

DIGITAL IMAGE PROCESSING

Assignment 3: Theoretical Questions

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I. Explain which assumptions lead to the “unexpected” border values in each image and why they are different for both methods.

Ans: The “unexpected” behavior is due to the border handling applied to convolution. As a part of convolution, in order to ensure that the size of output image matches that of the input image, the output borders are padded. This padding leads to black borders as their values are set to 0 leading to the “unexpected” behavior.

Frequency convolution is periodic and the said borders in frequency convolution are because the kernel cyclically shifted and written into a matrix same size as the input image. Their border behavior is different for both methods because border handling in spatial and frequency domain are different.

II. What steps are necessary for the convolution in spatial domain to produce the result in Fig. 1c)?

Ans: As mentioned above, frequency convolution is periodic and the said borders in frequency convolution are because the kernel cyclically shifted and written into a matrix same size as the input image. This doesn't require border padding as needed by the spatial convolution. So if kernel is made same size as the input image and cyclically rotated, spatial convolution can achieve same border results as the frequency convolution.

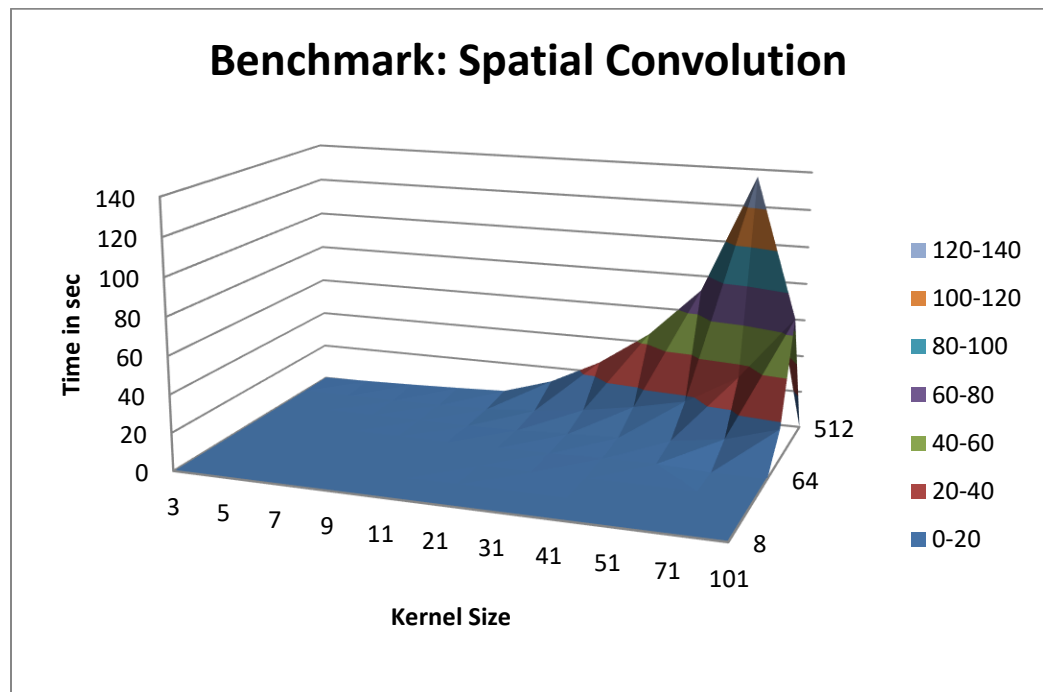
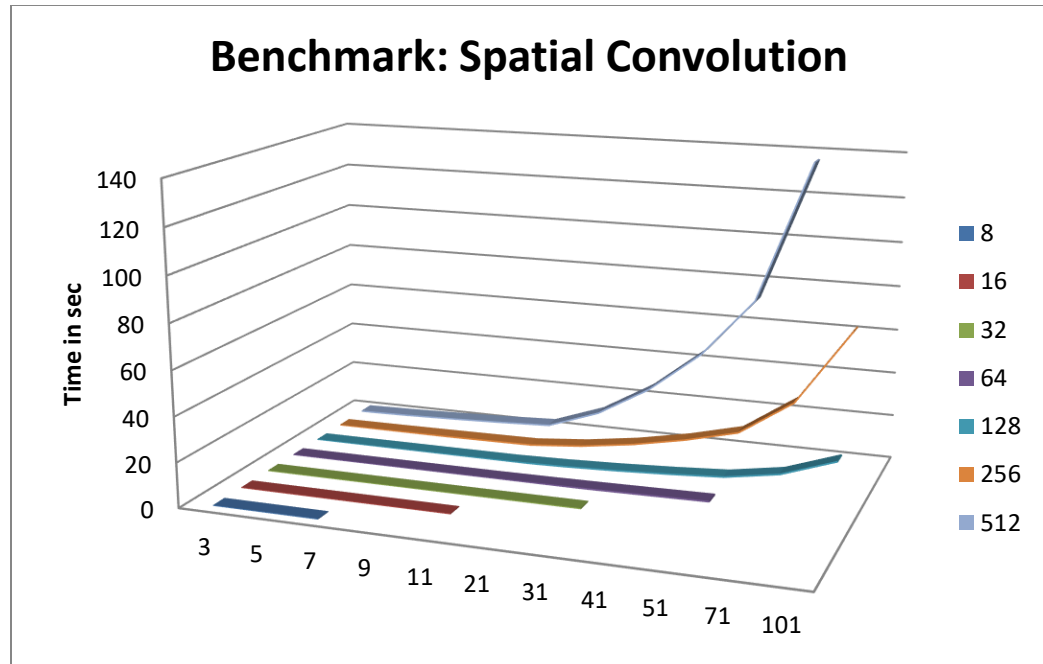
III. What steps are necessary for the convolution by multiplication in frequency domain to produce the result in Fig. 1b)?

Ans: Spatial convolution has borders due to padding at the borders. So if border padding is used in frequency convolution, the borders of spatial convolution can be achieved in the frequency convolution.

Time Behavior of convolution - Benchmarking

I. Spatial Convolution:

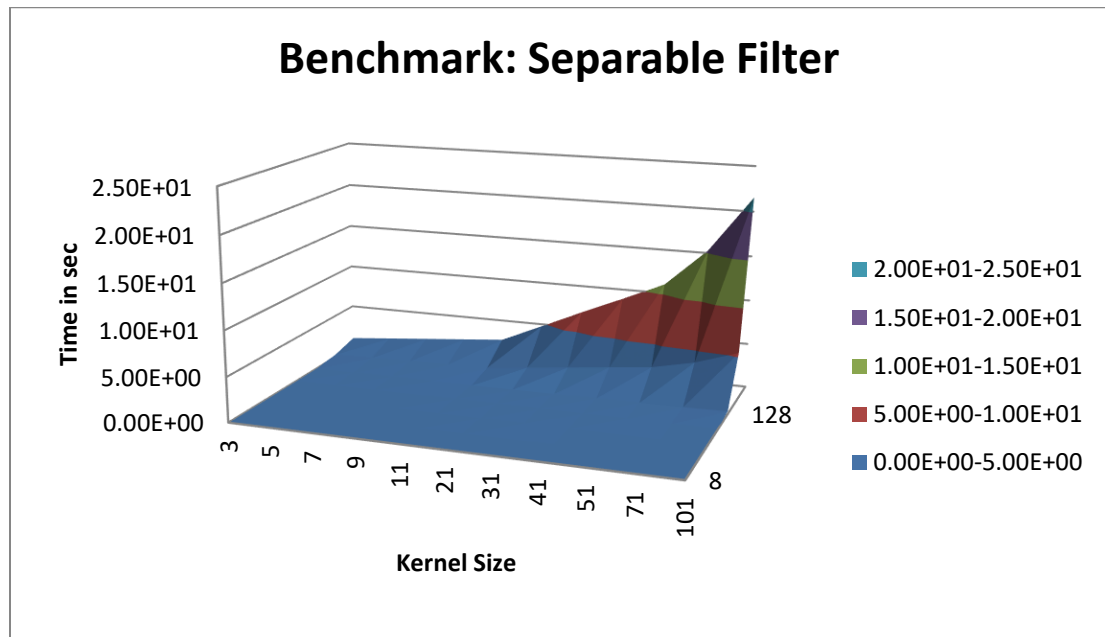
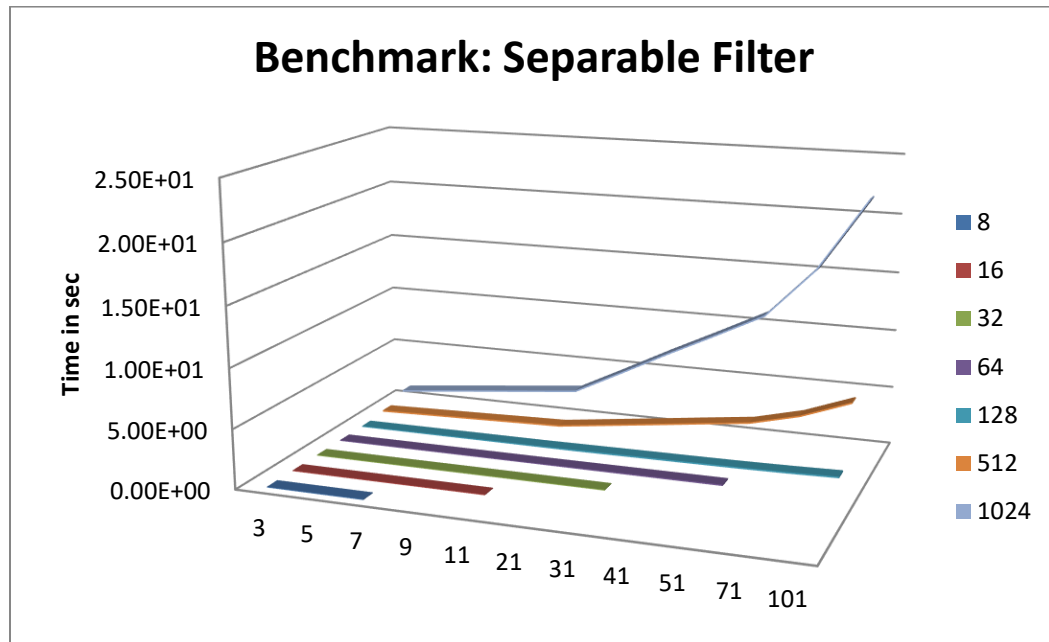
Spatial convolution is performed by shifting the kernel over the input image and multiplying them. The complexity of spatial convolution is of the order of $O(n^4)$. As the kernel and input image size goes on increasing, the computations needed increase and this causes the time for convolution to increase exponentially. This can be seen in the plot below, for larger kernel size and input image size, the time needed is the highest.



II. Separable Filter:

Separable filter performs 2 1-D convolutions horizontally and vertically respectively. The time complexity is $O(n)$. Hence the computation and time needed for the same is linear as can be seen in the plot below. The slope of the linearly increasing line depends on the size of

the kernel and the input image. As can be seen in the plots below, as the size of image and kernel increases, the slope increases.



III. Frequency Convolution:

For frequency convolution, the time complexity is $O(n \log(n))$. As can be seen from plots below, as the kernel and image size increases, the time needed for frequency convolution

increases. However, for a constant image size, as the kernel size changes, the time difference doesn't change a lot and remains fairly constant.

