CS425 Assignment-2

Anupam Chaudhary(20170)

February 16, 2024

Solution 1: The file named "**CRC.cpp**" can be compiled using the code "**g++ CRC.cpp -o CRC**". On executing this code using "**.\CRC**" the program generates a random 10 bits message and using the pattern(P = 110101) the program calculates the Frame to be transmitted(appending the message with CRC).Upon introduction of randomly generated 15 bits error, the received frame is generated and it is verified whether the received frame is accepted or rejected.

Solution 2: Let's say that the window size is $n = 2^k$, and suppose the sender sends frame 0 and the reciever sends ACK RR1,and then the sender sends frame $\{1, 2, \ldots, n-1, 0\}$.Now,the sender again receives another RR1, this could mean that the receiver has succesfully received the sent frames and RR1 is the cummulative ACK .

On the other hand this could also mean that the sent frames were lost or damaged and the receiver is requesting to retransmit the frames from the previous window.

To avoid this problem,the maximum window size in Go-back-N ARQ mechanism is 2^{k-1}.

Solution 3: Suppose there is a k-bit sequence number for selective reject.Let the window size be w and asssume two windows with, first window having frame numbers $f_1 = \{1, 2, \ldots, w-1\}$ and $f_2 = \{w, w+1, \ldots, 0, \ldots\}$(overlap betwenn f_1 and f_2.Sender sends f_1 and receiver receives f_2 and acknowledges with RRw.

Beacause of Noise burst RRw got lost or damaged,sender runs into timeout due to frame 0 and retransmits the previous window ,it will send frame 0 corresponding to f_1 again. But, as the receiver was expecting the next frame f_2, it will accept the frame 0 into its buffer, and assume that the preceding frames of the window f_2 were lost. Thus, we run into a problem when there is an overlap, because there is ambiguity regarding exactly what window an overlapping frame belongs to. Because of this ambiguity, we cannot have a window size which causes two consecutive windows to have an overlap.

When there is no overlap, we can safely assume that there shall be no ambiguities because we can know exactly to which window a frame belongs. For example, the first w frames belong to the window f_1, and the next w frames (with no overlap with the first w frames) belong strictly to window f_2.

So, it boils down to finding out what window sizes ensure no overlap. Since, two consecutive windows are disjoint, they contain exactly $w + w = 2w$ distinct frames with them. Moreover, with k-bit sequence numbers, the maximum number of frames is 2^k.Thus, we have, $2w \leq 2^k \implies w \leq 2^{k-1}$
i.e,

$$\text{Maximum Window Size} = 2^{k-1}$$

Solution 4: According to the question,

$$U \geq 50\% = \frac{1}{2}$$

$$U = \frac{1}{2a+1}$$

$$\frac{1}{2a+1} \geq \frac{1}{2}$$

$$2a \leq 1$$

$$a \leq \frac{1}{2}$$

$$\text{where, a} = \frac{\text{Propagation time}}{\text{Trasmission time}} = \frac{t_{prop}}{t_{frame}}$$

we are given $t_{prop} = 20\text{ms}$,

$$\frac{20\text{ms}}{t_{frame}} \leq \frac{1}{2}$$

$$t_{frame} \geq 40\text{ms}$$

Let **S** be the size of the frame,given that data rate = 4kbps,

$$t_{frame} = \frac{\text{S}}{4\text{kbps}} \geq 40\text{ms}$$

$$\frac{\text{S}}{4000\text{bps}} \geq 40 \times 10^{-3}\text{s}$$

$$\text{S} \geq 160\text{bits}$$

Therefore,the size of the frame should be atleast 160 bits in order for the efficiency to be atleast 50% in stop-and-wait.

Solution 5: Let F be the number of bits per frame i.e, F = 4(according to the question).p be the probability of bit error, p = 10^{-3}

a) P(E_i) be the probability if ith bit is flipped from its original bit.

$$\text{P}(\bar{E}) = 1 - p$$

Probability that there is no error in any bit is given as:

$$\mathrm{P}(\bar{E_1} \cap \bar{E_2} \cap \bar{E_3} \cap \bar{E_4})$$

Since each bit is independent,

$$\begin{aligned}
\mathrm{P}(\bar{E_1} \cap \bar{E_2} \cap \bar{E_3} \cap \bar{E_4}) &= \prod_{i=1}^{i=4} \mathrm{P}(E_i) \\
&= \prod_{i=1}^{i=4} (1-p) \\
&= (1-p)^4 \\
&= (1-10^{-3})^4 \\
&\approx 0.996
\end{aligned}$$

b) Let P_2 be the probability that the received frame contains atleat one error. So, P_2 is given by,

$$\mathrm{P}_2 = 1 - p$$
$$\mathrm{P}_2 = 1 - 0.996$$
$$\mathrm{P}_2 = 0.004 = 4 \times 10^{-3}$$

c) After adding a parit bit into the frame we will now receive 5 bits instead of 4 bits.However,with the parity check,error can not be detected if there are even number of bit ineversions.So,we need to calculate the probability of those cases where there are even number of bit are in error i.e, 2 or 4 bit are in error. Probability that the frame is in error and can not be detected is given by,

Probability when 2 bits are in error + Probability when 4 bits are in error

$$= \binom{5}{2} \times p^2 \times (1-p)^3 + \binom{5}{4} \times p^4 \times (1-p)$$

$$= \binom{5}{2} \times (10^{-3})^2 \times (1-10^{-3})^3 + \binom{5}{4} \times (10^{-3})^4 \times (1-10^{-3})$$

$$\approx 9.97 \times 10^{-6}$$

Solution 6: First append the message **M** with $5(|\mathrm{P}| - 1)$ zeroes on the right side. Therfore, Dividend = 1110001100000 and Divisor = 110011.

$$
\begin{array}{r|ccccccccccccc}
\multicolumn{1}{r}{} & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & & & & & \\
\cline{2-14}
110011 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\multicolumn{1}{r}{} & 1 & 1 & 0 & 0 & 1 & 1 & & & & & & & \\
\cline{2-7}
\multicolumn{1}{r}{} & & 0 & 1 & 0 & 1 & 1 & 1 & & & & & & \\
\multicolumn{1}{r}{} & & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
\cline{3-8}
\multicolumn{1}{r}{} & & & 1 & 0 & 1 & 1 & 1 & 1 & & & & & \\
\multicolumn{1}{r}{} & & & 1 & 1 & 0 & 0 & 1 & 1 & & & & & \\
\cline{4-9}
\multicolumn{1}{r}{} & & & & 1 & 1 & 1 & 0 & 0 & 0 & & & & \\
\multicolumn{1}{r}{} & & & & 1 & 1 & 0 & 0 & 1 & 1 & & & & \\
\cline{5-10}
\multicolumn{1}{r}{} & & & & & 0 & 1 & 0 & 1 & 1 & 0 & & & \\
\multicolumn{1}{r}{} & & & & & 0 & 0 & 0 & 0 & 0 & 0 & & & \\
\cline{6-11}
\multicolumn{1}{r}{} & & & & & & 1 & 0 & 1 & 1 & 0 & 0 & & \\
\multicolumn{1}{r}{} & & & & & & 1 & 1 & 0 & 0 & 1 & 1 & & \\
\cline{7-12}
\multicolumn{1}{r}{} & & & & & & & 1 & 1 & 1 & 1 & 1 & 0 & \\
\multicolumn{1}{r}{} & & & & & & & 1 & 1 & 0 & 0 & 1 & 1 & \\
\cline{8-13}
\multicolumn{1}{r}{} & & & & & & & & 0 & 1 & 1 & 0 & 1 & 0 \\
\multicolumn{1}{r}{} & & & & & & & & 0 & 0 & 0 & 0 & 0 & 0 \\
\cline{9-14}
\multicolumn{1}{r}{} & & & & & & & & & 1 & 1 & 0 & 1 & 0
\end{array}
$$

After dividing we get Remainder = 11010 and quotient = 10110110.Since CRC is the remainder, we have,

$$\text{CRC} = 11010$$

Solution 7:

a) We are given with Polynomial,

$$P(x) = X^4 + X + 1$$

converting P(x) into binary sequence, we get

$$\text{P} = 10011(5 \text{ bits}) \text{ and M} = 10010011011$$

Appending M with $4(|\text{P}| - 1)$ zeroes on the right side.
Therefore,Dividend = 100100110110000 and Divisor = 10011.

$$
\begin{array}{r|ccccccccccccccc}
 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & & & & & \\
\cline{2-16}
10011 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
 & 1 & 0 & 0 & 1 & 1 & & & & & & & & & & \\
\cline{2-7}
 & & 0 & 0 & 0 & 1 & 0 & & & & & & & & & \\
 & & 0 & 0 & 0 & 0 & 0 & & & & & & & & & \\
\cline{3-8}
 & & & 0 & 0 & 1 & 0 & 1 & & & & & & & & \\
 & & & 0 & 0 & 0 & 0 & 0 & & & & & & & & \\
\cline{4-9}
 & & & & 0 & 1 & 0 & 1 & 1 & & & & & & & \\
 & & & & 0 & 0 & 0 & 0 & 0 & & & & & & & \\
\cline{5-10}
 & & & & & 1 & 0 & 1 & 1 & 0 & & & & & & \\
 & & & & & 1 & 0 & 0 & 1 & 1 & & & & & & \\
\cline{6-11}
 & & & & & & 0 & 1 & 0 & 1 & 1 & & & & & \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & & & & & \\
\cline{7-12}
 & & & & & & & 1 & 0 & 1 & 1 & 1 & & & & \\
 & & & & & & & 1 & 0 & 0 & 1 & 1 & & & & \\
\cline{8-13}
 & & & & & & & & 0 & 1 & 0 & 0 & 0 & & & \\
 & & & & & & & & 0 & 0 & 0 & 0 & 0 & & & \\
\cline{9-14}
 & & & & & & & & & 1 & 0 & 0 & 0 & 0 & & \\
 & & & & & & & & & 1 & 0 & 0 & 1 & 1 & & \\
\cline{10-15}
 & & & & & & & & & & 0 & 0 & 1 & 1 & 0 & \\
 & & & & & & & & & & 0 & 0 & 0 & 0 & 0 & \\
\cline{11-16}
 & & & & & & & & & & & 0 & 1 & 1 & 0 & 0 \\
 & & & & & & & & & & & 0 & 0 & 0 & 0 & 0 \\
\cline{12-16}
 & & & & & & & & & & & & 1 & 1 & 0 & 0
\end{array}
$$

After dividing we get quotient = 10001010100 and remainder = 1100.Since,CRC is the remainder, we have,

$$\text{CRC} = 1100$$

To encode the message M, we concatenate CRC to this message.So, the encoding of 10010011011 is 100100110111100.

b)To obtain the received signal we simply flip the 1^{st} and 5^{th} bit of the intended signal(obtained in the previous part), we get,

$$\text{Received Signal} = \text{S} = 000110110111100$$

We could also have obtained **S** by taking XOR of **introduced error** with **intended signal**.

To detect for an error we have to check for reminder after dividing **S**(received signal) with P(x)(from previous part,P=10011).

$$
\begin{array}{r|ccccccccccccccc}
 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & & & & \\
\cline{2-16}
10011 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
 & & & & 1 & 0 & 0 & 1 & 1 & & & & & & & \\
\cline{5-9}
 & & & & & 1 & 0 & 0 & 0 & 0 & & & & & & \\
 & & & & & 1 & 0 & 0 & 1 & 1 & & & & & & \\
\cline{6-10}
 & & & & & & 0 & 0 & 1 & 1 & 1 & & & & & \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & & & & & \\
\cline{7-11}
 & & & & & & & 0 & 1 & 1 & 1 & 1 & & & & \\
 & & & & & & & 0 & 0 & 0 & 0 & 0 & & & & \\
\cline{8-12}
 & & & & & & & & 1 & 1 & 1 & 1 & 1 & & & \\
 & & & & & & & & 1 & 0 & 0 & 1 & 1 & & & \\
\cline{9-13}
 & & & & & & & & & 1 & 1 & 0 & 0 & 1 & & \\
 & & & & & & & & & 1 & 0 & 0 & 1 & 1 & & \\
\cline{10-14}
 & & & & & & & & & & 1 & 0 & 1 & 0 & 0 & \\
 & & & & & & & & & & 1 & 0 & 0 & 1 & 1 & \\
\cline{11-15}
 & & & & & & & & & & & 0 & 1 & 1 & 1 & 0 \\
 & & & & & & & & & & & 0 & 0 & 0 & 0 & 0 \\
\cline{12-16}
 & & & & & & & & & & & & 1 & 1 & 1 & 0
\end{array}
$$

Here,as we can see that remainder is non-zero.We can conclude that there is an error in the received signal.

From the divison we found remainer R = 1110 $\iff$ R(x) = $X^3 + X^2 + X$ which is non-zero.Hence,**error is detected**.

c) To obtain the received signal we have to take XOR of **error pattern** and the **intended signal**.

Received signal S is given by,

$$\mathrm{S} = (100100110111100) \bigoplus (100110000000000)$$

i.e, S = 000010110111100.And,now to detect error we follow the same procedure as in the previous part.Divide S by P.

$$
\begin{array}{r|ccccccccccccccc}
\multicolumn{1}{r}{} & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & & & & \\
\cline{2-16}
10011 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
\multicolumn{1}{r}{} & & & & & 1 & 0 & 0 & 1 & 1 & & & & & & \\
\cline{6-11}
\multicolumn{1}{r}{} & & & & & & 0 & 1 & 0 & 1 & 1 & & & & & \\
\multicolumn{1}{r}{} & & & & & & 0 & 0 & 0 & 0 & 0 & & & & & \\
\cline{7-12}
\multicolumn{1}{r}{} & & & & & & & 1 & 0 & 1 & 1 & 1 & & & & \\
\multicolumn{1}{r}{} & & & & & & & 1 & 0 & 0 & 1 & 1 & & & & \\
\cline{8-13}
\multicolumn{1}{r}{} & & & & & & & & 0 & 1 & 0 & 0 & 1 & & & \\
\multicolumn{1}{r}{} & & & & & & & & 0 & 0 & 0 & 0 & 0 & & & \\
\cline{9-14}
\multicolumn{1}{r}{} & & & & & & & & & 1 & 0 & 0 & 1 & 1 & & \\
\multicolumn{1}{r}{} & & & & & & & & & 1 & 0 & 0 & 1 & 1 & & \\
\cline{10-15}
\multicolumn{1}{r}{} & & & & & & & & & & 0 & 0 & 0 & 0 & 0 & \\
\multicolumn{1}{r}{} & & & & & & & & & & 0 & 0 & 0 & 0 & 0 & \\
\cline{11-16}
\multicolumn{1}{r}{} & & & & & & & & & & & 0 & 0 & 0 & 0 & 0 \\
\multicolumn{1}{r}{} & & & & & & & & & & & 0 & 0 & 0 & 0 & 0 \\
\cline{12-16}
\multicolumn{1}{r}{} & & & & & & & & & & & & 0 & 0 & 0 & 0
\end{array}
$$

From the divison we found that remainder R = 0 $\iff$ R(x) = 0, the **error can not be detected**.