$f(0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}$$

$$\boxed{f(0) = 0.24.}$$

b) $x = 1, \mu = 0, \sigma = 1$

$$f(1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}$$

$$\boxed{f(1) = 0.24}$$

c) $$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(x)\,dx = 0.3$$

$$P(x_1 \le X \le x_3) = \int_{x_1}^{x_3} f(x)\,dx = 0.45$$

$$P(x_2 \le X \le x_3) = \int_{x_2}^{x_3} f(x)\,dx$$

$$P(x_1 \le X \le x_2) + P(x_2 \le X \le x_3) = P(x_1 \le X \le x_3)$$

$$P(x_2 \le X \le x_3) = P(x_1 \le X \le x_3) - P(x_1 \le X \le x_2)$$

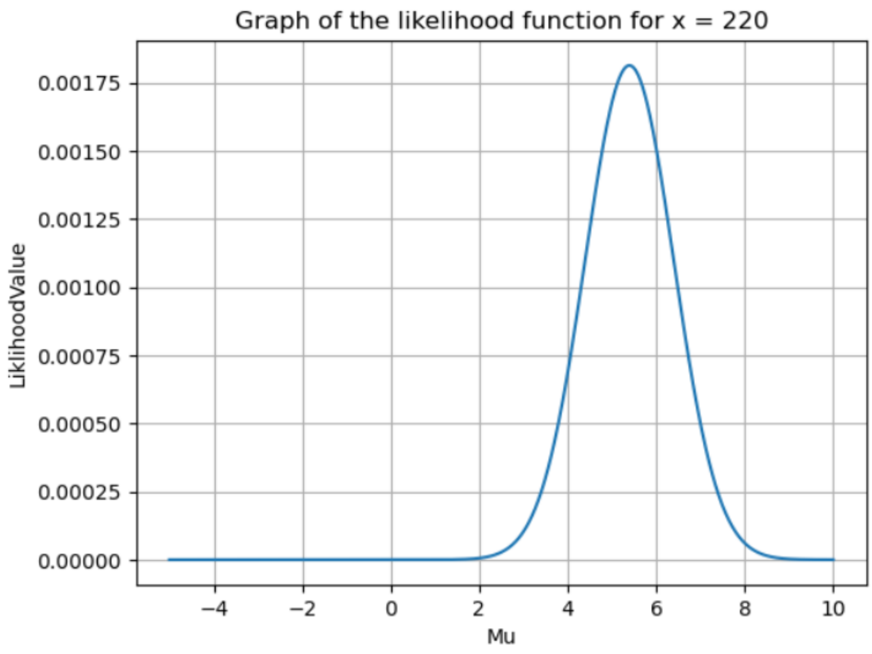
$$P(x_2 \le X \le x_3) = 0.45 - 0.3$$

$$\boxed{P(x_2 \le X \le x_3) = 0.15}$$

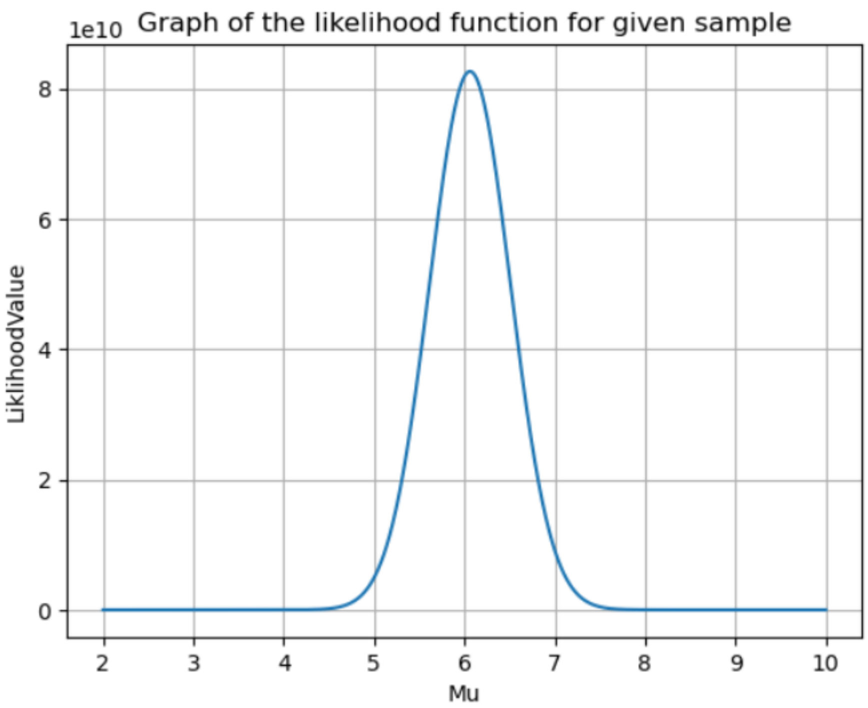
Part 4: The likelihood function

4.1

(a)



(b)



The likelihood function when x is (fixed as) the observed sample of recognition times i.e., 303.25, 443, 220, 560, 880.

Part 4:

4.1 c) $$f(x_{1:n}, \mu_1) = \frac{1}{\left(\prod_{i=1}^{n} x_i\right)\left(\sqrt{2\pi}\right)^n} e^{\frac{-\sum_{i=1}^{n}(\log x_i - \mu_1)^2}{2}}$$

$$\frac{df}{d\mu_1} = 0.$$

$$\frac{-2\sum_{i=1}^{n}(\log x_i - \mu_1)}{2} = 0.$$

$$n\mu_1 = \sum_{i=1}^{n} \log(x_i)$$

$$\boxed{\mu_1 = \frac{\sum_{i=1}^{n} \log(x_i)}{n}}$$

~~x = [303~~

$$x = [303, 443, 220, 560, 880]$$

$$\mu_1 = \frac{(\log 303 + \log 443 + \log 220 + \log 560 + \log 880)}{5}.$$

$$\boxed{\mu_1 = 6.08}$$