

Strongly Connected Components

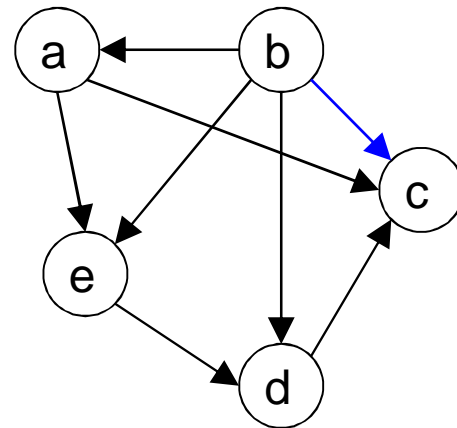
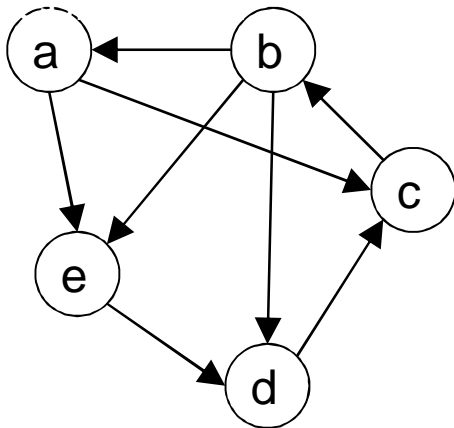
for Directed Graph

Outlines

- ✓ Introduction
- ✓ DFS and Post Fix Order
- ✓ Reverse Directed Graph
- ✓ Example

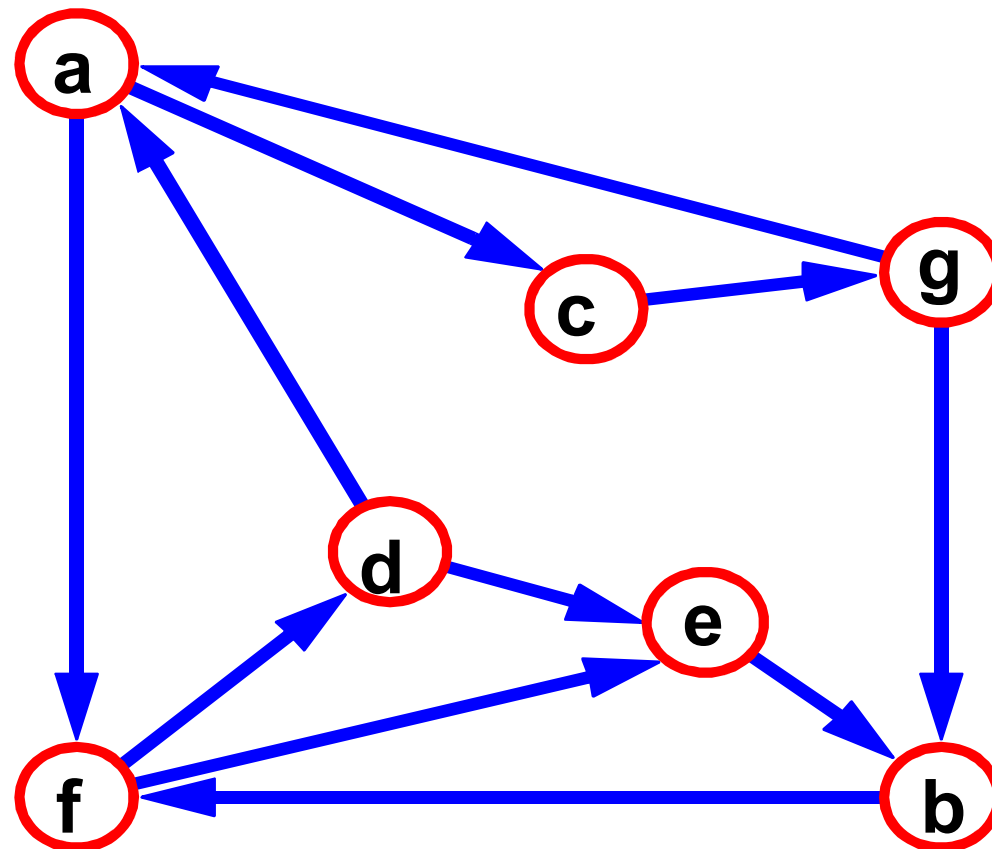
Strongly Connected Directed graphs

Graph G is ***strongly connected*** if for every u and v in V , there is some path from u to v and some path from v to u .



Strongly Connected Directed graphs

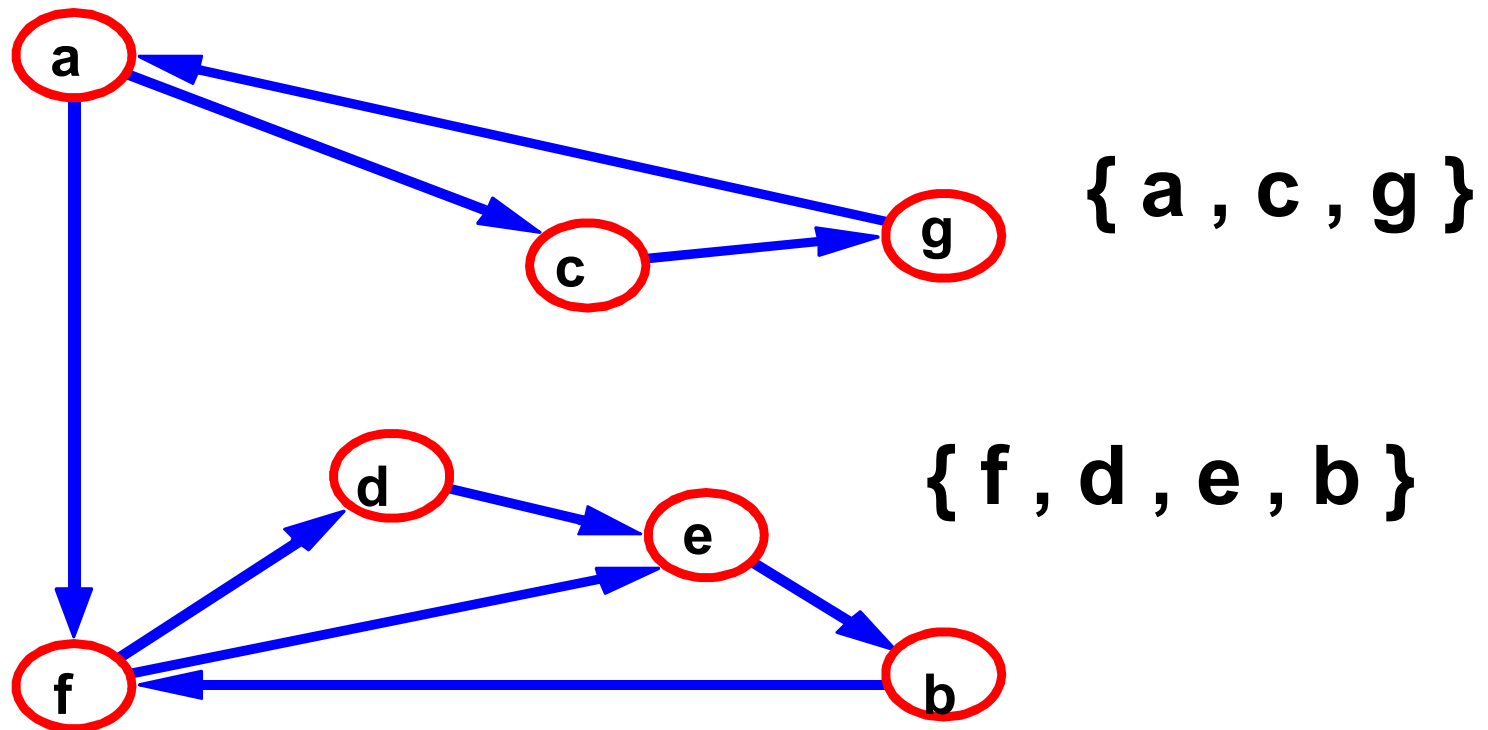
Every pair of vertices are reachable from each other.



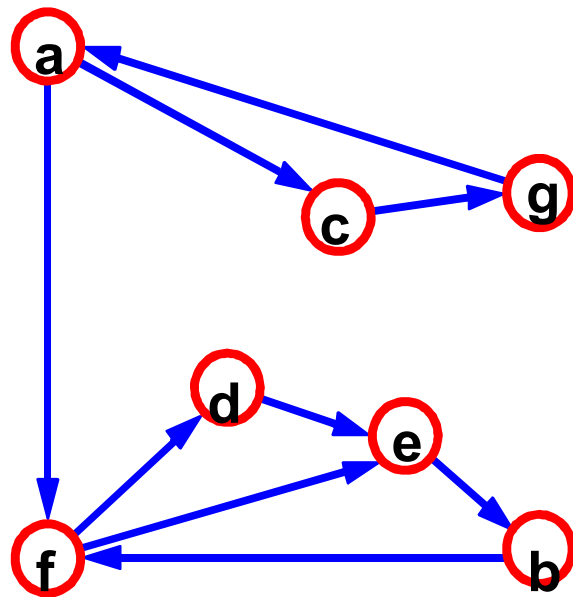
Strongly-Connected Components

A strongly connected ***component*** of a graph is a maximal subset of nodes (along with their associated edges) that is strongly connected. Nodes share a strongly connected component if they are **inter-reachable**.

Strongly Connected Components



Reduced Component Graph of Strongly Connected Components



$\{ a , c , g \}$
↓
 $\{ f , d , e , b \}$

Component graph $G^{SCC}=(V^{SCC}, E^{SCC})$: one vertex for each component

$(u, v) \in E^{SCC}$ if there exists at least one directed edge from the corresponding components

Algorithm

The following algorithm will determine if a directed graph contains strongly connected components:

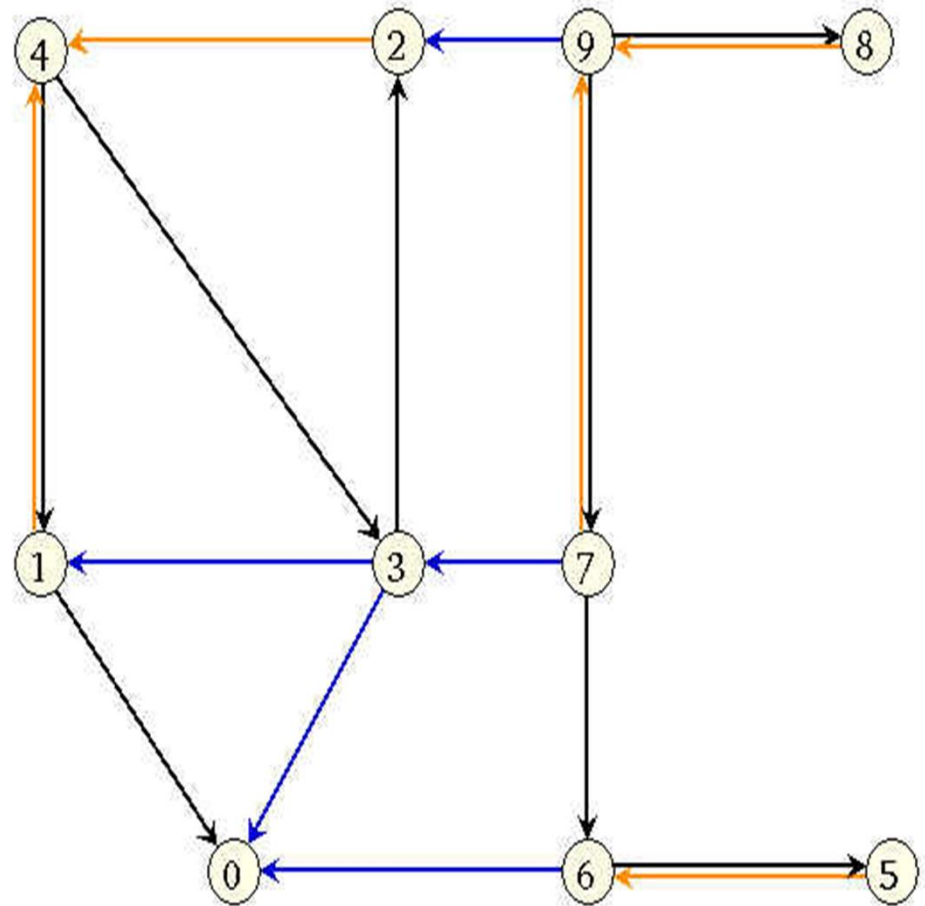
1. Perform depth first search (DFS) and label vertices in post fix order.
2. Compute reversed directed graph.
3. Perform depth first search on reversed graph.
4. Components of this forest are strongly connected components.

Depth First Search and Post Fix Order

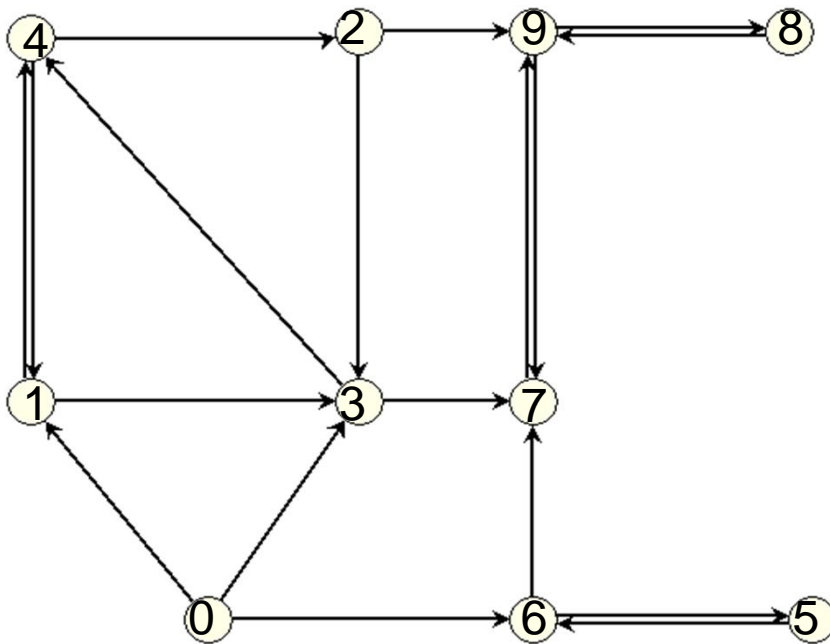
Perform depth first search on the graph: black – tree edge, blue - cross edge, orange - back edge.

Label nodes in post fix order.

Note: Start DFS at node 4, then node 9.



Reverse Directed Graph



Compute reversed directed graph (copy the graph but reverse the arrows).

Transpose of a Digraph

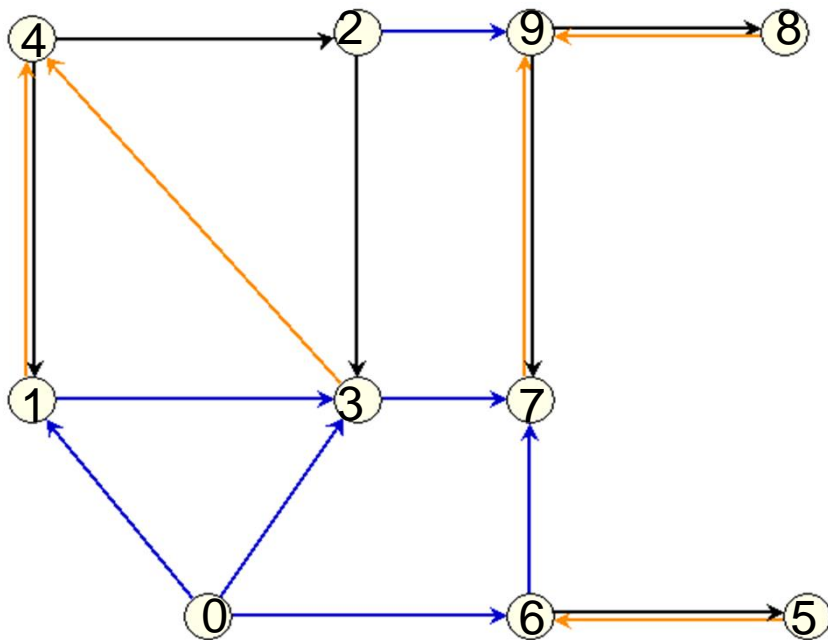
Transpose of $G = (V, E)$:

$G^T = (V, E^T)$, where $E^T = \{(u, v) : (v, u) \in E\}$

If G is a DAG then G^T is also a DAG

If we print the topological order of G in the reverse order, then it is a topological order of G^T

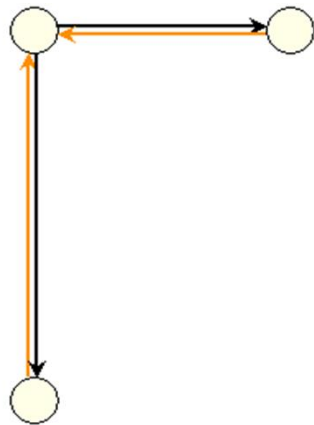
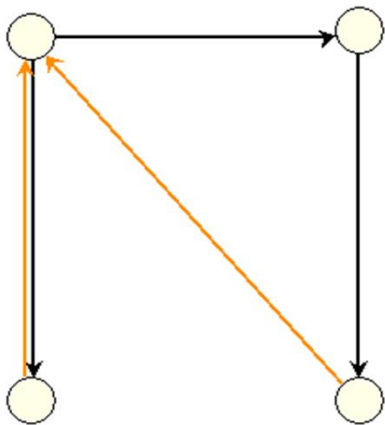
Depth First Search on Reversed Graph



Perform depth first search on reversed graph starting as highest post fix order vertex.

Continue until finished .

Strongly Connected Components



Components from the depth first search of the reversed graph compose the strongly connected components of the original graph

Runtime

Lines 1 and 3 are $\Theta(E+V)$ due to DFS

Line 2 involves creating an adjacency list or matrix, and it is also $O(E+V)$

Line 4 is constant time

So, $\text{SCC}(G)$ is $\Theta(E+V)$