Digital Image Processing

Berlin University of Technology (TUB),
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Image Restoration

- Image analysis often begins with pre-processing

Enhancement

- Contrast, brightness, sharpening etc.
- Working with information inherent in the signal

Restoration

- Sensor defects (noise, blur)
- Unfortunate conditions (moving objects)
- Ageing originals
- Restoring information that has been lost

Image Restoration

Image Restoration – Inverse Filter

Signal Model

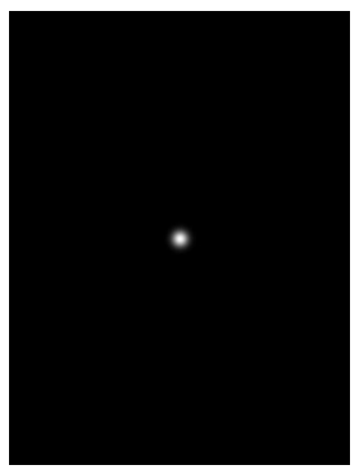
$$s_i = \sum_j o_j \, p_{i-j} \quad \xrightarrow{\text{FFT}} \quad S_i = O_i \cdot P_i$$

s_i: Observed signal	S_i: Spectrum of the observed signal
o_i: Original signal	O_i: Spectrum of the original signal
p_i: Impulse response	P_i: Spectrum of the impulse response

- E.g. camera with a small aperture (kernel p causes blurring)

o_i

$\otimes$



p_i

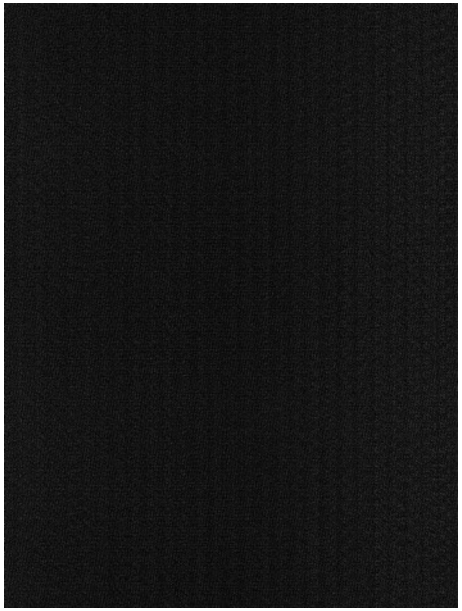
s_i

Image Restoration – Inverse Filter

- **Convolution Theorem**:
  - Convolution is equivalent to multiplication in the frequency domain
  - Multiplication is easly reversed by division!

$$S_i = O_i \cdot P_i \longrightarrow O_i = S_i \cdot 1/P_i \longrightarrow o_i = IFFT(1/P_i) * s$$
$$= IFFT(S_i/P_i)$$

$\otimes \quad IFFT(1/P_i) =$



?

Image Restoration – Inverse Filter

- **Problem**: P_i is practically equal to zero in some parts of the spectrum
  - Lowpass filters (blur) induce $P_i = 0$ at high frequencies
  - Inversion is not feasible due to limited numerical accuracy
  - E.g. small inaccuracies in the FFT are strongly amplified

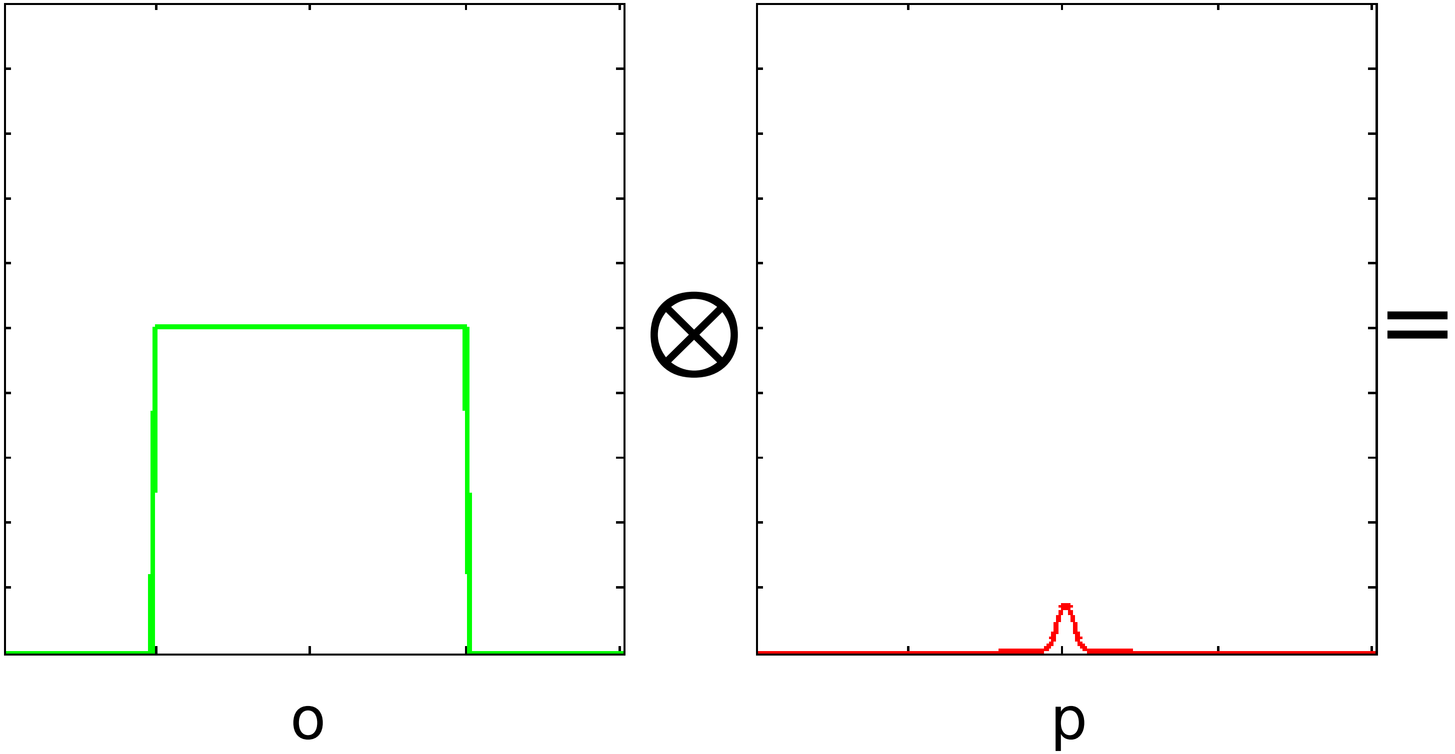


Image Restoration – Inverse Filter

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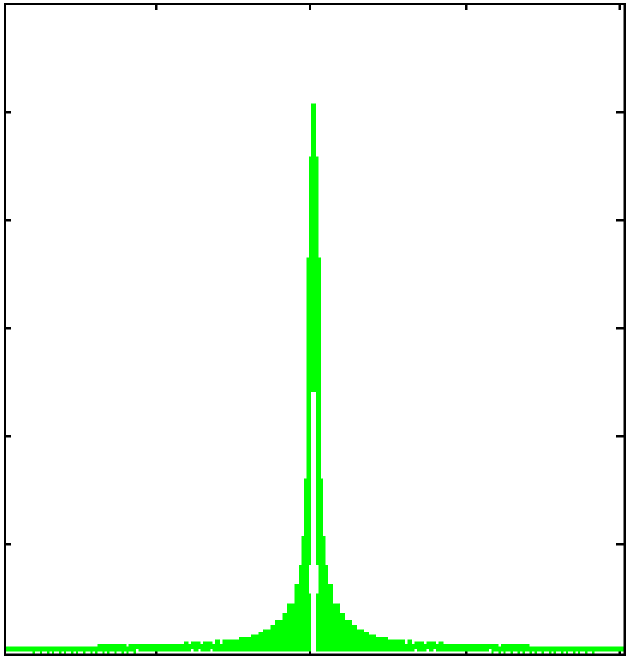
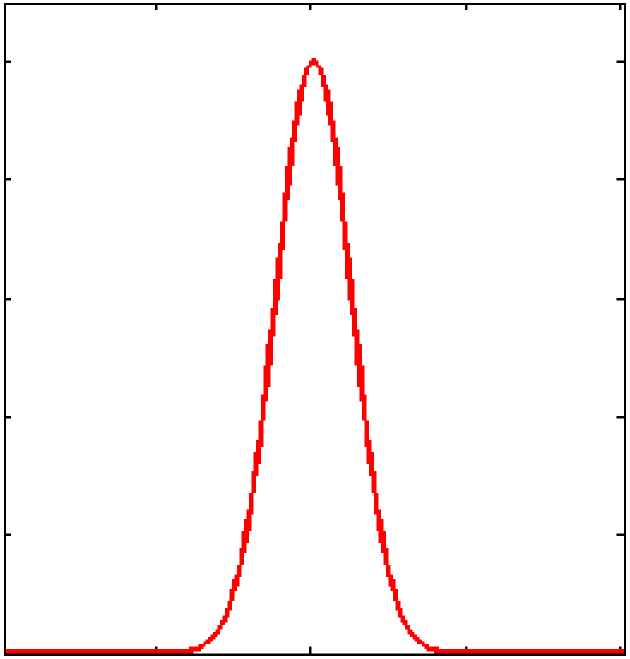


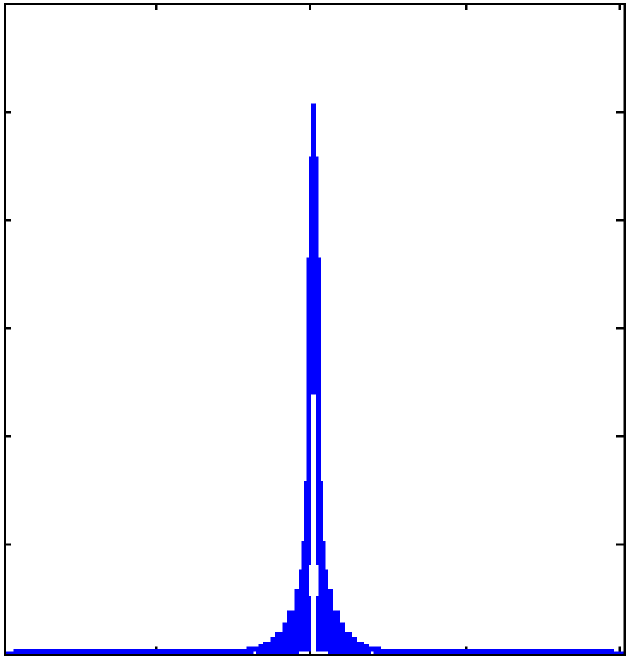
O = FFT(o)

$\cdot$



P = FFT(p)

=



S

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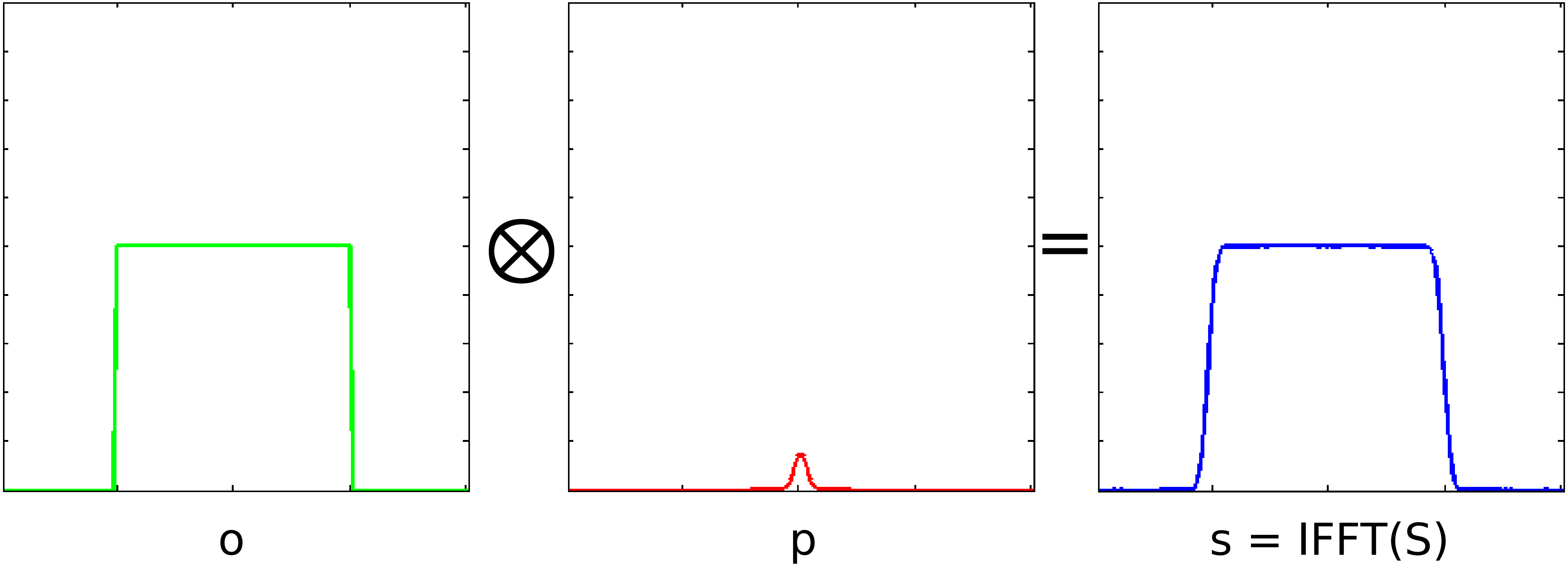

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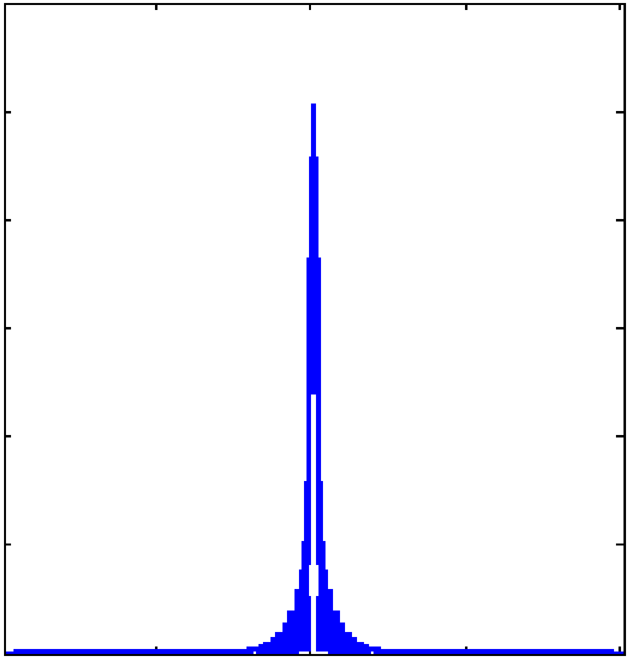
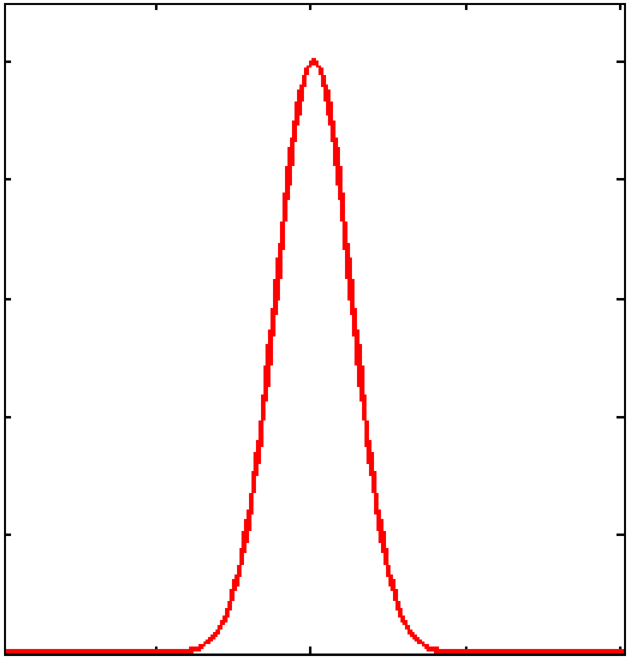


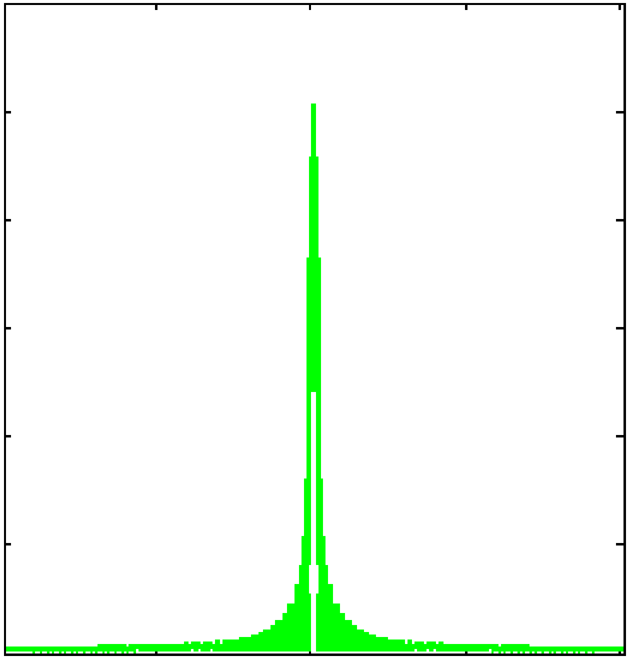
S = FFT(s)

/



P = FFT(p)

$\stackrel{!}{=}$



O

Image Restoration – Inverse Filter

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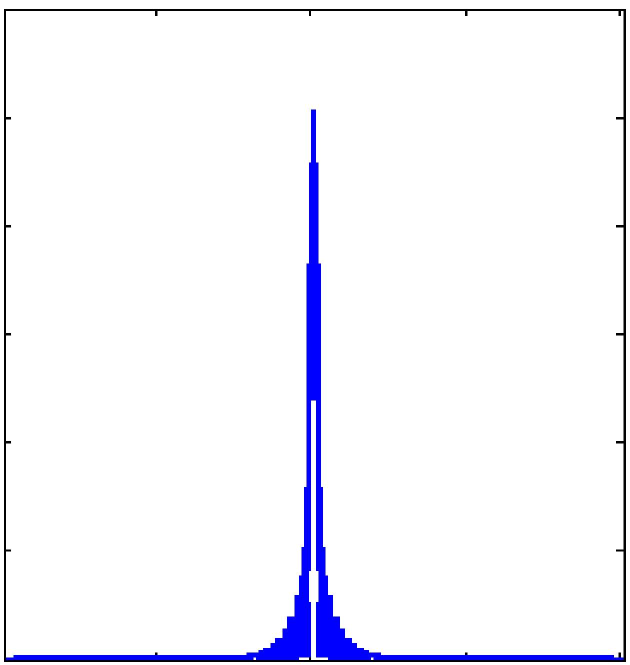
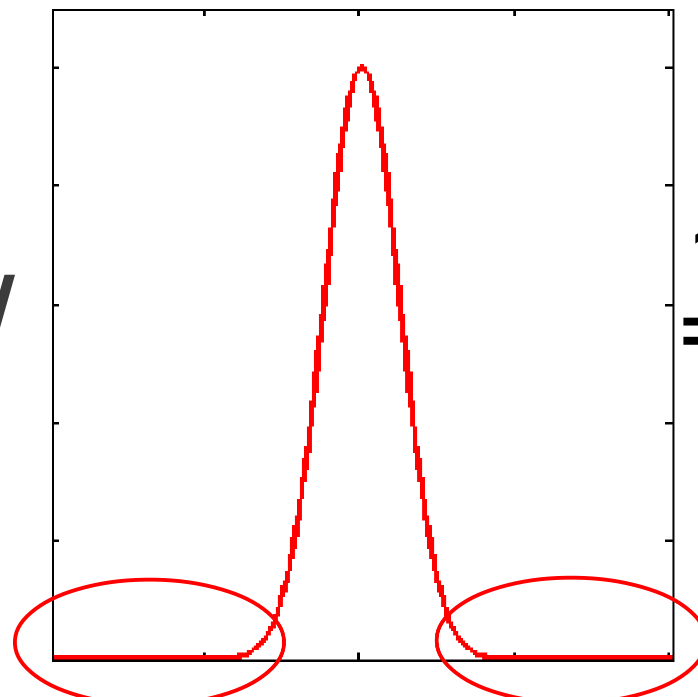


S = FFT(s)

/



P = FFT(p)

$\stackrel{?}{=}$

Image Restoration – Inverse Filter

- **Problem**: P_i is practically equal to zero in some parts of the spectrum
  - Lowpass filters (blur) induce $P_i = 0$ at high frequencies
  - Inversion is not feasible due to limited numerical accuracy
  - E.g. small inaccuracies in the FFT are strongly amplified
- **Solution**: Replace the inverse filter $1/P_i$ *by* Q_i

$$Q_i = \begin{cases} 1/P_i & , if \quad |P_i| \geq T \\ 1/T & , if \quad |P_i| < T \end{cases} \qquad T = \epsilon \, max_j(|P_j|)$$

$$\otimes \; IFFT(Q_i) =$$

Image Restoration – Wiener Filter

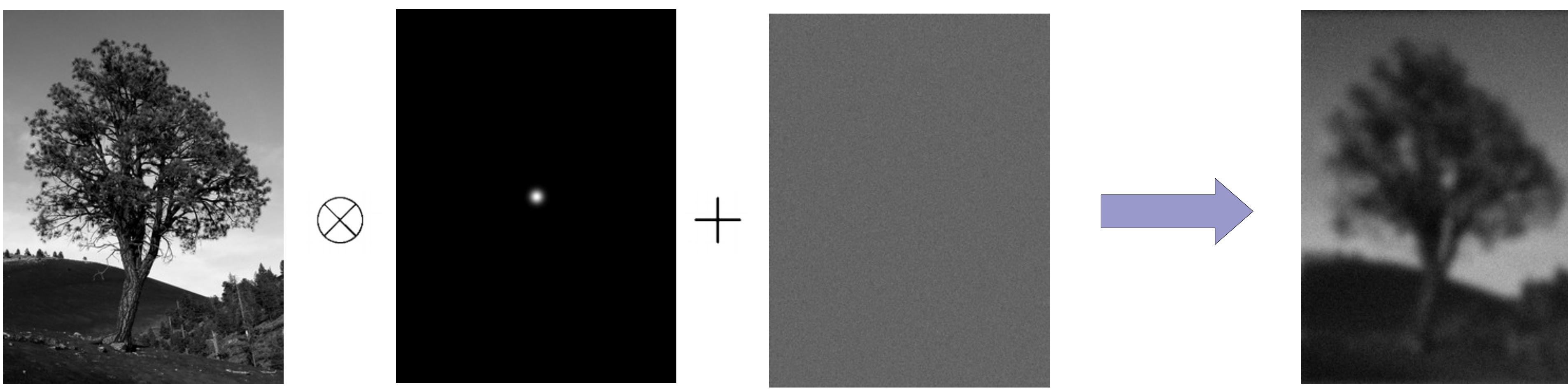
Signal Model

$$s_i = \sum_j o_j \, p_{i-j} + n_i \quad \xrightarrow{\text{FFT}} \quad S_i = O_i \cdot P_i + N_i$$

- s_i: Observed signal
- o_i: Original signal
- p_i: Impulse response
- n_i: Noise

- S_i: Spectrum of the observed signal
- O_i: Spectrum of the original signal
- P_i: Spectrum of the impulse response
- N_i: Noise spectrum

More realistic: real sensors are not perfect and numerical accuracy is limited



o_i $\otimes$ p_i $+$ n_i $\rightarrow$ s_i

Image Restoration – Wiener Filter

- Find a filter q_i that is convolved with signal s_i to approximate the original o_i
  - ➔ Minimize the difference between o and $s \otimes q$

$$e = E\left\{\sum (o - q \otimes s)^2\right\} = E\left\{\sum_i \left(o_i - \sum_j q_j s_{i-j}\right)^2\right\} = min$$

$$d_k = \frac{\partial}{\partial q_k} e = E\left\{2 \sum_i s_{i-k}\left(o_i - \sum_j q_j s_{i-j}\right)\right\} = E\left\{2\left(o - q \otimes s\right) \odot s\right\} \stackrel{!}{=} 0$$

$$D_k = E\left\{2\left(O_k - Q_k S_k\right) S_k^*\right\} = 2\, E\left\{O_k S_k^* - Q_k S_k S_k^*\right\} = 0$$

$$E\left\{O_k S_k^*\right\} = Q_k\, E\left\{S_k S_k^*\right\}$$

$$\boxed{Q_k = \frac{E\left\{S_k^* O_k\right\}}{E\left\{|S_k|^2\right\}} = \frac{E\left\{P_k^* |O_k|^2 + N_k^* O_k\right\}}{E\left\{|P_k|^2 |O_k|^2 + |N_k|^2 + P_k O_k N_k^* + P_k^* O_k^* N_k\right\}}}$$

(using $S_i = O_i \cdot P_i + N_i$)

Image Restoration – Wiener Filter

$$Q_k = \frac{E\{S_k^* O_k\}}{E\{|S_k|^2\}} = \frac{E\{P_k^* |O_k|^2 + N_k^* O_k\}}{E\{|P_k|^2 |O_k|^2 + |N_k|^2 + P_k O_k N_k^* + P_k^* O_k^* N_k\}}$$

$$Q_k = \frac{P_k^* E\{|O_k|^2\} + E\{N_k^* O_k\}}{|P_k|^2 E\{|O_k|^2\} + E\{|N_k|^2\} + P_k E\{O_k N_k^*\} + P_k^* E\{O_k^* N_k\}}$$

Image Restoration – Wiener Filter

$$Q_k = \frac{P_k^* E\{|O_k|^2\} + E\{N_k^* O_k\}}{|P_k|^2 E\{|O_k|^2\} + E\{|N_k|^2\} + P_k E\{O_k N_k^*\} + P_k^* E\{O_k^* N_k\}}$$

1. Signal o and noise n are not correlated

$$E\{O_k^* N_k\} = 0 \Rightarrow Q_k = \frac{P_k^* E\{|O_k|^2\}}{|P_k|^2 E\{|O_k|^2\} + E\{|N_k|^2\}}$$

$$Q_k = \frac{P_k^*}{|P_k|^2 + E\{|N_k|^2\} / E\{|O_k|^2\}}$$

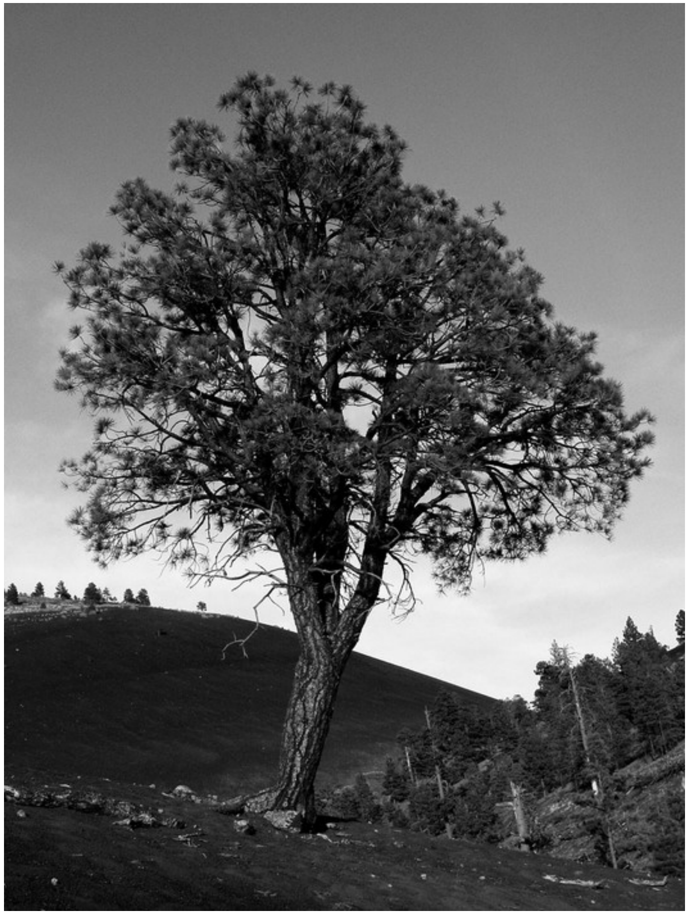
2. n and o are unknown

Signal to noise ratio $SNR_k^2 = \frac{E\{|O_k|^2\}}{E\{|N_k|^2\}} \Rightarrow \boxed{Q_k = \frac{P_k^*}{|P_k|^2 + 1/SNR^2}}$

$$SNR = \infty \Rightarrow Q_k = \frac{P_k^*}{|P_k|^2 + 0} = \frac{1}{P_k}$$

(Inverse filter!)

Image Restoration – Wiener Filter



Original o

Degraded s

$s = o \otimes p + n$

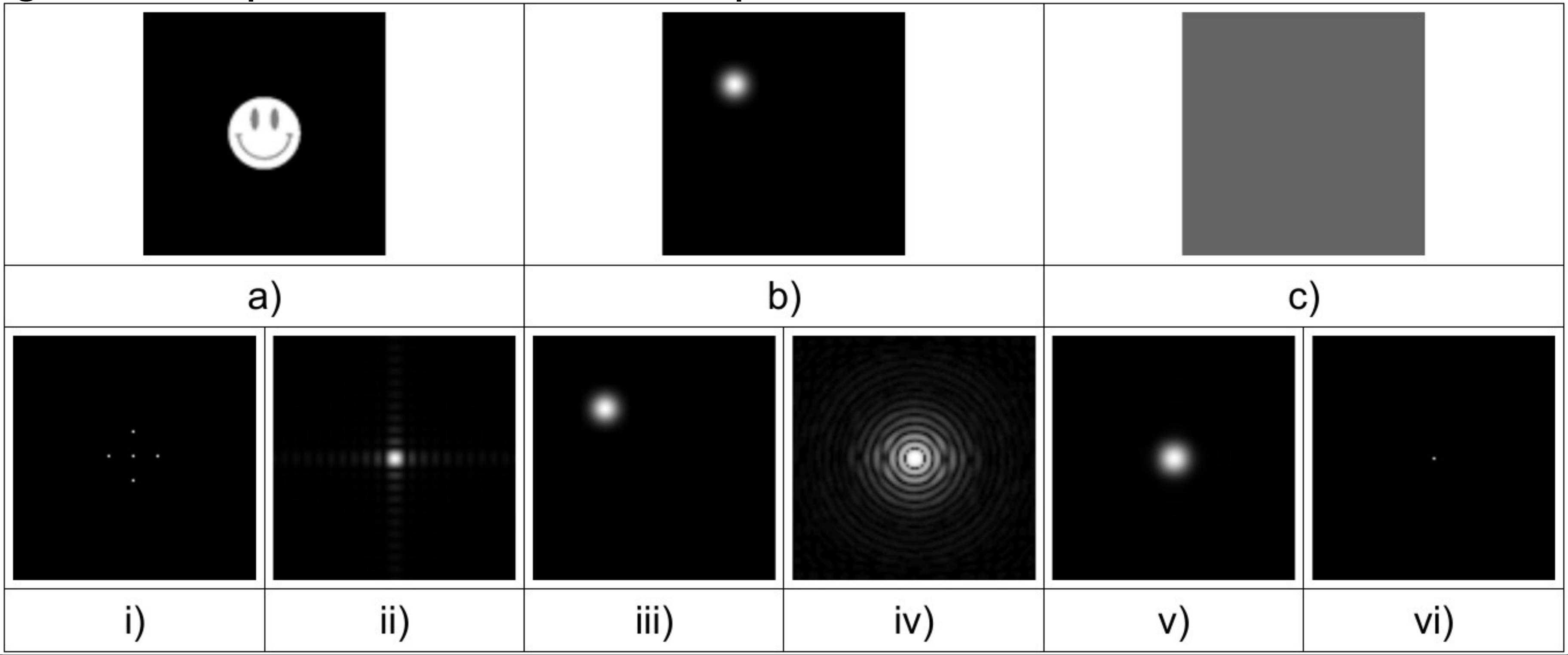
Restored

$s \otimes q$

4. Exercise - Theory

1. What is the **ringing effect** in the context of image filtering? How is it caused and how can it be avoided?

2. Figures a)-c) show three different images, while Figures i)-vi) depict the **amplitude** of six different Fourier spectra. State which of the given spectra corresponds to which of the images and why? Note: A spectrum can be assigned multiple times and not all spectra have to be used.



4. Exercise - Given

File: main.cpp

```
main(int argc, char** argv)
```

- Loads image, path given in argv[1]
- Adds distortion: Gaussian blur (stddev in argv[3]) and
  Gaussian noise (SNR in argv[2])
- Calls restoration functions
- Saves restored images

File: dip4.cpp

```
Mat degradeImage(Mat& img, Mat& degradedImg, double filterDev, double snr)
```

`img`	input image
`degradedImage`	output image
`filterDev`	standard deviation of Gaussian blur
`snr`	signal-to-noise ratio
`return`	filter kernel used for blurring

- Adds Gaussian blur and Gaussian noise

4. Exercise – To Do

```
Mat_<std::complex<float>> DFTReal2Complex(Mat_<float>& input)
```

`input` real valued input image
`return` complex valued spectrum of input image

Computes the DFT and returns a full size complex valued spectrum without the special packing that exploits the convex conjugate symmetry.

```
Mat_<float> IDFTComplex2Real(Mat_<std::complex<float>>& input)
```

`input` complex valued spectrum
`return` real valued image

Computes the inverse DFT and returns the real part. You can assume without testing, that the supplied spectrum is convex conjugate symmetric.

```
Mat_<std::complex<float>> applyFilter(Mat_<std::complex<float>>& input,
                                      Mat_<std::complex<float>>& filter)
```

`input` complex valued image spectrum
`filter` complex valued filter spectrum
`return` complex valued spectrum of filtered

Multiplies the spectrums.

4. Exercise – To Do

```
Mat_<std::complex<float>> computeInverseFilter(
                          Mat_<std::complex<float>>& input, float eps)
```

`input`	Spectrum of assumed blur filter
`eps`	Factor to compute threshold
`return`	`Spectrum of` inverse Filter

Computes the thresholded (clipped) inverse filter. Note that the threshold is computed as the eps-fraction of the maximum amplitude in the blur spectrum.

```
Mat_<float> inverseFilter(Mat_<float>& degraded,
                          Mat_<float>& filter, float eps)
```

`degraded`	input image
`filter`	filter that caused distortion
`eps`	Factor to compute threshold
`return`	restored image

Performs the thresholded (clipped) inverse filter to restore image:

- Zero-pad and circ-shift blur kernel
- Compute complex spectra of image and kernel
- Compute inverse filter
- Apply inverse filter and IDFT

4. Exercise – To Do

```
Mat_<std::complex<float>> computeWienerFilter(
                         Mat_<std::complex<float>>& input, float snr)
```

`input` Spectrum of assumed blur filter
`snr` Signal to noise ratio (non-logarithmic)
`return` `Spectrum of Wiener filter`

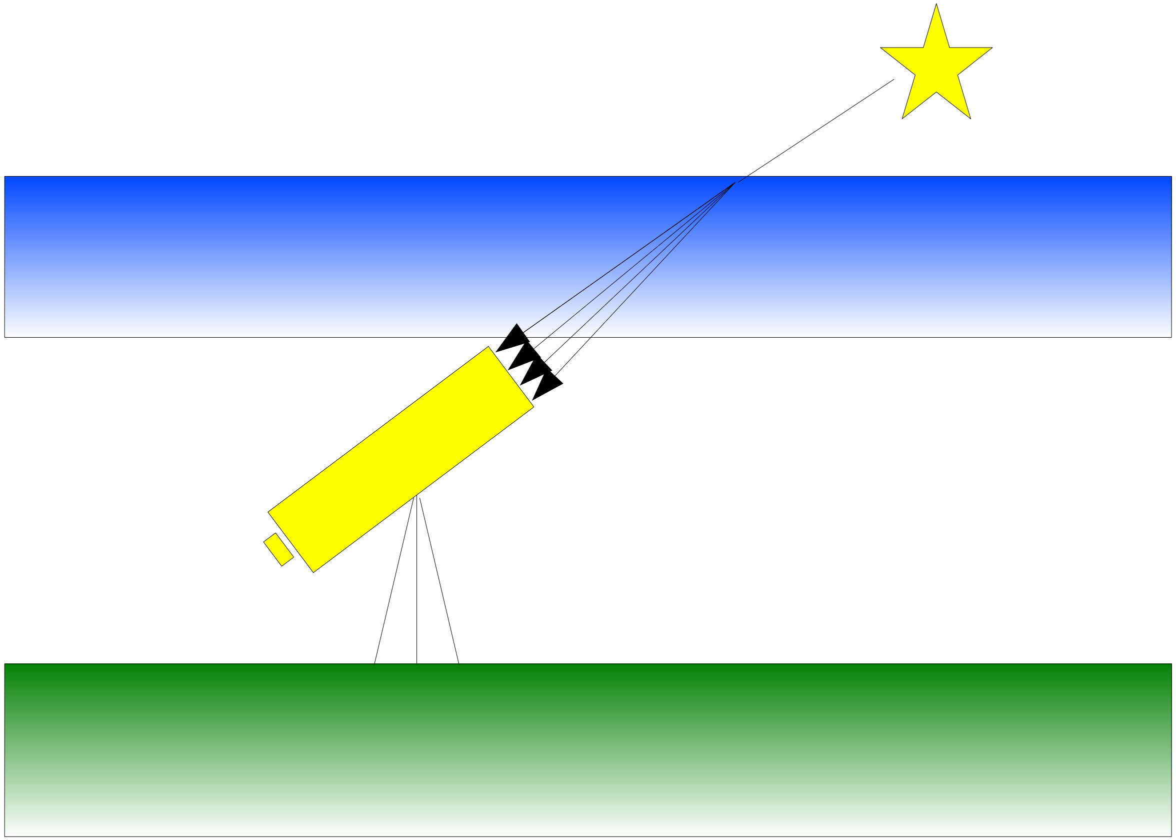
Computes the Wiener filter.

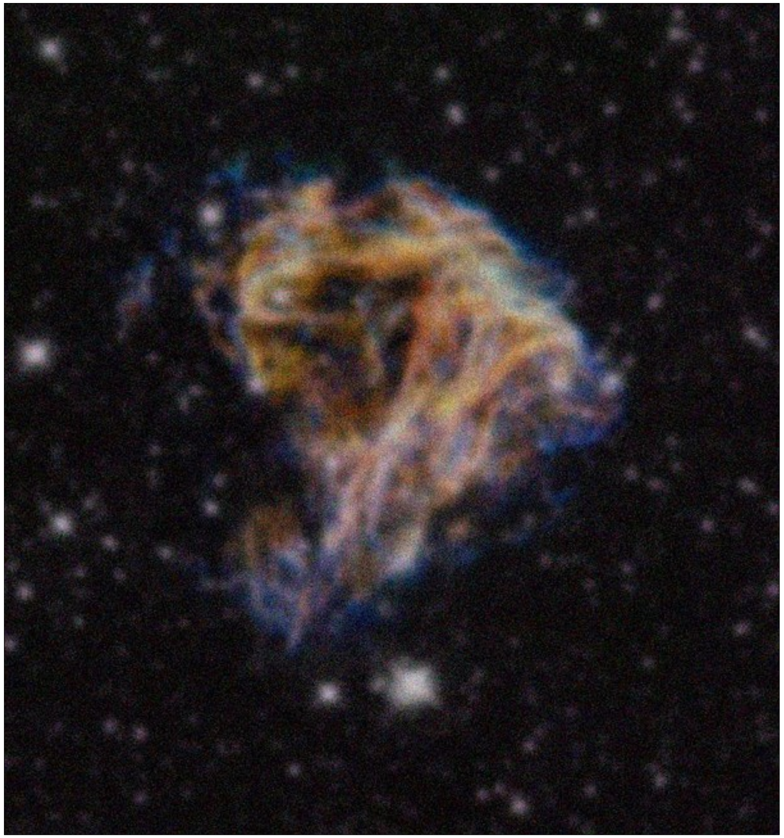
```
Mat_<float> wienerFilter(Mat_<float>& degraded,
                         Mat_<float>& filter, float snr)
```

`degraded` input image
`filter` filter that caused distortion
`snr` signal to noise ratio (non-logarithmic)
`return` restored image

Performs the Wiener filter to restore image:
- Zero-pad and circ-shift blur kernel
- Compute complex spectra of image and kernel
- Compute Wiener filter
- Apply Wiener filter and IDFT

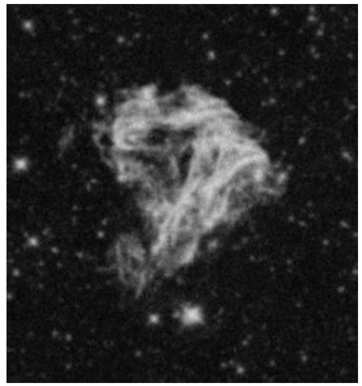
Image Restoration – Optional!





- Acquisitions by earth-based telescopes are often severely degraded
  - Atmosphere: refraction and anisotropy cause distortions (blurring)
  - CCD (sensor): Extremely dim objects imply significant thermal noise
  - **Wiener Signal Model**: Convolution (Atmosphere) + Noise (CCD)
  - However: SNR and impulse response p are unknown

Image Restoration



S(x,y)

T(x,y)

- Neither impulse response p_i nor SNR are known
- Variable with respect to atmospheric conditions

- **Automatic determination of the SNR**
  - Use threshold δ to separate foreground and background

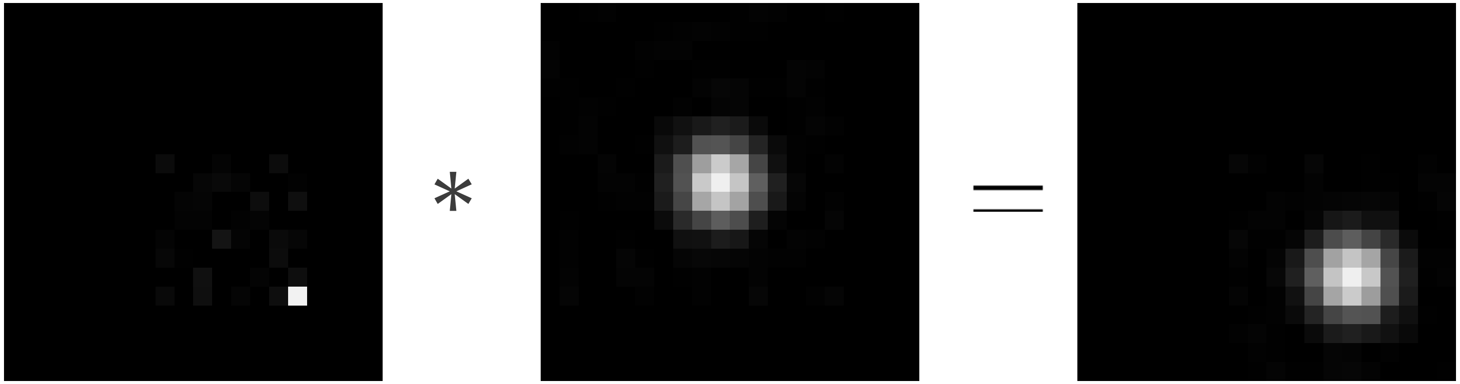
$$T(x,y)=\begin{cases}1 & S(x,y)\geq\delta \\ 0 & S(x,y)<\delta\end{cases} \qquad \delta=2\sigma[S(x,y)]$$

$\sigma[..]$: Standard deviation

  - The ratio of mean foreground to mean background intensity is the SNR:

$$SNR=\frac{\sum S(x,y)\cdot T(x,y)}{\sum T(x,y)} / \frac{\sum S(x,y)\cdot(1-T(x,y))}{\sum(1-T(x,y))}$$

Image Restoration



$$\delta_{a,b}(x,y) * p(x,y) = c \cdot p(x-a, y-b)$$

$$\delta_{a,b}(x,y) = \begin{cases} c & (x,y) = (a,b) \\ 0 & (x,y) \neq (a,b) \end{cases}$$

- Convolving a (linear) filter with a delta function yields:
  - ➔ The original impulse response centred at the delta impulse
- An image was convolved with an unknown kernel *p*
  - ➔ If the image contained a delta impulse, it will be replaced with *p*
  - ➔ The kernel can be established from the neighbourhood of the original delta!

Image Restoration

- Stars are (almost) delta impulses
- The neighborhood $N_S(x,y)$ of a star consists of p and thermal noise

$$N_S(x,y) = I \cdot p(x,y) + n(x,y) \quad -R \leq x,y \leq R$$

  - I: True intensity of the star
- Normalize intensity

$$N(x,y) = N_S(x,y) / max(N_S(x,y)) = p(x,y) + n(x,y) / I$$

- Averaging the response around numerous stars reduces noise

$$M(x,y) = \langle N(x,y) \rangle = p(x,y) + r$$

- **Estimating *p***: Eliminate offset r and normalize intensity

$$p(x,y) \approx (M(x,y) - min(M(x,y))) / \sum (M(x,y) - min(M(x,y)))$$

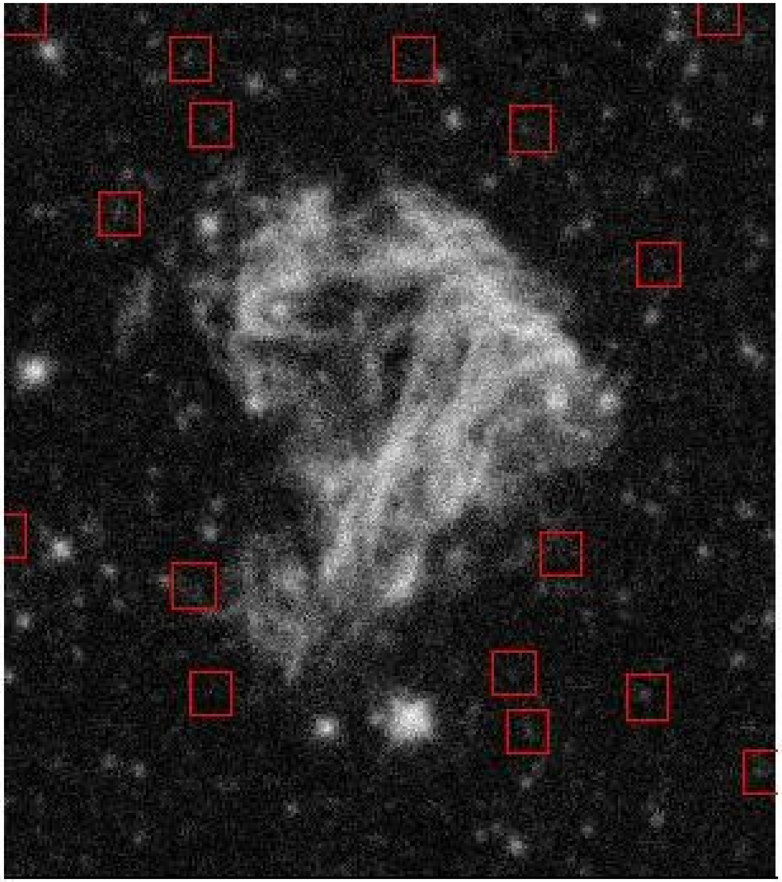
  - Normalize the kernel to unity (it should integrate to 1)

Image Restoration

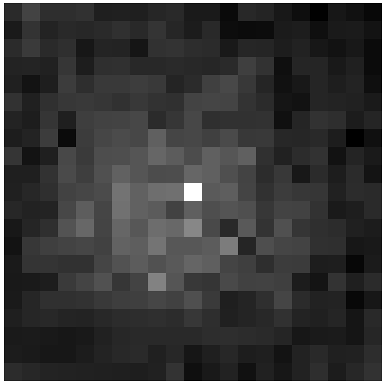
- Threshold: Separate foreground from background
- Enumerate foreground regions
- **For each region:**
  1. Determine indices of region
  2. Discard the region if it contains more than K pixels (galaxy or nebula?)
  3. Discard the region if there is another foreground structure in the vicinity (impulse responses overlap)
  4. Cut out the neighbourhood around the star (contains p)
  5. After scaling and averaging neighbourhoods, p is determined

- **Concerning Step 3**
  - Size of the neighbourhood: $2R+1$
  - Convolve T(x,y) with a mask of size $(2R+1)^2$ containing ones
  - This counts the number of foreground pixels in every neighbourhood in T
  - If the number of foreground pixels counted within a region exceeds the size of the region, there must be another star nearby

Image Restoration

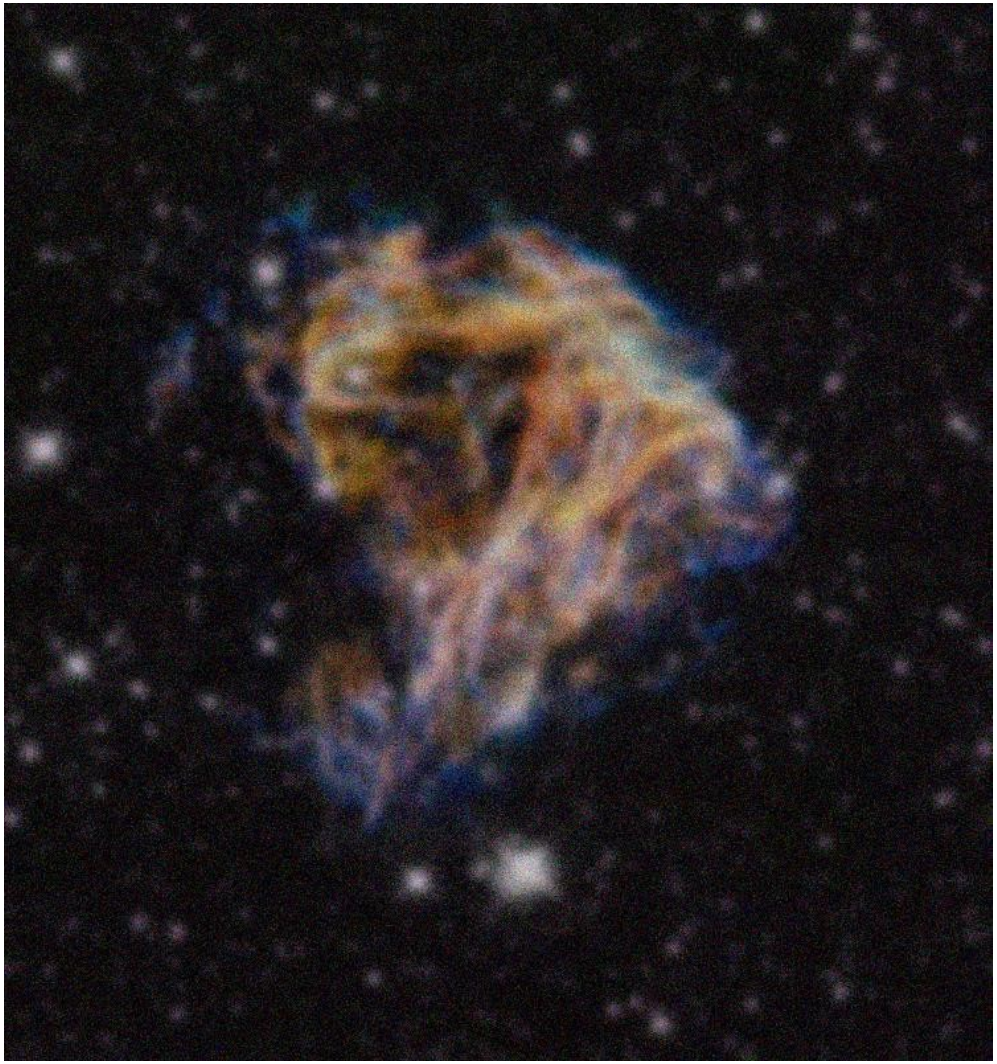


Detected Stars



Impulse Response *p*

Image Restoration



Deadline: 07.01.2020

Exams

- Mid-term
  - **Tuesday, <u>10.12.2019, 16:15pm, EW 201</u>**
  - Duration: 45 min
  - No grade, but pass is necessary to take part at the final exam

- Topics from lecture **and** exercise

- Questions in English. Answers? → Multiple choice!
- No books, no calculator, no script, no paper, …